EE 735: ASSIGNMENT 4

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PROBLEM 1

(a) Time Independent Part

SIMULATION APPROACH

Given,
Length of semiconductor $= 10\ \mu m$
Minority carrier lifetime, $\tau\ =\ 10^{-7}\ s$
Diffusivity, $D\ =\ 0.1\ cm^2/s$

The MATLAB code computes the concentration profile and flux profile by numerical method using finite difference matrix. After forming the finite difference matrix using the differential equation and boundary conditions, matrix inversion is done to calculate the concentration profile. The flux profile is obtained by calculating the gradient of the concentration profile.

The analytical solution for the two cases is given below:

Case 1 :
Boundary Conditions:
Concentration of particles at $A\ (x\ =\ 0):\ n_A = 10^{12}\ cm^{-3}$
Concentration of particles at $B\ (x\ =\ 10\ \mu m):\ n_B = 0$

Solving the differential equation : $D\frac{d^2n}{dx^2} = \frac{n}{\tau}$ using the given boundary conditions the concentration profile is, $n(x)\ =\ C_1 exp(-x/L_d) + C_2 exp(x/L_d)$

The flux profile is given by, $J(x)\ = -D\frac{dn(x)}{dx} = \frac{D}{L_d}[C_1 exp(-x/L_d) - C_2 exp(x/L_d)]$

where $C_1 = 10^{12}\ /cm^3, C_2 = -2.06115\ \times 10^3\ /cm^3, L_d = \sqrt{D\tau} = 10^{-4}\ cm$ and *x* is in *cm*.

Case 2 :
Boundary Conditions:
Concentration of particles at $A\ (x\ =\ 0):\ n_A = 10^{12}\ cm^{-3}$
Flux of particles at $B\ (x\ =\ 10\ \mu m):\ J_B = kn_B$ where $k\ =\ 10^3 cm/s$

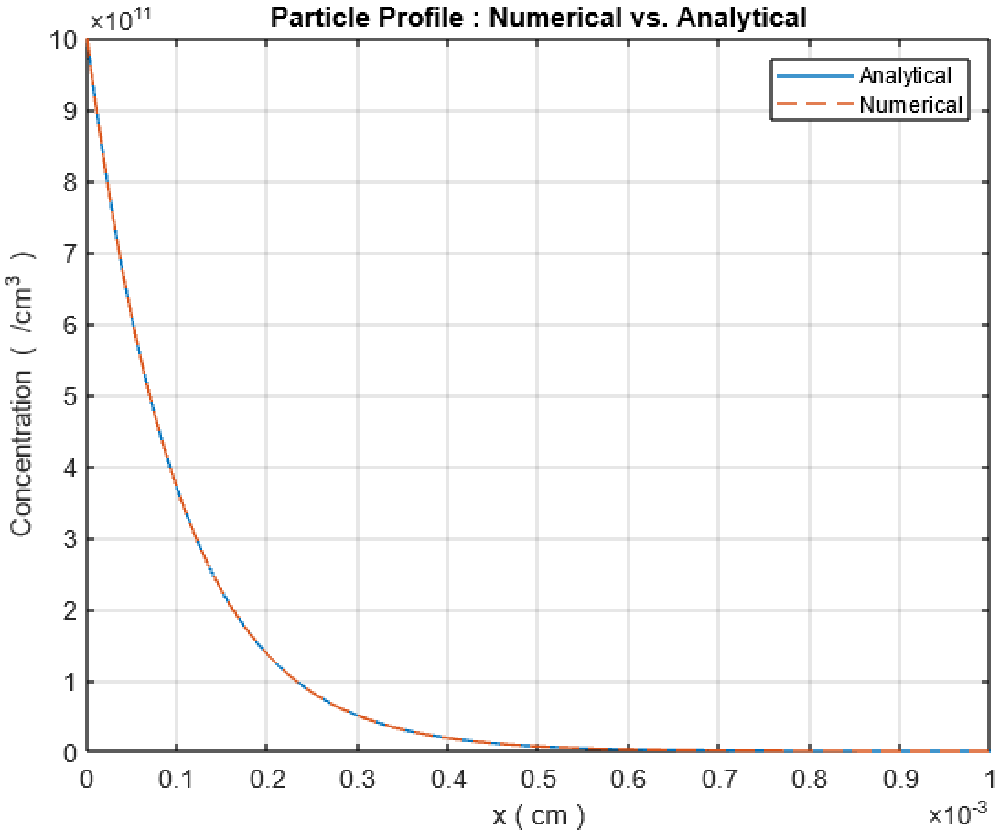
Solving the differential equation : $D\frac{d^2n}{dx^2} = \frac{n}{\tau}$ using the given boundary conditions the concentration profile is, $n(x)\ =\ C_1 exp(-x/L_d)$

