The Evaluation of a Horizontal Axis Wind Turbine in a Power Generation System

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Task 1 (I)

The goal of this task was to derive a closed-form equation based on the relationship provided for the power output of the rotor, W, in terms of the related parameters: n, ρ, v_1, $C_{L,i}$, K_h, R, σ, r_h dot, and λ. It's then compared to the Betz power output to obtain the Betz efficiency. The detailed derivation process was shown in Appendix I, and the results for the power output and efficiency were:

$$W = \frac{1.27}{3}\cdot\frac{n\rho\lambda K_h v_1^2}{R}\cdot\left[\sqrt{r^2+\left(\frac{2R}{3\lambda}\right)^2}\cdot\left(\left(r^2+\left(\frac{2R}{3\lambda}\right)^2\right)\left(\frac{\lambda v_1}{3R}\cdot\left(1+\frac{\sigma\hat{r}_h}{1-\hat{r}_h}\right)-\frac{\sigma r\lambda v_1}{4R^2(1-\hat{r}_h)}\right)+\frac{4\sigma r v_1}{72\lambda(1-\hat{r}_h)}\right)+\ln\left(r+\sqrt{r^2+\left(\frac{2R}{3\lambda}\right)^2}\right)\cdot\frac{2\sigma R^2 v_1}{81\lambda^3(1-\hat{r}_h)}\right]$$

$$\eta^* = \frac{1.27\cdot n\lambda}{\left(\frac{16}{27}\right)\left(\frac{3}{2}\right)\cdot\pi r^2}\cdot\left(\frac{K_h}{R}\right)\left[\sqrt{r^2+\left(\frac{2R}{3\lambda}\right)^2}\cdot\left(\left(r^2+\left(\frac{2R}{3\lambda}\right)^2\right)\left(\frac{\lambda}{3R}\cdot\left(1+\frac{\sigma\hat{r}_h}{1-\hat{r}_h}\right)-\frac{\sigma r\lambda}{4R^2(1-\hat{r}_h)}\right)+\frac{4\sigma r}{72\lambda(1-\hat{r}_h)}\right)+\ln\left(r+\sqrt{r^2+\left(\frac{2R}{3\lambda}\right)^2}\right)\cdot\frac{2\sigma R^2}{81\lambda^3(1-\hat{r}_h)}\right]$$

Task 2 (II)

The power and efficiency equations obtained from Task 1 was programmed in order to calculate them at specific condition. Based on the values given, W was 2.0032e+06 V, and η* was 0.8644. When the angle of attack was kept at 8°, the relationship between the setup angle ξ (in degrees) and radius between r_h and R (in meters) was also plotted in Fig.1. And it can be concluded that ξ increases as radius increases.

Task 3 (III)

Using task 2, we designed a horizontal axis wind turbine for the top of the Berkeley hills that will provide 1.5 kW of power at mean air density = 1.18 kg/m3 and mean wind speed (v) = 6.5 m/s. Based on provided info, we found R = 5.2000 m, W = 1.5468e+03 W, Sigma = 0.3000, Eff = 0.1916. When the angle of attack was kept at 8°, the relationship between the setup angle ξ (in degrees) and radius between r_h and R (in meters) was also plotted in Fig.1. And it can be concluded that ξ increases as radius increases.

Task 4 (IV)

For the first part of the problem, we assumed that the turbine generator nominally produces 12.0 V DC current and using that we determined the delivered current for the power output in task_3. We used P = VI, to find I, and we found it was 128.9 A.

For the second part of this problem, energy was first stored and then extracted from the H_2-O_2 fuel cell at the same current I = 3.5 A for an hour. The energy input was calculated by plugging in I into V_{cell} equation, which was found to be 1.4835 V. Similarly, the extracted power was obtained from V_L formula, and was measured to be 0.9765 V . Therefore, the fraction of energy extraction in the process was approximately 66%.

