High Performance Computing
Assignment 3

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Hardware Details

CPU Model:INTEL(R) Core i5-4590
No. of cores: 16
Memory:7.6 GiB
Compiler:gcc
Optimization flags if used: None
Precision: double

Prefix Sum(Exclusive Scan)

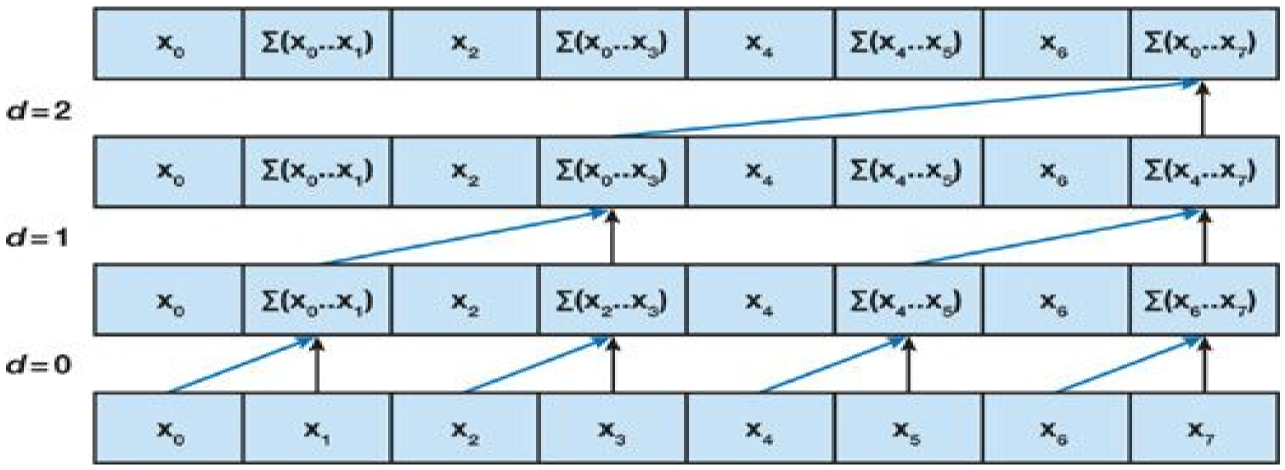
We have to calculate the prefix sum of the given array.

$$\text{prefix sum}[i] = \begin{cases} 0 & , i = 0 \\ \sum_{k=0}^{i-1} input[k] & , i > 0 \end{cases}$$

Complexity of the Problem(Serial)
Serial complexity of the problem is O(n), where n is the input size.
Span of Problem:
Parallel Span is O(logn), which is the height of the binary tree that we traverse twice.

Optimization Strategy:

Blelloch Algorithm is used for the parallel implementation of the problem. The algorithm is divided into two phases: Up sweep or Reduce Phase and Down sweep.



x_0 | $\Sigma(x_0..x_1)$ | x_2 | $\Sigma(x_0..x_3)$ | x_4 | $\Sigma(x_4..x_5)$ | x_6 | $\Sigma(x_0..x_7)$
$d=2$
x_0 | $\Sigma(x_0..x_1)$ | x_2 | $\Sigma(x_0..x_3)$ | x_4 | $\Sigma(x_4..x_5)$ | x_6 | $\Sigma(x_4..x_7)$
$d=1$
x_0 | $\Sigma(x_0..x_1)$ | x_2 | $\Sigma(x_2..x_3)$ | x_4 | $\Sigma(x_4..x_5)$ | x_6 | $\Sigma(x_6..x_7)$
$d=0$
x_0 | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | x_7

