**Formulation and Programming for estimation of Dynamic Coefficients of CylindricalJournal Bearings**

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**Abstract**

Journal bearings are widely used in many machinery for example, a turbine. It is very important to know the behavior of journal bearings under action of load. For that we estimate the dynamic coefficients of the journal bearings.

This project is a formulation for deriving the stiffness and damping coefficients of a cylindrical journal bearing. It is based on a perturbation solution of the Reynolds equation where the resulting partial differential equations are evaluated numerically by using finite difference method. A java program has been written to estimate the dynamic coefficients using this formulation.

1. **Introduction:**

A journal bearing is a metal sleeve made of appropriate material which supports a rotor or shaft which freely rotates inside the bearing.

Journal bearings are widely used in turbines. Journal are considered to be sliding bearings.

Journal bearings shown in fig. 1.

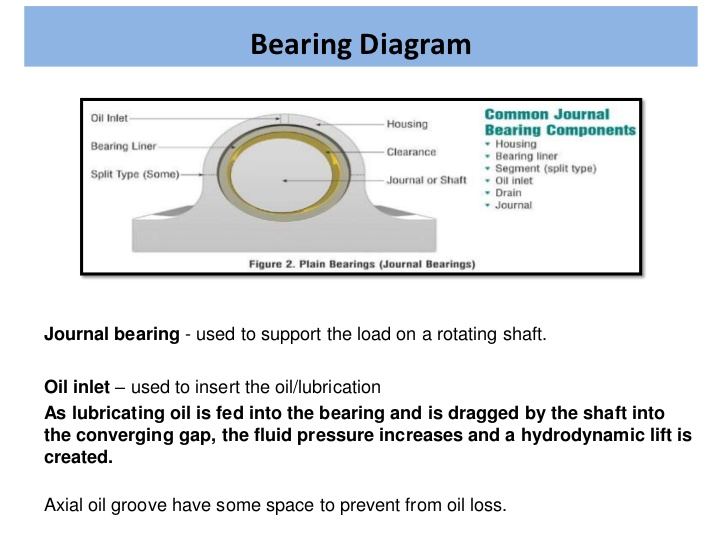


Fig 1. A cylindrical journal bearing with housing.

**Advantages:**

1. It can handle high load and velocities.

2. The bearing is very durable and long lasting.

3. The damping effect of oil film in the journal bearing helpsto support flexible shaft.

**Disadvantages:**

1. They require large supply of lubricating oil.

2. Suitable for relatively low temperature.

3. Starting resistance is much greater than the running resistance.

1. **Objectives of the present work:**
2. Formulation of governing equations for estimation of stiffness and damping coefficients of journal bearings.
3. Development of JAVA program for the estimation of stiffness and damping coefficients.
4. **Derivation of Reynolds equation:**

Reynolds equation is a specific case of Navier Stokes equations. Navier Stokes Equations for incompressible flow

Assumptions for the Reynolds euqation:

* Incompressible
* Constant viscosity
* Neglecting body forces
* Thin film geometry
* Neglecting inertial forces
* Laminar flow

are negligible , therefore they are neglected and their corresponding 2nd order terms.

After substituting these conditions in equations in 1 and 3 we get 4 and 5 respectively.

Integration of 4 and 5 with respect to dy results in 6 and 7 respectively.

No slip boundary conditions are considered as given below.

At the rotor u=U, w=0, y=0

At the bearing u=0, w=0, y=h

After substituting the no slip boundary conditions,

Equation 6 becomes

Equation 7 becomes

Continuity equation for incompressible

Integrating this equation from 0 to h w.r.t dy

Apply Leibniz rule.

When the velocity at the rotor end is and at the bearing end is then the rate of change of oil film thickness at particular angle is as given below.

After substituting the above, equation 10 becomes:

Here as we are assuming the bearing is uniform along the width. Therefore

1. **Reynolds equation for cylindrical journal bearings:**

The side view of cylindrical journal bearing is shown in Fig.2.

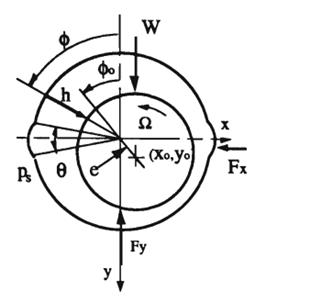
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Fig. 2. Side view of the journal bearing

The circumference of the bearing is unwrapped to represent the variation of film thickness as shown in Fig.3.

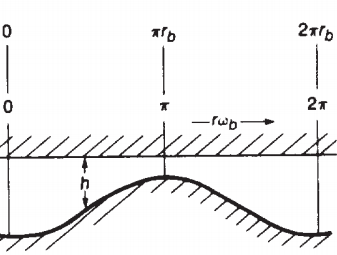


Fig. 3 Unwrapped view of oil film thickness along the circumference of the bearing

In this figure angle is measured from line joining rotor centre and bearing centre. Substitute in *Reynolds equation* results in the following.

The pressure depends on the displacement of rotor in x and y direction and velocity of rotor in x and y direction. Using Taylor expansion and neglecting second order terms, the pressure can be written as

is equilibrium distribution of *h* .

Substituting 12 and 13 in equation 11 and neglecting higher order termssuch as terms containing etc., results in the following.

From separation of variables

To non-dimensionalize the equation 11, let

,

where is angular frequency of rotor

By substituting equation 19 in equation 11 becomes equation 20 as given below.

Boundary conditions:

* Pressure at the bearing ends is zero

1. **Eccentricity:**

The geometry of the journal bearing is shown in figure 4.The shaft does not normally run concentric with the bearing. The displacement ofthe shaft center relative to the bearing center is known as the eccentricity (e in fig 4). Theshaft's eccentric position within the bearing clearance is influenced by the loadthat it carries. The amount of eccentricity adjusts itself until the load is balancedby the pressure generated in the converging lubricating film.