Graphs

Task 2:

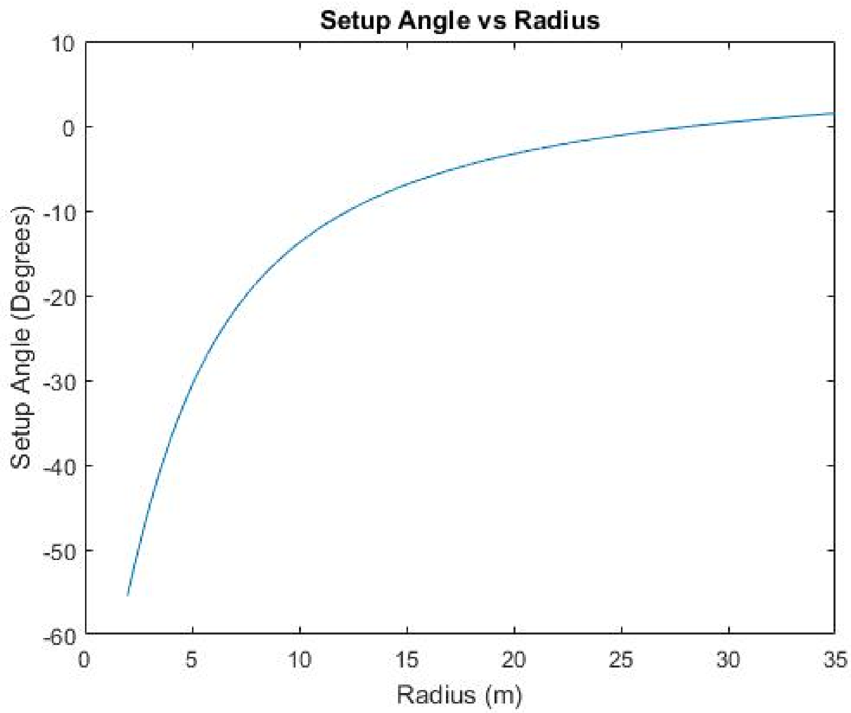

Figure 1. Plot of setup angle ξ with respect to radius changing between r_h and R. It demonstrates a positive trend of change in both variables.

Task 3

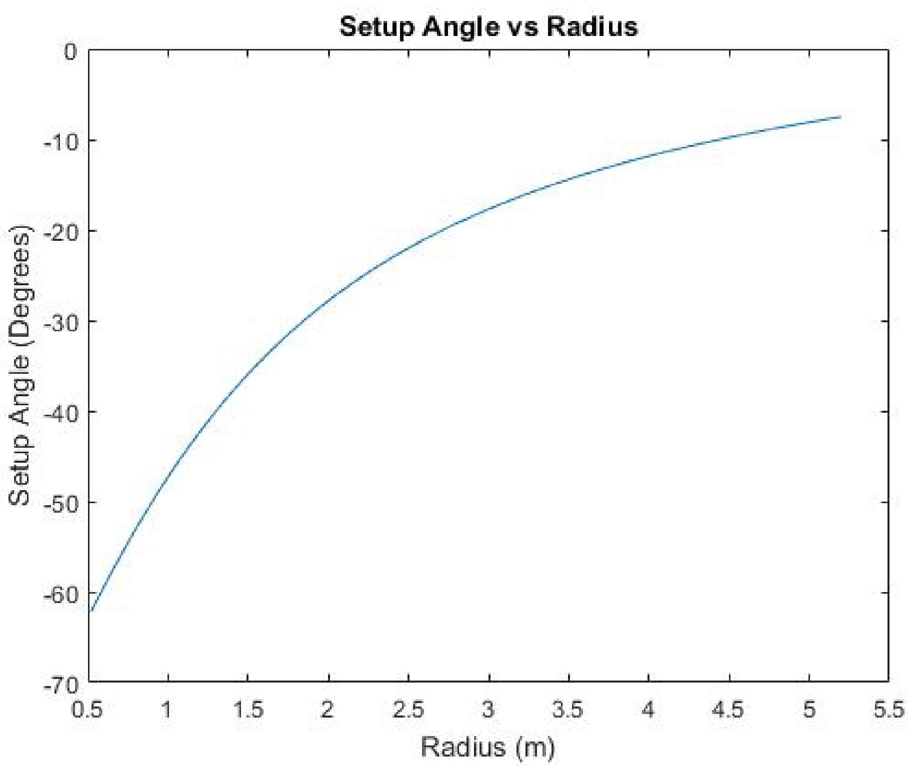

Figure 2. Plot of setup angle ξ with respect to radius changing between r_h and R. It demonstrates a positive trend of change in both variables.

