The Design and Analysis of a 3 Solar Collector System

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Introduction

Nowadays, the solar energy has become an active research subject in order to create a more sustainable environment for our current and future generations. One of the most common applications is the solar collectors. As the name implies, they're devices that convert the radiation from the Sun to thermal energy and primarily heat up the water for uses. In this project, the ultimate goal was to provide distinct economic and environmental analyses of the installation of the solar collectors, given a variety of real-life conditions. For an inclusive evaluation of the arrangement of the devices, it's split into 5 steps, and a wide range of factors were taken into consideration and illustrated (see Fig.1) below.

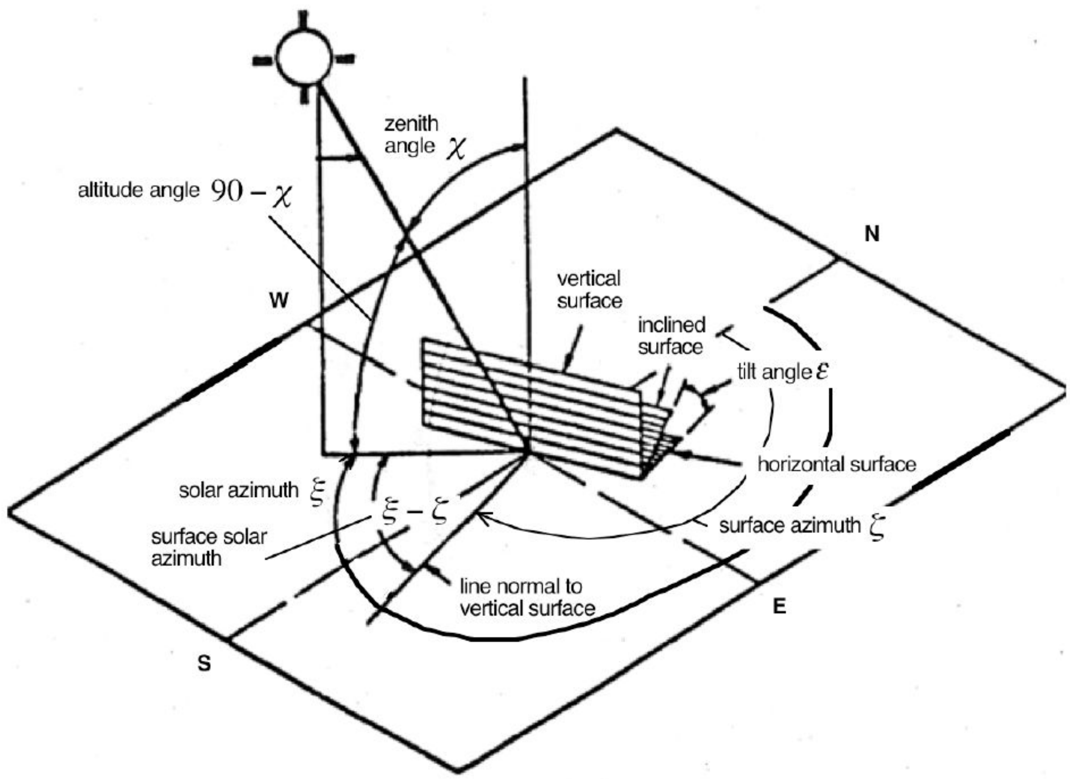

***Figure 1**. The illustration of variables needed to consider in relation to the design of the installation of the solar collectors*

- Latitude λ is the angular angle between the location and the Earth's equator and varies from 0° (at the equator) to 90° (at the North or South pole).
  Latitude =λ (= 37.9 °N for Berkeley, CA) **(Eqn. 1)**
- Surface azimuth angle ζ is a horizontal angle from the north line of the location and ranges from 0° to 360°.
  Surface azimuth angle =ζ **(Eqn. 2)**

- Hour angle α is linearly proportional to t, the local solar time in hours based on 24-hour clock.
  Hour angle $=\alpha = 15(t - 12)°$ **(Eqn. 3)**
- Surface inclination ε is the distance between the collector to the horizontal surface.
  Surface inclination from horizontal $=\varepsilon$ **(Eqn. 4)**
- Declination angle δ relies on d, the number of days in the year.
  Declination = $\delta = 23.44 \sin[\frac{360}{365.25}(d-80)]$ (d=number day of the year) **(Eqn. 5)**
- Solar azimuth angle ξ is the horizontal angle between the normal line to the surface of the collector and the Sun. It can be determined using the the signs of the hour angle α and its tangent.
  Solar azimuth angle ξ $\tan\xi = \frac{\sin\alpha}{\sin\lambda \cos\alpha - \cos\lambda \tan\delta}$ **(Eqn. 6)**

Table 1. The methods of determining ξ using the signs of α and $\tan\xi$

Sign(α)	Sign($\tan\xi$)	ξ
+	+	180° + arctan(tanξ)
+	-	360° + arctan(tanξ)
-	+	arctan(tanξ)
-	-	180° + arctan(tanξ)

- Solar zenith angle, χ is in a sinusoidal relationship with the latitude λ, the declication δ and the hour angle α.
  Solar zenith angle χ $\cos\chi = \sin\lambda\sin\delta + \cos\lambda\cos\delta\cos\alpha$ **(Eqn. 7)**
- The intensity of direct normal radiation, I_{DN}, is an exponential function in terms of solar zenith angle χ.
  Intensity of direct normal radiation: $I_{DN} = Ae^{-B/\sin(90-\chi)}$, A=1310, B=0.18 **(Eqn. 8)**
- Incident direct solar flux I_D is a result of the combination of the intensity of direct normal radiation I_{DN}, solar zenith angle χ, surface inclication ε, surface azimuth angle ζ, and solar azimuth angle ξ .
  Incident direct solar flux $I_D = I_{DN}[\cos\chi\cos\varepsilon + \sin\varepsilon\sin\chi\cos(\xi-\zeta)]$ **(Eqn. 9)**
- Solar collector efficiency, η_{coll}, is dependent on the heat removal factor F_R and the conductance U_{loss}.

  Collector efficiency $$\eta_{coll} = \overline{F}_R\left[\tau_g\alpha_c - \frac{U_{loss}}{I_D}(T_i - T_a)\right]$$ **(Eqn. 10)**

  Heat removal factor: $$\overline{F}_R = \frac{1-\exp\{-A_cU_{loss}/\dot{m}c_p\}}{A_cU_{loss}/\dot{m}c_p}$$ **(Eqn. 11)**

Total conductance: $$U_{loss}A_c = \left[\frac{1}{h_{conv,o}A_c} + \frac{\delta_g}{k_g A_c} + \frac{1}{h_{conv,i}A_c}\right]^{-1} + \left[\frac{1}{h_{conv,o}A_c} + \frac{\delta_{ins}}{k_{ins}A_c}\right]^{-1}$$ **(Eqn. 12)**

- A series of constants for other components were used for the convenience of the calculations below.
  Collector glazing transmissivity $\tau_g = 0.89$
  Collector absorber plate absorptivity $\alpha_C = 0.85$
  Outside air convective heat transfer coefficient $h_{conv,o} = 7$ W/m^{2}K
  Glazing thickness $\delta_g = 0.7$ cm = 0.007 m
  Glazing thermal conductivity $k_g = 1.3$ W/mK
  Convection coefficient between absorber and glazing (inside) $h_{conv,i} = 3.1$ W/m^{2}K
  Thickness of insulation on backside of collector $\delta_{ins} = 6$ cm = 0.06 m
  Insulation thermal conductivity $k_{ins} = 0.045$ W/mK
  Specific heat of water $c_p = 4186$ J/kg°C
  Mass flow rate of water m = 0.0267 kg/s

