Sublinear Time Recommendation Algorithms

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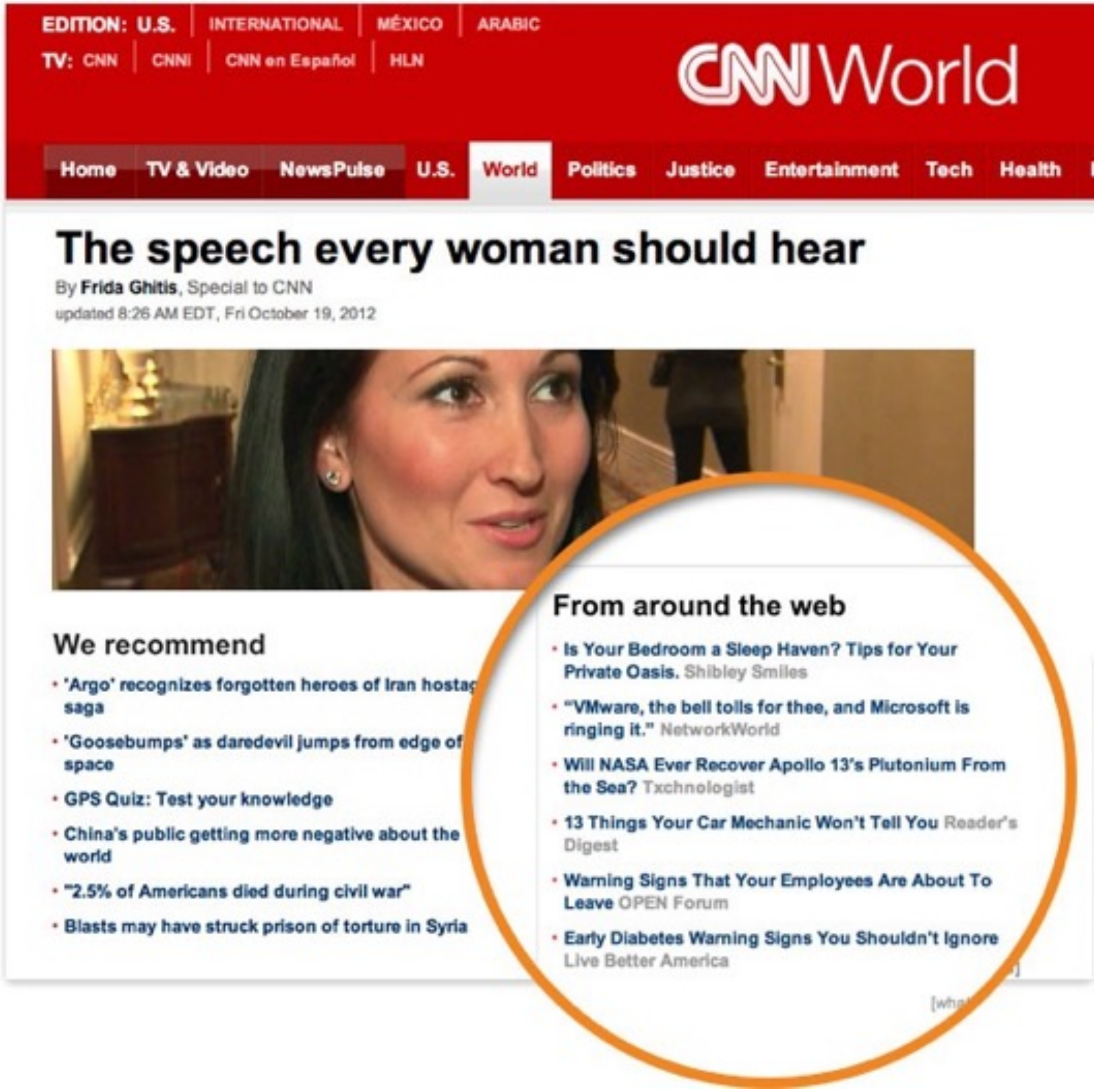
Motivation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The speech every woman should hear
By Frida Ghitis, Special to CNN
updated 8:26 AM EDT, Fri October 19, 2012
We recommend
'Argo' recognizes forgotten heroes of Iran hostag saga
'Goosebumps' as daredevil jumps from edge of space
GPS Quiz: Test your knowledge
China's public getting more negative about the world
"2.5% of Americans died during civil war"
Blasts may have struck prison of torture in Syria
From around the web
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"VMware, the bell tolls for thee, and Microsoft is ringing it." NetworkWorld
Will NASA Ever Recover Apollo 13's Plutonium From the Sea? Txchnologist
13 Things Your Car Mechanic Won't Tell You Reader's Digest
Warning Signs That Your Employees Are About To Leave OPEN Forum
Early Diabetes Warning Signs You Shouldn't Ignore Live Better America

Recommendations Today

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- Two key steps
  - *Learning:* estimate a predictive model using past data
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  - *Learning:* estimate a predictive model using past data
  - *Optimization:* make recommendation decisions in real-time
- **This Talk: Operationalize existing predictive models**
  - Formulate as optimization problem
  - Propose *sublinear* time algorithm with provable guarantees
  - Show substantial improvement on massive dataset from Outbrain

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Concrete Example: Spotify

- Learn a mapping from 10^8 products to $\mathbb{R}^{1000}$
- Good embedding: $\|v_i - v_j\|$ small if similar vis-a-vis 'co-occurrences'
- Many algorithms do this
  - Collaborative filtering, matrix factorization [Bernhardsson '14]
  - NLP models: word2vec on playlists [Johnson '15]
  - Neural networks: CNN on audio files [Dieleman '14]
- Customers: distributions on $v_1, \ldots, v_n$

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- Customers are random vectors $U \in \mathbb{S}^{d-1}$
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1. Sample $\hat{U} = \{u_1, ...u_s\}$ from U
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3. Greedy algorithm over customer samples and relevant products

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Assumptions

1. **Bounds on Customer Consumption**

   a. **Constant Upper Bound:** For any fixed user realization u in the support of U, expected number of conversions does not grow

   $$\mathsf{E}\left[\sum_{v} \mathbf{1}\left(v^{\top} u + \epsilon_v > \gamma\right)\right] = O(1)$$

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2. **Good Embeddings:** Product vectors are 'uniformly spread'

$$|B_{c\gamma}(u) \cap V| = O(c^d)|B_{\gamma}(u) \cap V|$$

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Goal: Sublinear time recommendation algorithm