The flux profile is given by, $J(x)\ = -D\frac{dn(x)}{dx}\ = \frac{DC_1}{L_d} exp(-x/L_d)$

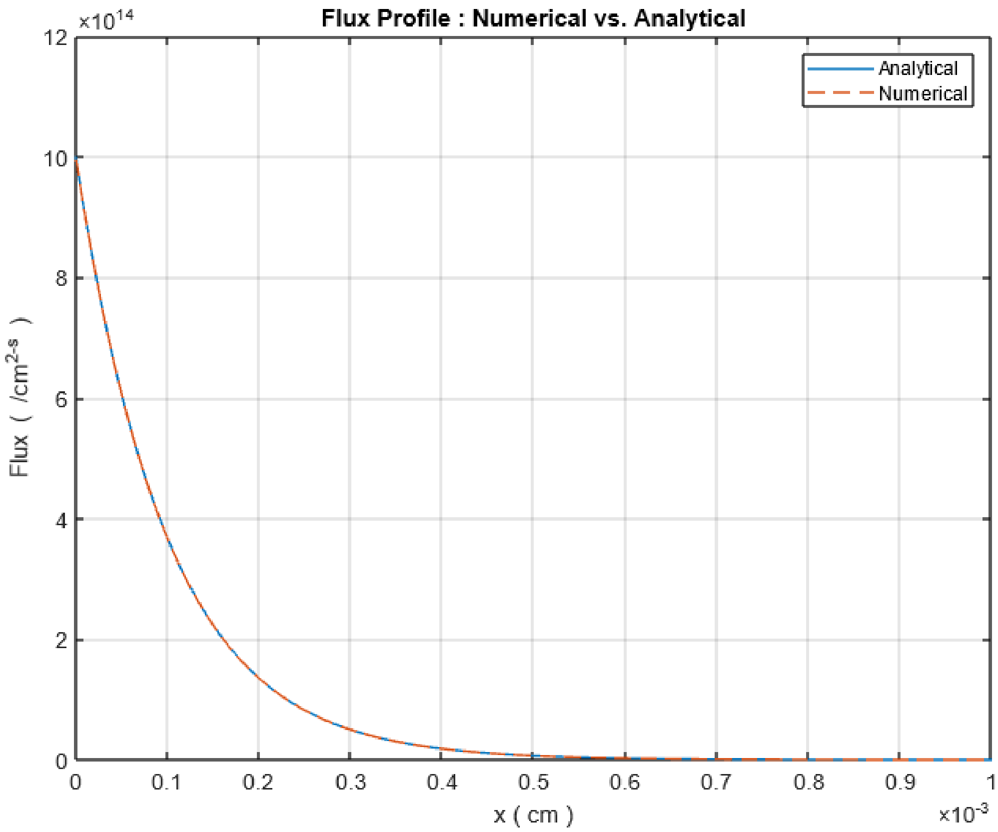
where $C_1 = 10^{12}\ /cm^3, L_d = \sqrt{D\tau} = 10^{-4}\ cm$ and *x* is in *cm*.

RESULT AND DISCUSSION

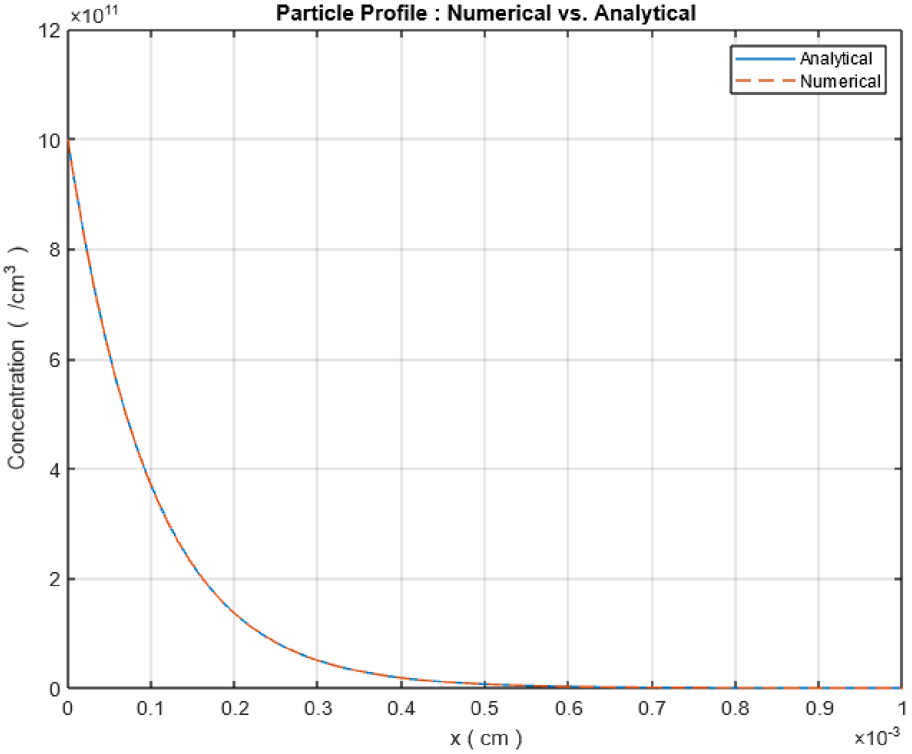
The concentration and flux profile obtained using numerical and analytical method are matching perfectly as given below, Concentration profile for Case 1 :