Task 1

The intention of this step was to set up a general equation to calculate the incident solar radiation flux I_D per square meter given 5 specific parameters: solar time t, the latitude λ, total days of the year d, surface azimuth angle ξ, and surface inclination ε. Looking at its Eqn.9, it's clear that the final answer required solving Eqn. 2, 3, 6, 7 and 8 first. Based on the information in Eqn. 3 and 6 and Table 1, in order to obtain surface azimuth angle ξ, an if statement was included to account for the principal value of arctan(ξ), which ranges from -90° to 90°, with respect to the change of location of the Sun in a given period of time. Similarly, the single value of solar zenith angle χ was calculated using arccosine of Eqn. 7, and plugged into Eqn.8 to yield an expression for intensity of direct normal radiation I_D. Then, the incident direct solar flux in Eqn.9 can be evaluated by substituting the variables. In this case, the surface had a azimuth angle of 200° and a tilt angle of 36° on April 30, 2018 at 1:00 PM solar time in Berkeley. The code for Task 1 can be seen in Appendix I. Using the code and the appropriate assumed values for the variables listed above, the incident solar radiation at 1pm on April 30th, 2018, with an azimuth of 200 degrees and an inclination of 36 degrees in Berkeley, CA, was found to be 1048.3 W/m^2.

Task 2

Similar as the previous problem, this one targets the total energy incident in response to the same 5 factors over a given period of time. The corresponding function was the product of the area A_c and the incident direct solar flux I_D per square meter . The former was assumed to be 1 m^2 and the latter was developed in Task 1. To account for less than 2% change over the time period in $I_{D,}$ the solar time interval (between the initial and final time) was separated into per second.

Then the trapezoidal rule was employed to yield best approximation of the total energy incidence within the designated time interval. The code for Task 2 can be seen in Appendix II. Therefore, given a solar collector surface that had an azimuth of 200 degrees and an inclination of 36 degrees in Berkeley, CA on April 30th, 2018 and other appropriate assumed values, the total energy incident from 10:00 AM to 4:00 PM was calculated to be 2.103 $*10^7$ W.

Task 3

Like many machines in real life, the solar collectors aren't perfectly efficient. Hence, the purpose of this step was to determine solar collector efficiency η_{coll} and measure its change with respect to the change in the insulation thermal conductivity k_{ins}. The efficiency element η_{col}, found in Eqn. 10, entailed Eqn. 9, 11 and 12, where the latter 2 were solvable using the constant values provided and the assumed area value 1 m^2 for A_c. The further dependence on Eqn. 9 explicitly implied that there's 1 particular η_{coll} for each total energy incident I_D (since A_c was assumed to be $1m^2$) calculated by the 5 specific parameters. Therefore, based on the same time interval created in Task 2 and the product of $I_D\eta_{coll}$, the actual total energy incident was approximated using the trapezoidal method. Then as for the efficiency η_{coll} itself, it's the ratio of the real total energy incident to the theoretical total energy incident. The code for Task 3 can be seen in Appendix III. After plugging the assumed and know inputs, for a collector surface that has a 200 degree azimuth, a 36 degree tilt angle, a 15 Celsius water inlet temperature, and a 12 Celsius ambient temperature on April 30th, 2018 between 2:00 and 3:00 PM in Berkeley, the efficiency turned out to be 0.7350 with a total incident energy of $3.2791*10^6$ W. In addition, if the insulation thermal conductivity was decreased to 0.018 W/mK, the change in the efficiency was found to be 0.7379. This is because of the decreased U_{loss} which increased the η_{coll}.

Task 4

Solar collectors are usually used in combination with other collectors to increase the exit temperature of the water. After figuring out the efficiency of the solar collector, the next step of the problem was to use the efficiency and the input energy to find the exit temperature of the water depending on the mass flow rate m, surface azimuth angle ζ, tilt angle ε and the flow arrangement of the collectors.

In this problem we assumed that there were 3 solar collectors on top of a residential apartment building in Berkeley, CA. The constants used for this problem were stated in the Introduction. We assumed that the area of each collector, A_c, was 3.25 m^2. We again assumed that the day was April 30th, the 120th solar day of the year, and the collectors were active for a total of 6 hours from 10:00 AM and 4:00 PM. To make ambient conditions more realistic, we assumed that the ambient temperature varied linearly from 9 to 16 Celsius in the period of 6 hours. Shown below are examples of parallel and series collectors, respectively (Fig. 2 and 3).

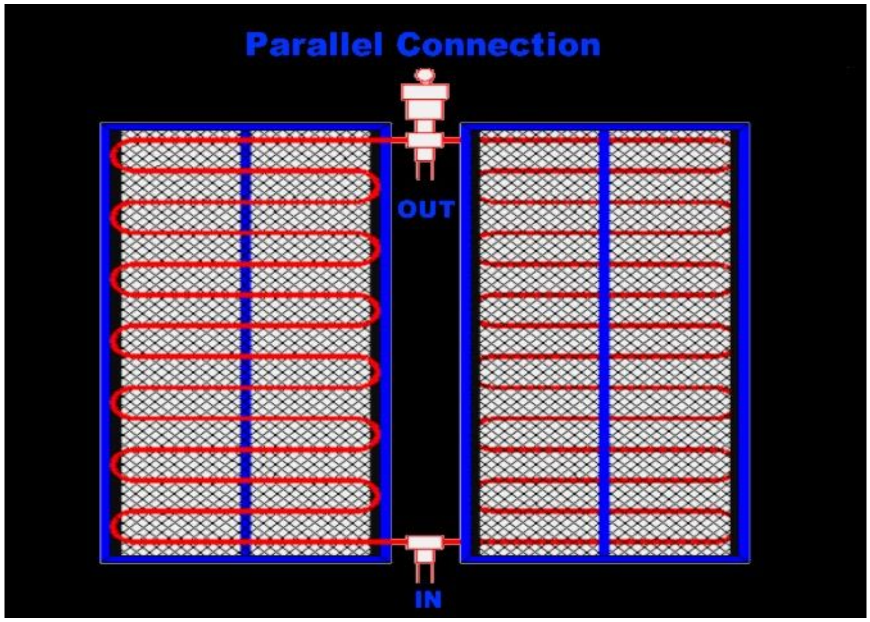

Figure 2. *The design of parallel collectors. It's similar to our design, except we didn't assume a serpentine pattern of pipes and had 3 solar collectors.*

