Finite Blocklength Rates over a Fading Channel with an Energy Harvesting Transmitter

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***Abstract*—We consider a fading channel with Additive White Gaussian noise with an energy harvesting transmitter. A finite blocklength analysis of the channel capacity is provided. This work combines known results for fading channels and energy harvesting AWGN channels that have been studied recently in the finite blocklength regime.**

***Index Terms*—Fading channel, energy harvesting, finite blocklength, channel capacity**

I. Introduction

Finite blocklength analysis for channels with fading under various power control schemes is available in [1].

A. Main Contributions

II. Model and Notation

Consider a wireless communication system powered by an energy harvesting transmitter, as depicted in Figure 1. The energy harvesting system consists of an energy buffer or battery that provides power to transmit symbols. The buffer harvests energy $E_i \geq 0$ at time instant i from the ambient environment. We assume these energy arrivals are independent and identically distributed (i.i.d.) with mean $\overline{E}$ and variance σ_E^2. Further details on the transmitter are discussed in Section II-B.

The transmitter selects a message S uniformly randomly from the set $[1 : M]$ (in general, we denote the set of j consecutive natural numbers starting from ℓ as $[\ell : \ell + j - 1]$). The encoder maps this message to an n length codeword $\mathbf{X}' \triangleq (X_1', X_2', \ldots, X_n')$, where $X_i' \in \mathbb{C}$ (the set of complex numbers) for $i \in [1 : n]$. This codeword is to be transmitted across a wireless channel and hence some processing may be carried out (in order to meet channel constraints etc.) on X_i'. Therefore, let X_i denote the corresponding input to the channel after processing. Next, we describe our channel model.

A. Channel Model

Let τ_c (in appropriate time units) denote the coherence time of the underlying wireless channel. We refer to any *coherence period* of duration τ_c as a *block*. Let D denote the maximum delay permissible at the physical layer (as dictated by the application). For convenience, assume that D is an integer multiple of τ_c. Then, $B \triangleq D/\tau_c$ denotes the number of blocks over which the communication happens. Let n_c denote the number channel uses in each block. We consider a discrete time block fading channel in which the fading gain $H_b \in \mathbb{C}$ in block $b \in [1 : B]$ remains constant for all n_c uses within the block. The fading gains are independently and identically distributed (i.i.d.) across blocks. Let F_H (assumed to be known to both the transmitter and the receiver) denote the cumulative distribution distribution of H_b, $b \in [1 : B]$. We assume F_H is such that $\int_{\mathbb{C}} |h|^2 dF_H(h) \equiv \sigma_H^2 < \infty$, where $|h|$ denotes the absolute value of the complex number h.

For $k \in [1 : n_c]$, k^{th} transmission in b^{th} block corresponds to the transmission of $((b-1)n_c + k)^{\text{th}}$ codeword symbol. For convenience, let $[b, k] \triangleq (b-1)n_c + k$. Note that $[b, 0] = [b-1, n_c]$ and $[b, n_c + 1] = [b+1, 1]$. Then, the additive receiver noise corresponding to k^{th} transmission in b^{th} block is denoted as $Z_{[b,k]}$. We assume that $\left\{Z_{[b,k]}, k \in [1 : n_c], b \in [1 : B]\right\}$ is independent and identically distributed according to $\mathcal{CN}\left(0, \sigma_N^2\right)$, where $\mathcal{CN}\left(0, \sigma_N^2\right)$ the probability density function of a circularly symmetric, complex Gaussian random variable with mean vector $[0, 0]$ and covariance matrix $\frac{\sigma_N^2}{2} I_2$ with I_2 being the 2×2 identity matrix.

B. Transmitter Model

We assume that the transmitter is equipped with a battery (equivalently, a buffer or a *cell*) with infinite storage capacity. Let $A_{[b,k]}$ correspond to the energy available in the cell at the beginning of k $^{\text{th}}$ transmission in b^{th} block. Let $E_{[b,k]}$ denote the energy harvested at the beginning of k^{th} transmission in b^{th} block. We assume that the total energy available at the beginning of k^{th} transmission in b^{th} block is $E_{[b,k]} + A_{[b,k]}$ and that the energy buffer is initially empty i.e. $E_0 = 0$.