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- Output: $\mathcal{V} \subset \{v_1, \ldots, v_n\}$ such that

$$\mathbb{P}(v_j \in \mathcal{V}) = p(\|v_j - u\|)$$

- Goal: construct $\mathcal{V}$ in sublinear (amortized) runtime

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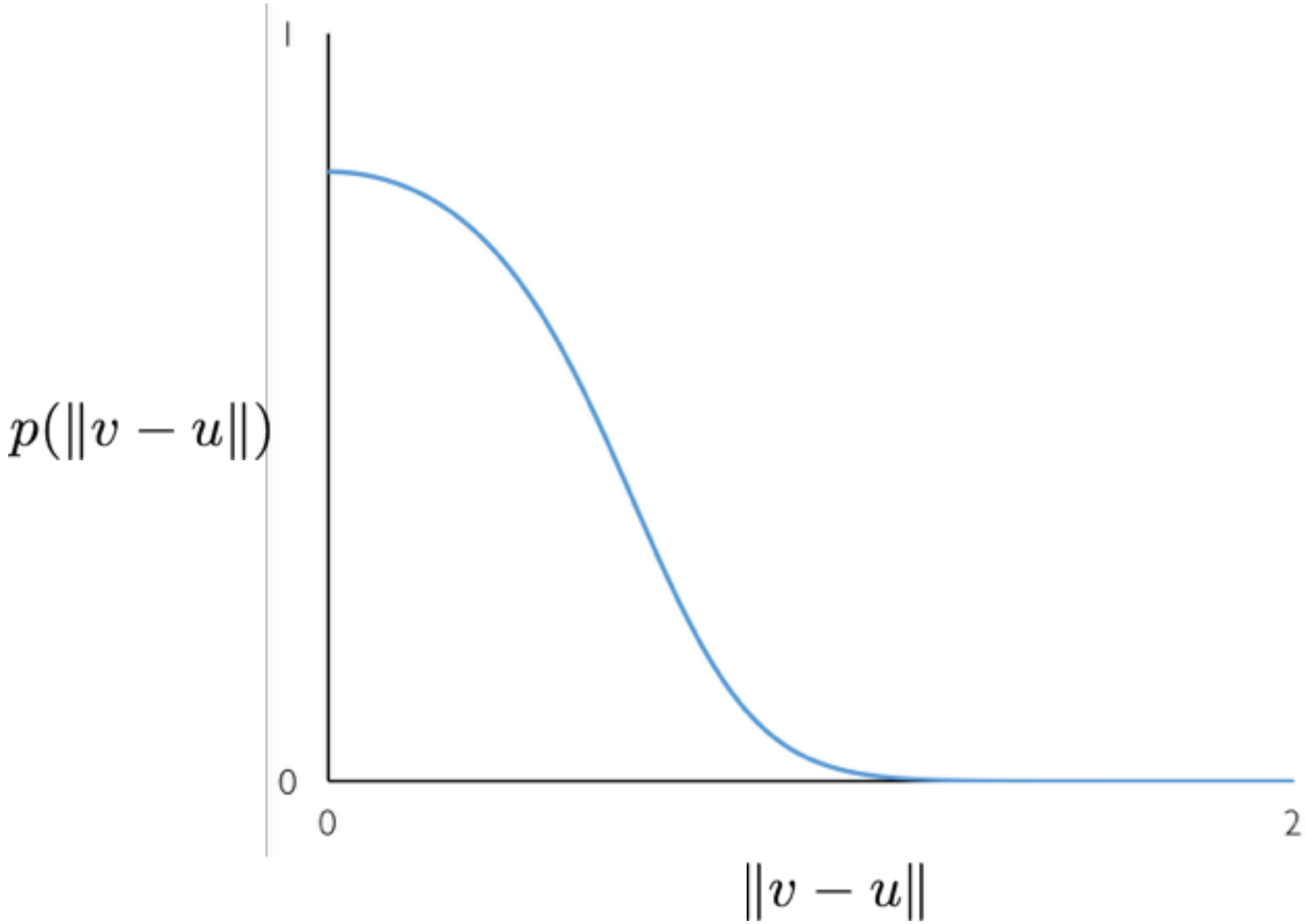
with expected amortized running time

$$\tilde{O}\left(\log(1/\epsilon)\, n^{2/3}\right)$$

The Key Problem in Picture



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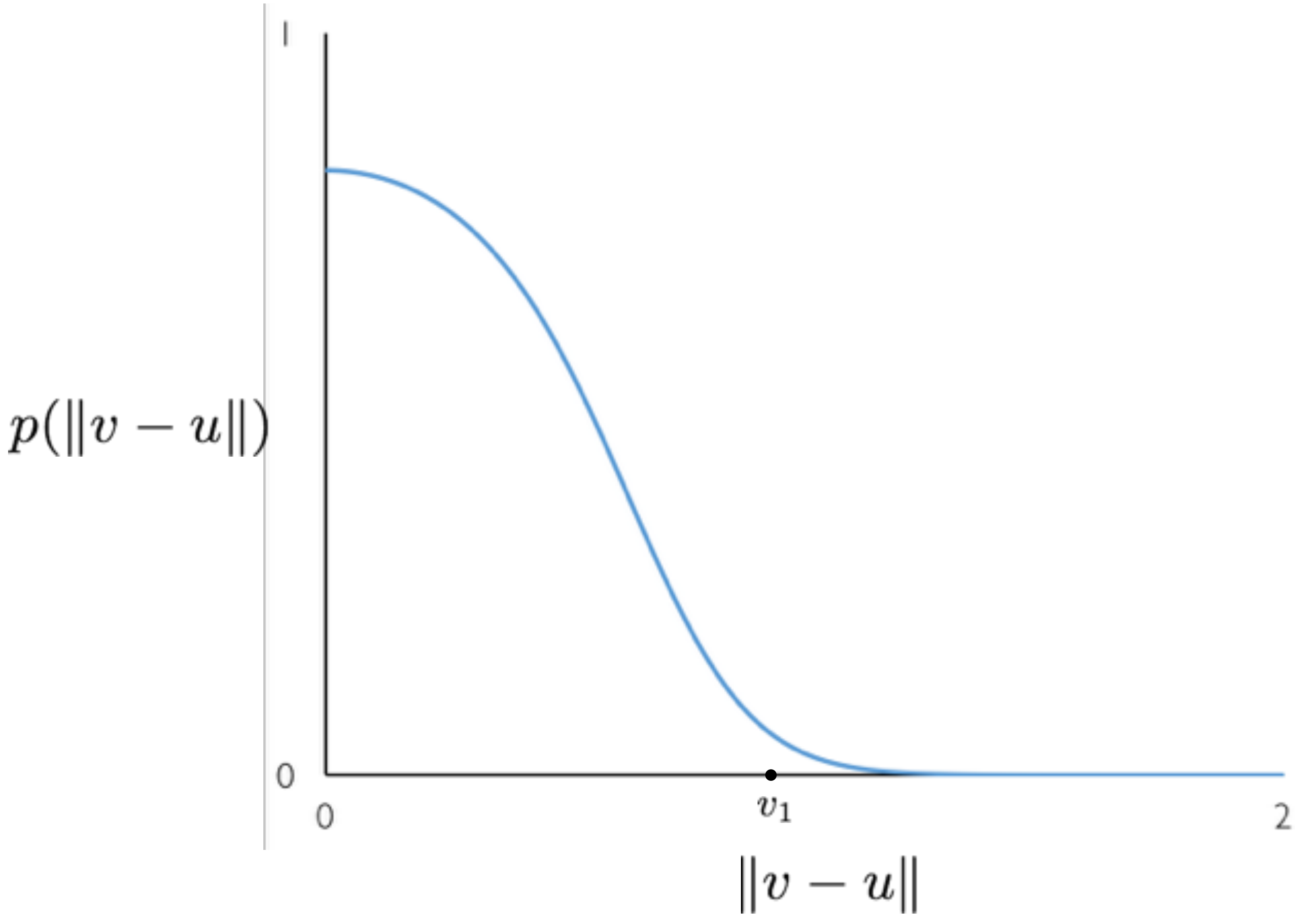