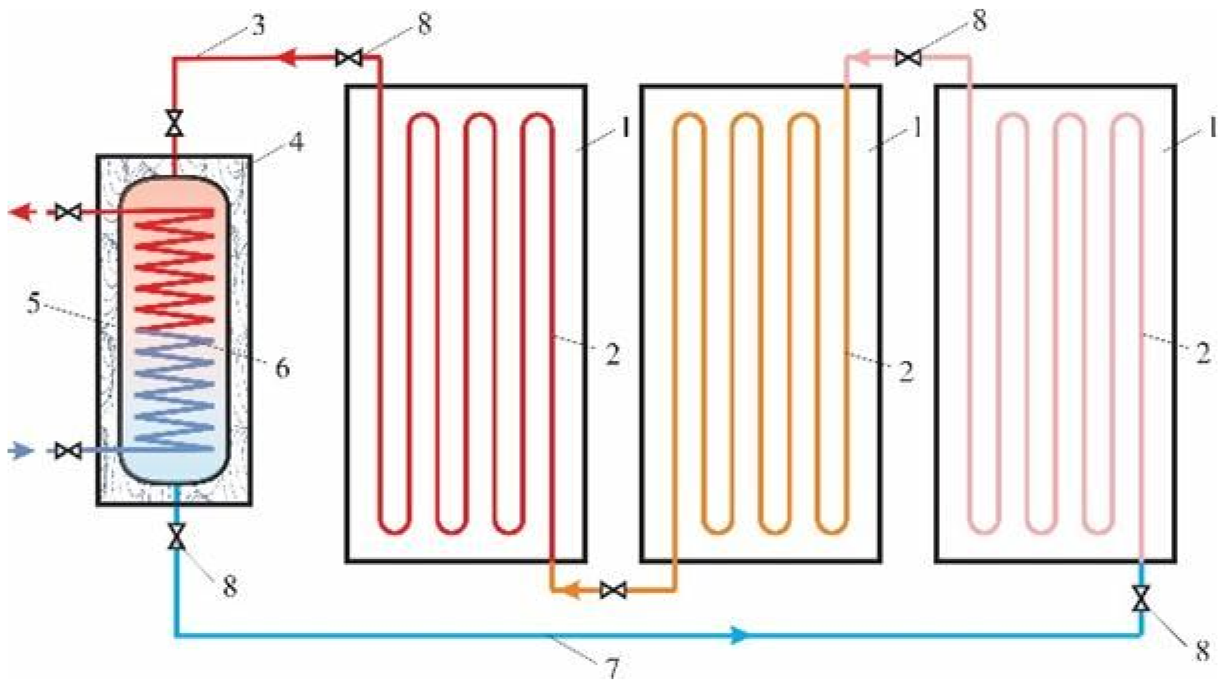

Figure 3*. The design of series collectors. The serpentine design of pipes were not assumed, but flow from one collector to another was the same in our solar collector system.*

Part 4a:

To make a good estimate for a residential area, we had to determine a good system to heat the water to 65 Celsius using all the information provided above. For the sake of convenience, we assumed that the input temperature of water was 16 Celsius throughout the day. Our approach to this was to use Eqn. 13 for finding the exit temperature after finding the useful energy absorbed.

Q= $m * c_P * (T_{out} - T_{in}) = 3 * I_D * \eta_{coll} * A_C$ **(Eqn 13)**

The factor of 3 was because of for parallel collectors, the mass flow rate per collector was 1/3rd of the total mass flow rate and for series collectors, the area of the collectors was 3 times the area of one collector.

Using Eqn. 13, we can find the exit temperature. Approach to our code was to use the code in Task 3 to find out $I_D \eta coll$ per second of time interval between 10:00 AM and 4:00 PM and then to use Eqn 13 again, to find the T_{out} per second of water passing through the collector. The mass flow rate was assumed to have units of kg/s from Task 3 and hence the calculations were done per second. For accounting for the linearly changing ambient temperature, we found a line in the form of $y = mx + b$ where $m = 7/6$ and b = -8/3 using the data points which we were given, i.e. 9 Celsius at 10:00 AM and 16 Celsius at 4:00 PM. More specifically, y described the temperature while x was the time input. Using the per second time, we used a for loop to find the ambient temperature per second. Please refer to Appendix IV for the code for the function task_4.

For calculating parallel system flow parameters: We know that mass flow rate per collector was 1/3rd of the total mass flow rate and that we need to consider in the effects of changing the solar azimuth angle and the surface tilt angle for the overall system. So we defined a script containing all of our assumed parameters and defined our minimum and maximum mass flow rates. Maximum flow rate choice will be discussed in further detail in part b. Now we defined a nested for loop with a nested for loop to account for the three factors and used the function called task_4 (Appendix IV) for finding the T_out of the collector per second. Since the T_out per collector is the same, the task wasn't repeated for collectors 2 and three. After finding the collector mass, azimuth angle and tilt angle dependances, we sought out to look for the T_out closest to 65 Celsius. We calculated the average temperature and then found the min of the difference in the absolute values of T_avg and 65. Using that value of T_avg, we found the zeta, epsilon and mass flow rate of the found T_avg closest to 65 Celsius. Our findings are presented below. The code can be viewed in Appendix V-(i).

T_average = 64.9679 Celcius
Zeta = 140
Epsilon = 20
Mass flow rate = 0.0267 kg/s
M_total = 152.35 Gallons.

This isn't the only good design and we had 49 options to choose from, but this design heated the water closest to 65 Celsius. The other 48 designs had T_out averages between 64 and

66 Celsius. If we were to choose this design, we would choose a parallel collector combination with a azimuth angle of 140 and a surface tilt angle of 20 and a mass flow rate of 0.0267 kg/s. Shown below is the variance of T_avg vs the three parameters: mass, azimuth angle and tilt angle. As we can see that the effect of azimuth angle is negligible on the Average temperature. A small effect is seen when the azimuth angle is between 140 and 240. When the zeta were looked at using the limits that T_avg <66 and T_avg >64, they ranged from 0 to 360, hence proving that the azimuth angle dependance was minimal.

The effect of epsilon or the tilt angle on the other hand was pretty big as seen in the second subplot below. The tilt angles that made the temperature go up to 65 Celsius were only from 0 to 80. The higher values of tilt angles resulted in either low temperatures or even cooling of water. This can be understood by visualizing higher tilt angles which result in the solar collector to look away from the sun and look at the ground and away from it. With no heat input to the collector, the water might cool off, hence being the best way to explain the cooling of water to -30 Celsius at higher tilt angles. The tilt angles from the other 48 options for our answer were between 0 and 80, hence showing the large dependance of exit temperature on the tilt angle.