Flux profile for Case 1 :



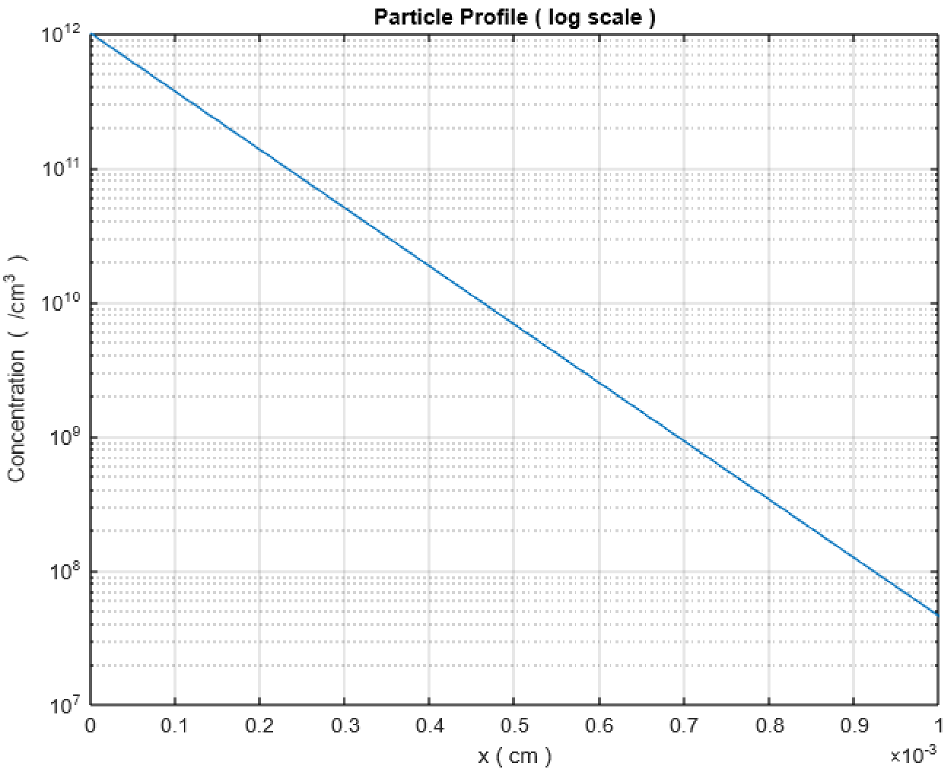
Concentration profile for Case 2 :



The value of flux at position B for **Case 1** is $9.0805 \times 10^{10}\ /cm^2 - s$

The value of flux at position B for **Case 2** is $4.5516 \times 10^{10}\ /cm^2 - s$

Due to the change in boundary condition, there is a decrease in flux at position B and the particle concentration at position B for **Case 2** is not zero as shown in plot below.



(b) Continuation

<u>SIMULATION APPROACH</u>

Given,
Length of semiconductor $= 10\ \mu m$
Minority carrier lifetime, $\tau\ =\ 10^{-7}\ s$
Diffusivity, $D\ =\ 0.1\ cm^2/s$
Particle flux introduced at $x =\ 5.5\ \mu m$ is $J_0 = 10^{13}\ /cm^2 - s$

The MATLAB code computes the concentration profile and flux profile by numerical method using finite difference matrix. After forming the finite difference matrix using the differential equation and boundary conditions, matrix inversion is done to calculate the concentration profile.

