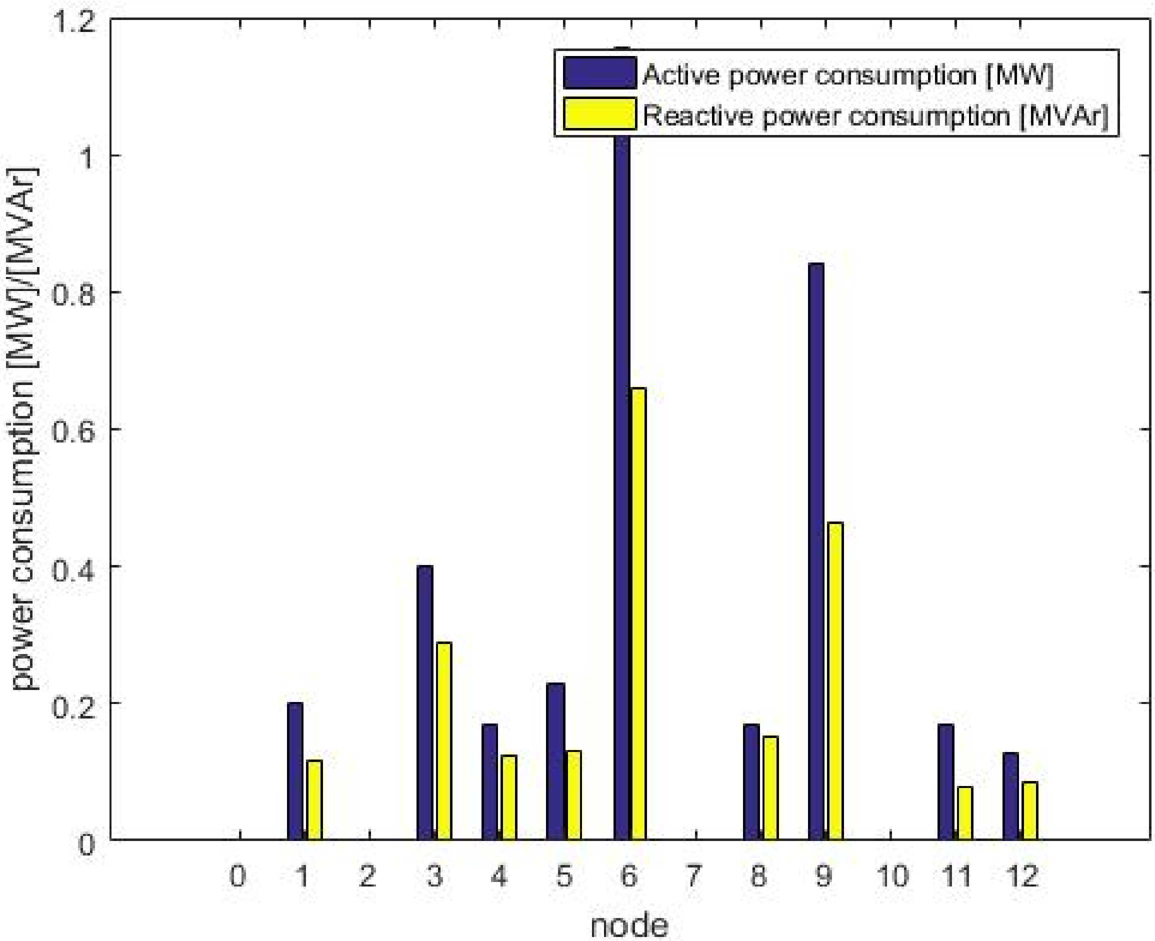
1 Problem 1: Network Parameters



(a)

(b) $$\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

2 Problem 2: Balancing Supply Demand without a Network

(a) Our optimization variables are: p_i- The active power generated at node i in MW

q_i- The reactive power generated at node i in MVAr

s_i- The apparent power generated at node i in MVA

(b) Our Objective function is

$\min\limits_{s} \sum_{j=0}^{12} \mathrm{c}_j * s_j = \min\limits_{s}(\mathrm{c}^T * s)$