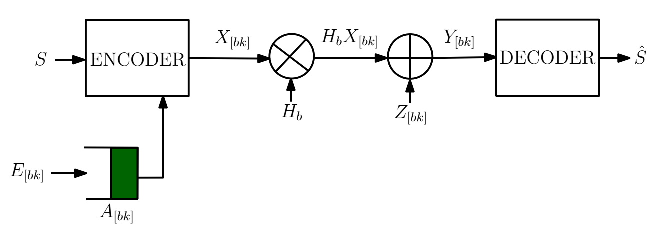

Fig. 1. Block diagram of a wireless communication system with energy harvesting transmitter

The buffer state $A_{[b,k]}$ evolves according to the equation

$$A_{[b,k+1]} = \max\{A_{[b,k]} + E_{[b,k+1]} - X^2_{[b,k+1]}, 0\}, \quad (1)$$

since the buffer cannot be negative. If at any instant the symbol to be transmitted requires more energy than what is available (including harvested energy), we refer to it as an energy outage event. It is worth observing that if there was sufficient energy to transmit every symbol (for a given input distribution), then it would be equivalent to studying the channel alone for that input. Unfortunately this is an unlikely event for several useful distributions (including capacity achieving distributions) and hence, a suitable transmission scheme or policy is required. Two well known and studied schemes are the Save and Transmit scheme as well as the Best Effort scheme. In this paper, we focus on the former as it is known to provide the optimal second order term (not necessarily the coefficient).

C. Save and Transmit Scheme

This scheme consists of two phases, namely the saving phase and the transmission phase. During the saving phase, no symbols are transmitted and the energy buffer is allowed to build up. During the transmission phase, we start transmitting symbols but now, the buffer contains some energy. This initial energy helps to lower, and hence control, the probability of energy outage. The tradeoff here is the number of slots where we don't transmit anything which is, as far as information transfer is concerned, wasted. However by carefully choosing the duration, we can achieve a balance of sorts.

D. Known Results and Useful Lemmas

We recall known finite blocklength bounds for the fading channel with additive Gaussian noise with *long term* power constraint [1], [5].

Lemma 1. *Consider a EH-AWGN channel with noise variance σ^2, channel capacity $C_{EG} = \frac{1}{2}\log\left(1 + \frac{E}{\sigma^2}\right)$, with energy harvesting process $\{E_i\}$, i.i.d. at the encoder, with mean $\mathbb{E}[E_1]$ and variance $\sigma_E^2 < \infty$.*

1) *(Achievable bound) Given maximal probability of error $\varepsilon > 0$ and any $0 < \lambda < 1$, for sufficiently large blocklength n, the maximum size of the code, $M^*(n,\varepsilon)$, is lower bounded by*

$$\log M^*(n,\varepsilon) \geq nC_{EG} + \sqrt{n}\left[\sqrt{V_{EG}}\Phi^{-1}(\lambda\varepsilon) - K_{\varepsilon,\lambda}C_{EG}\right] - \frac{1}{2}\log n + O(1), \quad (2)$$

where $V_{EG} = \frac{\mathbb{E}[E_1]}{\mathbb{E}[E_1]+\sigma^2}\log_2^2(e)$, $K_{\varepsilon,\lambda} = \sqrt{\frac{4(2\mathbb{E}[E_1]^2+\sigma_E^2)}{(1-\lambda)\varepsilon\mathbb{E}[E_1]^2}}$.

2) *(Converse Bound) The upper bound on $M^*(n,\varepsilon)$ is given by*

$$\log M^*(n,\varepsilon) \leq nC_{EG} + \sqrt{nV_{EG2}}\Phi^{-1}(\varepsilon) + \frac{1}{2}\log n + O(1) \quad (3)$$

where $V_{EG2} = \frac{\mathbb{E}[E_1]^2+\mathbb{E}[E_1^2]+4\sigma^2\mathbb{E}[E_1]}{4(\mathbb{E}[E_1]+\sigma^2)^2}\log_2^2(e)$.

To summarize, $\log M^*(n,\varepsilon) = nC + \Theta(\sqrt{n})$. Due to energy harvesting, some of the coefficients will get affected but they are restricted to the second order and above terms. This was observed when analyzing the energy harvesting AWGN channel [2]–[4].

The following lemma from [2] will aid in tackling the energy harvesting achievability part by decoupling the channel statistics from the energy harvesting analysis.

Lemma 2. *Consider a channel with an energy harvesting setup. Suppose the input to the energy harvesting channel is generated i.i.d. with zero mean and variance $\overline{E}$ and the energy arrivals are i.i.d. with mean $\overline{E}$ and variance σ_E^2. Let $0 < \varepsilon < 1$ be the allowed probability of error for the system. Then, given $0 < \lambda < 1$, there exists a Save and Transmit scheme, whose saving phase lasts $K_{\varepsilon,\lambda}\sqrt{n}$ samples and transmit phase lasts n samples such that the channel probability of error alone ε_n (non energy harvesting errors) is bounded as*

$$\varepsilon_n \leq \lambda\varepsilon - \frac{4\sigma_E^2}{K_{\varepsilon,\lambda}\overline{E}^2\sqrt{n}}, \quad (4)$$

for $K_{\varepsilon,\lambda} = \sqrt{\frac{4(2\overline{E}^2+\sigma_E^2)}{(1-\lambda)\varepsilon\overline{E}^2}}$.

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