<u>EE 735: ASSIGNMENT 1</u>

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PROBLEM 1

QUESTION

Consider a system of two parallel plates as shown in Figures 1 & 2:

A. Find out the capacitance (per unit width) of the structure by numerically solving the 2d Poisson's equation.

B. Plot the electrostatic potential and equipotential surfaces.

C. Plot the 2d electric field profile.

D. Compare the simulated capacitance with the theoretical value $\left(C_{th} = \frac{\epsilon L}{d}\right)$. Which one is smaller/larger and why?

E. Now lower half of the region is replaced with another dielectric material (dielectric constant of 5) as shown in figure 2. Repeat parts A, B, C, D.

(Given : t = 1 nm, d = 8 nm, L = 800 nm)

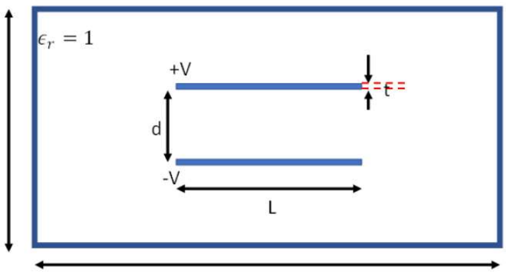


Figure 1

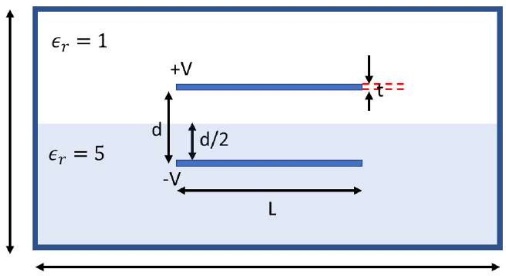


Figure 2

SIMULATION APPROACH

The capacitance of the given structures is calculated using MATLAB. The MATLAB code implemented uses FDM. The FDM or finite difference method is a simple numerical technique which solves the 2D Laplace Equation (special case of Poisson's equation) and computes the electric potential surrounding the two capacitor plates.

The gradient function of MATLAB provides the 2D Electric field intensity due to the capacitor plates. The energy density (Energy per unit volume) due to electric field is given by $\frac{1}{2}\epsilon E^2$. The total energy per unit width of the system is the sum of the magnitude of the electric field over the 2D array multiplied by $\frac{1}{2}\epsilon$.

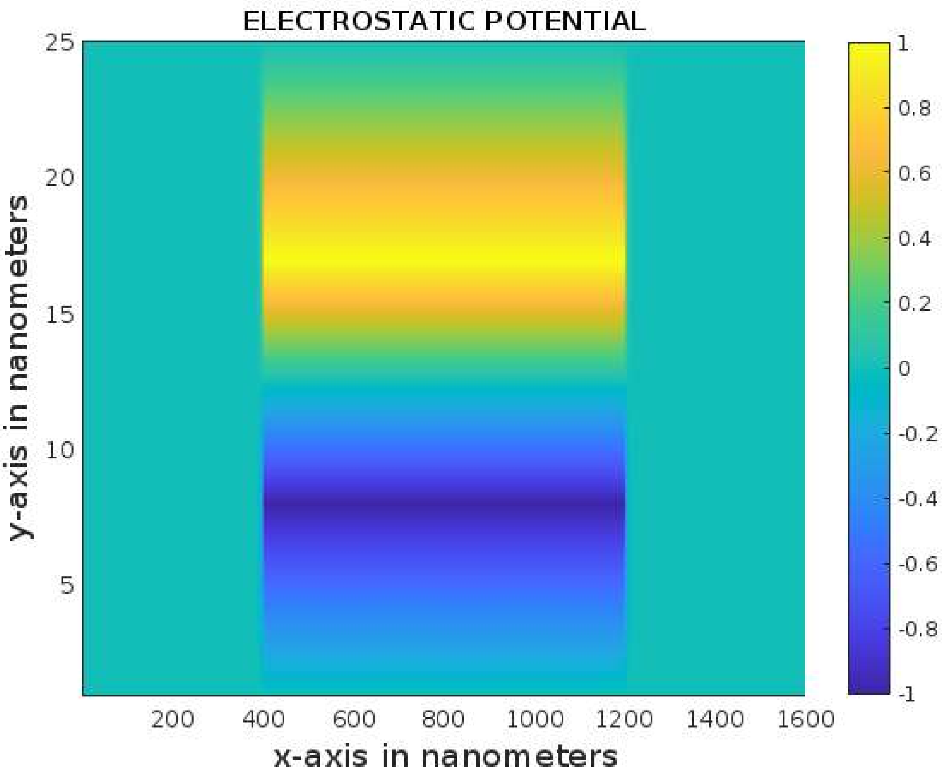
The total energy per unit width (W) is calculated as, $W = \frac{1}{2}\epsilon \sum E^2$. Now, energy stored in the capacitor is given as, $W = \frac{1}{2} C V_d^{\,2}$ where C is Capacitance per unit width and V_d is the potential difference between the two plates.