(c) Our Constraints are:

(a) Apparent Power Balance: $\sum_{j=0}^{12} s_i = \sum_{j=0}^{12} \sqrt{(p_i)^2 + (q_i)^2}$

(b) Non negative Power generation: $p_j \geq 0 \quad q_j \geq 0$

(c) Apparent Power Generation Capacity: $s_j \leq s_{j,max}$

(d) Apparent Power Formula: $s_j = \sqrt{p_j^2 + q_j^2}$

(d) Because of the 4th/dth constraint in part c, this program is none of the provided options. Constraints 1, 2, 3 are linear and the objective function is linear as well.

If we relax equation 6, and convert it to $s_j \geq \sqrt{p_j^2 + q_j^2}$, then this problem becomes a version of convex programs, specifically SOCP. Our optimal solution will minimize cost and satisfy these new equality conditions.

(e)

```
Status: Solved
Optimal value (cvx_optval): +256.251

------------------- PROBLEM 2 --------------------
--------------------------------------------------
Minimum Generating Cost : 256.25 USD

Node 0 [Grid]  Gen Power : p_0 = 0.439 MW | q_0 = 0.439 MW | s_0 = 1.063 MW
Node 3 [Gas]   Gen Power : p_3 = 0.000 MW | q_3 = 0.000 MW | s_3 = 0.000 MW
Node 9 [Solar] Gen Power : p_9 = 1.134 MW | q_9 = 1.134 MW | s_9 = 3.000 MW

Total active power   : 3.466 MW   consumed | 1.572 MW   generated
Total reactive power : 2.102 MVAr consumed | 1.572 MVAr generated
Total apparent power : 4.063 MVA  consumed | 4.063 MVA  generated
```

3 Problem 3: Add Line Power Flows

(a) Optimization variables are: s_j = The apparent power generated at node j ,j=0,,12,in MVA

p_j: The active power generated at node j, j=0,,12 in MW

q_j: The reactive power generated at node j, j=0,,12 in MVAR

P_{ij}: The active power flow in line i,j where i = p(j) j= 1, ,12

Q_{ij}: The reactive power flow in line i,j , where i = p(j) and j= 1, ,12

(b) (a) Non negative Power generation: $p_j \geq 0$ $q_j \geq 0$

(b) Apparent Power Generation Capacity: $s_j \leq s_{j,max}$

(c) Apparent Power Formula: $s_j = \sqrt{p_j^2 + q_j^2}$

(d) Active Power Flow Conservation: $P_{ij} = (l_j^P p_j) + r_{ij} L_{ij} + \sum_{k \in N} A_{jk} P_{jk}$ where $\forall j \in N, i = (j)$. Here the first term on the RHS corresponds to power consumed at node, the third corresponds to power flow from the node to others and the second term is zero. The term on the LHS describes power flow to the node.

(e) Reactive Power Flow Conservation: $Q_{ij} = (l_j^Q q_j) + r_{ij} L_{ij} + \sum_{k \in N} A_{jk} Q_{jk}$ where $\forall j \in N, i = (j)$.

(c)

```
Status: Solved
Optimal value (cvx_optval): +255.359

------------------- PROBLEM 3 --------------------
--------------------------------------------------
Minimum Generating Cost : 255.36 USD

Node 0 [Grid]  Gen Power : p_0 = 0.901 MW | q_0 = 0.546 MW | s_0 = 1.054 MW || mu_s0 =   0 USD/MW
Node 3 [Gas]   Gen Power : p_3 = 0.000 MW | q_3 = 0.000 MW | s_3 = 0.000 MW || mu_s3 =   0 USD/MW
Node 9 [Solar] Gen Power : p_9 = 2.565 MW | q_9 = 1.556 MW | s_9 = 3.000 MW || mu_s9 =  50 USD/MW

Total active power   : 3.466 MW   consumed | 3.466 MW   generated
Total reactive power : 2.102 MVAr consumed | 2.102 MVAr generated
Total apparent power : 4.063 MVA  consumed | 4.054 MVA  generated
```

The answers should be nearly the same as Problem 2 as the optimization constraints and objective function are equivalent for the 2 problems.

