CIVE 295 Energy System and Control
Spring 2019 Scott Moura and Dong Zhang

HW 01

1 Problem 1: Review Submission Procedure from HW0 (ungraded)

Reviewed

2 Problem 2: Reading

A lot of issues have risen in the past few years which have spurred the growth and interest in developing a smart grid. Some of these issues have been the growing carbon footprint, the uncertainties in fossil fuels which have spurred the market for electrification of transportation. Moreover, there has been a growing demand of quality power without any outages and to transport this power without losses. These have all created a need for a smart grid. Modeling, Controls and optimization and modeling bring together the technology which will be used to make this Smart grid work. This Smart Grid will be driven by a cyber infrastructure in which where power flow is enabled through wide-area sensing, advanced communications, control, and distributed actuation. All the issues today with the electric grid and sustainable energy have a controls problem hidden in them. The article gives examples of control of active power in a wind farm, coordinated control of variable generation using FACTS, and reactive power control using distributed renewables.

3 Problem 3: Black-box vs. White-box Modeling

Advantages of Black Box modeling:

1. No need to understand the fundamental forces or internal workings
2. Specificity of the model is seen
3. Easy to create test cases
4. Can usually be automated

Disadvantages of Black Box Modeling:

1. Usually does not capture all scenarios
2. Doesn't tell anything about the internal functioning.
3. Not thorough
4. Extrapolation is a weakness.

Advantages of White Box modeling:

1. Need to understand workings and the technical stuff
2. Explanatory
3. Highly systematic / easy to reproduce
4. Can uncover errors early in the development process
5. Thorough

Disadvantages of White Box Modeling:

1. complexity
2. Some conditions can go untested
3. Missing functionality may not be discovered.

4 Problem 4: Mathematical Modeling Uses

1. Analysis: simulation or prediction. Given a future trajectory of $u(t), x(0)$ at the present, and the system model σ,predict the future of $y(t)$
2. State Estimation: Given a system σ with time histories $u(t)$ and y(t), find x that is consistent with σ,u,y. This is the monitoring problem. That is, you cannot measure every state, yet you wish to monitor every state.
3. System Design or Planning: Given u(t)and some desired y(t), find σ such that u(t)acting on the system will produce y(t). Most engineering disciplines deal with design synthesis. Traditionally,one might create various physical prototypes to synthesize a desired system.
4. Model Identification: Given time histories u(t)and y(t), usually obtained from experimental data, determine a model and its parameter values that are consistent with u and y.
5. Control Synthesis. Given a system with current state x(0)and some desired y(t), find u(t) such that the system will produce y(t). Energy management problem.

5 Problem 5: Mathematical Modeling

(a) Stocks of the model: State of charge of the battery, Energy stored in the battery.

State Variables: $z(t)$ $V_c(t)$

(b) (a) KVL for the circuit gives us: $V(t) - OCV(z(t)) - V_c(t) - R_1 I(t) = 0$.
Rearranging for V(t) gives us: $V(t) = OCV(z(t)) + V_c(t) + R_1 I(t)$

(b) KCL for the circuit gives us: $I(t) = I_c(t) + I_2(t)$
Substituting for $I_c(t) = C\frac{dV_c(t)}{dt}$ and $I_2(t) = \frac{V_c(t)}{R_2}$ gives us:
$I(t) = C\frac{dV_c(t)}{dt} + \frac{V_c(t)}{R_2}$

(c) Given equation for z(t) is $\dot{z}(t) = \frac{1}{Q}I(t)$

(c) Parameters of the model, θ are: R_1, R_2 and C

(d) State Equations: Equation 1: $\dot{z}(t) = \frac{1}{Q}I(t)$

Equation 2: $\frac{dV_c(t)}{dt} = \frac{1}{C}(I(t) - \frac{V_c(t)}{R_2})$

Combined: $\frac{d}{dt}\begin{bmatrix} z(t) \\ V_c(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{CR_2} \end{bmatrix} \begin{bmatrix} z(t) \\ V_c(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{Q} \\ \frac{1}{C} \end{bmatrix} I(t)$

Output Equation is:

$$V(t) = OCV(z(t)) + V_c(t) + R_1 I(t)$$

(e) The state equations in part d are of a linear system. The A and B matrices for these equations are: A = $\begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{CR_2} \end{bmatrix}$ and B= $\begin{bmatrix} \frac{1}{Q} \\ \frac{1}{C} \end{bmatrix}$

The output equation is not linear as OCV(z(t)) is not linear with z(t) and hence that is the term that produces the non linearity.