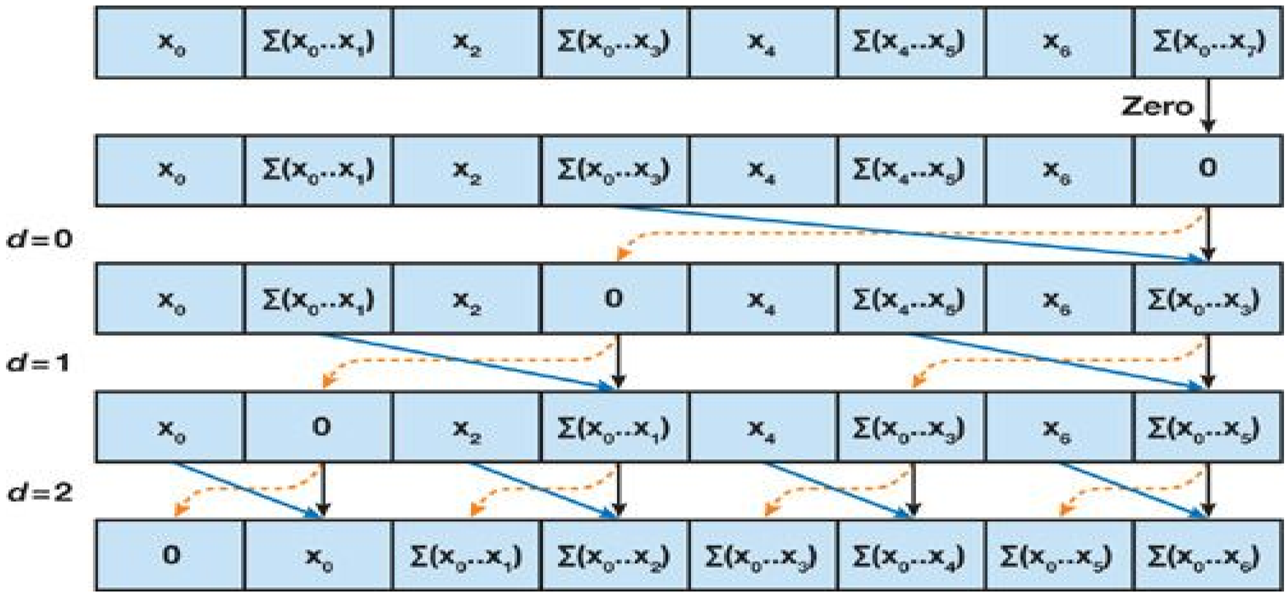
Up Sweep

Up sweep :

In the reduce phase, we traverse the tree from leaves to root computing partial sums at internal nodes of the tree. At the end of this phase, the last element contains sum of all the elements of the array.

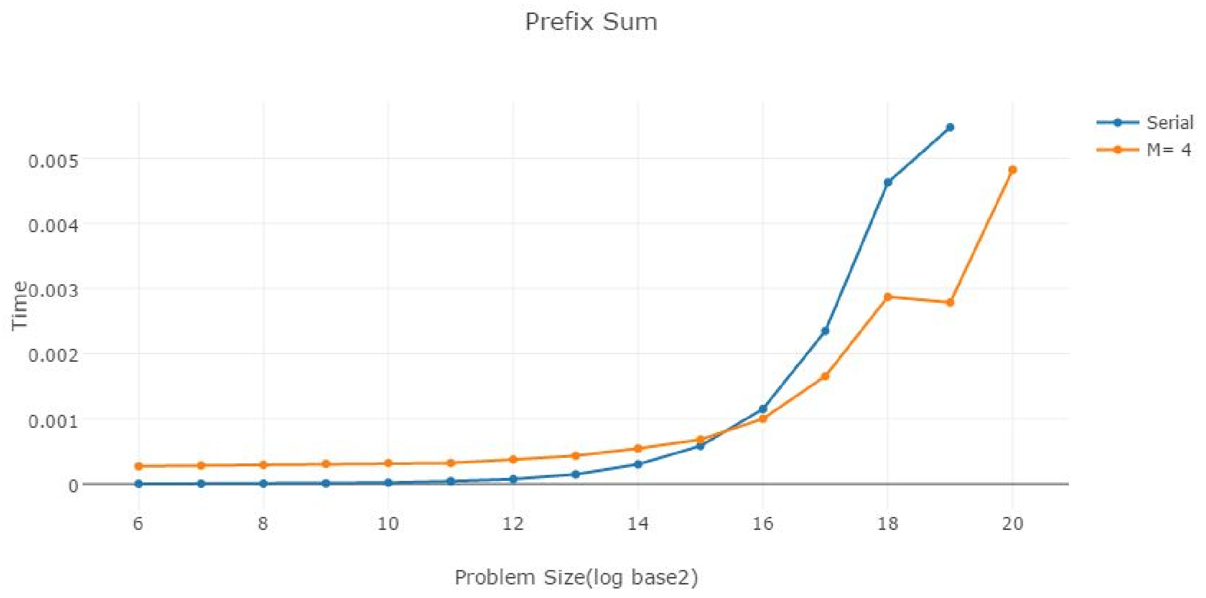
Down sweep:

We start by inserting zero at the root of the tree(last element of the array), and on each step, each node at the current level passes its own value to its left child, and the sum of its value and the former value of its left child to its right child.



x_0 | $\Sigma(x_0..x_1)$ | x_2 | $\Sigma(x_0..x_3)$ | x_4 | $\Sigma(x_4..x_5)$ | x_6 | $\Sigma(x_0..x_7)$
Zero
x_0 | $\Sigma(x_0..x_1)$ | x_2 | $\Sigma(x_0..x_3)$ | x_4 | $\Sigma(x_4..x_5)$ | x_6 | 0
$d=0$
x_0 | $\Sigma(x_0..x_1)$ | x_2 | 0 | x_4 | $\Sigma(x_4..x_5)$ | x_6 | $\Sigma(x_0..x_3)$
$d=1$
x_0 | 0 | x_2 | $\Sigma(x_0..x_1)$ | x_4 | $\Sigma(x_0..x_3)$ | x_6 | $\Sigma(x_0..x_5)$
$d=2$
0 | x_0 | $\Sigma(x_0..x_1)$ | $\Sigma(x_0..x_2)$ | $\Sigma(x_0..x_3)$ | $\Sigma(x_0..x_4)$ | $\Sigma(x_0..x_5)$ | $\Sigma(x_0..x_6)$

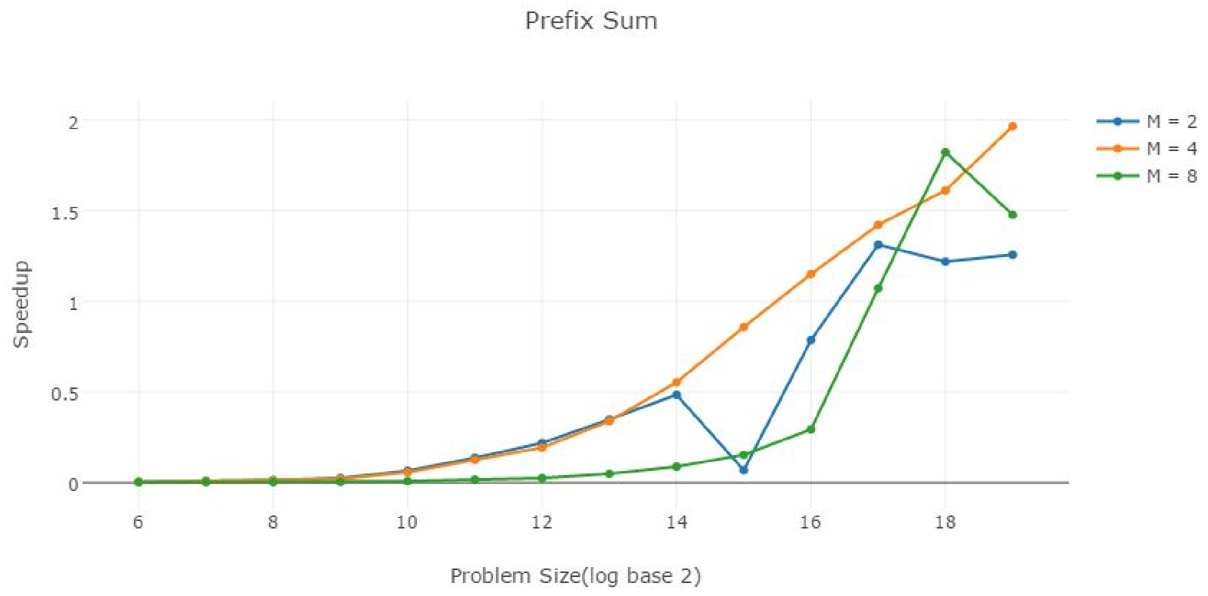
Down sweep



Problem Size vs Time(Serial and Parallel)