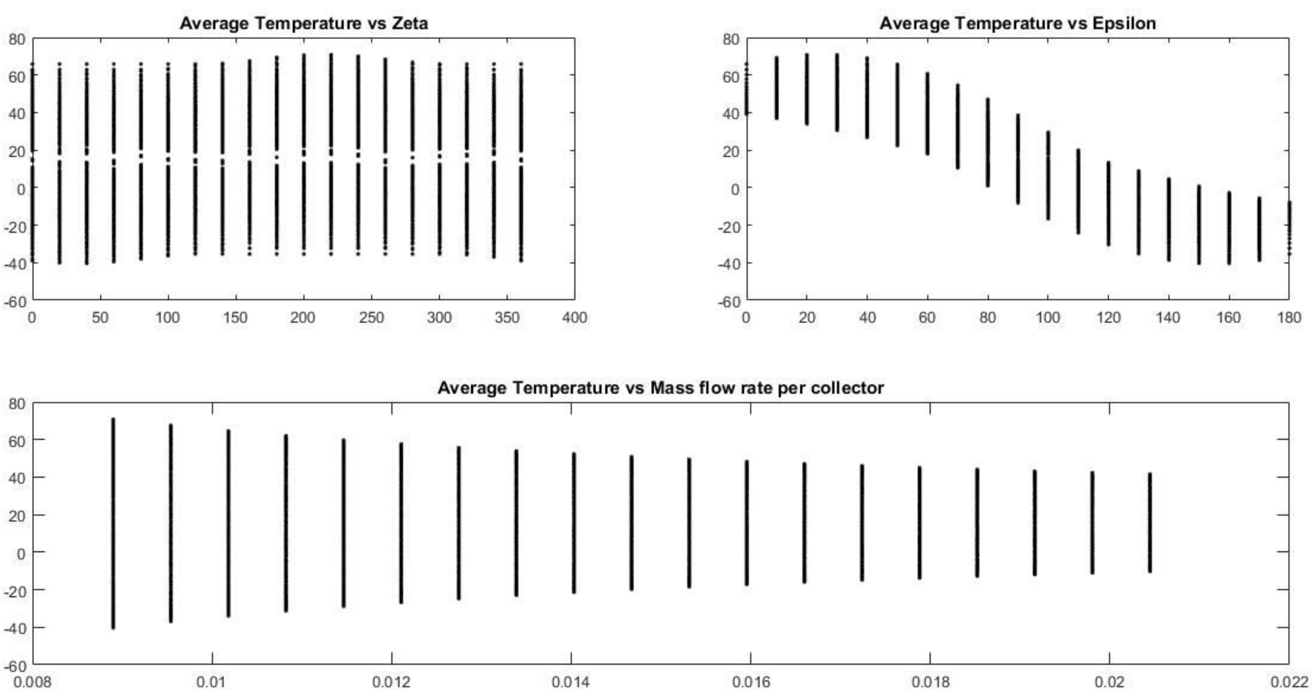

Figure 4: *Subplots of average exit temperature points vs 3 parameters: mass flow rate, tilt angle and azimuth angle for a parallel 3 collector configuration.*

For the dependance of Average Exit Temperature on the mass flow rate, that can be seen by the third subplot of the above figure. Higher the mass flow rate, lower the chance of average exit temperature reaching 65 degree celsius. The lower values of mass flow rate only produce 65 Celsius exit temperatures and this can be explained by the transfer of heat per collector being most efficient for smaller masses of water. For our answer and the 48 other options, mass flow rates were between 0.0089 and 0.011. For the three collectors, the total mass of water heated over 6 hours turned out to be 152.35 gallons. This can be calculated by converting kg/s to L/s to Gallons/s to Gallons per 6 hours. For water, 1 kg/s = 1 L/s. Hence we used equation 14 to convert the kg/s to Gallons/6 hrs.

$$m_{total} = m_{in}(kg/s) * 3 * 21600(s/6\ hrs)/(3.78541\ (L/Gallon)) \qquad \textbf{(Eqn 14)}$$

For calculating series system flow parameters: We know that mass flow rate was same per collector and that the T_out for the first collector will be the T_in for the second collector and the T_out of the second collector would be the T_in for the third one. We also need to consider in the effects of changing the surface azimuth angle and the surface tilt angle for the overall system. So we defined a script containing all of our assumed parameters and defined our minimum and maximum mass flow rates. Maximum flow rate choice will be discussed in further detail in part b. Now we defined a nested for loop with a nested for loop to account for the three factors and used the function called task_4 (Appendix IV) for finding the T_out of each collector per second. Using the logic in the first statement of the paragraph, we used three lines of code to figure out our final T_out. After finding the collector mass, azimuth angle and tilt angle dependances, we sought out to look for the T_out closest to 65 Celsius. We calculated the average temperature and then found the min of the difference in the absolute values of T_avg and 65. Using that value of T_avg, we found the zeta, epsilon and mass flow rate of the found T_avg closest to 65 Celsius. Our findings are presented below. The code can be viewed in Appendix V-(ii)

T_average = 65.0163 Celcius
Zeta = 200
Epsilon = 30
Mass flow rate = 0.0286 kg/s
M_total = 163.337 Gallons

This isn't the only good design and we had 64 options to choose from, but this design heated the water closest to 65 Celsius. The other 63 designs had T_out averages between 64 and 66 Celsius. If we were to choose this design, we would choose a parallel collector combination with a azimuth angle of 200 and a surface tilt angle of 30 and a mass flow rate of 0.0286 kg/s. Shown below is the variance of T_avg vs the three parameters: mass, azimuth angle and tilt angle.
As we can see that the effect of azimuth angle is negligible on the Average temperature. A small effect is seen when the azimuth angle is between 140 and 240. When the zeta were looked at using the limits that T_avg <66 and T_avg >64, they ranged from 0 to 360, but most of them were between 140 and 260, hence proving that the azimuth angle dependance was minimal but

observable.

The effect of epsilon or the tilt angle on the other hand is pretty big as seen in the second subplot below. The tilt angles that made the temperature go up to 65 Celsius were only from 0 to 60. The higher values of tilt angles resulted in either low temperatures or even cooling of water. This can be understood by visualizing higher tilt angles which result in the solar collector to look away from the sun and look at the ground and away from it. With no heat input to the collector, the water might cool off, hence being the best way to explain the cooling of water to -30 Celsius at higher tilt angles. The tilt angles from the other 63 options for our answer were between 0 and 60, hence showing the large dependance of exit temperature on the tilt angle. As compared to the parallel configuration, the variance of T_average vs epsilon is larger in the series combination than the parallel one.

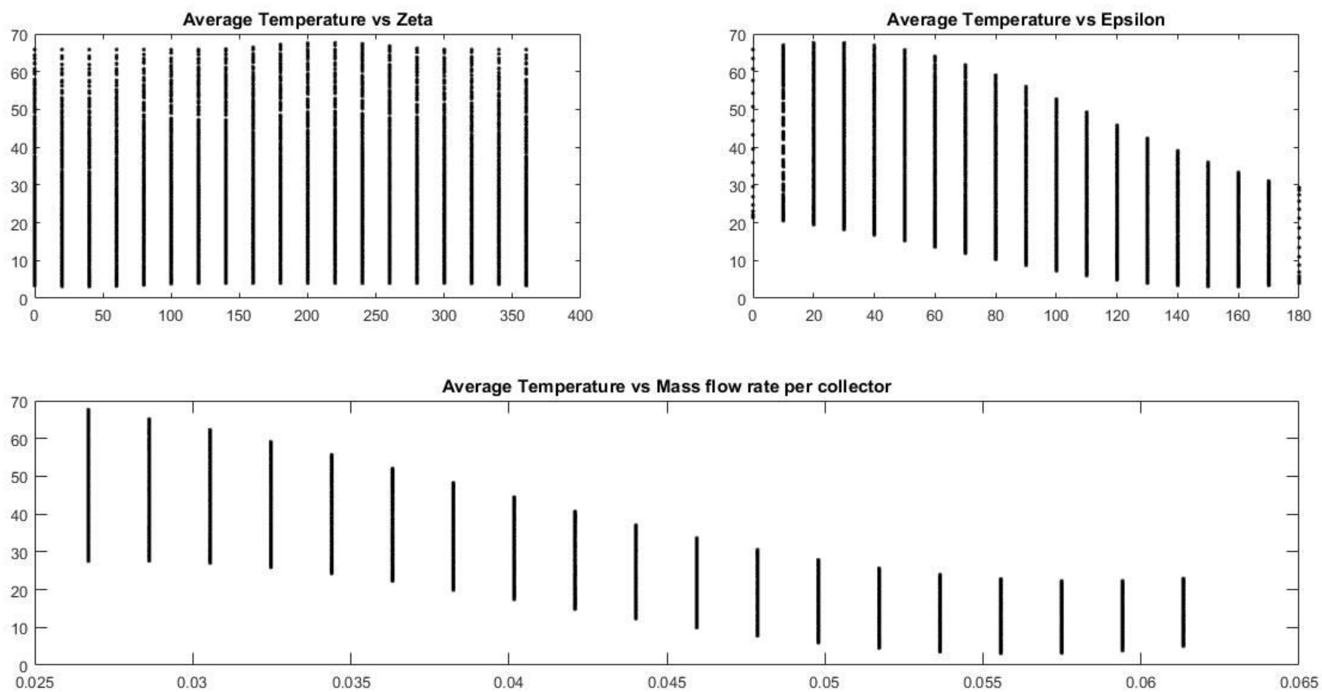

Figure 5: *Subplots of average exit temperature points vs 3 parameters: mass flow rate, tilt angle and azimuth angle for a series 3 collector configuration.*