$$dW = \frac{1}{3} n \rho\, w(r)\, v_1\, C_L(r)\, K(r)\, \omega\, r\, dr$$

$$= \frac{1}{3} n \rho \cdot \sqrt{\left(\frac{2}{3}v_1\right)^2 + \omega^2 r^2} \cdot v_1 \cdot (1.27) \cdot K_h \left[1 - \sigma\left(\frac{r-r_h}{R-r_h}\right)\right] \omega\, r\, dr$$

$$= \underbrace{\frac{1.27}{3} n \rho v_1 K_h \omega}_{b} \int \left[r - r\sigma\left(\frac{r-r_h}{R-r_h}\right)\right] \sqrt{\left(\frac{2}{3}v_1\right)^2 + \omega^2 r^2}\, dr$$

$$= b\left(\int r\sqrt{\left(\tfrac{2}{3}v_1\right)^2 + \omega^2 r^2}\, dr - \int r\sigma\left(\frac{r-r_h}{R-r_h}\right)\sqrt{\left(\tfrac{2}{3}v_1\right)^2 + \omega^2 r^2}\, dr\right)$$

$\because \lambda = \frac{\omega R}{v_1}$

$\therefore v_1 = \frac{\omega R}{\lambda}$

$$\Rightarrow = b\left(\omega \int r\sqrt{\left(\tfrac{2R}{3\lambda}\right)^2 + r^2}\, dr - \frac{\sigma\omega}{R-r_h}\int (r^2 - r r_h)\sqrt{\left(\tfrac{2R}{3\lambda}\right)^2 + r^2}\, dr\right)$$

$$= b\left(\omega \int r\sqrt{\left(\tfrac{2R}{3\lambda}\right)^2 + r^2}\, dr - \frac{\sigma\omega}{R-r_h}\int r^2\sqrt{\left(\tfrac{2R}{3\lambda}\right)^2 + r^2}\, dr + \frac{\sigma\omega r_h}{R-r_h}\int r\sqrt{\left(\tfrac{2R}{3\lambda}\right)^2 + r^2}\, dr\right)$$

$$= b\left(\omega \cdot \frac{1}{3}\left[r^2 + \left(\tfrac{2R}{3\lambda}\right)^2\right]^{3/2} - \frac{\sigma\omega}{R-r_h}\cdot\left[\left(\frac{r}{4}\right)\sqrt{\left(r^2 + \left(\tfrac{2R}{3\lambda}\right)^2\right)^3} - \left(\left(\tfrac{2R}{3\lambda}\right)^2 \frac{r}{8}\right)\sqrt{r^2 + \left(\tfrac{2R}{3\lambda}\right)^2} - \frac{1}{8}\left(\tfrac{2R}{3\lambda}\right)^4 \ln\left(r + \sqrt{r^2 + \left(\tfrac{2R}{3\lambda}\right)^2}\right)\right]\right.$$
$$\left. + \frac{\sigma\omega r_h}{R-r_h}\cdot\left(\frac{1}{3}\right)\left(r^2 + \left(\tfrac{2R}{3\lambda}\right)^2\right)^{3/2}\right)$$

let $a = \frac{2R}{3\lambda}$

$$\Rightarrow = b\left(\frac{\omega}{3}(r^2+a^2)^{3/2}\left(1 + \frac{\sigma r_h}{R-r_h}\right) - \frac{\sigma\omega}{R-r_h}\left[\left(\frac{r}{4}\right)(r^2+a^2)\sqrt{r^2+a^2} - \frac{a^2 r}{8}\sqrt{r^2+a^2} - \frac{a^4}{8}\ln\left(r + \sqrt{r^2+a^2}\right)\right]\right)$$

let $c = \sqrt{r^2+a^2}$

$$\Rightarrow = b\left(c\left(\frac{\omega}{3}\cdot\left(1 + \frac{\sigma r_h}{R-r_h}\right)c^2 - \frac{\sigma\omega r}{4(R-r_h)}c^2 + \frac{\sigma\omega \cdot a^2 r}{8(R-r_h)}\right) + \frac{\sigma\omega a^4}{8(R-r_h)}\ln(r+c)\right)$$