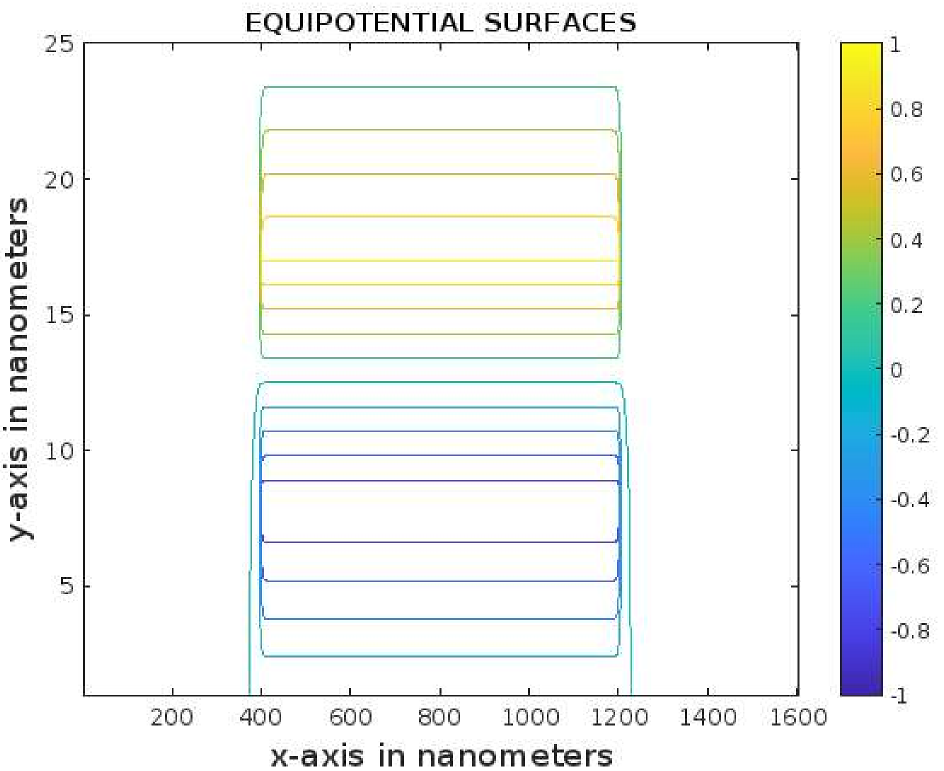
In problem 1, $V_d = V - (-V) = 2V$. Thus, Capacitance (per unit width), $C = \frac{W}{2V^2}$.

RESULTS

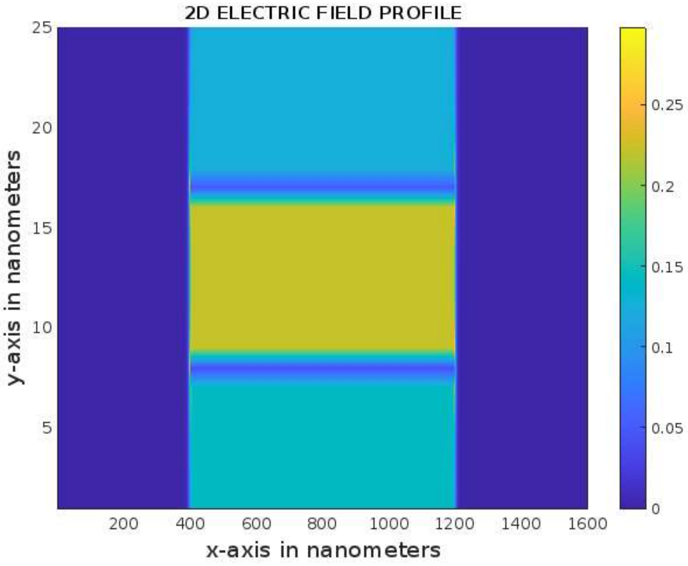
A. The capacitance (per unit width) for the structure given in figure 1 is $1.1897 \times 10^{-9}\ \mathrm{F/m}$ (or) $1.1897\ \mathrm{nF/m}$.

B. The plots of electrostatic potential and equipotential surfaces are given below.





C. The plot of 2D electric field profile is given below.



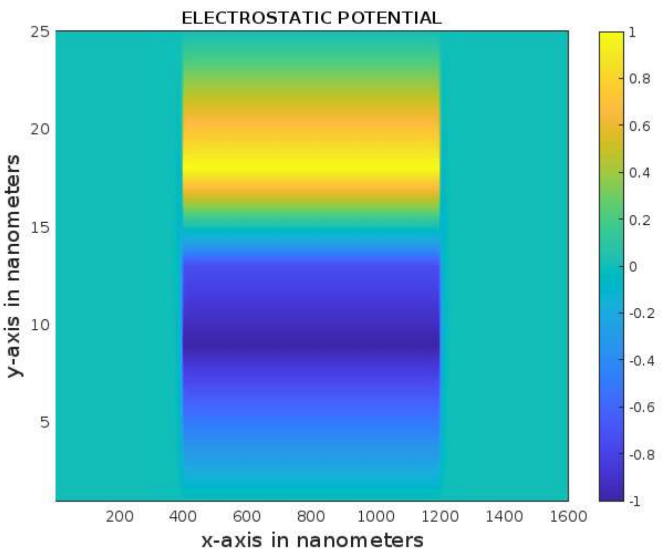
D. The theoretical value of the capacitance (per unit width) is,

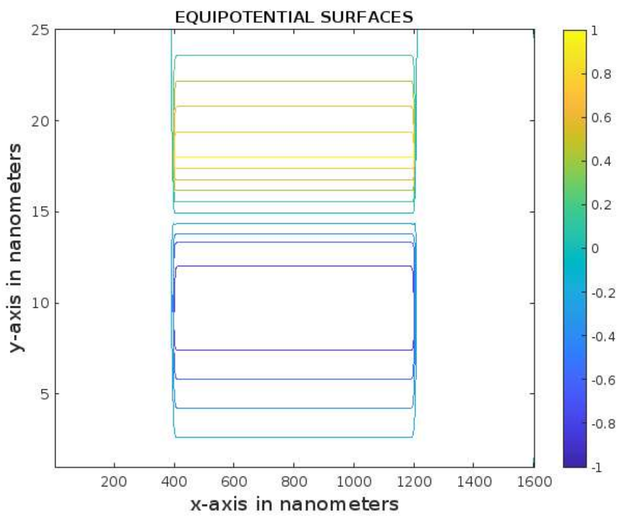
$$C_{th} = \frac{\epsilon L}{d} = \frac{\epsilon_r \epsilon_0 L}{d} = \frac{1 \times 8.854 \times 10^{-1} \times 800}{8} = 8.854 \times 10^{-10}\ \mathrm{F/m} = 0.8854\ \mathrm{nF/m}$$

The simulated value of the capacitance is larger than the theoretical value of the capacitance as the simulation considers the effect of non-uniform electric field lines around the edge of the capacitor plates. The effect of non-uniform electric field lines around the edge of the capacitor plates is known as Fringing effect which is considered negligible for theoretical calculation.