For the dependance of Average Exit Temperature on the mass flow rate, that can be seen by the third subplot of the above figure. Higher the mass flow rate, lower the chance of average exit temperature reaching 65 degree celsius. The lower values of mass flow rate only produce 65 Celsius exit temperatures and this can be explained by the transfer of heat per collector being most efficient for smaller masses of water. For our answer and the 48 other options, mass flow rates were between 0.0267 kg/s and 0.0286 kg/s. For the three collectors, the total mass of water heated over 6 hours turned out to be 163.337 gallons. This can be calculated by converting kg/s to L/s to

Gallons/s to Gallons per 6 hours. For water, 1 kg/s = 1 L/s. Hence we used Eqn. 14 to convert the kg/s to Gallons/6 hrs. The factor of three is removed since the mass flow rate is same for all collectors.

$m_{total} = m_{in}(kg/s) * 21600(s/6\ hrs)/(3.78541\ (L/Gallon))$

Assumptions made for the series calculations: Water exists in all three collectors at the same time. Water can be heated up to 65 Celsius from 16 Celsius in an instant.

Design recommendation: Accounting for the higher mass flow rate and hence more water heated, the series combination is recommended and is described below:

T_average = 65.0163 Celcius
Azimuth angle = 200
Tilt angle = 30
Mass flow rate = 0.0286 kg/s
M_total for 6 hours = 163.337 Gallons

Part b:

Instead of determining a system to raise the water to 65C and then calculating the mass of water heated, we thought of determining a system that will raise the temperature of 350 gallons of water from 14 Celsius input to 65 Celsius output. If there doesn't exist such a system then we have to determine the amount of CH4 to be burned to cover up for the increase in temperature at 90% efficiency of burning. We got the lower heating value of natural gas (CH4) as 50,050 kJ/kg. For figuring out the mass flow rate of this system, we use Eqn. 14 again.

$m_{total} = m_{in}(kg/s) * 3 * 21600(s/6\ hrs)/(3.78541\ (L/Gallon)) \Rightarrow m_{in} = 350 * 3.78541/(3 * 21600)$
$= 0.0204\ kg/s$ (For parallel)
$m_{total} = m_{in}(kg/s) * 3 * 21600(s/6\ hrs)/(3.78541\ (L/Gallon)) \Rightarrow m_{in} = 350 * 3.78541/21600$
$= 0.0612\ kg/s$ (For series).

This was assumed to be the maximum in part a for both series and parallel and actually turned out to be a good choice since the results showed us that we cannot achieve 65C using the flow-rates given above. To determine the max average temperature reached, we altered the code for task_4a in Appendix V-(i) and V-(ii). We removed a for loop for mass and only set one mass flow rate depending upon configuration. Using the same strategy we found out the data for our series and parallel best configurations. The data is presented below and the code can be seen in Task VI-(i) and (ii).

Series:
max T_avg = 39.6965 C

Parallel
Max T_avg = 22.8709;
No possible combination to raise temp to 65 C.
Using these temperatures, we plug it into equation 15, given below
$m * c_P * (T_{out} - T_{in}) = \eta_{burn} * E_{burn}(J/kg) * 1000$ **(Eqn. 15)**
Which then for series comes out to be:
350*4186*(65-39.6965)= .9*50050*x*1000;
Where **x = 0.855 kg** and hence the amount of CH4 to be burned is 0.855 kg to reach 65 C for our series combination.
For the parallel combination:
350*4186*(65-22.8709)= 0.9*50050*x*1000;
Where **x= 1.3703 kg** is the amount of CH4 to be burned to reach 65C for our parallel combination.

Part c:

In Part a, it was determined that the series system with appropriate parameters would result the desired outlet temperature of 65 Celsius. By converting solar radiation to to the thermal energy, the combination of solar collectors prevented the emission of carbon dioxide (CO_2) into the atmosphere from the combustion of natural gas. Using Eqn. 15, the total amount of thermal energy can be found.

Q=
$$m * c_P * (T_{out} - T_{in}) = \frac{0.0286\ kg}{s} * \frac{4.186\ kJ}{kg.\ C} * (65.0163 - 16)C * \frac{3600s}{hr} * \frac{6\ hr}{day} * \frac{200\ days}{year} = 2.535 * 10^7\ \frac{kJ}{year}$$

Given the lower heating value of natural gas, which is 50050 kJ/kg, the amount of CH_4 needed to generate the equivalent amount of heat can be obtained. Then, the amount of CO_2 was measured based on the stoichiometry in the combustion equation of the natural gas.

CH_4: $2.535 * 10^7\ \frac{kJ}{year} * \frac{kg}{50050\ kJ} = 506.5 \frac{kg}{year}$

CO_2: $506.5 \frac{kg}{year} * \frac{1000\ g}{kg} * \frac{mol\ CH4}{16\ g\ CH4} * \frac{mol\ CO2}{mol\ CH4} * \frac{48\ g\ CO2}{mol\ CO2} * \frac{kg}{1000\ g} = 1519.5\ \frac{kg}{year}$

Therefore, if 3 solar collectors were arranged in series, had an inlet water line at 16 Celsius and exit temperature of 65 Celsius, and operated from 10 AM to 4 PM for 200 days of the year, the reduced CO_2 emission into the atmosphere was calculated to be 1519.5 kg per year.

Task 5a

For Task 5, we tried to make conditions more realistic. We assumed that the owner of a neighboring property has developed plans to build a tall adjacent building that will cast a shadow that blocks sunlight from reaching the collectors in our system before 11:00 AM in the morning at all times of the year. We also assumed that during the year, on the average, 46% of the hours between 10:00 AM and 4:00 PM are cloudy and provide no solar energy input.

First, we thought of the percent of energy collected from the previous system. Since 10-11 AM provided no energy whatsoever, then that part of the 46% from 10-4 or 1 hour from the 2.76 hours would result in no energy gained. Hence from 11-4, only 1.76 hours or 35.2% of the hours gave NO energy. Hence, energy lost would be the total energy gained over the year multiplied by 35.2%. We separated the year into a 2 week interval period getting 27 data points from 1 to 365 with a 14 day interval. We then put our system into task_5a, seen in Appendix VII, and calculated the total energy lost, average temperatures over the days and also the amount of CH4 to be burned for the 27 days. We then use trapezoidal rule of integration to find the total energy lost, and the amount of CH4 to be burned per year and the money per year required to burn that amount of methane. When the script was run,

a. The total energy lost per year was found out to be $3.8021 * 10^9$ W.
b. The Amount of natural gas to be burned at 90% efficiency was found to be 60.1066 kg of CH4.
c. The amount of money spent on the CH4 was found to be $30.5778. This is based upon the $9.45 dollars per 1000 cubic foot for residential areas[3]. This 1000 cubic foot was converted to cubic meters> 28.3168 m3. This was then multiplied by the density of methane, 0.656 kg/m3 to get the mass of methane for $9.45. The mass found was then divided by mass for $9.45 dollars and then multiplied by 9.45 to get the amount of money required.