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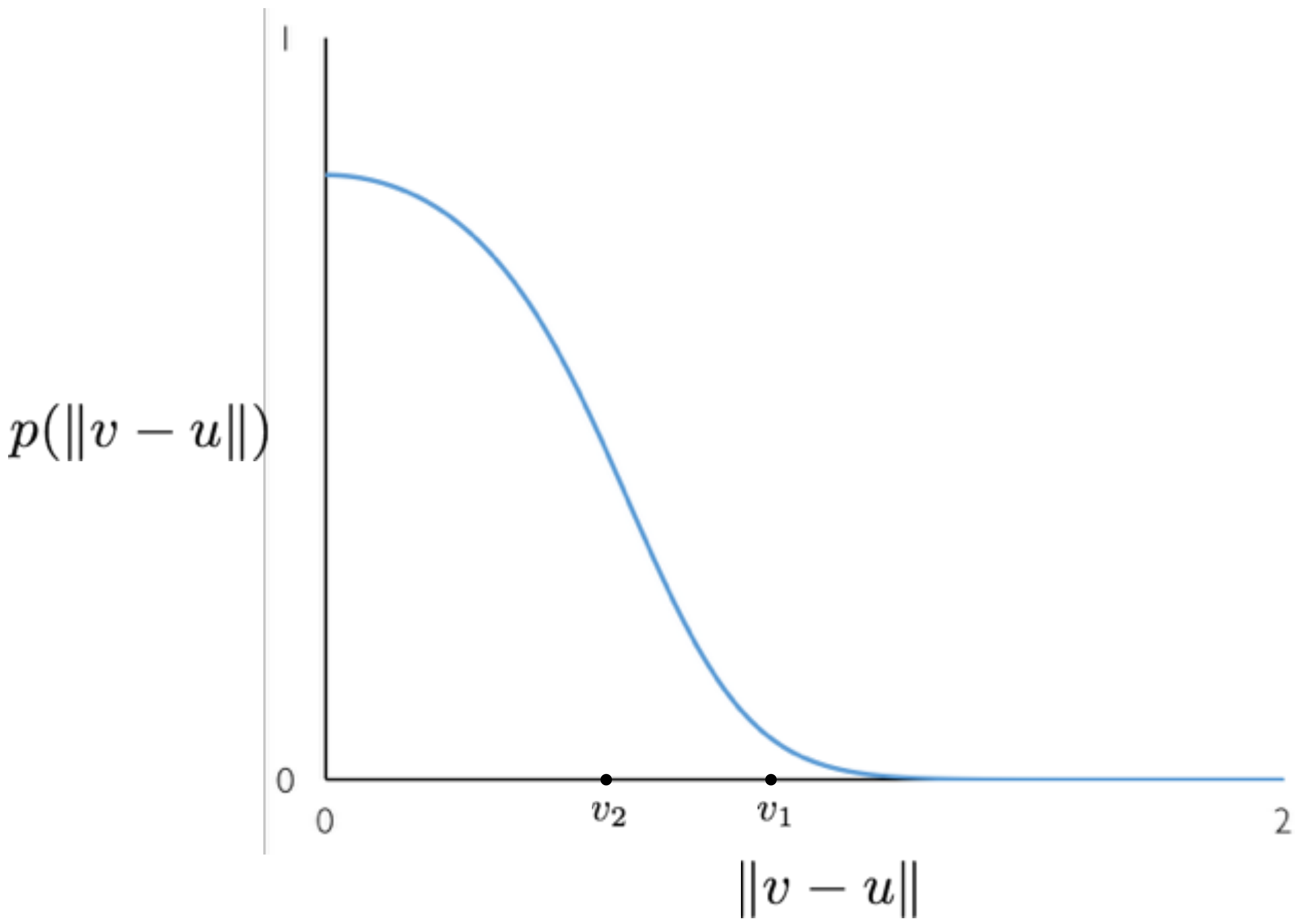

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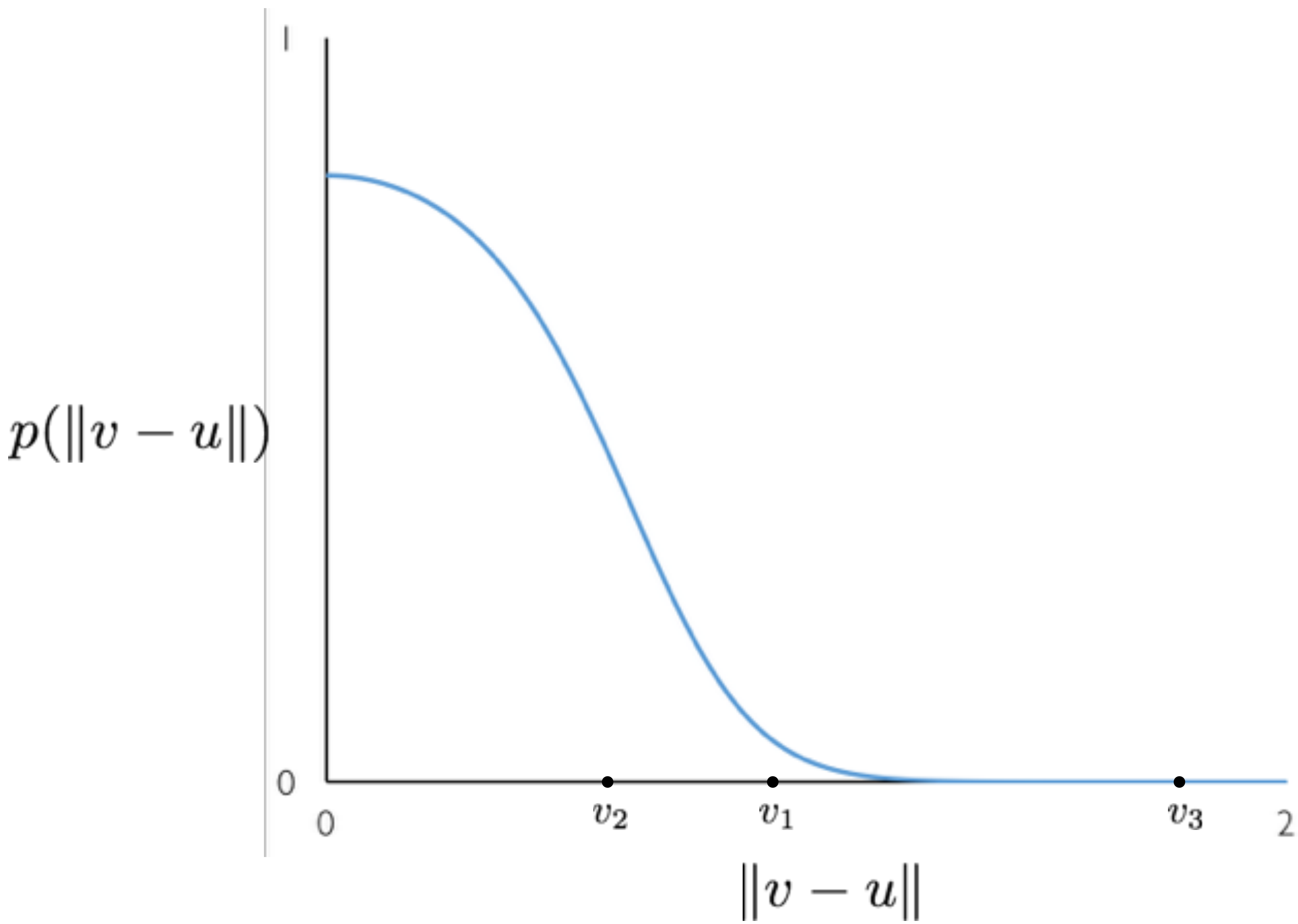

