CIVE 295 — Energy Systems and Control
Spring 2019 — Scott Moura and Dong Zhang — HW 02

1 Problem 1: Dynamic System Modeling

(a) To develop a model of a drill in order to estimate the drill bit speed using measured values of the drill table speed in order to prevent oscillations in the drill string and prevent mechanical fatigue to the bit itself.

Controllable Inputs: Table Torque,

Uncontrollable Inputs: Frictional Bit Torque

Measured Outputs: ω_T

Performance Outputs: ω_B

Parameters: J_B, J_T

(b) For the top portion:

$$T(t) - k(\theta_T(t) - \theta_B(t)) - b\omega_T(t) = J_T\dot{\omega}_T(t) \tag{1}$$

For the bit portion:

$$T_f(t) + k(\theta_T(t) - \theta_B(t)) - b\omega_B(t) = J_T\dot{\omega}_B(t) \tag{2}$$

(c) Combining this into one input and one output equation gives us:

$$\frac{d}{dt}\begin{bmatrix}\theta_T(t)\\ \theta_B(t)\\ \omega_T(t)\\ \omega_B(t)\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{-k}{J_T} & \frac{k}{J_T} & \frac{-b}{J_T} & 0\\ \frac{k}{J_B} & \frac{-k}{J_B} & 0 & \frac{-b}{J_B}\end{bmatrix}\begin{bmatrix}\theta_T(t)\\ \theta_B(t)\\ \omega_T(t)\\ \omega_B(t)\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ \frac{1}{J_T} & 0\\ 0 & \frac{-1}{J_B}\end{bmatrix}\begin{bmatrix}T(t)\\ T_f(t)\end{bmatrix}$$ Output Equation is

$$y(t) = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\theta_T(t)\\ \theta_B(t)\\ \omega_T(t)\\ \omega_B(t)\end{bmatrix}$$

Where the state space matrices are:

$$A = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{-k}{J_T} & \frac{k}{J_T} & \frac{-b}{J_T} & 0\\ \frac{k}{J_B} & \frac{-k}{J_B} & 0 & \frac{-b}{J_B}\end{bmatrix} B = \begin{bmatrix}0 & 0\\ 0 & 0\\ \frac{1}{J_T} & 0\\ 0 & \frac{-1}{J_B}\end{bmatrix} C = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}$$

2 Problem 2: Observability Analysis

We have $J_T = 100, J_B = 25, k = 2, b = 5$

(a) Since we have 4 states: $O = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -0.02 & 0.02 & -0.05 & 0 \\ 0.001 & -0.0010 & -0.0175 & 0.02 \\ 0.002 & -0.002 & 0.0019 & -0.005 \end{bmatrix}$

Here the rank of the Observability matrix is 3, which is because there are 2 columns which are linearly dependent on each other. All states are hence not observable from the measurements of table velocity.

(b) Now we have a new state: $\theta(t) = \theta_T(t) - \theta_B(t)$

Let our x = $\begin{bmatrix} \theta(t) \\ \omega_T(t) \\ \omega_B(t) \end{bmatrix}$ This gives us our new A, B, C matrices as:

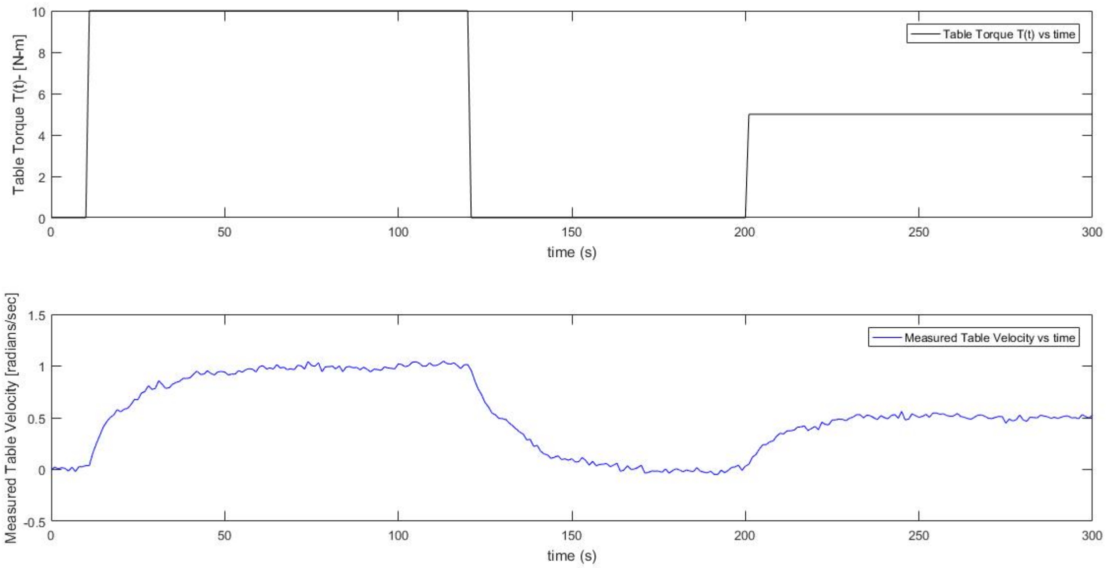
A = $\begin{bmatrix} 0 & 1 & -1 \\ \frac{-k}{J_T} & \frac{-b}{J_T} & 0 \\ \frac{k}{J_B} & 0 & \frac{-b}{J_B} \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ \frac{1}{J_T} & 0 \\ 0 & \frac{-1}{J_B} \end{bmatrix}$ $C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

(c) New Observability Matrix is

O = $\begin{bmatrix} 0 & 1.0000 & 0 \\ -0.0200 & -0.0500 & 0 \\ 0.0010 & -0.0175 & 0.0200 \end{bmatrix}$

The rank of this matrix is 3, aka all columns are independent which means that all the states are observable from the measurements of table velocity.

3 Problem 3: Measurement Data



4 Problem 4: Luenberger Observer

(a) Eigenvalues of the A matrix are $\begin{bmatrix} -0.0834 + 0.2986i \\ -0.0834 - 0.2986i \\ -0.0832 + 0.0000i \end{bmatrix}$

Luenberger Observer Equations:

$\dot{\hat{x}}$ = A$\hat{x}$+ Bu(t)+ L(y(t) - $\hat{y}(t)$ = (A-LC)$\hat{x}$+ $\begin{bmatrix} B & L \end{bmatrix} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}$

$$\hat{y} = C\hat{x} \tag{3}$$

D = 0 from earlier

(b) Selected eigenvalues are: $\begin{bmatrix} -0.2502 + 0.8958i \\ -0.2502 - 0.8958i \\ -0.2497 + 0.0000i \end{bmatrix}$

These are 3 times the eigenvalues of matrix A. I chose this number after trying different values of the factor to see which number gave least RMSE and least noise in the graph.

Using the *place* command, we get L as $\begin{bmatrix} -38.9994 \\ 0.5001 \\ 0.6 \end{bmatrix}$

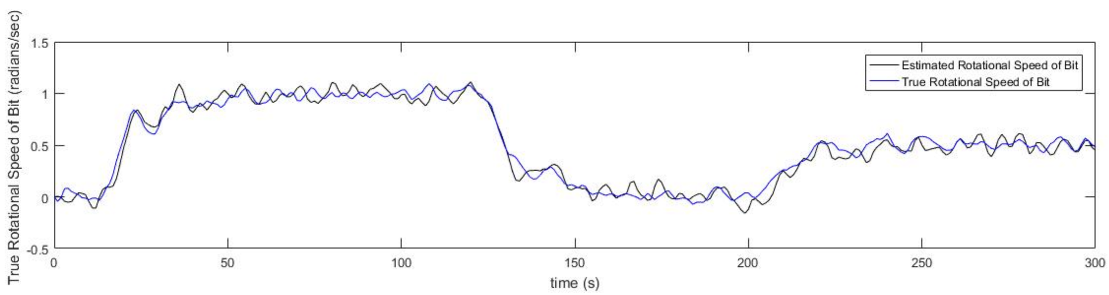
(c) Since Frictional torque is not included, the state space matrices for the Luenberger Observer are:

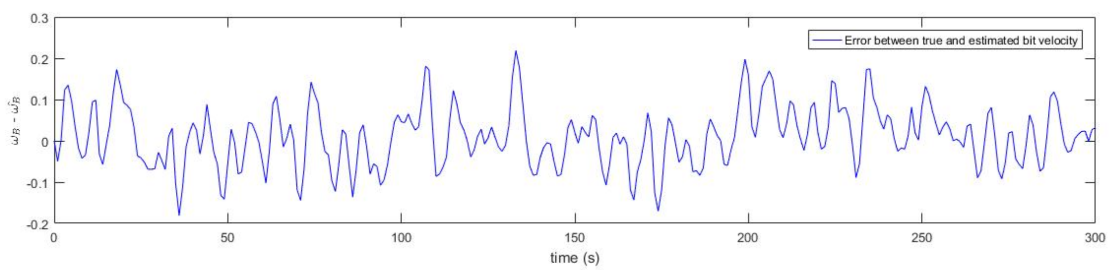
$$A_{lobs}=\begin{bmatrix} 0 & 39.9994 & -1.0000 \\ -0.0200 & -0.5501 & 0 \\ 0.0800 & -0.6000 & -0.2000 \end{bmatrix} \quad B_{lobs}=\begin{bmatrix} 0 & -38.9994 \\ 0.0100 & 0.5001 \\ 0 & 0.6000 \end{bmatrix} \quad C_{lobs}=\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

(d) RMSE found for 7*eig of A-LC= 0.599

RMSE found for 3.5*eig of A-LC = 0.0835

RMSE found for 3*eig of A-LC = 0.0725





5 Problem 5: Kalman Filter (KF) Design

(a) Along with the equations in the question, $\dot{\hat{x}} = A(\hat{x}) + Bu(t) + L(t)(y_m - \hat{y})$

$\hat{y} = C\hat{x}(t)$

$L(t) = \Sigma(t) * C' * T(t) * N^{-1}$

$\dot{\Sigma}(t) = \Sigma(t)A(t)T + A(t)\Sigma(t) +$ W - $\Sigma(t)C'T(t)N^{-1}C(t)\Sigma(t)$, $\Sigma(0) = \Sigma 0$

(b) W = diag[0, 0, 0.1505]. I tried different values of W_{33} to see how the RMSE changed and how exactly did the noise change in the plot. It is diagonal since the 3 different values correspond to the 3 different variables in xhat.