The analytical solution for the problem is given below,
Boundary Conditions:
Concentration of particles at $A\ (x\ =\ 0) :\ n_A = 0$
Concentration of particles at $B\ (x\ =\ 10\ \mu m) :\ n_B = 0$
Flux at $x =\ 5.5\ \mu m$ is $J_0 = 10^{13}\ /cm^2 - s$

Solving the differential equation : $D\frac{d^2n}{dx^2} = \frac{n}{\tau}$ using the given boundary conditions the concentration profile is,

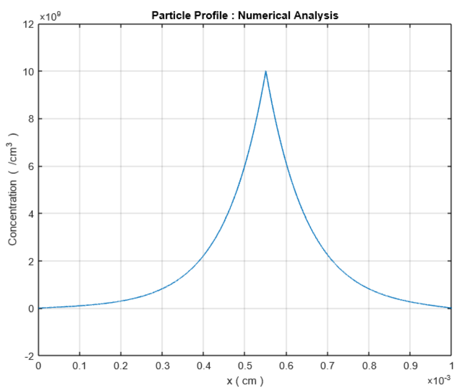
$$n(x) = C_1 exp(-x/L_d) + C_2 exp(x/L_d) \qquad 0 \le x \le 5.5 \times 10^{-4}$$

$$n(x) = C_3 exp[-(x - 5.5 \times 10^{-4})/L_d)] + C_4 exp[(x - 5.5 \times 10^{-4})/L_d)] \qquad 5.5 \times 10^{-4} \le x \le 10 \times 10^{-4}$$

where,
$C_1 = -4.0867 \times 10^7\ /cm^3$, $\quad C_2 = 4.0867\ \times 10^7\ /cm^3$, $\quad C_3 = 9.9988 \times 10^9\ /cm^3$
$C_4 = -1.2339\ \times 10^6\ /cm^3$, $\quad L_d = \sqrt{D\tau} = 10^{-4}\ cm$ and x is in cm.

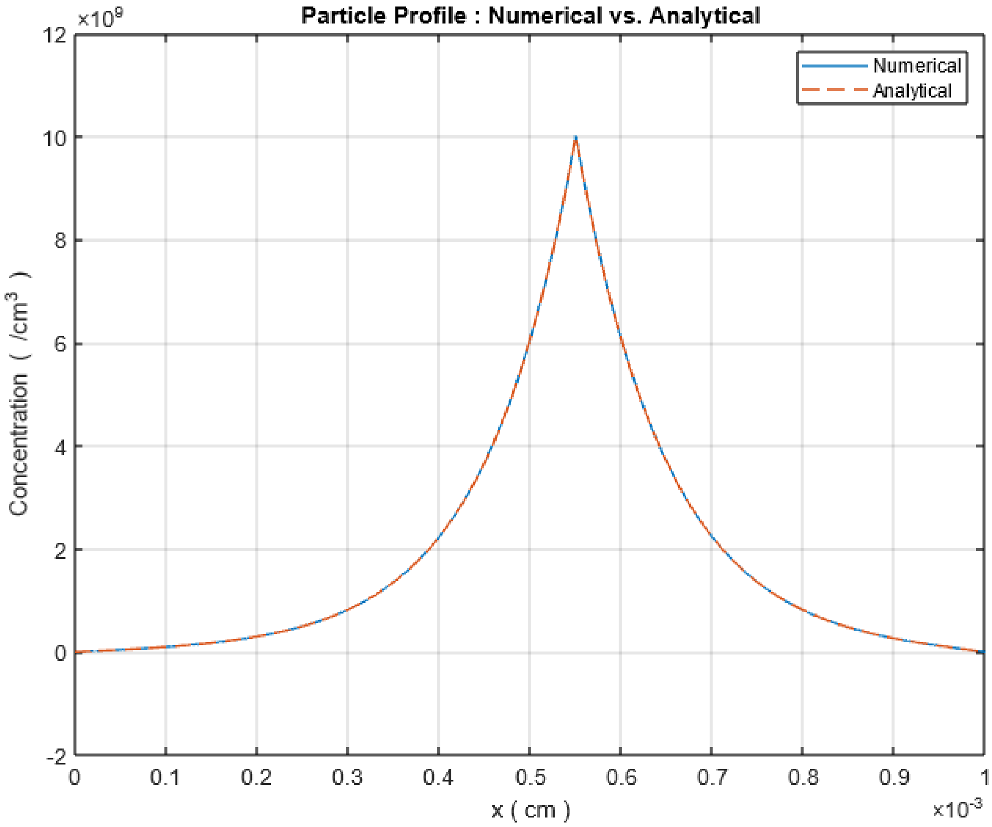
<u>RESULT AND DISCUSSION</u>

The particle profile obtained using numerical method is given below,



Particle flux at A is $-8.1766 \times 10^{10}\ /cm^2 - s$
Particle flux at B is $2.2229 \times 10^{11}\ /cm^2 - s$

<u>Comparison of Numerical and Analytical Profile</u> :



(c) Time Dependent Part

<u>SIMULATION APPROACH</u>

Given,
Length of semiconductor $= 10\ \mu m$
Diffusivity, $D\ =\ 10^{-4}\ cm^2/s$
Delta function particle injection at $x =\ 5\ \mu m$ of $n_0 = 36 \times 10^6\ /cm^3$

The MATLAB code computes the concentration profile using Implicit or Backward Euler Method and Explicit or Forward Euler Method for different time instances.