Conclusion

The overarching purpose of this project was to analyze the performance of a 3 solar collector system in the distinct residential neighborhood settings and was split into 5 detailed steps. Both students discussed and contributed to the design and correction of almost all 5 tasks and the final report together throughout the 2 week period.

Through Task 1 to 3, the general equations were established in order to find incident solar radiation flux, total energy incidence, and the efficiency of the solar collectors for the specific set of parameters. It was found that the series design was better in accounting for the higher mass flow rate. Task 4 further explored different arrangement and designs of a 3 solar collector system so that certain desired outputs can be achieved for various purposes. The amount of carbon dioxide saved from being released into the atmosphere were also measured. The last task, combined the real life conditions with the system, such as the preference of the property owner and weather change, scaled the setting in the previous question to an annual basis. Additionally, it clearly demonstrated the economic advantage of adopting the solar system compared to burning natural gas. Hence, the total energy loss was measured to be $3.8021 * 10^9$ W, the amount of natural gas needed to manage the system at 90% efficiency is 60.1066 kg and cost $30.5778 annually.

Therefore, the solar collector system was an overall beneficial design and should be adopted, if possible. The specific design of each shall be carefully evaluated in order to maximize the energy and benefit.

Bibliography:

1. **http://www.jc-solarhomes.com/collector-flow-dynamics.htm**
2. **http://article.sciencepublishinggroup.com/html/10.11648.j.ijepe.20160504.11.html**
3. **http://www.ppinys.org/reports/jtf2004/naturalgas.htm**
4.

Appendix I


```
% Setting up variables and values
t= 13;
lambda=37.9;
d= 120;
zeta= 200;
epsilon = 36;
% Using equations 2, 3, 6, 7, 8, 9 and solving them to get different
% parameters
%Finding Hour angle
alpha = 15*(t-12);
%Finding Declination Angle
del = 23.44*sind((360/365.25)*(d-80));
%Finding Zenith angle
cos_zen_ang= sind(lambda)*sind(del)+ cosd(lambda)*cosd(del)*cosd(alpha);
chi = acosd(cos_zen_ang);
%finding Solar Azimuth angle
tan_xi = sind(alpha) /((sind(lambda)*cosd(alpha)-cosd(lambda)*tand(del)));
if alpha >= 0 && tan_xi >= 0
    xi = 180 + atand(tan_xi);
elseif alpha >= 0 &&  tan_xi < 0
    xi = 360 + atand(tan_xi);
elseif alpha < 0 && tan_xi >= 0
    xi = atand(tan_xi);
elseif alpha<0 && tan_xi < 0
    xi = 180+ atand(tan_xi);
end
A = 1310;
B = 0.18;
%Finding Direct Normal Radiation
I_dn = A*exp(-B/(sind(90-chi)));
%Finding Direct Solar Flux per unit area
Id = I_dn*(cos_zen_ang*cosd(epsilon)+sind(epsilon)*sind(chi)*cosd(xi - zeta));
Id
```

Appendix II


```
%Setting up parameters
t_0= 10;
t_i=16;
lambda=37.9;
d= 120;
zeta= 200;
epsilon = 36;
%Setting up time as a per second change
t = t_0:(1/3600):t_i;
%Setting up Parameters from task 1
for i = 1:length(t)
    alpha(i) = 15*(t(i)-12);
end
del = 23.44*sind((360/365.25)*(d-80));
cos_zen_ang = zeros(0,length(alpha));
chi = zeros(0,length(alpha));
tan_xi = zeros(0,length(alpha));
for i = 1:length(t)
cos_zen_ang(i)= sind(lambda)*sind(del)+ cosd(lambda)*cosd(del)*cosd(alpha(i));
chi(i) = acosd(cos_zen_ang(i));
end
for i = 1:length(t)
    tan_xi(i) = (sind(alpha(i))) /((sind(lambda)*cosd(alpha(i))-cosd(lambda)*tand(del)));
end
for i = 1:length(t)
    if alpha(i) > 0 && tan_xi(i) > 0
        xi(i) = 180 + atand(tan_xi(i));
    elseif alpha(i) > 0 &&  tan_xi(i) < 0
        xi(i) = 360 + atand(tan_xi(i));
    elseif alpha(i) < 0 && tan_xi(i) >0
        xi(i) = atand(tan_xi(i));
    elseif alpha(i)<0 && tan_xi(i) < 0
        xi(i) = 180+ atand(tan_xi(i));
    end
end
A = 1310;
B = 0.18;
for i = 1:length(chi)
    I_dn(i) = A*exp(-B/(sind(90-chi(i))));
end
%Finding Direct solar flux per unit area per second
for i = 1:length(t)
     Id_i(i) = I_dn(i)*(cos_zen_ang(i)*cosd(epsilon)+sind(epsilon).*sind(chi(i))*cosd(xi(i) -↙
zeta));
end
%Using the trapezoidal rule for finding the total Incident radiation
Id_f = 0;
for i = 1:length(Id_i)-1
Id_f = ((((t_i-t_0)*3600)/length(t))*(((Id_i(i)) + Id_i(i+1))/2))+ Id_f;
end
Id_f
```

```
function [n_colll,Id] = task_3(Ac,cp,m,t_0,t_i,lambda,d, zeta,epsilon, alpha_c, tau_g, h_convo,↙
del_g,kg,h_convi, del_ins, k_ins, T_i,T_a)
%Setting up time and given variables
t = t_0:1/3600:t_i;
for i = 1:length(t)
    alpha(i) = 15*(t(i)-12);
end
del = 23.44*sind((360/365.25)*(d-80));
cos_zen_ang = zeros(0,length(alpha));
for i = 1:length(t)
cos_zen_ang(i)= sind(lambda)*sind(del)+ cosd(lambda)*cosd(del)*cosd(alpha(i));
chi(i) = acosd(cos_zen_ang(i));
end
for i = 1:length(t)
    tan_xi(i) = sind(alpha(i)) /((sind(lambda)*cosd(alpha(i))-cosd(lambda)*tand(del)));
end
for i = 1:length(t)
    if alpha(i) > 0 && tan_xi(i) > 0
        xi(i) = 180 + atand(tan_xi(i));
    elseif alpha(i) > 0 &&  tan_xi(i) < 0
        xi(i) = 360 + atand(tan_xi(i));
    elseif alpha(i) < 0 && tan_xi(i) >0
        xi(i) = atand(tan_xi(i));
    elseif alpha(i)<0 && tan_xi(i) < 0
        xi(i) = 180+ atand(tan_xi(i));
    end
end
A = 1310;
B = 0.18;
for i = 1:length(chi)
    I_dn(i) = A*exp(-B/(sind(90-chi(i))));
    Id_i(i) = I_dn(i)*(cos_zen_ang(i)*cosd(epsilon)+sind(epsilon)*sind(chi(i))*cosd(xi(i) -↙
zeta));
end
%Finding the conductance, heat removal factors and collector efficiencies
%per second
U_loss = ((1/(h_convo*Ac) + del_g/(kg*Ac) + 1/(h_convi*Ac))^-1 + (1/(h_convo*Ac) + (del_ins/↙
(k_ins*Ac)))^-1)/Ac;
F_r = (1- exp(-Ac*U_loss/(m*cp)))/(Ac*U_loss/(m*cp));
Id = 0;
for i = 1:length(t)-1
Id = (((t_i-t_0)*3600/length(t))*(((Id_i(i)) + Id_i(i+1))/2))+Id;
end
n_coll = F_r*(tau_g *alpha_c - (U_loss./Id_i)*(T_i-T_a));
%Finding total energy absorbed in watts
Id_ncol = 0;
for i = 1:length(t)-1
Id_ncol = (((t_i-t_0)*3600/length(t))*(((Id_i(i))*n_coll(i) + n_coll(i)*Id_i(i+1))/2))+Id_ncol;
end
n_colll = Id_ncol/Id;
```

