FIFTH INTERNATIONAL WORKSHOP ON INDOOR AND OUTDOOR SMALL CELLS

Load Dependent Optimal ON-OFF Policies in Cellular Heterogeneous Networks

Deeksha Sinha [1]

Indian Institute of Technology Bombay

May 12, 2014

[1] Co-authors: V. Kavitha and Abhay Karandikar

Outline

Outline

1. Motivation
2. System Description
3. Problem Statement
4. Optimal Operational Policies
5. Conclusion and Future Work

Motivation

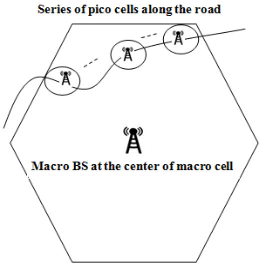
- Exponential increase in mobile subscriptions
- **Heterogeneous networks** proposed to meet demands
- ICT responsible for 2% of world's carbon emissions
- Improvements can reduce GHG emission by 16.5% by 2020
- Make cellular networks energy efficient
- Sleep-wake up mechanisms for BSs

Sleep-Wake Up Mechanisms

- No. of users varies throughout the day
- BS deployment meets peak demand
- Deployment of small cells further adds to the redundancy
- BSs consume about 80% of total power
- BSs consume significant power even while serving no or very few users
- Appropriate to switch OFF BSs at times of low load

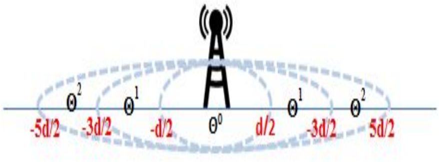
Outline

System Layout



- Busy road/Metro line passing through a macro cell
- Load of MBS shared by series of PBSs (separated by d) along the road
- PBSs can be switched OFF during low traffic
- Fraction of PBSs switched OFF η determined by traffic and QoS requirements

Pico Base Station



- Road divided into cells of length d
- Rate regions for each PBS of fixed transfer rate
- Rate θ^k for users k cells away
- $\theta^0 > \theta^1 > \cdots > \theta^i > \theta^{i+1} > \cdots$

Macro Base Station

- MBS serves all users at θ^M
- Users in a cell served at same rate both by MBS and PBS
- Modelled as queue of users
- Arrival process of users in each queue is IID

Definitions and Notations

- Activation vector **a** determines which PBSs are OFF
  - $a_i = 0$ if ith PBS is ON, otherwise 1 ($a_0 = 0 \forall \mathbf{a}$)
- J-association policy for user-base station association
  - Connect to MBS if all J neighbouring PBSs are OFF else connect to the nearest ON PBS
- QoS metric is the customer average of the expected waiting times
  - $\overline{W}(\mathbf{a}, J) = \limsup_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} W_n(\mathbf{a}, J)$

Outline

The Optimization Problem

Maximize switch-OFF ratio maintaining QoS within prescribed limits

$$\sup_{\mathbf{a},J} \quad \eta \quad \text{subject to} \tag{1}$$

$$\overline{W}(\mathbf{a}, J) \leq \overline{W}_{\text{QoS}} \quad \text{and} \qquad \liminf_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} a_n \geq \eta.$$

Two control policies -

- PBS ON-OFF policy i.e. **a**
- User-base station association policy i.e. J

Solution Outline

1) For given η, determine $(\mathbf{a}^*(\eta), J^*(\eta))$ which minimizes $\overline{W}(\mathbf{a}, J)$ i.e.

$$\min_{\mathbf{a},J} \limsup_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} W_n(\mathbf{a}, J)$$

$$\text{subject to} \quad \liminf_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} a_n = \eta. \tag{2}$$

2) Determine minimum avg. waiting time for every η i.e. $\overline{W}(\mathbf{a}^*(\eta), J^*(\eta))$.

3) Determine maximum η for which $\overline{W}(\mathbf{a}^*(\eta), J^*(\eta)) \leq W_{QoS}$ and show that this η is the solution of (1).

Outline

Optimal Policies for a fixed η

Theorem

For $J < \bar{J}$, the function $f_n(\mathbf{a}_1^n) := W_{n-J}(\mathbf{a}, J)$ is Multimodular $\forall$ n where $\bar{J} = \max_k \{k : \theta^k > \theta^M\}$.

Theorem

*$(\mathbf{a}^*(\eta), \bar{J})$ is the minimizer of the optimization problem (2) i.e. $\overline{W}(\mathbf{a}, J) \geq \overline{W}(\mathbf{a}^*(\eta), \bar{J}) \;\; \forall$ J and for all activation vectors **a** in which the fraction of base stations that are switched OFF is η where*

$$\mathbf{a}^*(\eta) := \{a_n\}$$
$$a_n = \lfloor n\eta + \beta \rfloor - \lfloor (n-1)\eta + \beta \rfloor$$

Optimal Policies for a fixed η

$$a_n = \lfloor n\eta + \beta \rfloor - \lfloor (n-1)\eta + \beta \rfloor$$

η	$\mathbf{a}^*(\eta)$
0.2	0 0 0 0 1 0 0 0 0 1
0.3	0 0 0 1 0 0 1 0 0 1
0.6	0 1 0 1 1 0 1 0 1 1

- $\mathbf{a}^*(\eta)$ has a very simple form and can be easily calculated
- $\mathbf{a}^*(\eta)$ is periodic when η is rational
- $\mathbf{a}^*(\eta)$ depends only on η (not on J, θ^i, θ^M etc)
- $J^*(\eta) = \bar{J} \;\forall\; \eta$

Structural properties of $\mathbf{a}^*(\eta)$

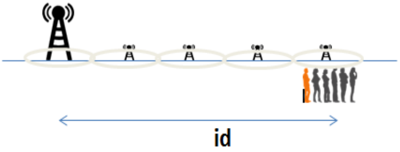


Figure: Queue of type i

Theorem

When $\eta = k_1/k_2$ and $h(r-1) < \eta \leq h(r)$ for some r, with $\gamma := \min\{r-1, \bar{J}\}$ we have the following in a block of k_2 consecutive PBSs:

1. $(1-\eta)$ fraction of the PBSs are ON.