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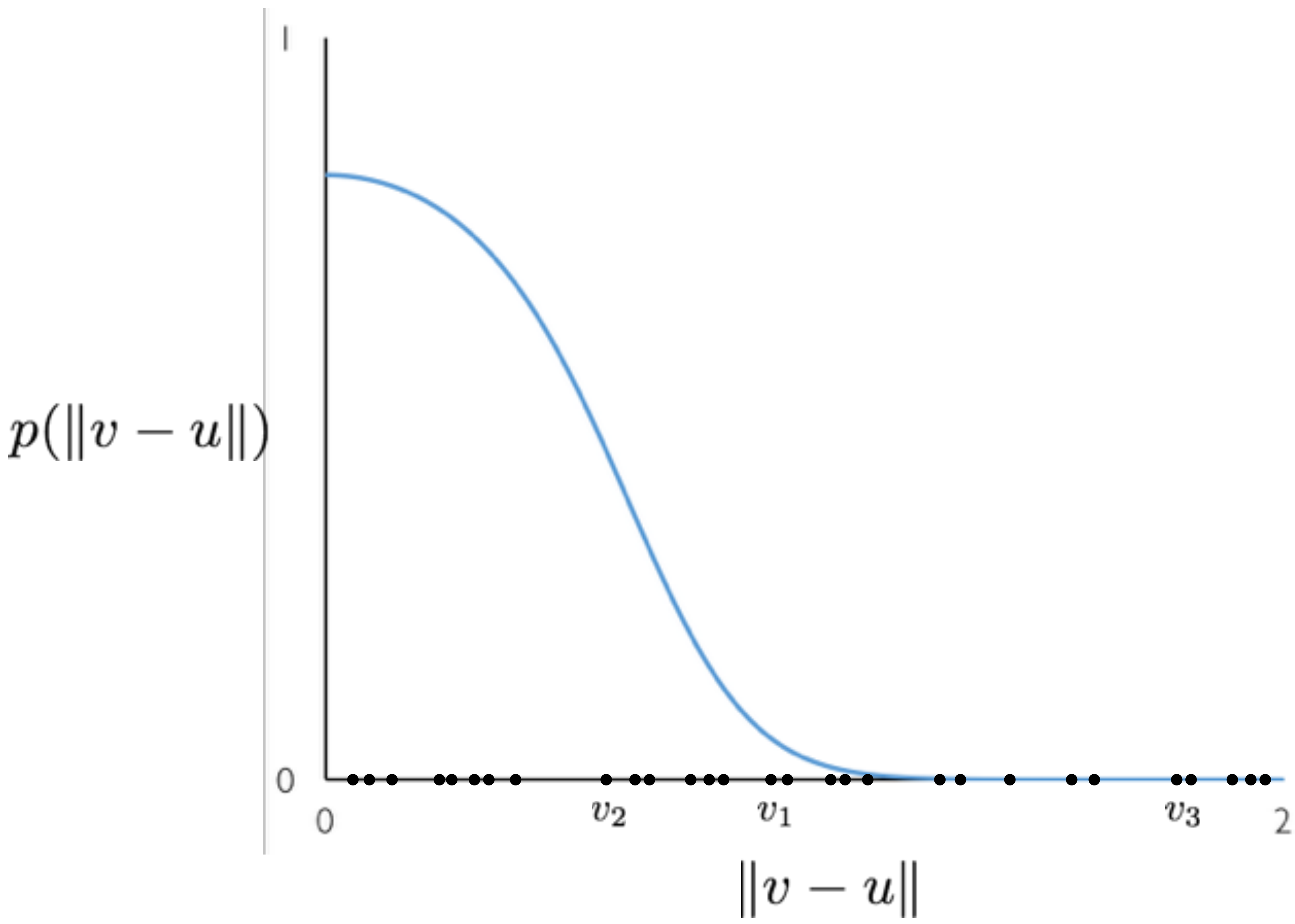