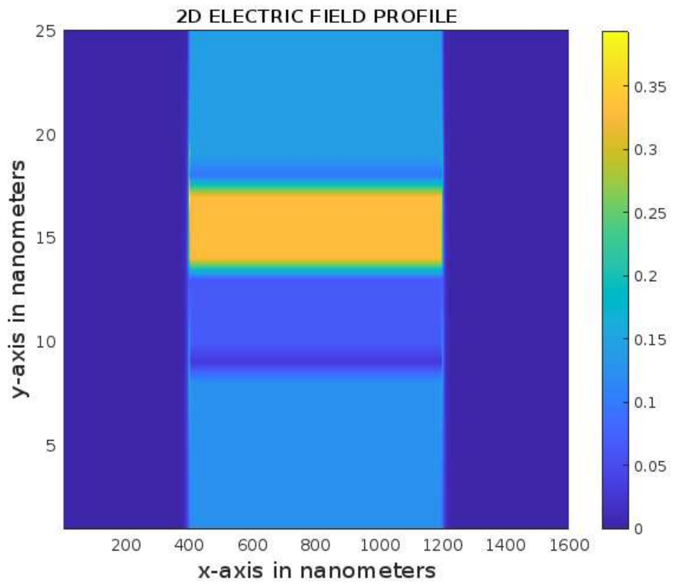
E. The capacitance (per unit width) for the structure given in figure. 2 is $2.3038 \times 10^{-9}\ \mathrm{F/m}$ (or) $2.3038\ \mathrm{nF/m}$.

The plots of electrostatic potential and equipotential surfaces are given below.





The plot of 2D electric field profile is given below.



The theoretical capacitance due to two dielectric medium system in figure 2 is,

$$C_{th} = \frac{2\epsilon_0 L}{d}\left(\frac{\epsilon_{r1}\epsilon_{r2}}{\epsilon_{r1}+\epsilon_{r2}}\right) = \frac{2 \times 8.854 \times 10^{-12} \times 800}{8}\left(\frac{1 \times 5}{1+5}\right) = 1.4757 \times 10^{-9}\ \mathrm{F/m}$$

Since the simulation takes Fringing effect into consideration, the simulated capacitance value is higher than theoretical value of capacitance.

PROBLEM 2

QUESTION

Now consider a system as shown in Figure 3:

There are three plates (P_1, P_2, P_3) placed as shown in Figure 3. The length (L_1) of plate P_1 is 500 nm which is fixed for this system. The lengths of P_2 and P_3 are the same, $L_2 = L_3$. Now d_2 is the variable distance between P_2 and P_3 such that $L_2+L_3+d_2 = L_1$ and d_1 = 5nm (fixed). P_2 is at 5 V, and P_1 is at -5 V for this entire exercise. Starting with d_2 = 30 nm, increase it till 90 nm in 30 nm steps with plate P_3 at (V_3) 0 V, +5 V, and -5 V.

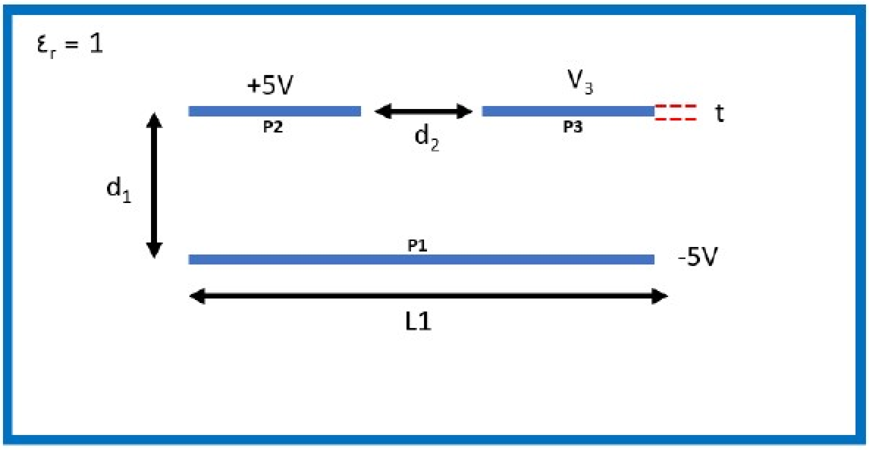
A. Find out the capacitance (per unit width) of the structure by numerically solving the 2d Poisson's equation for all 9 cases. (Hint: First fix the voltage level of P_3 and change d_2 or vice-versa)

B. Plot the electrostatic potential and equipotential surfaces for all cases.

C. Plot the 2d electric field profile and find out the position of the maximum electric field for all cases.

D. Note and explain the observations from your simulation results of A, B, and C.

(Given: t = 1nm)



SIMULATION APPROACH

The given system consists of three plates thereby having three capacitance values pairwise between P_1P_2, P_2P_3 and P_3P_1. The capacitance of the above system depends on the configuration of the metal plates and independent of the voltage.

Now, $W = \frac{1}{2}C_{12}V_{12}^2 + \frac{1}{2}C_{23}V_{23}^2 + \frac{1}{2}C_{31}V_{31}^2$ where, W is the energy stored per unit width and C_{12}, C_{23}, C_{31} are pairwise capacitance per unit width between the three plates.