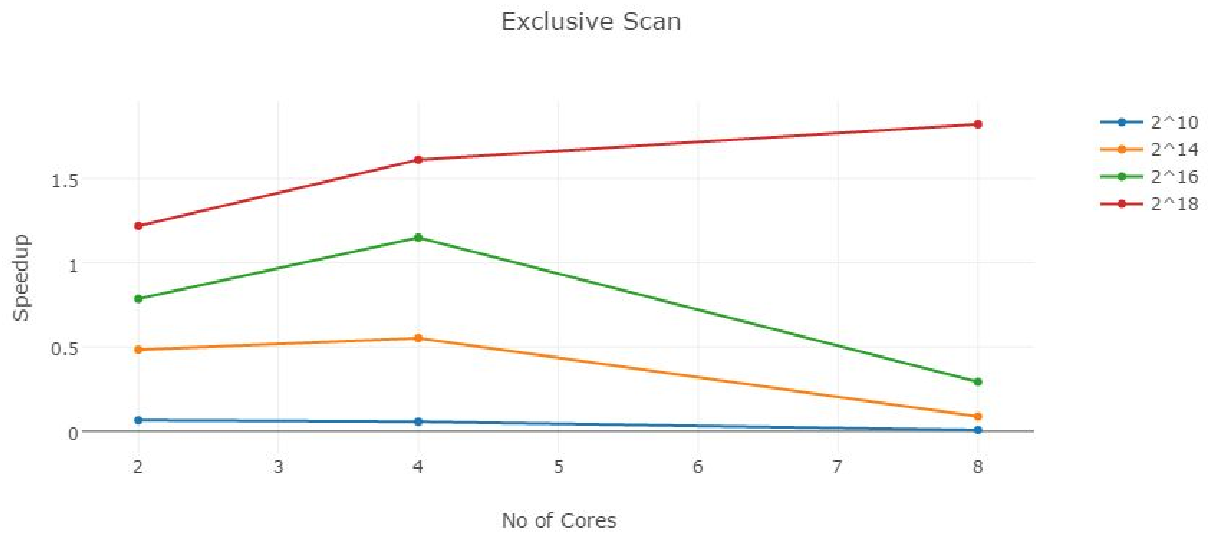
Observation:

For small input sizes(upto 2^{15}), the parallel code takes more time than the serial code because of the overhead of dividing work among the threads surpasses the reduced parallelized time. For the larger input sizes parallel code performs better.



Problem Size vs Speedup for 2, 4 and 8 cores

Observation:
With the increase in the problem size, speedup increases. For larger input sizes, the reduced parallelized time compensates for the overhead.For the small input sizes, 2 cores perform better than 4 cores, but with the increase in problem size, 4 cores perform much better than 2 cores.



No of cores vs Speedup

Observation:
For a particular number of cores as the input size increases (becomes greater than 2^{16})then the speedup tends to increase that is parallel code becomes more efficient as compared to serial code .As number of cores increases from M=2 to M = 4 the speedup tends to increase for larger value of the input parameter but its more efficient for M=4 as compared to M=8 . (possibly due to overheads)

Problems faced due to parallelization:

1. For input ($N>2^{20}$) the code output shows Segmentation Fault,primarily because of the memory bound constraints.
2. For some larger values of N (2^{17}) the code performs better for M=4 than for M=8 , the possible reasons might be that the number of overheads overpower the amount of time saved due to parallelization.
3. The code shows correct value for only those N that can be represented in power of 2 as the up sweep and down sweep treats array as a binary tree and sum the numbers in pair , it is better to give input in form of 2^x.

Inclusive Scan

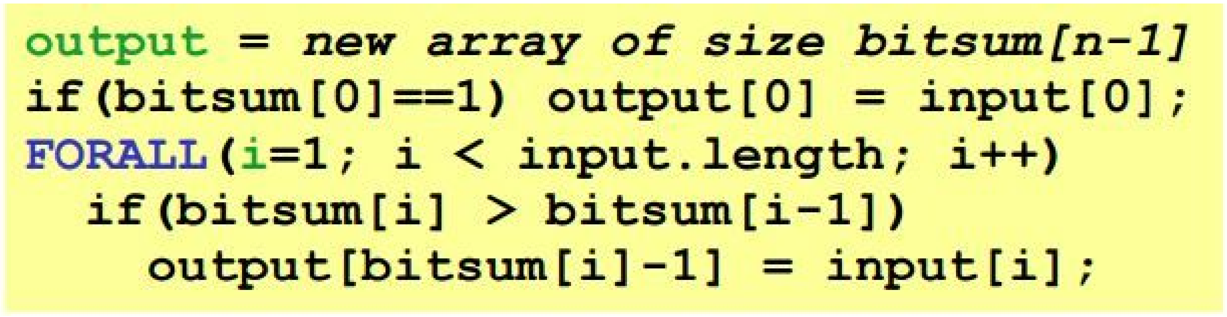
We can convert the exclusive scan to inclusive scan using array of size 1 more than the input size. At the end of the up sweep phase the last element would be containing the sum of all the elements of the array, we copy that value to another location and perform the down sweep.