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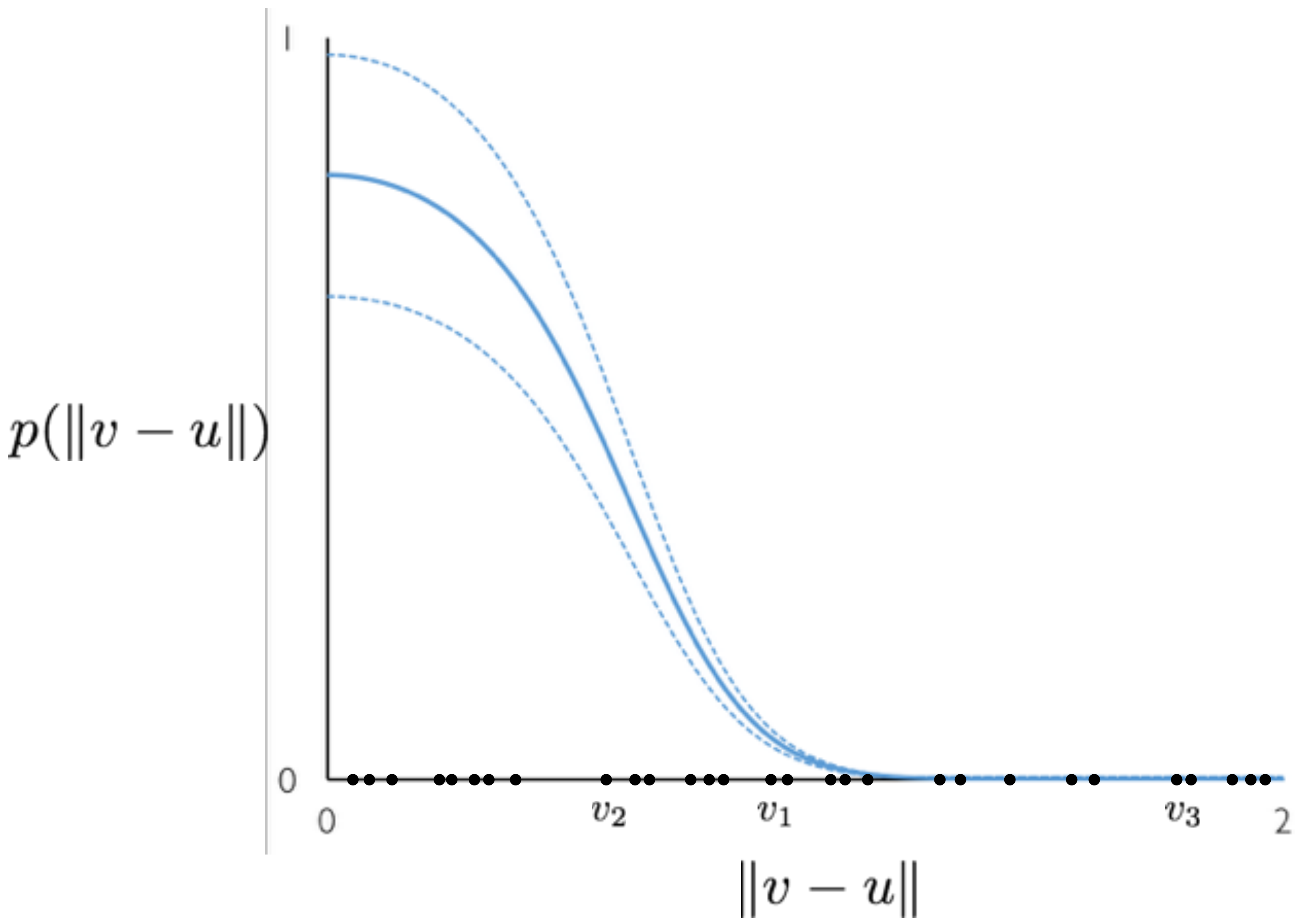
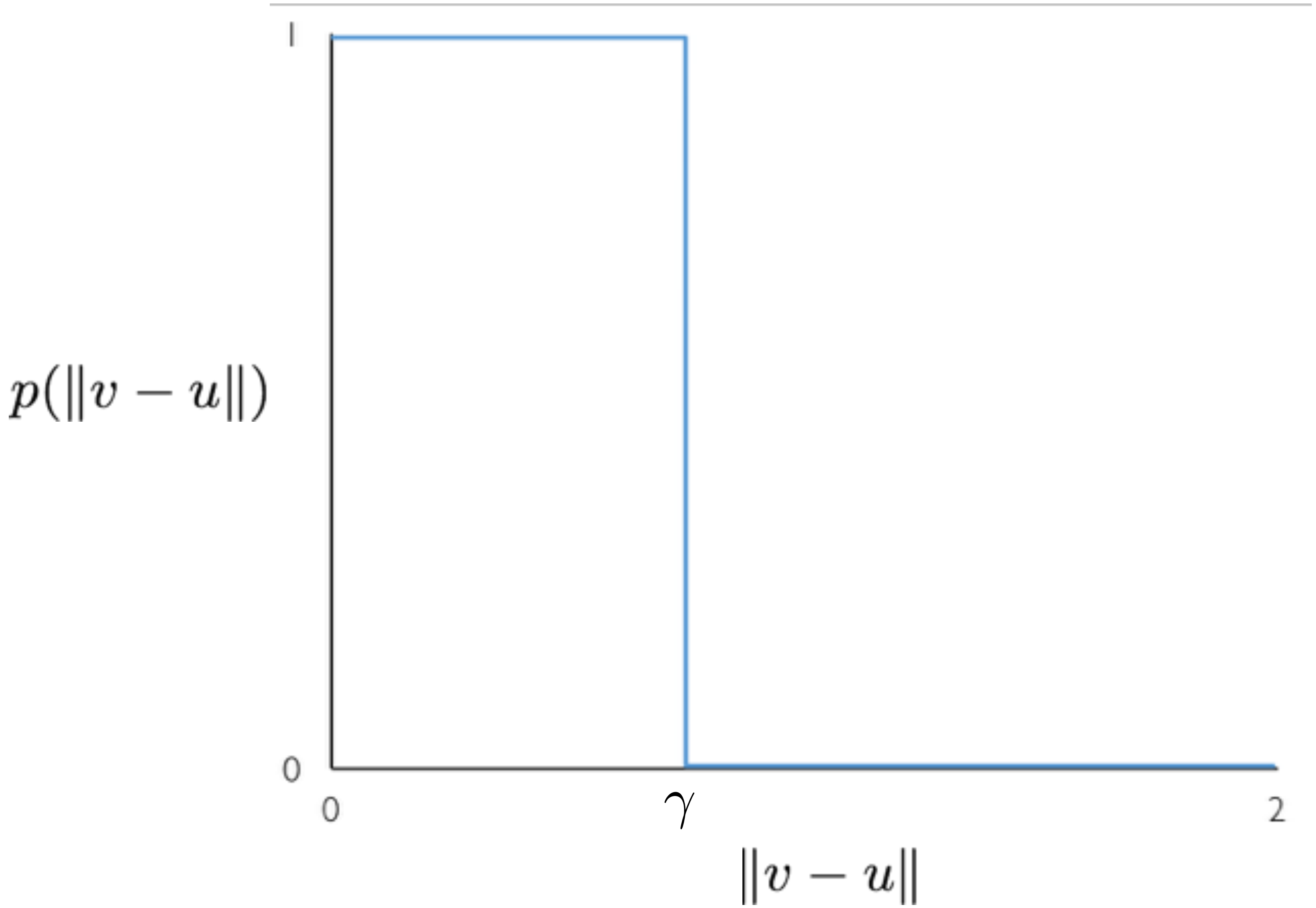


Example: Near Neighbor

- Return all $v \in \{v_1, \ldots, v_n\}$ s.t. $\|v - u\| \leq \gamma$



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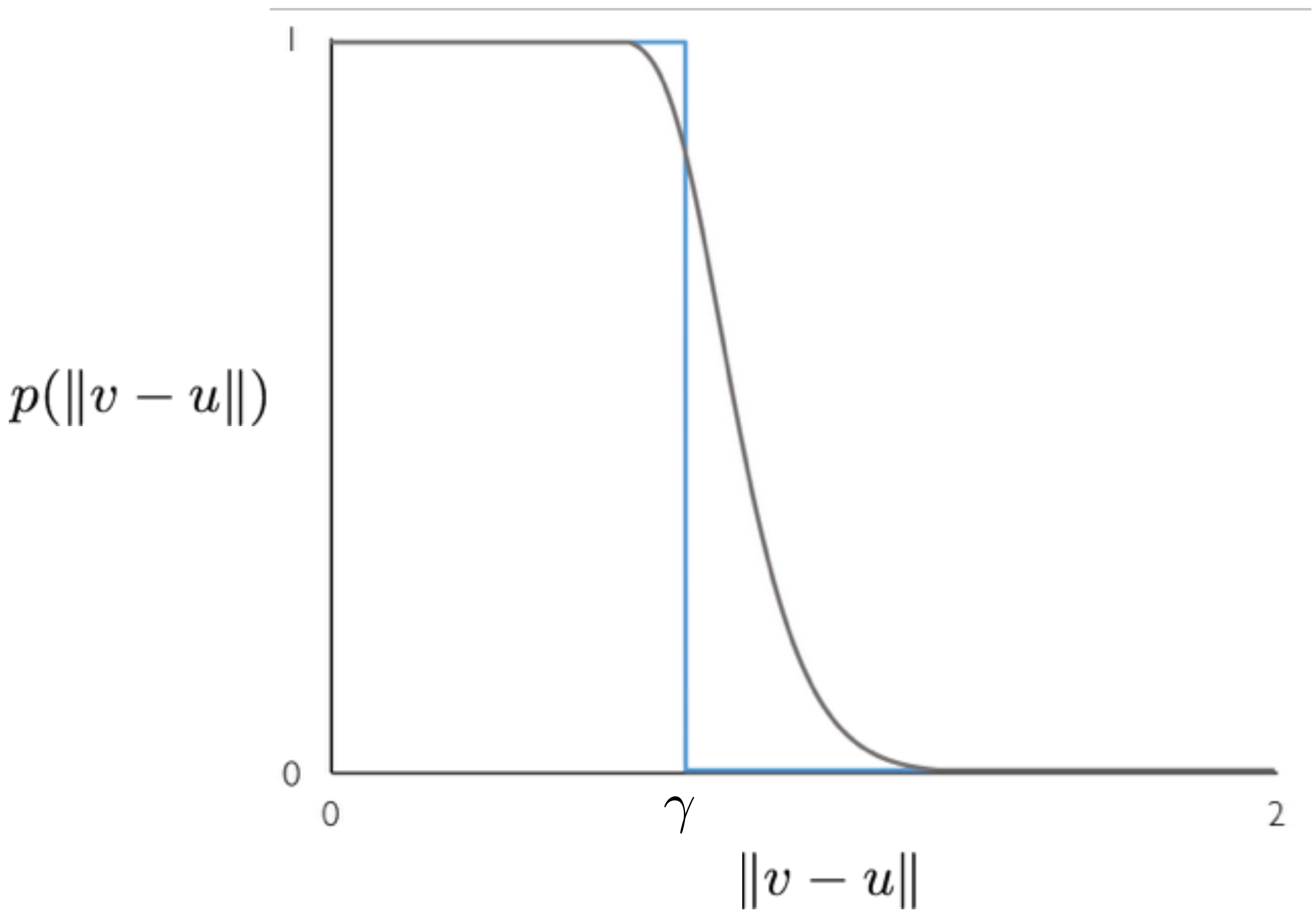
- State of the art: Locality-Sensitive Hashing (LSH)