$$= \frac{1.27}{3} n\rho v_1 K_h \cdot \frac{\lambda v_1}{R}\cdot\left[\sqrt{r^2+\left(\tfrac{2R}{3\lambda}\right)^2}\left(\frac{1}{3}\cdot\frac{\lambda v_1}{R}\cdot\left(1 + \frac{\sigma \cdot \hat{r}_h \cancel{R}}{\cancel{R} - \hat{r}_h\cancel{R}}\right)\left(r^2 + \left(\tfrac{2R}{3\lambda}\right)^2\right) - \frac{\sigma r}{4(R-\hat{r}_h R)}\cdot\frac{\lambda v_1}{R}\cdot\left(r^2 + \left(\tfrac{2R}{3\lambda}\right)^2\right) + \frac{\sigma r}{8(R-\hat{r}_h R)}\cdot\frac{4R^{\cancel{2}}}{9\cancel{\lambda^2}_{\lambda}}\cdot\frac{\cancel{\lambda} v_1}{\cancel{R}}\right) + \right.$$
$$\left.\frac{\sigma}{8(R-\hat{r}_h R)}\cdot\frac{16R^{\cancel{4}}\,R^2}{81\lambda^{\cancel{4}}{}_3}\cdot\frac{\cancel{\lambda} v_1}{\cancel{R}}\cdot\ln\left(r+\sqrt{r^2+\left(\tfrac{2R}{3\lambda}\right)^2}\right)\right]$$

$$\boxed{W = \frac{1.27}{3}\cdot\frac{n\rho\lambda K_h v_1^2}{R}\cdot\left[\sqrt{r^2+\left(\tfrac{2R}{3\lambda}\right)^2}\cdot\left(\left(r^2+\left(\tfrac{2R}{3\lambda}\right)^2\right)\left(\frac{\lambda v_1}{3R}\cdot\left(1+\frac{\sigma\hat{r}_h}{1-\hat{r}_h}\right) - \frac{\sigma r\lambda v_1}{4R^2(1-\hat{r}_h)}\right) + \frac{4\sigma r v_1}{72\lambda(1-\hat{r}_h)}\right) + \ln\left(r+\sqrt{r^2+\left(\tfrac{2R}{3\lambda}\right)^2}\right)\cdot\frac{2\sigma R^2 v_1}{81\lambda^3(1-\hat{r}_h)}\right]}$$

$$W_{Betz} = \frac{16}{27}\cdot\left(\frac{1}{2}\right)\rho v_1^3 (2\pi r\,\Delta r) \qquad \text{from lecture}$$
$$= \frac{16}{27}\cdot\left(\frac{1}{2}\right)\rho v_1^3 (\pi r^2)$$