The analytical solution for the problem is given below,
Boundary Conditions:
Concentration of particles at $A\ (x\ =\ 0) :\ n_A = 0$
Concentration of particles at $B\ (x\ =\ 10\ \mu m) :\ n_B = 0$
Delta function particle injection at $x =\ 5\ \mu m$ of $n_0 = 36 \times 10^6\ /cm^3$

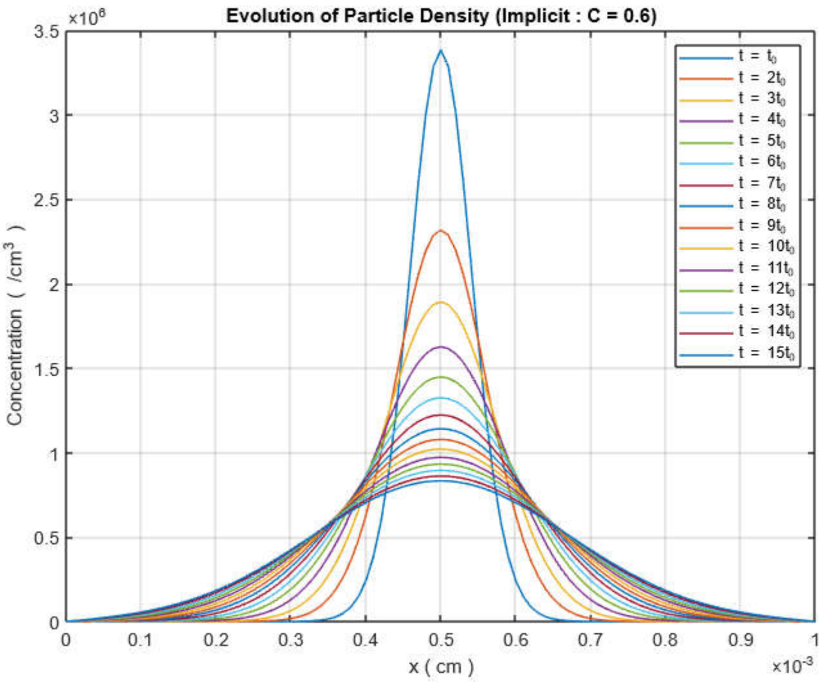
Solving the differential equation : $\frac{d^2n}{dx^2} = \frac{1}{D}\frac{dn}{dt}$ using the given boundary conditions the concentration profile as a function of time and space is given by,

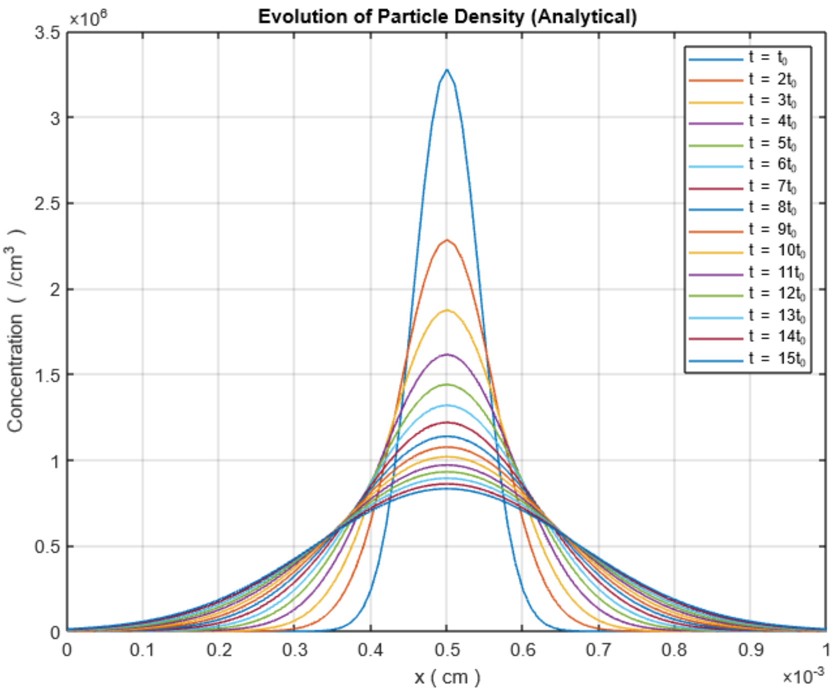
$$n(x,t) = \frac{Q}{2\sqrt{\pi D t}} exp\left[\frac{-(x - 5\times10^{-4})^2}{4Dt}\right]$$