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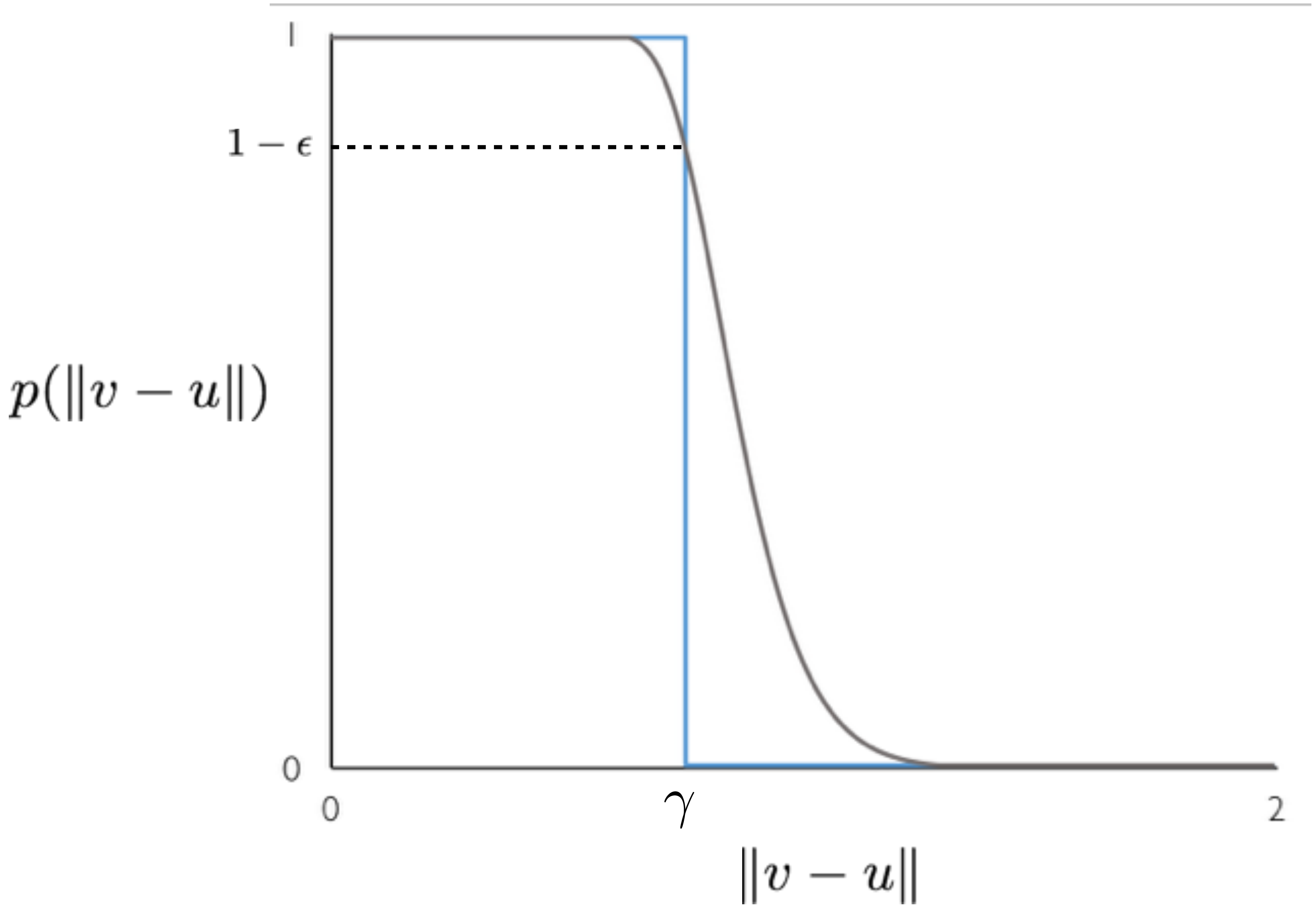

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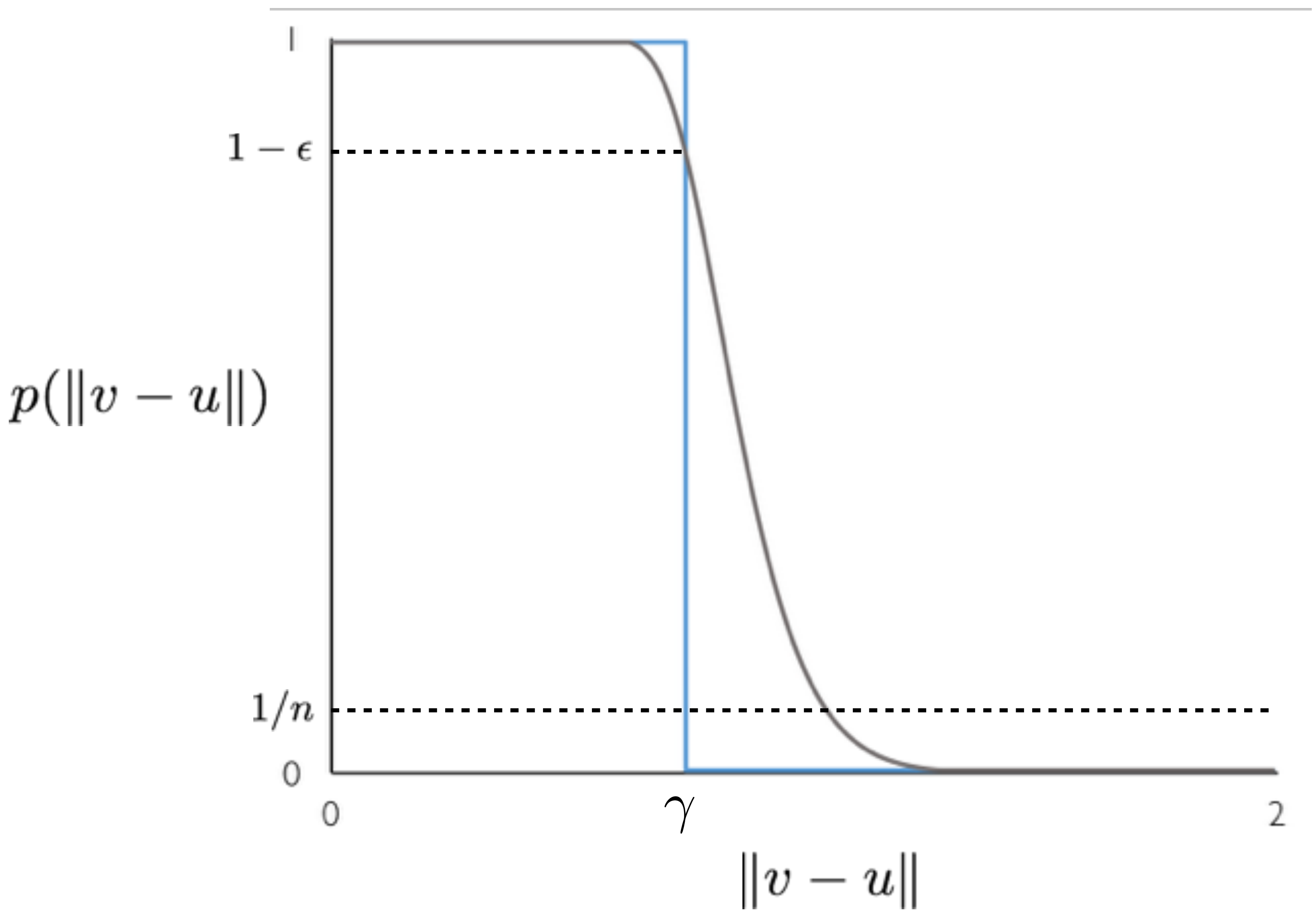