6 Problem 6: Stability and Linearization

(a) For zero input current, our state equations transform into:
$\dot{x}(t) = \frac{d}{dt}\begin{bmatrix} z(t) \\ V_c(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{CR_2} \end{bmatrix}\begin{bmatrix} z(t) \\ V_c(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -0.4 \end{bmatrix}\begin{bmatrix} z(t) \\ V_c(t) \end{bmatrix}$. The eigenvalues of A are 0 and -0.4 as A is diagonal. This shows that the Re($\lambda_i \leq 0$) for i = 1,2. Thus this system is marginally stable.

(b) To linearize, we first take:
$y(t) = V(t) = V(V_c(t), OCV(z(t)), I(t)) = OCV(z(t)) + V_c(t) + R_1 I(t)$
Now we linearly approximate using taylor series:

$$\begin{aligned} V(V_c(t), z(t), I(t)) = \quad & V(V_c^{\text{eq}}, I^{\text{eq}}, z^{eq}) + \\ & \frac{\partial V}{\partial V_c}(V_c, z, I)|_{V_c^{eq} z^{eq}}^{I^{eq}}(V_c(t) - V_c^{eq}) \\ & + \frac{\partial V}{\partial I}(V_c, z, I)|_{V_c^{eq} z^{eq}}^{I^{eq}}(I(t) - I^{eq}) + \\ & \frac{\partial V}{\partial z}(V_c, z, I)|_{V_c^{eq} z^{eq}}^{I^{eq}}(z(t) - z^{eq}) \end{aligned}$$

Let us calculate the RHS, term by term:

(a) V(V_c^{eq}, I^{eq}, z^{eq}) $= V_c^{eq} + I^{eq} + OCV(z^{eq}) = 0 + 0 + p_0 + p_1 * 0.5 + p_2 * 0.5^2 + p_3 * 0.5^3 = p_0 + 0.5p_1 + 0.25p_2 + 0.125p_3$

(b) $\frac{\partial V}{\partial V_c}(V_c, z, I)|_{V_c^{eq} z^{eq}}^{I^{eq}} = 1$

(c) $\frac{\partial V}{\partial I}(V_c, z, I)|_{V_c^{eq} z^{eq}}^{I^{eq}}(I(t) - I^{eq}) = R_1$

(d) $\frac{\partial V}{\partial z}(V_c, z, I)|_{V_c^{eq} z^{eq}}^{I^{eq}}(z(t) - z^{eq}) = p_1 + 2p_2 Z^{eq} + 3p_3 z^{eq2}$

Recombination:
$V(V_c(t), z(t), I(t)) = p_0 + 0.5p_1 + 0.25p_2 + 0.125p_3 + 1(V_c(t) - V_c^{eq}) + R_1(I(t) - I^{eq}) + (p_1 + 2p_2 Z^{eq} + 3p_3 z^{eq2})(z(t) - z^{eq}) = p_0 + 0.5p_1 + 0.25p_2 + 0.125p_3 + V_c(t) + R_1 I(t) + (p_1 + p_2 + 0.75p_3)(z(t) - 0.5)$

7 Problem 7: Simulation and Analysis

Part c: The linear model moves away from the non linear one because the linear model only considers the first two terms of the taylor series for approximating the function. At higher values, this approximation is off from the non linear model and hence the linearized model incurs errors.

Looking at the graph the linearized version would go off track when the OCV plateaued and then went down. The linear model incurs errors because of this change of direction of the second derivative and it has only consider the first derivative. The OCV went from concave up to down.

Linearized models can only help us estimate a system up to a certain point. This is evident by the fact that there were errors when the OCV changed directions and second derivatives.