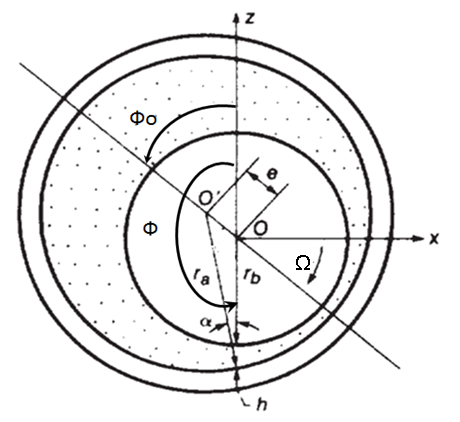


Fig 4. Side view of the bearing which reveals eccentricity

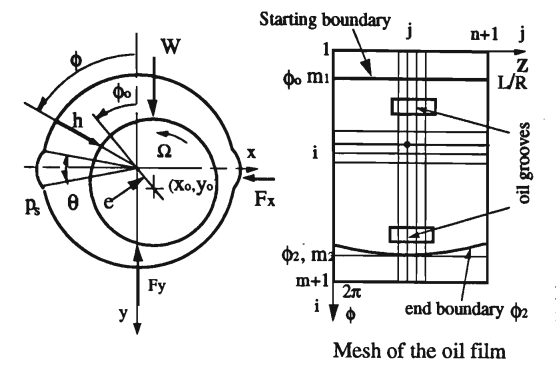
From sine rule

From binomial expansion and after neglecting terms with and higher power since is in the order, we get

where

1. **Attitude angle:**

The angle between load axis (i.e., y axis in fig 5) and line joining bearing center and rotor center is called as attitude angle (i.e., in fig 5). At attitude angle the film thickness is minimum and pressure is maximum.



**Fig. 5 Side view of the journal bearing Fig. 6 Developed view of the oil film with**

**indicating the forces some of the nodes marked**

So we define an error function

When from eq 21 then is satisfied.

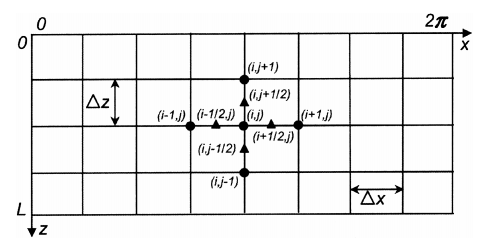
Using Newton-Raphson method is calculated by starting with an initial guess value.

If We accept as attitude angle.

Here

1. **Bearing forces:**

Fx is in horizontal direction and Fy is in vertical direction.



**Fig. 7 Developed view of the oil film indicating the nodes with properly marked axes**

Here

From fig. 7,

1. **Computation by Finite Difference Method-Infinitesimal perturbation:**

So

After applying Leibniz rule

After non dimensionalizing

Similarly for other coefficients

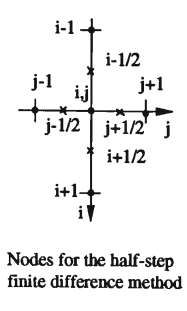
are perturbing pressures

1. **Reynolds equation solution:**

After Non dimensionalizing equations 14,15,16,17,18 from eq 19 we get the following equations

These equations can solved by finite difference method.

We are using forward difference.



**Fig. 8 Nodes for half step finite difference method**

The oil film is divided into a grid of desired size.

So now

⇒

Similarly

Where

Now the Reynolds equation is of the form

Where

for

for

for

for

for

After rewriting

Pressure can calculated by iterating using over relaxation method

For , during iteration if < 0 then is set to zero.

Iteration is repeated until

After is calculated for is calculatedand boundary nodes are noted and then the same equations are solved iteratively to get . After that they are integrated to get the dynamic coefficients

1. **Summary:**

First, Reynolds equation for journal bearings (eq. 11) is derived from Navier Stokes equations (eq. 1, 2, 3).Then, eccentricity is defined. Attitude angle is calculated using Newton Raphson formula by starting with a guess value. The Reynolds equation is non-dimesionalised (eq. 20). Reynolds equation is then solved using finite difference method. The dynamic coefficients are formulated in terms of the perturbed pressures. The perturbed pressures are calculated using equations 23,24,25,26. The dynamic coefficients are calculated by integrating the perturbed pressures.

1. **References**:
2. Zhi Ling Qiu(1995)“A theoretical and experimental study on dynamic characteristics of journal bearings”
3. “Calculation of Journal Bearing Dynamic  
   Characteristics Including Journal Misalignment  
   and Bearing Structural Deformation” OMIDREZA EBRAT , ZISSIMOS P. MOURELATOS , NICKOLAS VLAHOPOULOS & KUMAR VAIDYANATHAN(2010)
4. “Fundamentals of Fluid Film Lubrication”, Bernard J. Hamrock, The Ohio State University, Columbus, Ohio, NASA Reference Publication 1255, 1991

**Flow Chart for the function pCalc** : with grooves between

Start,Counter=0

is calculated for phi=0,counter++

End

yes

no

yes

no

no

yes

no

no

no

no

no

no

yes

yes

yes

no

no

yes

yes

no

no

temp array to store values. if then it is set to zero in the case of Po

If j<size

i++;

If j<size

J++;

If

j=0

I=0

all pressures set to zero

phi =5 degrees and is calculated for phi=0

If j<size

i++;

yes

If j<size

J++;

temp array to store values. if then it is set to zero in the case of Po

If

j=0

I=0

Phi=0

yes

no

If counter=0