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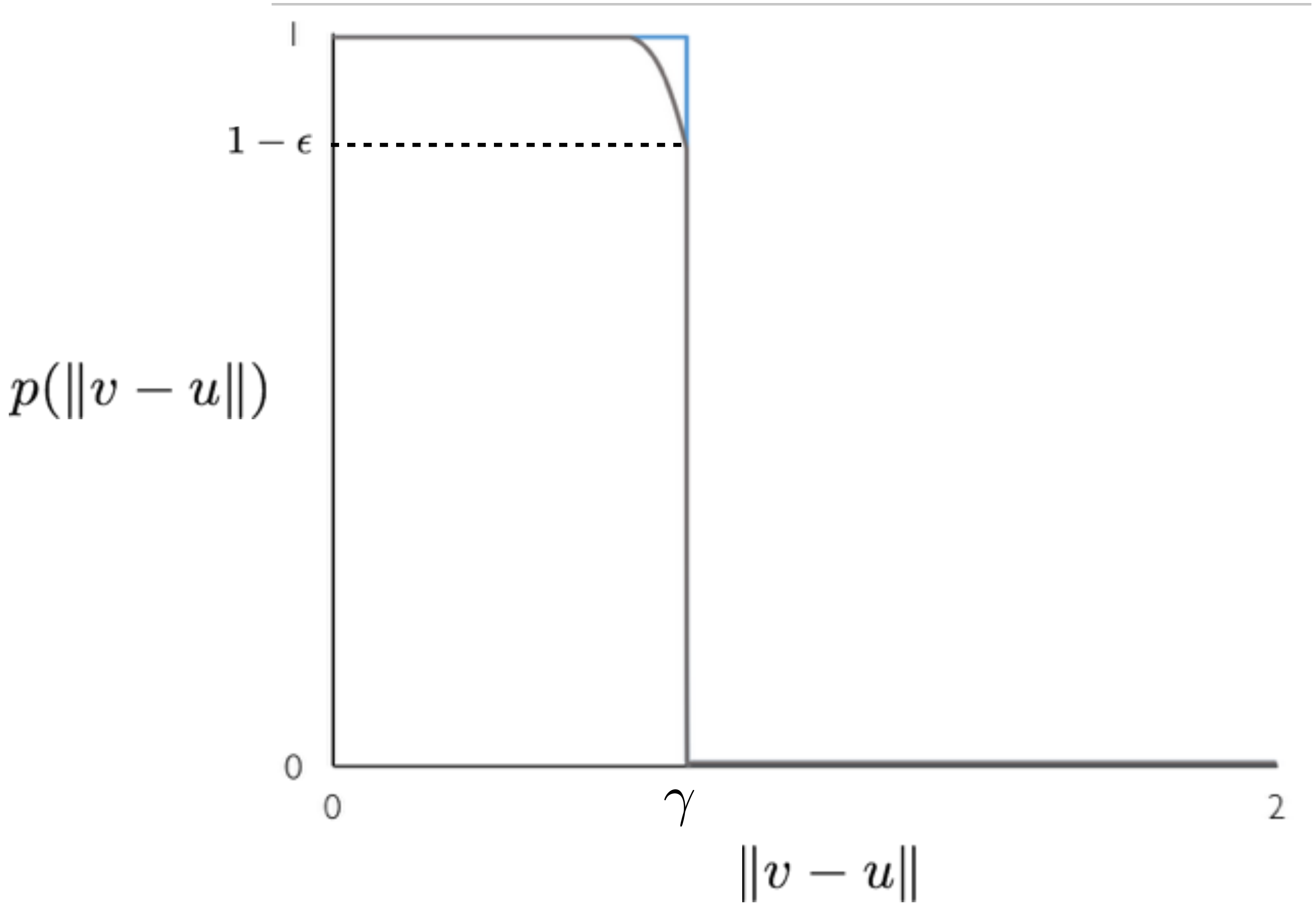

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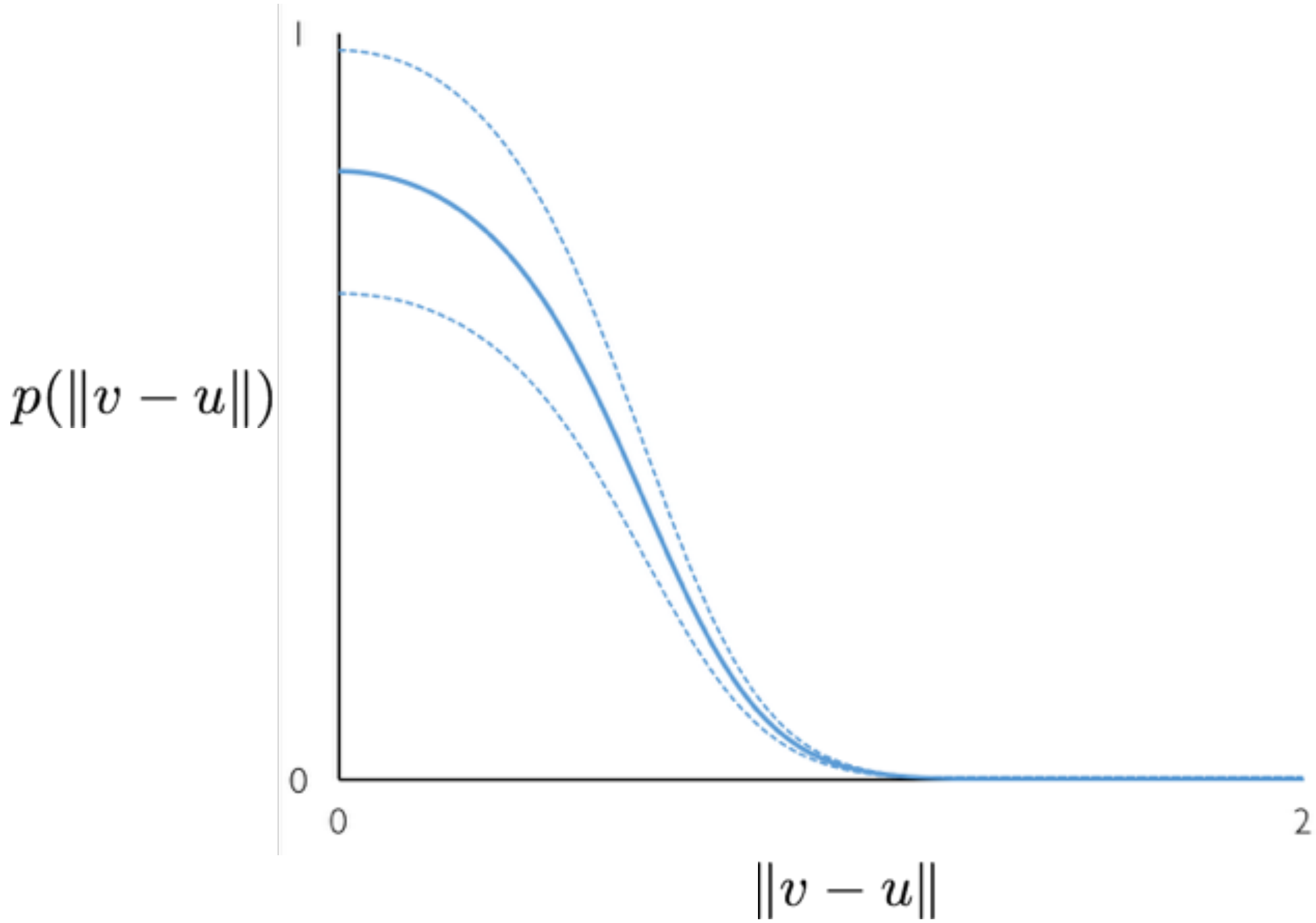