Efficient Filter

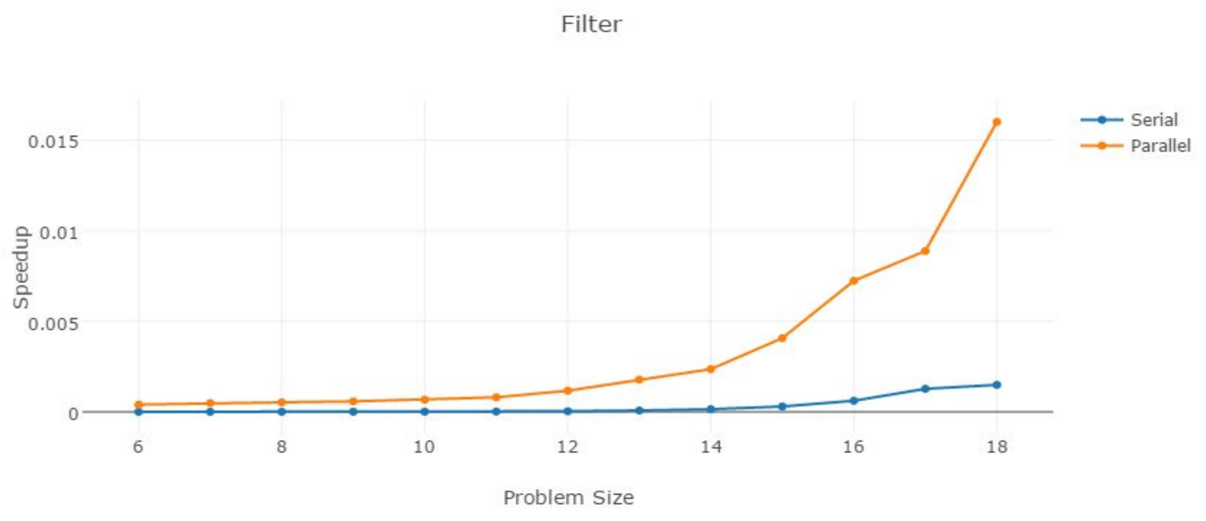
Given an input array we have to filter out the elements which satisfy the given condition.

Serial Complexity:O(n),where n is size of the input array .

Optimization Strategy :
We linearly traverse the whole array in sequential fashion and check which value is greater than given number and store the value in another array.Thus the running time will be O(n).
Parallel:We traverse the input parallely to compute a bit-vector for true elements. The corresponding element in the bit vector will be 1 if the filter is true, otherwise it will be 1. This takes O(n) time. Then we use prefix scan sum algorithm implemented above is applied on bit vector taking O(logn) time. From the prefix 'scanned' bit vector we can create the output array that contains the filter elements in the same order as they were in input array.