Appendix IV


```
function [T_out, Id_ncol] = task_4(t_0,t_i,d, zeta,epsilon, T_i,m)
%Setting up parameters as in previous tasks
lambda = 37.9;
t = t_0:1/3600:t_i;
for i = 1:length(t)
    T_a(i)= (7/6)*t(i)-(8/3);
end
Ac = 3.25;
alpha_c =0.85;
tau_g = 0.89;
h_convo = 7;
del_g = 0.007;
kg = 1.3;
h_convi = 3.1;
del_ins = .06;
k_ins = 0.045;
cp = 4186;
T_in = [];
for i = 1:length(t)
    alpha(i) = 15*(t(i)-12);
%Converting Tin to the same length as T_ambient for task 4
end
if length(T_i)==1
    for i =1:length(t)
        T_in(end+1) = T_i;
    end
else
    T_in = T_i;
end;
del = 23.44*sind((360/365.25)*(d-80));
cos_zen_ang = zeros(0,length(alpha));
for i = 1:length(t)
cos_zen_ang(i)= sind(lambda)*sind(del)+ cosd(lambda)*cosd(del)*cosd(alpha(i));
chi(i) = acosd(cos_zen_ang(i));
end
for i = 1:length(t)
    tan_xi(i) = sind(alpha(i)) /((sind(lambda)*cosd(alpha(i))-cosd(lambda)*tand(del)));
end
for i = 1:length(t)
    if alpha(i) > 0 && tan_xi(i) > 0
        xi(i) = 180 + atand(tan_xi(i));
    elseif alpha(i) > 0 &&  tan_xi(i) < 0
        xi(i) = 360 + atand(tan_xi(i));
    elseif alpha(i) < 0 && tan_xi(i) >0
        xi(i) = atand(tan_xi(i));
    elseif alpha(i)<0 && tan_xi(i) < 0
        xi(i) = 180+ atand(tan_xi(i));
    end
end
A = 1310;
B = 0.18;
for i = 1:length(chi)
    I_dn(i) = A*exp(-B/(sind(90-chi(i))));
    Id_i(i) = I_dn(i)*(cos_zen_ang(i)*cosd(epsilon)+sind(epsilon)*sind(chi(i))*cosd(xi(i) -
zeta));
end
```

```
U_loss = ((1/(h_convo*Ac) + del_g/(kg*Ac) + 1/(h_convi*Ac))^-1 + (1/(h_convo*Ac) + (del_ins/↙
(k_ins*Ac)))^-1)/Ac;
F_r = (1- exp(-Ac*U_loss/(m*cp)))/(Ac*U_loss/(m*cp));
Id = 0;
for i = 1:length(t)-1
Id = (((t_i-t_0)*3600/length(t))*(((Id_i(i)) + Id_i(i+1))/2))+Id;
end
for i =1: length(t)
n_colli(i) = F_r*(tau_g *alpha_c - (U_loss./Id_i(i)).*(T_in(i)-T_a(i)));
end
Id_ncol = 0;
for i = 1:length(t)-1
Id_ncol = (((t_i-t_0)*3600/length(t))*(((Id_i(i))*n_colli(i) + n_colli(i+1)*Id_i(i+1))/2))↙
+Id_ncol;
end
n_colll = Id_ncol/Id;
%Finding T_out per second using equation 13
for i = 1:length(t)
T_out(i)= (Id_i(i)*n_colli(i)*Ac)/(cp*m)+ T_in(i);
end
%Finding T_avg
T_out_avg = sum(T_out(:))/length(T_out);
```

Appendix V-(ii)


```
%Setting up parameters as in previous tasks.
t_0 = 10;
t_i = 16;
T_i = 16;
d =120;
Ac = 3.25;
alpha_c =0.85;
tau_g = 0.89;
h_convo = 7;
del_g = 0.007;
kg = 1.3;
h_convi = 3.1;
del_ins = .06;
k_ins = 0.045;
cp = 4186;
%Setting up data points for different m measurements.
m_min = 0.0267;   % mini m kg/s
m_max= (350*3.78541)/21600;  % max m kg/s if the total flow is 350 gallons over 6 hrs
m_range= linspace(m_min,m_max,19);
%Setting up data points for Zeta, Epsilon for loops
zet = (0:20:360);
eps = (0:10:180);
T_out_ser = [];
T_out_2 = [];
T_out_3 = [];
comb_2 = [];
%series arrangement
%Three nested for loops to determine the T_out of the third collector
for i = 1:length(m_range)
    x(i)=m_range(i);
    for j= 1:length(zet)
        y(j)=zet(j);
        for k=1:length(eps)
            z(k)=eps(k);
               [T_out_ser(:,end+1),Id_ncol] = task_4(t_0,t_i,d,y(j),z(k),T_i,x(i));
               [T_out_2(:,end+1),Id_ncol_2] = task_4(t_0,t_i,d,y(j),z(k),T_out_ser(:,i),x(i));
               [T_out_3(:,end+1),Id_ncol_3] = task_4(t_0,t_i,d,y(j),z(k),T_out_2(:,i),x(i));
                comb_2(:,end+1) = [y(j),z(k),x(i),Id_ncol];
        end;
    end;
end;
%finding the average temperatures
for i = 1:length(T_out_3(1,:))
T_avg_2(i) = sum(T_out_3(:,i))/length(T_out_3);
end;
%Finding temperatures between 64 and 66 and finding the closest one to 65
a= find(T_avg_2<66 & T_avg_2>64);
[m,l] = min(abs(T_avg_2(a)-65));
%Using all the inforamation to find the total mass, zeta, and epsilon of
%series configuration.
m_total_ser=(comb_2(3,a(l))*21600)/3.78541;
zeta_series = comb_2(1,a(l));
eps_series = comb_2(2,a(l));
%Plotting the Average temperature vs the 3 parameters changed
figure;
subplot(2,2,1);
```

```
plot(comb_2(1,:),T_avg_2,'.k')
title('Average Temperature vs Zeta');
subplot(2,2,2);
plot(comb_2(2,:),T_avg_2, '.k');
title('Average Temperature vs Epsilon');
subplot(2,2,[3,4]);
plot(comb_2(3,:),T_avg_2,'.k');
title('Average Temperature vs Mass flow rate per collector');

%end
```