$$\therefore \eta^* = \frac{W}{W_{Betz}}$$

$$\boxed{\eta^* = \frac{1.27\cdot n\lambda}{\left(\frac{16}{27}\right)\left(\frac{3}{2}\right)\cdot\pi r^2}\cdot\left(\frac{K_h}{R}\right)\left[\sqrt{r^2+\left(\tfrac{2R}{3\lambda}\right)^2}\cdot\left(\left(r^2+\left(\tfrac{2R}{3\lambda}\right)^2\right)\left(\frac{\lambda}{3R}\cdot\left(1+\frac{\sigma\hat{r}_h}{1-\hat{r}_h}\right) - \frac{\sigma r\lambda}{4R^2(1-\hat{r}_h)}\right) + \frac{4\sigma r}{72\lambda(1-\hat{r}_h)}\right) + \ln\left(r+\sqrt{r^2+\left(\tfrac{2R}{3\lambda}\right)^2}\right)\cdot\frac{2\sigma R^2}{81\lambda^3(1-\hat{r}_h)}\right]}$$

```
function [w_tur, eta]=task_1(rou, v1, alpha, Cl, n, Kh,sigma, rh, R,omega )

lambda = omega*R/v1;
rh_bar= rh/R;
a=2*R/3/lambda;
b=1-rh_bar;

syms r;
w = Cl/3*(n*rou*lambda*Kh*(v1^2)/R) * ((r^2+a^2)^(0.5) *((r^2+a^2)*(lambda*v1/3/R*
(1+sigma*rh_bar/b) - sigma*r*lambda*v1/4/(R^2)/b) + log(r+(r^2+a^2)^(0.5))*(2*sigma*R^2*v1)/
(81*lambda^3*b)));
w_b = 16/27*0.5*rou*v1^3*pi *r^2;

%turbine work
ans = subs(w,r, [rh R]);
w_tur =double(ans(2)-ans(1));

%betz work
ans1 = subs(w_b,r, [rh R]);
w_betz = double(ans1(2)-ans1(1));

eta=w_tur/w_betz;

c=rh:0.001:R;
xi = alpha*pi/180 - atan(2*R./3./lambda./c);
xi1=xi*180/pi;
plot(c, xi1)
title ('Setup Angle vs. Radius');
xlabel('Radius (m)', 'FontWeight','bold');
ylabel('Setup Angle (degrees)', 'FontWeight','bold');


end
```

```
function [overall_des_R,overall_des_w_tur, overall_des_sigma, overall_des_eff]=task_3( rou, v1,
alpha, Cl, n, omega)
%setting R, sigma, and other variables
R1=linspace(1,22,51);
sigma1=linspace(0,1,51);
Kh1 = 0.085*R1;
rh1 = 0.1*R1;
%Using task_1 to find W_tur and Efficiency for all different R and sigma
for i = 1:length(R1)
    disp(i);
    for j = 1:length(sigma1)
        [w_tur(i,j),eta(i,j)]=task_1(rou, v1, alpha, Cl, n, Kh1(i),sigma1(j), rh1(i), R1(i),omega
);
    end
end
%finding a value closest to 1500W
[a,b]= find(w_tur<1550 & w_tur>=1500);
for l = 1:length(a)
    des_2_w_tur(l) = w_tur(a(l),b(l));
    des_2_eff(l) = eta(a(l),b(l));
end
[p,q] = max(des_2_eff);
des_2_eff = p;
des_2_w_tur = des_2_w_tur(q);
%finding a value of efficiency closest to 1
[c,d]= find(eta<1.00 &eta>0.98);
for l = 1:length(c)
    des_1_w_tur(l) = w_tur(c(l),d(l));
    des_1_eff(l) = eta(c(l),d(l));
end
[e,f] = max(des_1_eff);
des_1_eff = e;
des_1_w_tur = des_1_w_tur(f);
des_1_R= R1(c(f));
des_1_sigma = sigma1(d(f));
des_2_R = R1(a(q));
des_2_sigma = sigma1(b(q));
des_R = [des_1_R,des_2_R];
des_eff = [des_1_eff,des_2_eff];
des_w_tur = [des_1_w_tur,des_2_w_tur];
%choosing between two alternatives for min sigma
[overall_des_sigma,index] = min([des_1_sigma,des_2_sigma]);
overall_des_R= des_R(index);
overall_des_w_tur = des_w_tur(index);
overall_des_eff = des_eff(index);
%plotting change in setup angle vs R
des_ov_rh = overall_des_R*0.1;
lambda = omega*overall_des_R/v1;
des_ov_c = des_ov_rh:0.001:overall_des_R;
des_ov_xi = alpha*pi/180 - atan(2*overall_des_R./3./lambda./des_ov_c);
des_ov_xi1=des_ov_xi*180/pi;
plot(des_ov_c, des_ov_xi1)
xlabel('Radius (m)')
ylabel('Setup Angle (Degrees)')
title('Setup Angle vs Radius')
```

```
end

function [w_tur, eta]=task_1(rou, v1, alpha, Cl, n, Kh,sigma, rh, R,omega )

lambda = omega*R/v1;
rh_bar= rh/R;
a=2*R/3/lambda;
b=1-rh_bar;

syms r;
w = Cl/3*(n*rou*lambda*Kh*(v1^2)/R) * ((r^2+a^2)^(0.5) *((r^2+a^2)*(lambda*v1/3/R*
(1+sigma*rh_bar/b) - sigma*r*lambda*v1/4/(R^2)/b) + log(r+(r^2+a^2)^(0.5))*(2*sigma*R^2*v1)/
(81*lambda^3*b)));
w_b = 16/27*0.5*rou*v1^3*pi *r^2;

%turbine work
ans = subs(w,r, [rh R]);
w_tur =double(ans(2)-ans(1));

%betz work
ans1 = subs(w_b,r, [rh R]);
w_betz = double(ans1(2)-ans1(1));

eta=w_tur/w_betz;

%c=rh:0.001:R;
%xi = alpha*pi/180 - atan(2*R./3./lambda./c);
%xi1=xi*180/pi;
%plot(c, xi1)


end
```

```
P = 1.5468e+03;
V = 12;
I = P/V;
```

```
function [fract]=task_4b(I_L, I_cell, V_rev)
V_L = V_rev - 0.11 - 0.041*I_L;
V_cell = V_rev + 0.11 + 0.041*I_cell;
fract = V_L/V_cell;
end
```