where, $Q = 360\ /cm^2, D = 10^{-4}\ cm^2/s$ and x is in cm

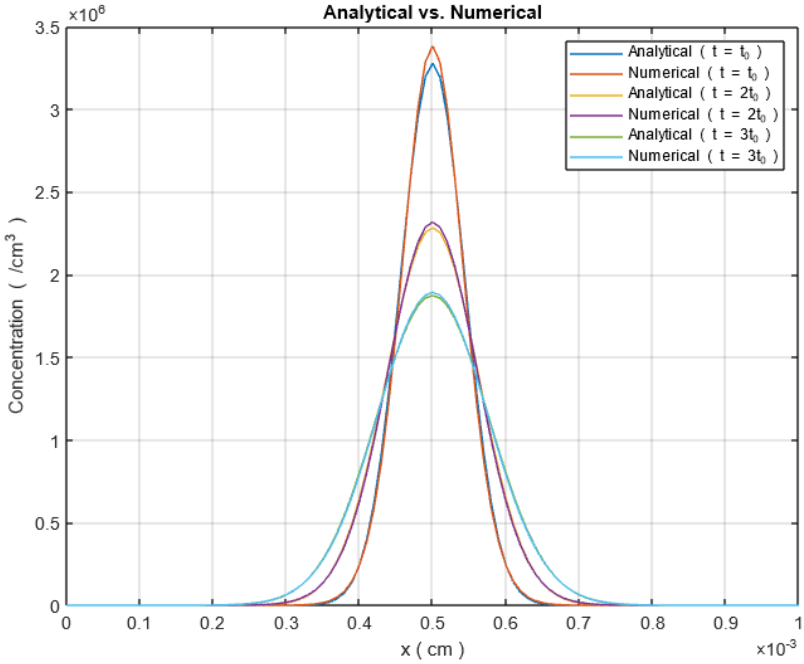
RESULT AND DISCUSSION

The evolution of particle density with time obtained using Implicit Euler method and analytical method are given below,