For three values of V_3 three linear equations are formed with the three capacitances as unknowns. The capacitances are calculated by solving the three equations.

By repeating the above calculation three sets of capacitance values are calculated for three values of d_2.

RESULTS

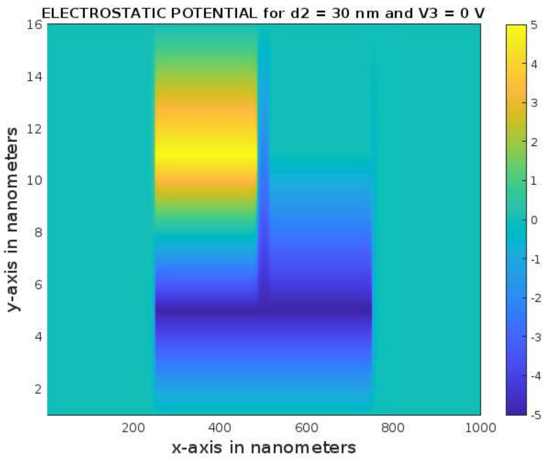
A. The capacitance values are given below.

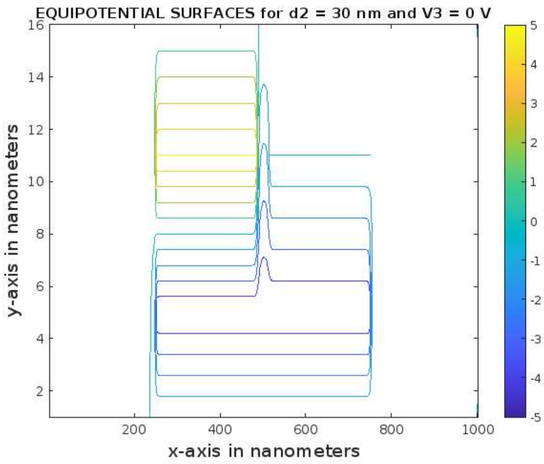
For d_2 = 30 nm: C_{12} = 0.5801 nF/m, C_{23} = 0.2254 nF/m and C_{31} = 0.5061 nF/m.

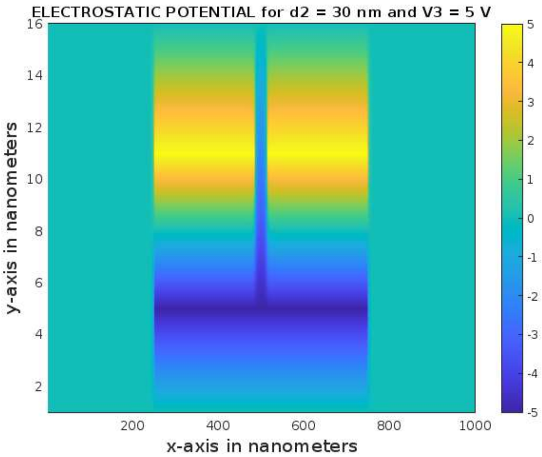
For d_2 = 60 nm: C_{12} = 0.5679 nF/m, C_{23} = 0.2113 nF/m and C_{31} = 0.4742 nF/m.

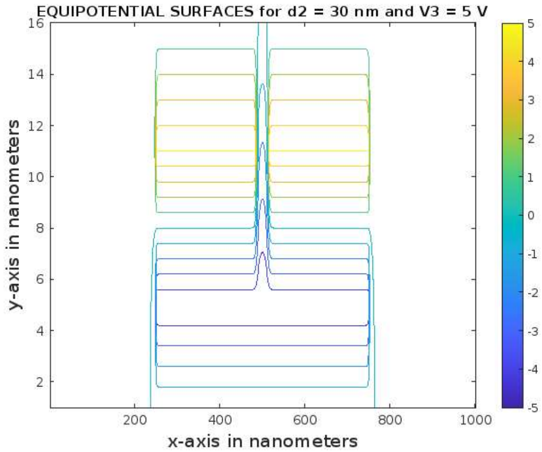
For d_2 = 90 nm: C_{12} = 0.5556 nF/m, C_{23} = 0.1972 nF/m and C_{31} = 0.4423 nF/m.

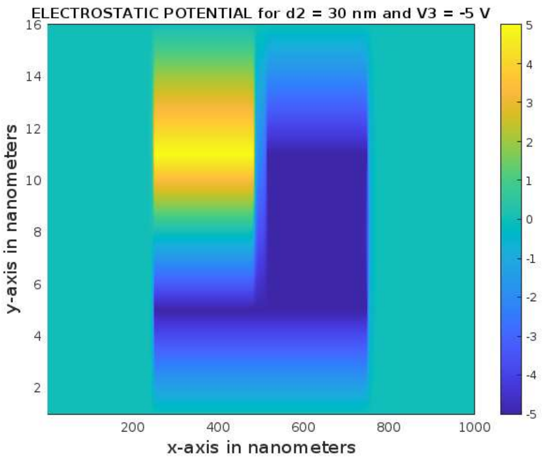
B. The plots of electrostatic potential and equipotential surfaces are given below.