(d) From the picture above we can see that mu$_s 9is50USD/MWwhichindicatesthatifweincreasedthecapacityofnode9by1$

4 Problem 4: The Complete Optimal Economic Dispatch with DistFlow Equations

(a) Optimization variables are: s_j = The apparent power generated at node j ,j=0,,12,in MVA

p_j: The active power generated at node j, j=0,,12 in MW

q_j: The reactive power generated at node j, j=0,,12 in MVAR

P_{ij}: The active power flow in line i,j where i = p(j) j= 1, ,12

Q_{ij}: The reactive power flow in line i,j , where i = p(j) and j= 1, ,12

L_{ij} = The squared magnitude of complex current on line (i,j) , where i = p(j) and j= 1, ,12

V_i = The squared voltage magnitude at node i, where i = 1, ... , 12

(b) (a) Non negative Power generation: $p_j \geq 0 \quad q_j \geq 0$

(b) Apparent Power Generation Capacity: $s_j \leq s_{j,max}$

(c) Apparent Power Formula: $s_j = \sqrt{p_j^2 + q_j^2}$

(d) Active Power Flow Conservation: $P_{ij} = (l_j^P p_j) + r_{ij} L_{ij} + \sum_{k \in N} A_{jk} P_{jk}$ where $\forall j \in N, i = (j)$. Here the first term on the RHS corresponds to power consumed at node, the third corresponds to power flow from the node to others and the second term corresponds to power loss. The term on the LHS describes power flow to the node.

(e) Reactive Power Flow Conservation: $Q_{ij} = (l_j^Q q_j) + r_{ij} L_{ij} + \sum_{k \in N} A_{jk} Q_{jk}$ where $\forall j \in N, i = (j)$.

(f) Voltage drops and relationships between node i and other connected nodes.
$V_j = V_i + (r_{ij}^2 + x_{ij}^2) L_{ij} - 2(r_{ij} P_{ij} + x_{ij} Q_{ij})$ where $\forall j \in N, i = (j)$.

(g) Current in line (i,j): $L_{ij} = (P_{ij}^2 + Q_{ij}^2)/V_j$.

(h) Voltage and current limitation: $v_{min}^2 \leq V_j \leq v_{max}^2$ and $L_{ij} \leq I_{ij,max}^2$

(c) No it is not a convex program. This is because constraint g from part b is not affine. We can relax this constraint by converting it to an equality:

$-L_{ij} + (P_{ij}^2 + Q_{ij}^2)/V_j \leq 0$.

$-L_{ij} + (P_{ij}^2 + Q_{ij}^2)/V_j$. is convex.

(d) Solution is not equal because of new constraints. There is power loss, voltage and current limitations which result in increase of cost as compared to problem 3.

```
------------------- PROBLEM 4 --------------------
--------------------------------------------------
Minimum Generating Cost : 299.69 USD

Node 0 [Grid]  Gen Power : p_0 = 1.568 MW | q_0 = 0.985 MW | s_0 = 1.852 MW || mu_s0 =   0 USD/MW
Node 3 [Gas]   Gen Power : p_3 = 0.000 MW | q_3 = 0.000 MW | s_3 = 0.000 MW || mu_s3 =   0 USD/MW
Node 9 [Solar] Gen Power : p_9 = 1.941 MW | q_9 = 1.216 MW | s_9 = 2.290 MW || mu_s9 =   0 USD/MW

Total active power   : 3.466 MW   consumed | 3.509 MW   generated
Total reactive power : 2.102 MVAr consumed | 2.201 MVAr generated
Total apparent power : 4.063 MVA  consumed | 4.142 MVA  generated

Node  1 Voltage : 1.000 p.u.
Node  2 Voltage : 0.967 p.u.
Node  3 Voltage : 0.963 p.u.
Node  4 Voltage : 0.963 p.u.
Node  5 Voltage : 0.962 p.u.
Node  6 Voltage : 0.960 p.u.
Node  7 Voltage : 0.957 p.u.
Node  8 Voltage : 0.957 p.u.
Node  9 Voltage : 0.957 p.u.
Node 10 Voltage : 0.964 p.u.
Node 11 Voltage : 0.955 p.u.
Node 12 Voltage : 0.954 p.u.
Node 13 Voltage : 0.953 p.u.
```