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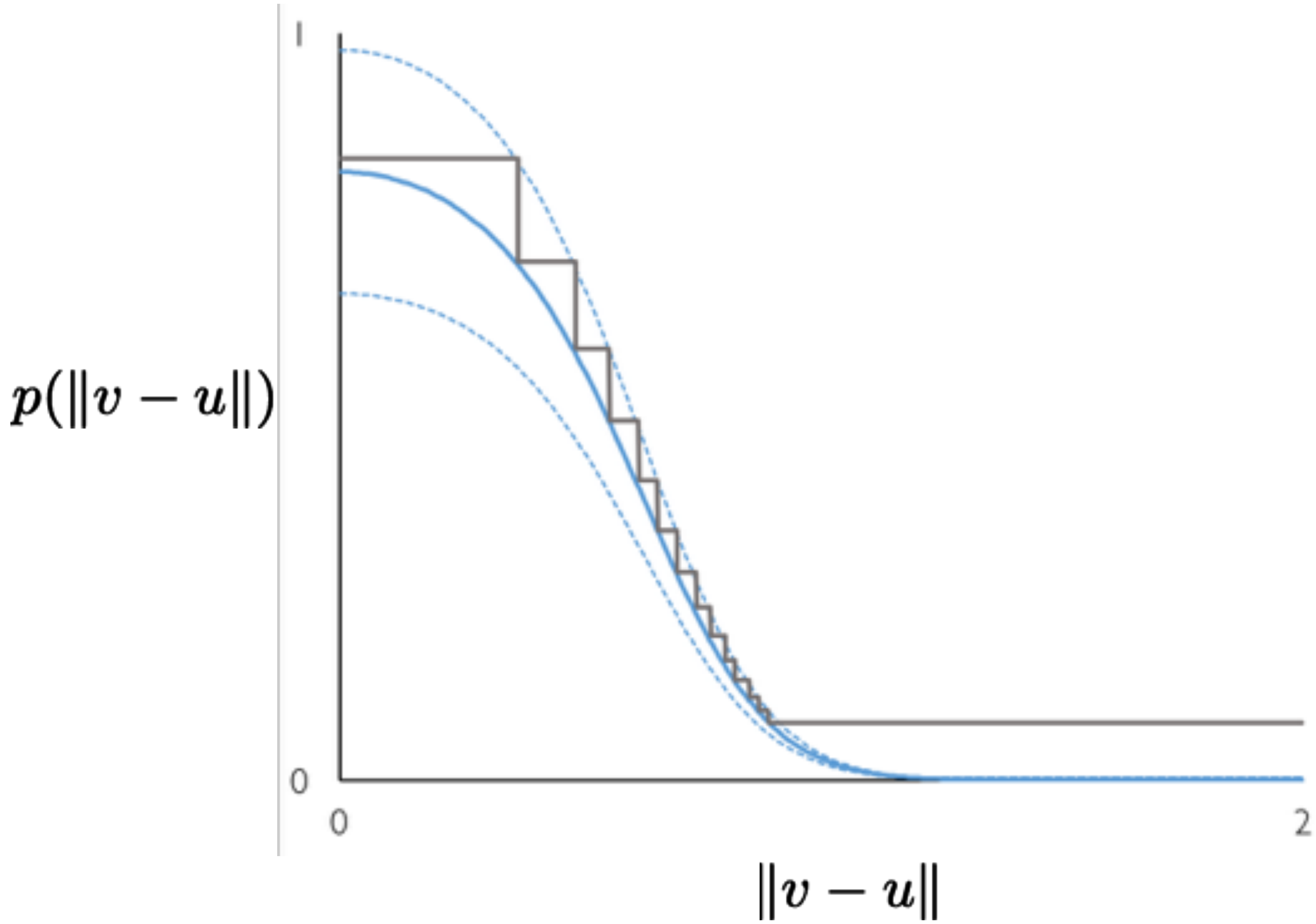

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A Sublinear Result

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- Dataset contains sample of pages viewed by users on multiple publisher sites in US between 14 - 28 June 2016
  - ~ 2 billion page views, 700 million unique users
- Construction of embedding:
  - For each user, make chronological list of page views
  - Construct embedding of articles in $\mathbb{R}^{100}$ using word2vec

Structure of Experiment

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- First 10 page views are taken as the samples from the distribution U
- Compared the performance of LSF with two benchmarks:
  - *Last viewed* - Recommends nearest neighbors of the last viewed page
  - *Mean* - Recommends nearest neighbors of the mean of the samples

Experiments: Results

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28.5% improvement corresponds to revenue increase of $\sim$ \$68 million

Thanks!

LSH

- LSH is a hashing scheme such that nearby points are more likely to have same hash value
- LSH algorithm:
  - Pre-process points in search space to obtain their hash values
  - Find hash value of the query and return input points having same
- LSH algorithm can determine if there exists a near neighbor with some probability in sub-linear time

Details on Near Neighbor

$$\hat{v} = argmax_{v \in V} \frac{1}{s} \sum_{i=1}^{s} \mathbf{1}(v^T u_i > \gamma)$$

- Reframe the problem as Near Neighbor queries
- Define Near Neighbor $NN(u) = \{v : v^T u > \gamma\}$
- Search space for $\hat{v}$ is narrowed from V to $\cup_{i=1}^{s} NN(u_i)$
- If the NN queries run in sublinear time and size of the set $\cup_{i=1}^{s} NN(u_i)$

  is sublinear, then $\hat{v}$ can be calculated in sublinear time

  - NN queries can be answered approximately in sublinear time using Locality Sensitive Hashing (LSH)