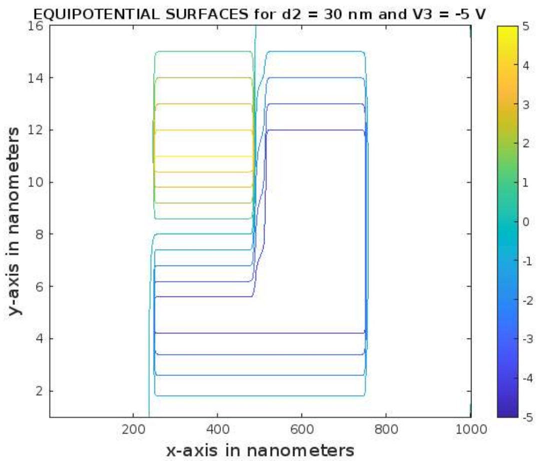


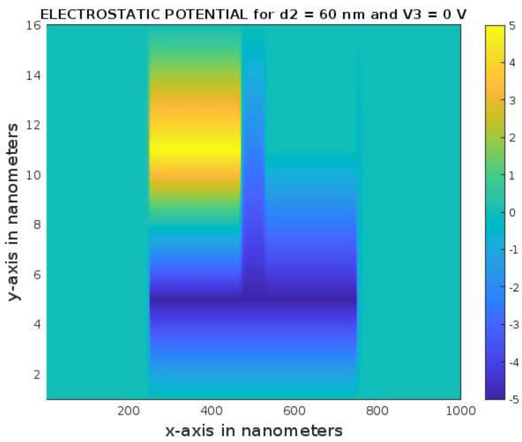


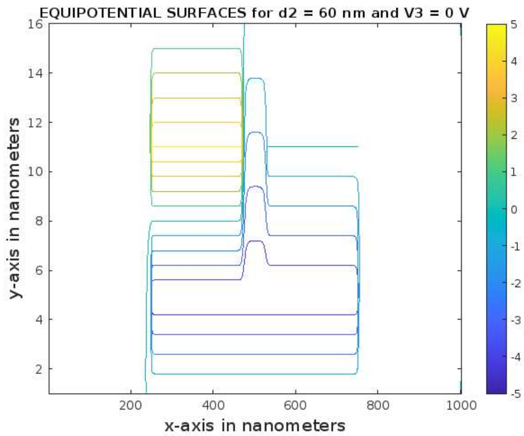


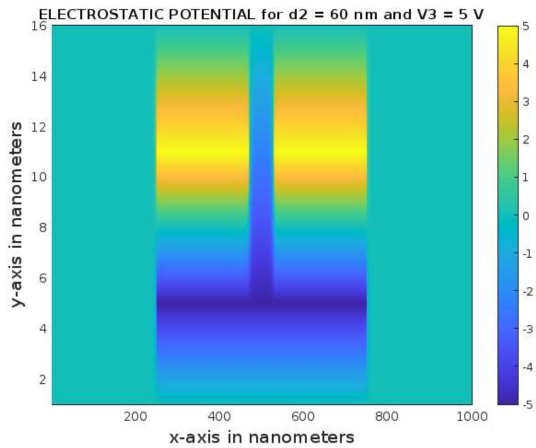


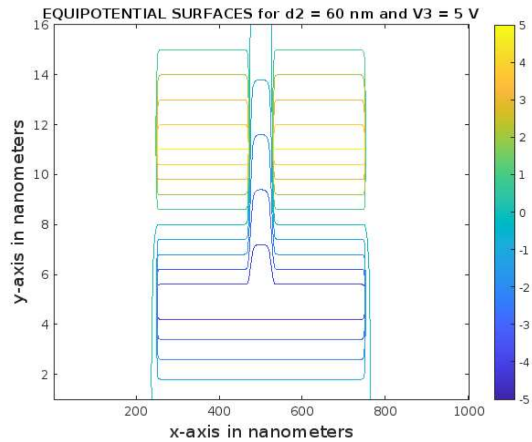


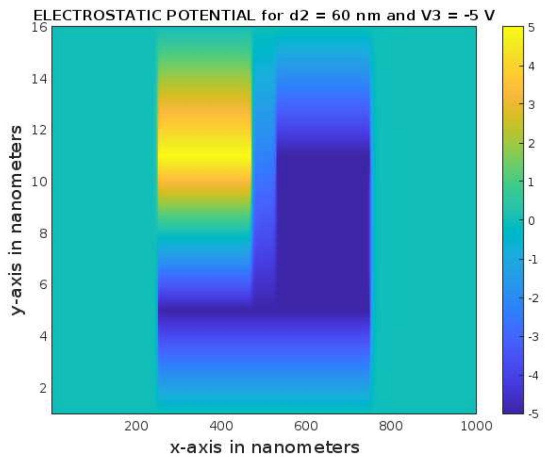


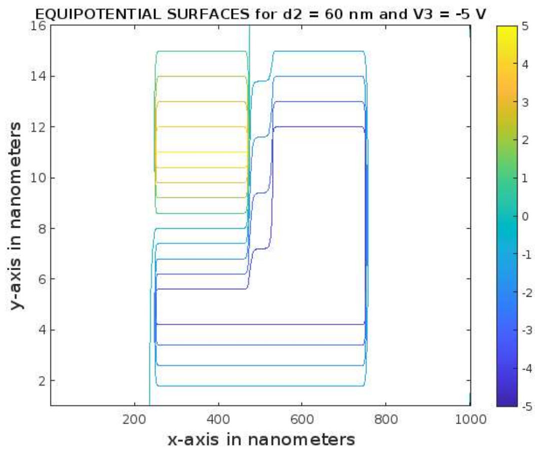

ELECTROSTATIC POTENTIAL for d2 = 60 nm and V3 = 0 V
y-axis in nanometers
x-axis in nanometers

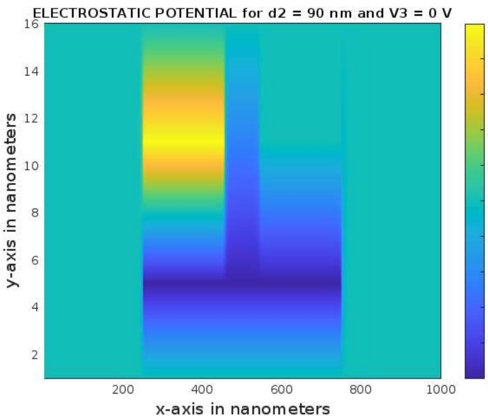

EQUIPOTENTIAL SURFACES for d2 = 60 nm and V3 = 0 V
y-axis in nanometers
x-axis in nanometers