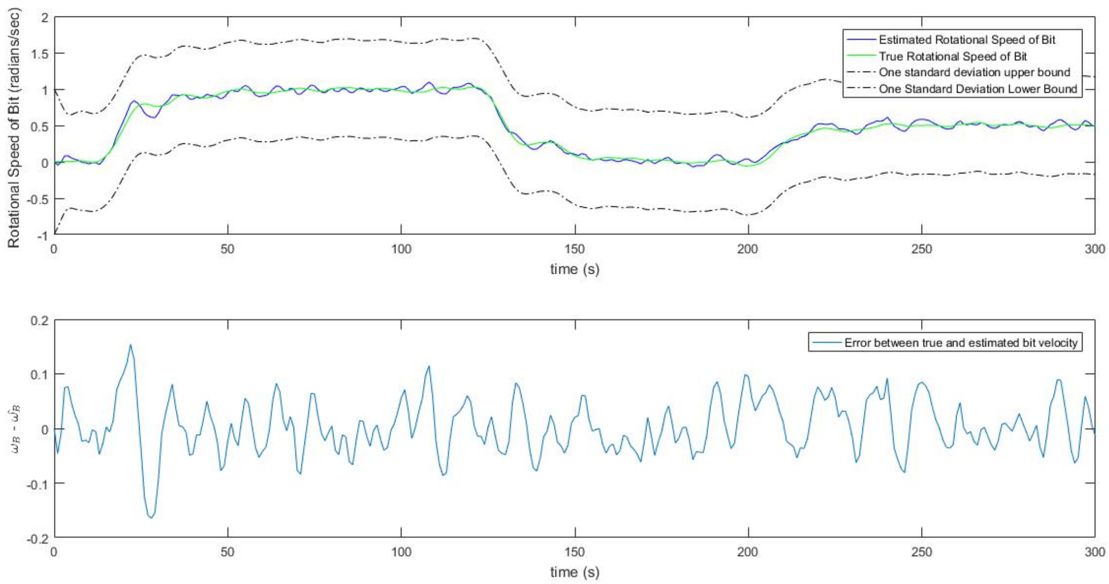
(1,1) corresponds to x1

(2,2) corresponds to x2

(3,3) corresponds to x3.

Hence, we can see that only W_{33} is non zero. The increasing value of this term meant the increasing amount of standard deviation of velocity estimate.

(c) Plot is:



RMSE for $W_{33} = 0.505$ is 0.0541 rad/s

RMSE for $W_{33} = 0.2505$ is 0.0508 rad/s

RMSE for $W_{33} = 0.0505$ is 0.0506 rad/s

RMSE for $W_{33} = 0.1505$ is 0.0507 rad/s

(d) Eigenvalues are $\begin{bmatrix} -0.1692 + 0.3331i \\ -0.1692 - 0.3331i \\ -0.4818 + 0.0000i \end{bmatrix}$

Magnitude wise, the real part of the KF A-LC matrix is higher than the Luenberger Observer. This is because of the extra factor of the Frictional Torque in the KF.

6 Problem 6: Extended Kalman Filter (EKF) Design

(a) Our plant model equation becomes:

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 & -1 \\ \frac{-k}{J_T} & \frac{-b}{J_T} & 0 \\ \frac{k}{J_B} & 0 & \frac{-b}{J_B} \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ -k_2 & \frac{1}{J_T} & 0 \\ k_2 & 0 & \frac{-1}{J_B} \end{bmatrix} \begin{bmatrix} \theta^3(t) \\ T(t) \\ T_f(t) \end{bmatrix} \tag{4}$$

and

$$\hat{y} = C\hat{x} \tag{5}$$

We can see that the second equation is linear in w.r.t x and hence we can write our jacobian H as: $H(t) = \frac{dh}{dt}(\hat{x}(t), u(t)) = C$.

On the other hand our F is not linear w.r.t x and hence our jacobian F is:

$$F(t) = \frac{df}{dt}(\hat{x}(t), u(t)) = \begin{bmatrix} 0 & 1 & -1 \\ \frac{-k_1 - 3k_2\theta^2(t)}{J_T} & \frac{-b}{J_T} & 0 \\ \frac{k_1 + 3k_2\theta^2(t)}{J_B} & 0 & \frac{-b}{J_B} \end{bmatrix}$$

(b) $\dot{\hat{x}} = f(\hat{x}, u) + L(t)(y_m - h(\hat{x}, u))$

$L(t) = \Sigma(t) * H * T(t) * N^{-1}$

$\dot{\Sigma(t)} = \Sigma(t)F(t)T + F(t)\Sigma(t)$ + W - $\Sigma(t)H'T(t)N^{-1}H(t)\Sigma(t)$, $\Sigma(0) = \Sigma 0$

(c) RMSE = 0.0472 rad/s