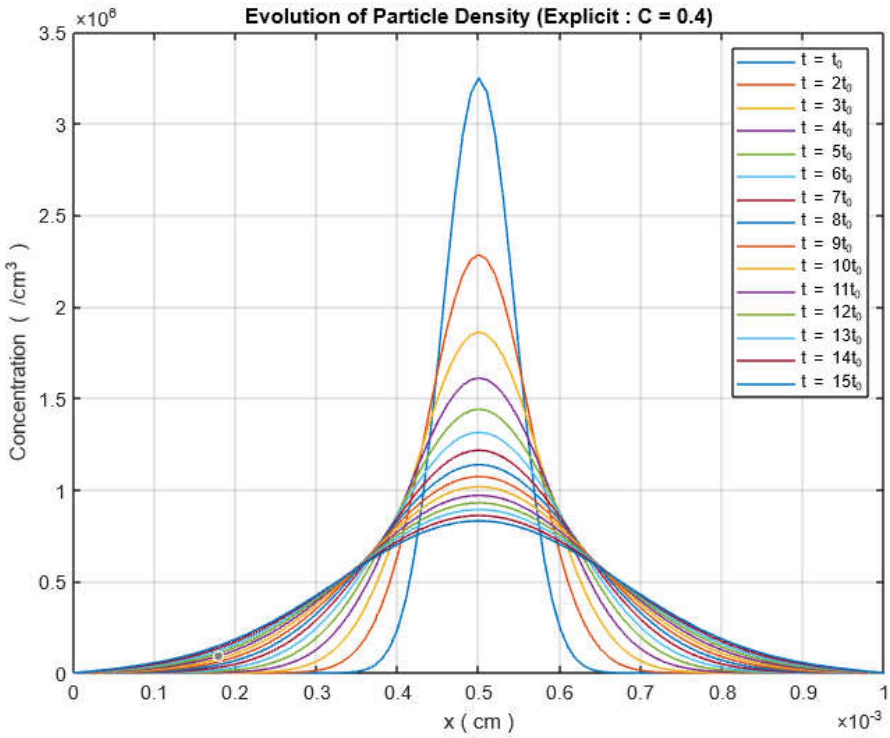


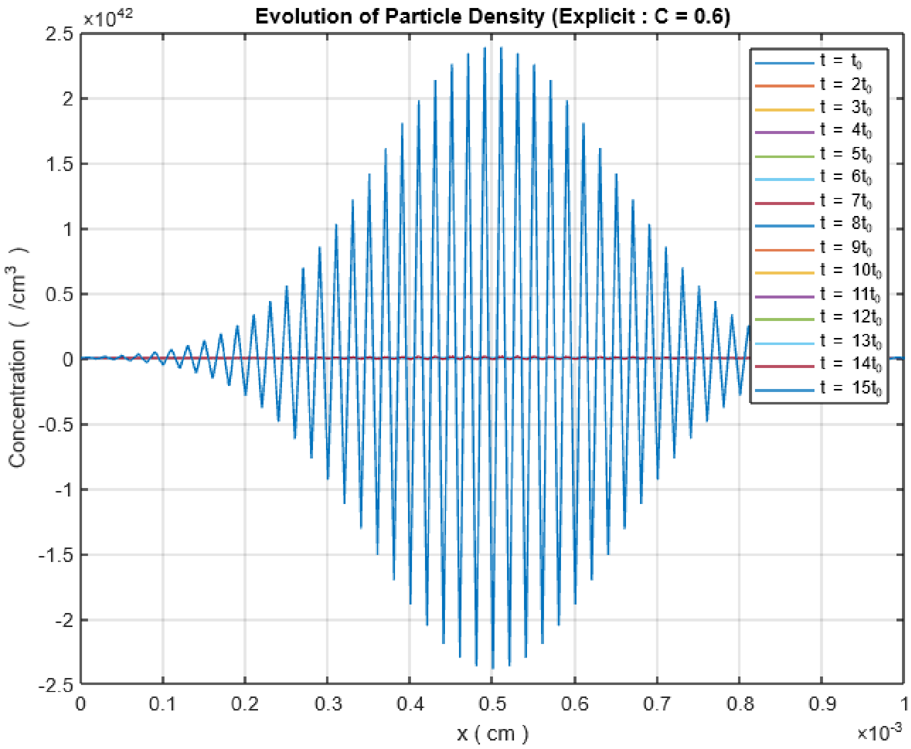
The comparison of numerical and analytical method is given below. The profile obtained using numerical method has a higher peak than its analytical counterpart.



The value of $\sqrt{Dt}$ provides an approximate measure of how par the particle has diffused from the point of injection on both sides.

The evolution of particle density with time obtained using Explicit Euler method are given below,





As expected, Explicit Euler method is unable to converge for C > 0.5.

PROBLEM 2

SIMULATION APPROACH

Given,
Length of semiconductor $= 100\ \mu m$
Diffusivity, $D = 10^{-4}\ cm^2/s$
Constant particle concentration at $x = 0$ of $n(x=0,t) = 2000\ /cm^3$

The MATLAB code computes the concentration profile using Implicit or Backward Euler Method.

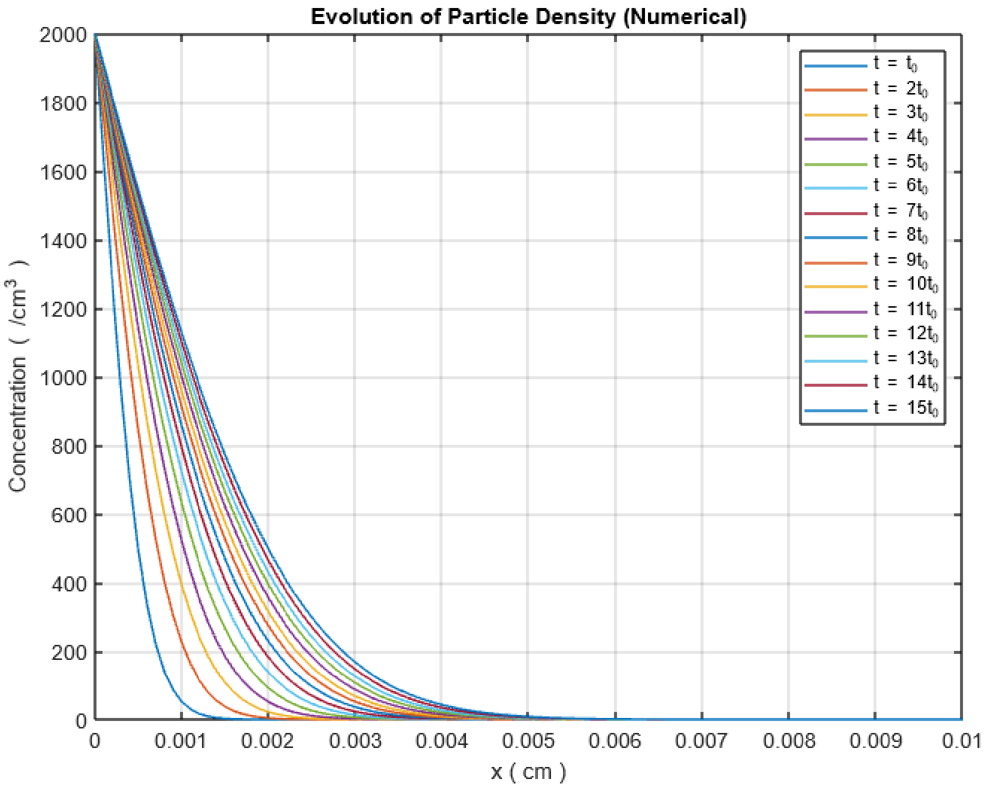
The analytical solution for the problem is given below,
Boundary Conditions:
Constant particle Concentration at $x = 0 : n_0 = 2000\ /\text{cm}^3$
Concentration of particles at $x = 100\ \mu m : n_{100} = 0$