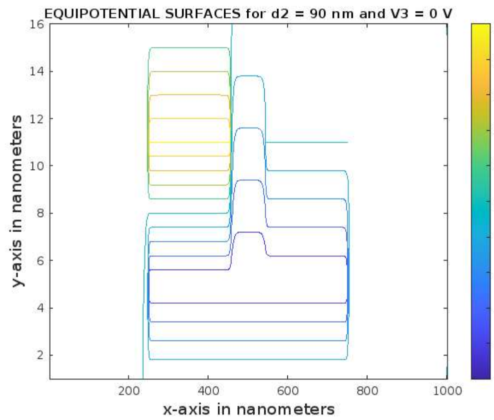

ELECTROSTATIC POTENTIAL for d2 = 60 nm and V3 = 5 V
y-axis in nanometers
x-axis in nanometers

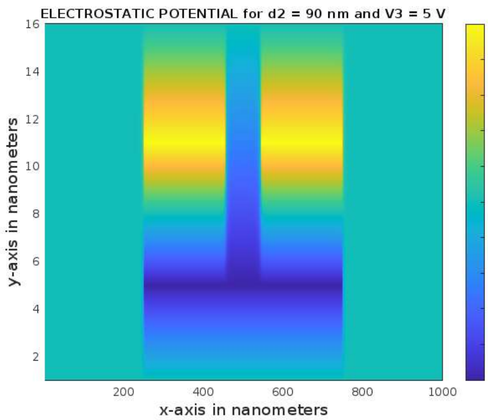

EQUIPOTENTIAL SURFACES for d2 = 60 nm and V3 = 5 V
y-axis in nanometers
x-axis in nanometers

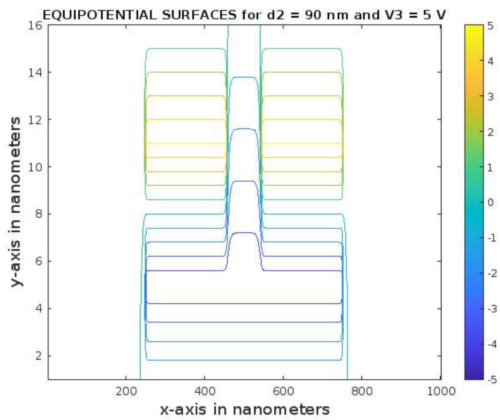

ELECTROSTATIC POTENTIAL for d2 = 60 nm and V3 = -5 V
y-axis in nanometers
x-axis in nanometers

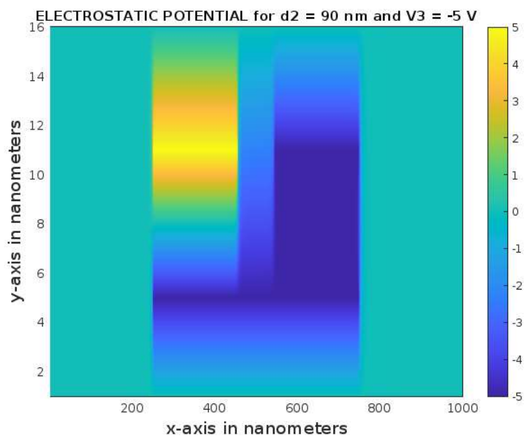

EQUIPOTENTIAL SURFACES for d2 = 60 nm and V3 = -5 V
y-axis in nanometers
x-axis in nanometers

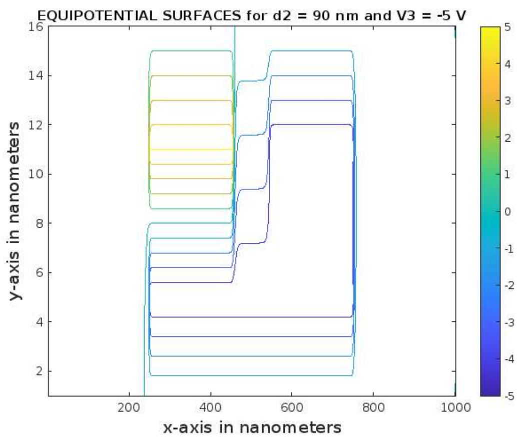

ELECTROSTATIC POTENTIAL for d2 = 90 nm and V3 = 0 V
y-axis in nanometers
x-axis in nanometers


EQUIPOTENTIAL SURFACES for d2 = 90 nm and V3 = 0 V
y-axis in nanometers
x-axis in nanometers

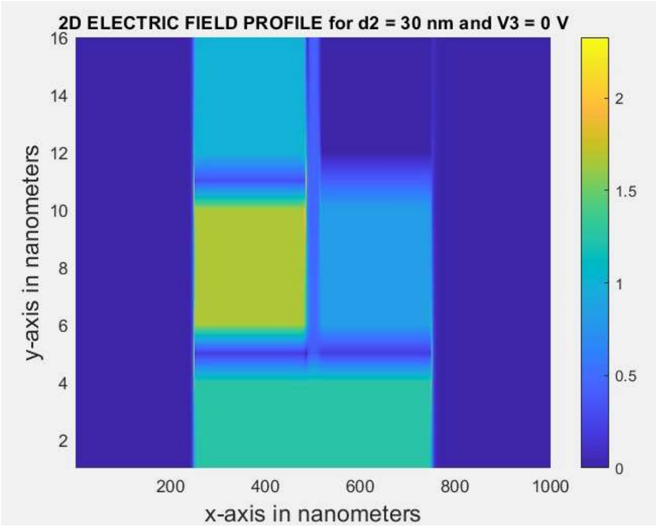

ELECTROSTATIC POTENTIAL for d2 = 90 nm and V3 = 5 V
y-axis in nanometers
x-axis in nanometers