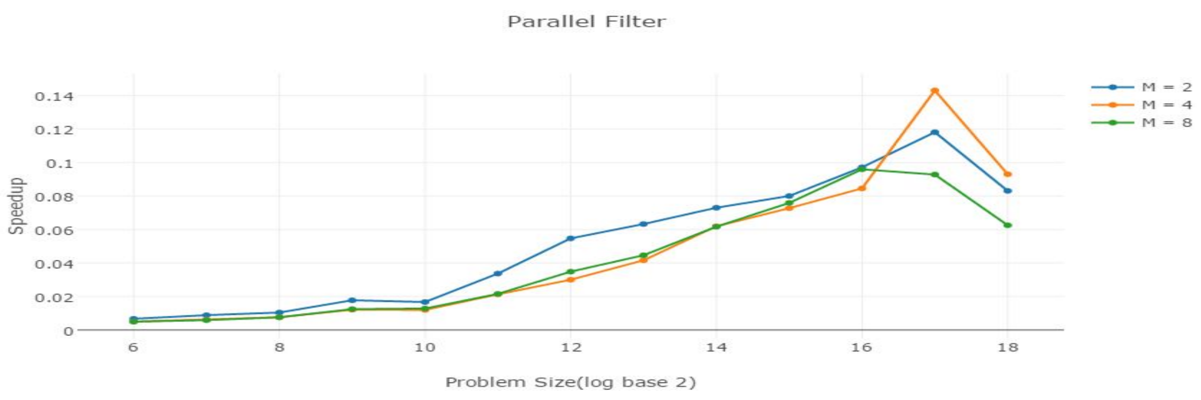


```
output = new array of size bitsum[n-1]
if(bitsum[0]==1) output[0] = input[0];
FORALL(i=1; i < input.length; i++)
  if(bitsum[i] > bitsum[i-1])
    output[bitsum[i]-1] = input[i];
```



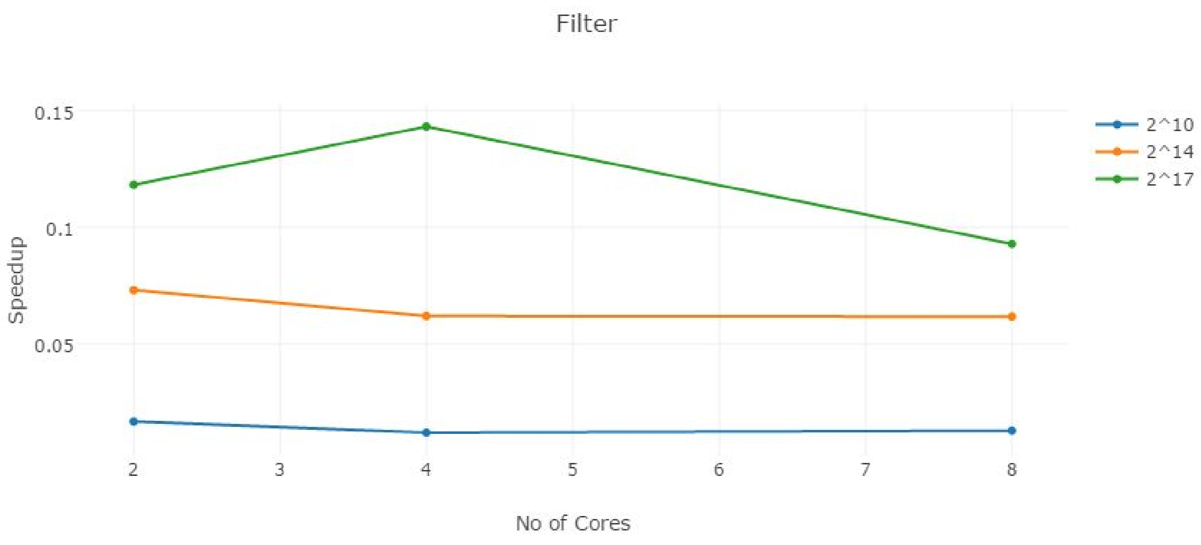
Problem size vs Time(in s)

Observation:
For all the values of input size, parallel code takes more time than the serial code. This is possibly due to synchronisation overhead.



Problem Size vs Speedup

Observation:
For small problem sizes M =2 cores performs better than M = 4 and 8 cores. For larger values of the input size, M = 4 cores perform the best.



No of Cores vs Speedup

Observation:
As the input array grows larger in size the parallel time becomes less and speedup tends to increase but is still less than 1.For M=2 the speedup although being less than 1 is still better than the speedup when M=4 or 8 (due to overhead).

Problems faced in parallelization:

1. The serial code is much more efficient when N is small ,the running time of serial code is better than that of parallel code using prefix scan.The running time for serial code is O(n) whereas for parallel it is O(n+logn)that implies serial is efficient.
2. Moreover as N increases,due to overhead and synchronization problems (as number of cores increase) the parallel code saves time but however these factor overpower the saved time and therefore speedup is still less than 1 .