2. $2(1-\eta)$ fraction of queues are of type i for each $1 \leq i \leq \gamma$.

3. Remaining are either of type r (if $r-1 < \bar{J}$) or are connected to the MBS.

Minimum Waiting Time

Theorem

$$\overline{W}^*(\eta) = w(\theta^x) - (1-\eta)\left(\sum_{k=0}^{\min(r-1,\bar{J})} w(\theta^k) b_{r,k} + w(\theta^x)\left(1 + 2\min(r-1,\bar{J})\right)\right)$$

$$where\ x = \begin{cases} r\ \text{if}\ r-1 < \bar{J} \\ M\ \text{if}\ r-1 \geq \bar{J} \end{cases} \quad and\ b_{r,k} = \begin{cases} -1\ \text{if}\ k = 0 \\ -2\ \text{if}\ 1 \leq k \leq \min(r-1,\bar{J}) \end{cases}$$

Overall Solution

When $w(\theta^0) \leq \overline{W}_{\text{QoS}} \leq w(\theta^M)$, η' which satisfies $\overline{W}^*(\eta') = \overline{W}_{\text{QoS}}$ is :

$$\eta' = 1 - \frac{w(\theta^x) - \overline{W}_{\text{QoS}}}{w(\theta^x)\left(1 + 2\min(r'-1, \bar{J}\right) + \sum_{k=0}^{\min(r'-1,\bar{J})} w(\theta^k) b_{r',k}}$$

with $\overline{W}^*(h(r'-1)) < \overline{W}_{\text{QoS}} \leq \overline{W}^*(h(r'))$ and $h(r) = \dfrac{2r}{1+2r}$.

Theorem

η' is the solution to the optimization problem (1).

Outline

Conclusion

- Bracket sequences are optimal for the activation vector of PBSs
- Optimal user-base station association policy is the J-association policy with $J = \bar{J}$
- $(a^*, \bar{J})$ is a pair of optimal policies which minimize the customer average of the waiting time for a fixed η
- We obtained explicit expression for average waiting time under these optimal policies and thus determined the maximum fraction of PBSs that can be switched OFF while maintaining QoS

Future Work

- Consider fading in the signals of MBS and PBSs
- Consider decentralized policies i.e. each PBS decides the ON-OFF policy by itself
- Also, consider policies based on current number of users and not just the arrival rates

Thank You

Multimodularity

- A function $f : \{0,1\}^n \to R$ is Multimodular if

$$f_n(\mathbf{a}+\mathbf{v}) + f_n(\mathbf{a}+\mathbf{u}) \geq f_n(\mathbf{a}) + f_n(\mathbf{a}+\mathbf{u}+\mathbf{v})$$

for all $\mathbf{a} \in \{0,1\}^n$ *and for all* $\mathbf{u}, \mathbf{v} \in F$ *(the Multimodular base) with* $\mathbf{u} \neq \mathbf{v}$ *and such that* $\mathbf{a}+\mathbf{u},\ \mathbf{a}+\mathbf{v},\ \mathbf{a}+\mathbf{u}+\mathbf{v} \in \{0,1\}^n$.
F contains the vectors $-e_1, s_2, s_3, \cdots, s_n, e_n$, where

$$\begin{aligned} -e_1 &= (\text{-1 } 0\ 0\ 0\ 0 \cdots\ 0\ 0) \\ s_2 &= (1 \text{ -1 } 0\ 0\ 0 \cdots\ 0\ 0) \\ s_3 &= (0\ 1 \text{ -1 } 0\ 0 \cdots\ 0\ 0) \\ &\vdots \\ s_N &= (0\ 0\ 0\ 0\ 0 \cdots\ 1 \text{ -1}) \textit{ and} \\ e_N &= (0\ 0\ 0\ 0\ 0 \cdots\ 0\ 1) \end{aligned}$$

- Bracket sequence $\mathbf{a^*} := \{a_n\}$ with given rate $\rho \in [0,1)$ and initial phase θ is

$$a_n = \lfloor n\rho + \theta \rfloor - \lfloor (n-1)\rho + \theta \rfloor$$

Multimodularity

Theorem

A bracket sequence $\mathbf{a}(\rho, \theta)$ *for any* $\theta \in [0, 1)$ *minimizes cost*

$$\limsup_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f_n(a_1, \cdots, a_n)$$

over all the sequences that satisfy

$$\liminf_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} a_n \geq \rho$$

where $\rho \in [0, 1)$*, under the following assumptions:*

1. f_n *is Multimodular* $\forall n$.
2. $f_n(a_1, \cdots, a_n) \geq f_{n-1}(a_2, \cdots, a_n) \forall n > 1$
3. *for any sequence* $\{a_n\}$, $\exists$ *a sequence* $\{b_n\}$ *such that* $\forall n, m$ *with* $n > m$,

$$f_n(b_1, \cdots, b_{n-m}, a_1, \cdots, a_m) = f_m(a_1, \cdots, a_m)$$

4. *for every* n, *the functions* $f_n(a_1, \cdots, a_n)$ *are increasing in* a_i *for every* i.