(e) When we look at *muLub*, we can see that the dual variables for current are mostly zero except for the flow between node 8(i) and node 9(j) which can be seen below. Hence at these nodes, the current limitation is active.

```
mu_Lub =

  13×1 cell array

    [3.1064e+05]
    [3.0792e-09]
    [9.8785e-09]
    [4.5277e-09]
    [2.5634e-08]
    [2.0441e-08]
    [1.9389e-09]
    [1.8603e-09]
    [5.0898e-09]
    [   22.5184]
    [2.1784e-08]
    [1.9567e-08]
    [1.0480e-08]
```

When we look at $muVub$ and

$muVlb$, we see that dual variables for voltage are zero/close to zero. Hence the constraints for voltage are not active.

```
mu_Vlb =

   1.0e-05 *

    0.0177
    0.0536
    0.0685
    0.0685
    0.0764
    0.0889
    0.1253
    0.1253
    0.1253
    0.0640
    0.1777
    0.2293
    0.2908
```

```
mu_Vub =

   1.0e-06 *

    0.1682
    0.1027
    0.0986
    0.0986
    0.0972
    0.0954
    0.0926
    0.0926
    0.0926
    0.0996
    0.0906
    0.0896
    0.0888
```

(f) The solution is given below.

On further inspection of the limits, we can see that there is a new limit which is activated, the Lower Bound for the voltage is activated for a node and this results in the change of power transfer and power loss, changing the minimum generating cost. The current limit is still active for the same node as above.

```
------------------ PROBLEM 4 --------------------
-------------------------------------------------
Minimum Generating Cost : 348.67 USD

Node 0 [Grid]  Gen Power : p_0 = 0.940 MW | q_0 = 0.174 MW | s_0 = 0.956 MW  | mu_s0 =   0 USD/MW
Node 3 [Gas]   Gen Power : p_3 = 0.760 MW | q_3 = 0.518 MW | s_3 = 0.920 MW  | mu_s3 =   0 USD/MW
Node 9 [Solar] Gen Power : p_9 = 1.789 MW | q_9 = 1.450 MW | s_9 = 2.303 MW  | mu_s9 =   0 USD/MW

Total active power   : 3.466 MW   consumed | 3.489 MW   generated
Total reactive power : 2.102 MVAr consumed | 2.142 MVAr generated
Total apparent power : 4.063 MVA  consumed | 4.178 MVA  generated

Node  1 Voltage : 1.000 p.u.
Node  2 Voltage : 0.989 p.u.
Node  3 Voltage : 0.992 p.u.
Node  4 Voltage : 0.992 p.u.
Node  5 Voltage : 0.984 p.u.
Node  6 Voltage : 0.983 p.u.
Node  7 Voltage : 0.984 p.u.
Node  8 Voltage : 0.984 p.u.
Node  9 Voltage : 0.984 p.u.
Node 10 Voltage : 0.991 p.u.
Node 11 Voltage : 0.982 p.u.
Node 12 Voltage : 0.981 p.u.
Node 13 Voltage : 0.980 p.u.
```

```
mu_Vlb =

   1.0e+03 *

    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    1.3705
```

```
mu_Lub =

  13×1 cell array

    [2.8134e+05]
    [2.2740e-09]
    [1.0372e-08]
    [4.9363e-09]
    [2.7711e-08]
    [2.2377e-08]
    [2.1594e-09]
    [2.0449e-09]
    [5.6020e-09]
    [   59.6697]
    [2.3737e-08]
    [2.1458e-08]
    [1.1511e-08]
```