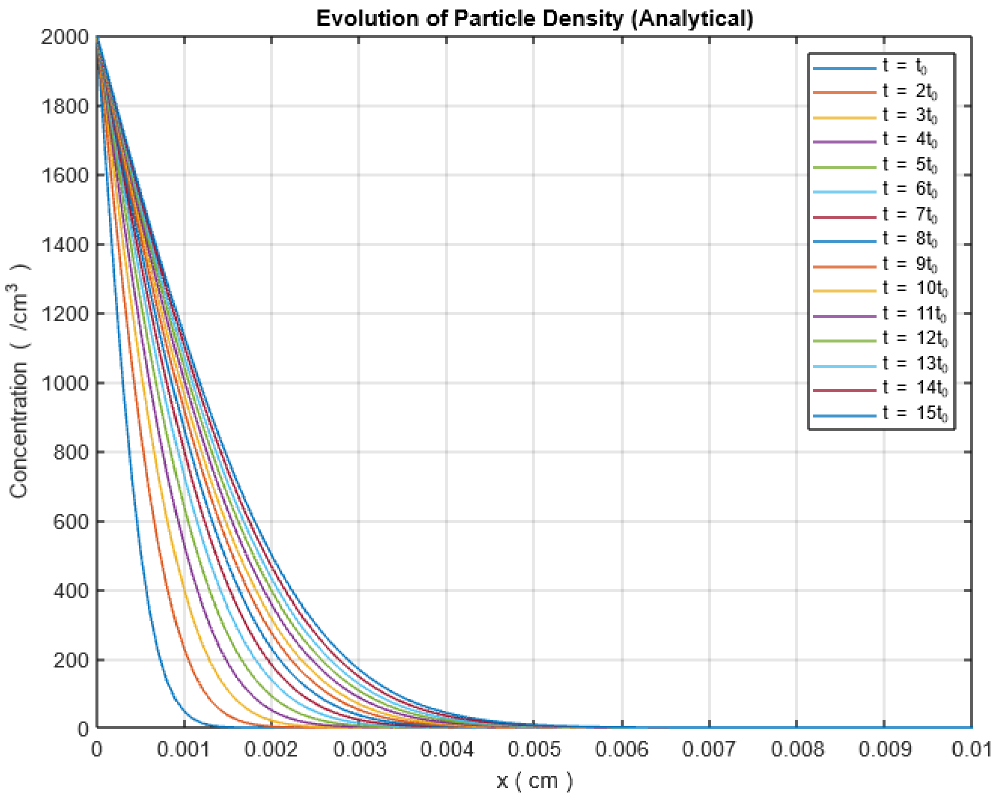
Solving the differential equation : $\frac{d^2n}{dx^2} = \frac{1}{D}\frac{dn}{dt}$ using the given boundary conditions the concentration profile as a function of time and space is given by,

$n(x,t) = n_0\left[erfc\left(\frac{x}{2\sqrt{Dt}}\right)\right]$ where, $n_0 = 2000\ /cm^3, D = 10^{-4}\ cm^2/s$ and x is in cm

RESULT AND DISCUSSION

The evolution of particle density with time obtained using Implicit Euler method and analytical method are given below,





The profile obtained using numerical and analytical are almost matching and equal as given below,