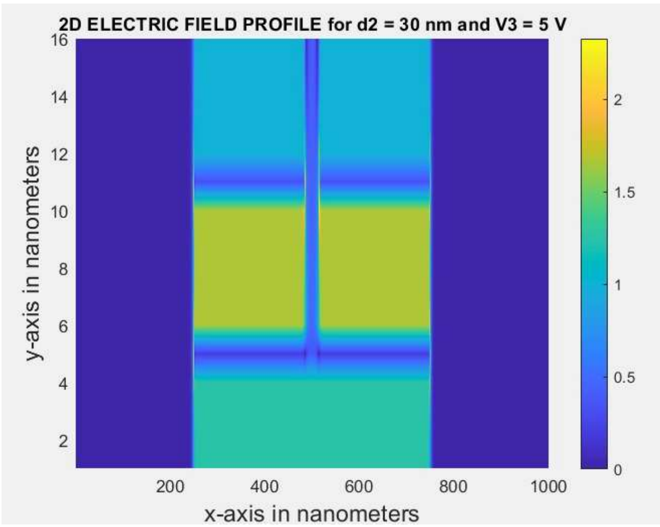

EQUIPOTENTIAL SURFACES for d2 = 90 nm and V3 = 5 V
y-axis in nanometers
x-axis in nanometers

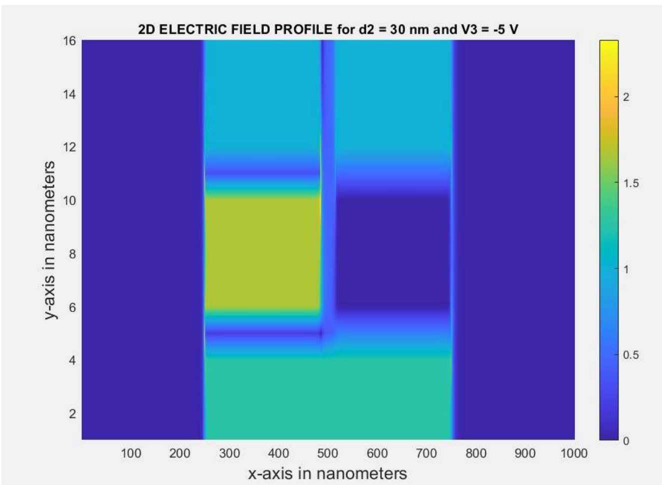

ELECTROSTATIC POTENTIAL for d2 = 90 nm and V3 = -5 V
y-axis in nanometers
x-axis in nanometers

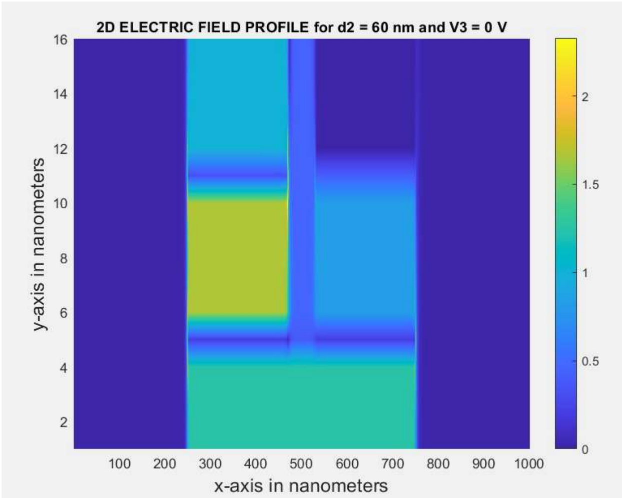

EQUIPOTENTIAL SURFACES for d2 = 90 nm and V3 = -5 V
y-axis in nanometers
x-axis in nanometers

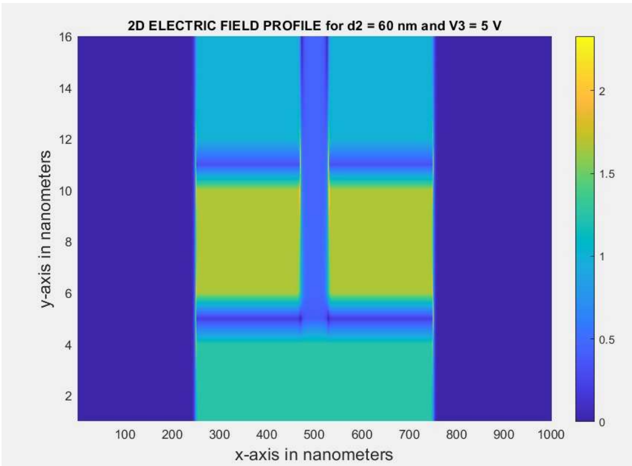
C. The plot of 2D electric field profile is given below.