Appendix V-(i)


```
%Setting up parameters as in previous tasks.
t_0 = 10;
t_i = 16;
T_i = 16;
d =120;
Ac = 3.25;
alpha_c =0.85;
tau_g = 0.89;
h_convo = 7;
del_g = 0.007;
kg = 1.3;
h_convi = 3.1;
del_ins = .06;
k_ins = 0.045;
cp = 4186;
%Setting up data points for different m measurements.
m_min = 0.0267;   % mini m kg/s
m_max= (350*3.78541)/21600;  % max m kg/s if the total flow is 350 gallons over 6 hrs
m_range= linspace(m_min,m_max,19);
%Setting up data points for Zeta, Epsilon for loops
zet = (0:20:360);
eps = (0:10:180);
T_out= [];
comb = [];
%parallel configuration
%Three nested for loops to determine the T_out of the third collector
for i = 1:length(m_range)
    x(i)=m_range(i)/3;
    for j= 1:length(zet)
        y(j)=zet(j);
        for k=1:length(eps)
            z(k)=eps(k);
            [T_out(:,end+1),Id_ncol] = task_4(t_0,t_i,d,y(j),z(k),T_i,x(i));
            comb(:,end+1) = [y(j),z(k),x(i),Id_ncol];
        end
    end
end
%finding the average temperatures
for i = 1:length(T_out(1,:))
T_avg(i) = sum(T_out(:,i))/length(T_out);
end;
%Finding the closest temp to 65C
I = find(T_avg<66 & T_avg>64);
[M,L] = min(abs(T_avg(I)-65));
%Finding total mass flow rate, Zeta, Epsilon for the parallel configuration
m_total=(3*comb(3,I(L))*21600)/3.78541;
zeta_parallel = comb(1,I(L));
eps_paralled = comb(2,I(L));
%Plotting the Average Temp vs the 3 parameters changed
subplot(2,2,1);
plot(comb(1,:),T_avg,'.k')
title('Average Temperature vs Zeta');
subplot(2,2,2);
plot(comb(2,:),T_avg, '.k');
title('Average Temperature vs Epsilon');
subplot(2,2,[3,4]);
```

```
plot(comb(3,:),T_avg,'.k');
title('Average Temperature vs Mass flow rate per collector');
```

Appendix VI(i)


```
%Setting up the parameters, constants
t_0 = 10;
t_i = 16;
T_i = 14;
d =120;
Ac = 3.25;
alpha_c =0.85;
tau_g = 0.89;
h_convo = 7;
del_g = 0.007;
kg = 1.3;
h_convi = 3.1;
del_ins = .06;
k_ins = 0.045;
cp = 4186;
m_max= (350*3.78541)/21600;  % max m kg/s if the total flow is 350 gallons over 6 hrs
zet = (0:20:360);
eps = (0:10:180);
T_out= [];
comb = [];
%Finding the T_out for different zeta and epsilon configurations
    for j= 1:length(zet)
        y(j)=zet(j);
        for k=1:length(eps)
            z(k)=eps(k);
            [T_out(:,end+1),Id_ncol] = task_4(t_0,t_i,d,y(j),z(k),T_i,m_max);
            comb(:,end+1) = [y(j),z(k),Id_ncol];
        end
    end
%Finding average temperature
for i = 1:length(T_out(1,:))
T_avg(i) = sum(T_out(:,i))/length(T_out);
end;
%Finding closest avg temp to 65C
I = find(T_avg<66 & T_avg>64);
[M,L] = min(abs(T_avg(I)-65));
%Finding mass flow rate, zeta, eps
m_total=(3*comb(3,I(L))*21600)/3.78541;
zeta_parallel = comb(1,I(L));
eps_paralled = comb(2,I(L));
%returning max avg temp
max(T_avg);
```

Appendix VI(ii)


```
%setting up parameters.
t_0 = 10;
t_i = 16;
T_i = 14;
d =120;
Ac = 3.25;
alpha_c =0.85;
tau_g = 0.89;
h_convo = 7;
del_g = 0.007;
kg = 1.3;
h_convi = 3.1;
del_ins = .06;
k_ins = 0.045;
cp = 4186;
m_max= (350*3.78541)/21600;  % max m kg/s if the total flow is 350 gallons over 6 hrs
t=linspace(t_0,t_i,216);
zet = (0:20:360);
eps = (0:10:180);
T_out_ser = [];
T_out_2 = [];
T_out_3 = [];
comb_2 = [];
T_avg_2=[];
%FInding the temp out of the third
%series arrangement
    for j= 1:length(zet)
        y(j)=zet(j);
        for k=1:length(eps)
            z(k)=eps(k);
               [T_out_ser(:,end+1),Id_ncol] = task_4(t_0,t_i,d,y(j),z(k),T_i,m_max);
               [T_out_2(:,end+1),Id_ncol_2] = task_4(t_0,t_i,d,y(j),z(k),T_out_ser(:,end),m_max);
               [T_out_3(:,end+1),Id_ncol_3] = task_4(t_0,t_i,d,y(j),z(k),T_out_2(:,end),m_max);
                comb_2(:,end+1) = [y(j),z(k),Id_ncol];
        end;
    end;
for i = 1:length(T_out_3(1,:))
T_avg_2(i) = sum(T_out_3(:,i))/length(T_out_3);
end;
%Finding closest temp to 65, mass flow rate and zeta, and epsilon.
a= find(T_avg_2<66 & T_avg_2>64);
[m,l] = min(abs(T_avg_2(a)-65));
m_total_ser=(comb_2(3,a(l))*21600)/3.78541;
zeta_series = comb_2(1,a(l));
eps_series = comb_2(2,a(l));
%returning max temp
max(T_avg_2);
```

```
%Setting up parameters,constants
t_0 = 11;
t_i = 16;
T_i = 16;
d =1:14:365;
Ac = 3.25;
alpha_c =0.85;
tau_g = 0.89;
h_convo = 7;
del_g = 0.007;
kg = 1.3;
h_convi = 3.1;
del_ins = .06;
k_ins = 0.045;
cp = 4186;
m_in = 0.0286;   % mini m kg/s
zet = (200);
eps = 30;
T_out_ser = [];
T_out_3 = [];
T_out_2 = [];
T_avg_out = [];
comb_2 = [];
Id_ncol=[];
Id_ncol_1=[];
Id_ncol_2=[];
%Using series to find T_out_3
for i= 1:length(d)
    [T_out_ser(:,end+1),Id_ncol(:,end+1)]= task_4(t_0,t_i,d(i),zet,eps,T_i,m_in);
    [T_out_2(:,end+1),Id_ncol_1(:,end+1)]= task_4(t_0,t_i,d(i),zet,eps,T_out_ser,m_in);
    [T_out_3(:,end+1),Id_ncol_2(:,end+1)]= task_4(t_0,t_i,d(i),zet,eps,T_out_2,m_in);
end
%Finding the total energy and lost energy for the 27 days
for i = 1:length(Id_ncol)
    True_Energy(i)=(Id_ncol(i)+Id_ncol_1(i)+Id_ncol_2(i))*0.648;
    Lost_energy(i) =(Id_ncol(i)+Id_ncol_1(i)+Id_ncol_2(i))*0.352;
end
%Finding total energy lost for the year using trapezoidal rule
Total_lost_energy = 0;
for i =1:length(d)-1
Total_lost_energy = ((365-1)/27)*(Lost_energy(i)+Lost_energy(i+1))/2+Total_lost_energy;
end
%Finding the amount of natural gas amount burned at 90% eff., and T_Avg
Natural_gas = [];
for i = 1:length(T_out_3(1,:))
T_avg_out(i) = sum(T_out_3(:,i))/length(T_out_3);
Natural_gas(end+1)= (163.337*4186*(65-T_avg_out(i)))/(0.9*50050*1000);
end
Total_CH4 = 0;
%Finding total amount burned over year
for i =1:length(d)-1
Total_CH4 = ((365-1)/27)*((Natural_gas(i+1)+Natural_gas(i))/2)+Total_CH4;
end
%Finding total cost of methane burned
Total_cost = Total_CH4*9.45/(28.3168*.656)
```