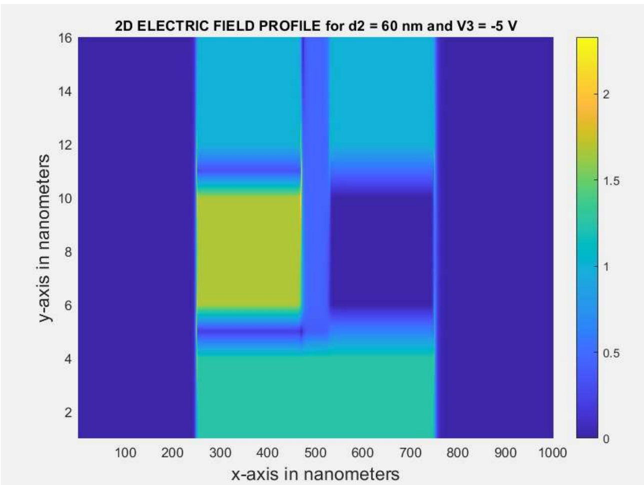


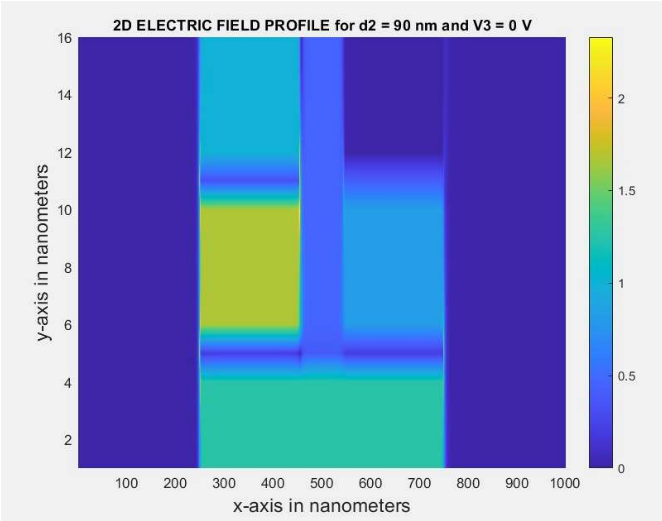


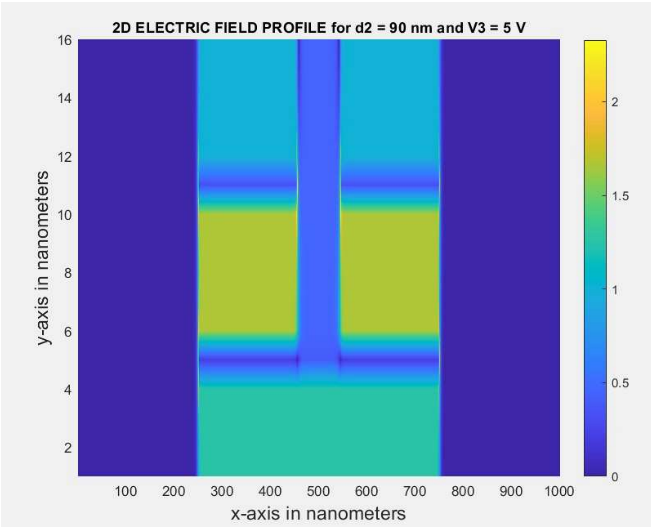


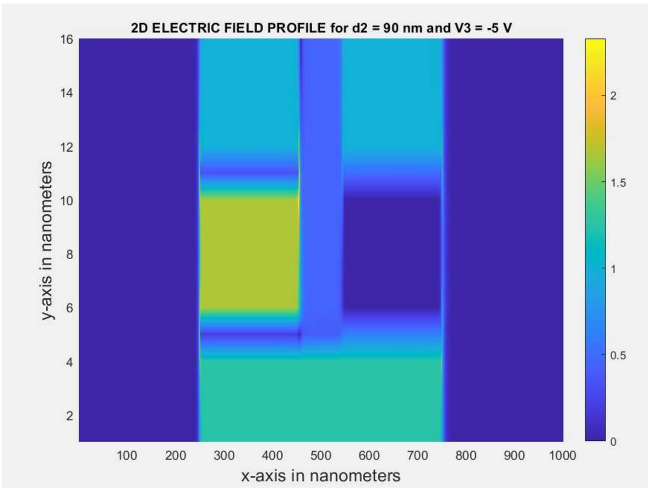

2D ELECTRIC FIELD PROFILE for d2 = 60 nm and V3 = 0 V
y-axis in nanometers
x-axis in nanometers
16
14
12
10
8
6
4
2
100 200 300 400 500 600 700 800 900 1000
2
1.5
1
0.5
0


2D ELECTRIC FIELD PROFILE for d2 = 60 nm and V3 = 5 V
y-axis in nanometers
x-axis in nanometers
16
14
12
10
8
6
4
2
100 200 300 400 500 600 700 800 900 1000
2
1.5
1
0.5
0


2D ELECTRIC FIELD PROFILE for d2 = 60 nm and V3 = -5 V
y-axis in nanometers
x-axis in nanometers
16
14
12
10
8
6
4
2
100 200 300 400 500 600 700 800 900 1000
2
1.5
1
0.5
0







The green(■) shaded region is the region with maximum electric field which lies primarily between the plates P_1 and P_2.

D. Since, $C_{12} = C_{31} = \epsilon\frac{(L - \frac{d_2}{2})}{d_1}$ where $L_2 = L_3 = L$ and $C_{23} = \epsilon\frac{t}{d_2}$ as the distance d_2 increases the capacitances C_{12} and C_{31} decreases as the effective length of the plates P_2 and P_3 decreases and in case of capacitance C_{23} it decreases because the distance d_2 between the plates P_2 and P_3 increases.

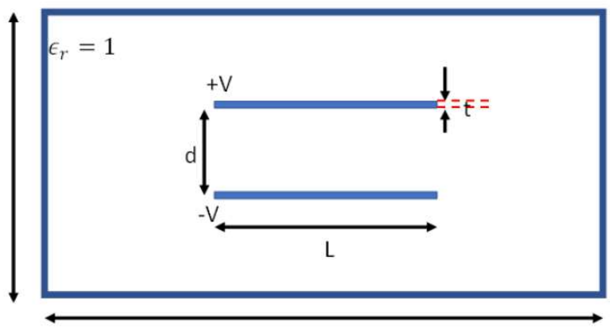
PROBLEM 3

QUESTION

For the Figure shown:

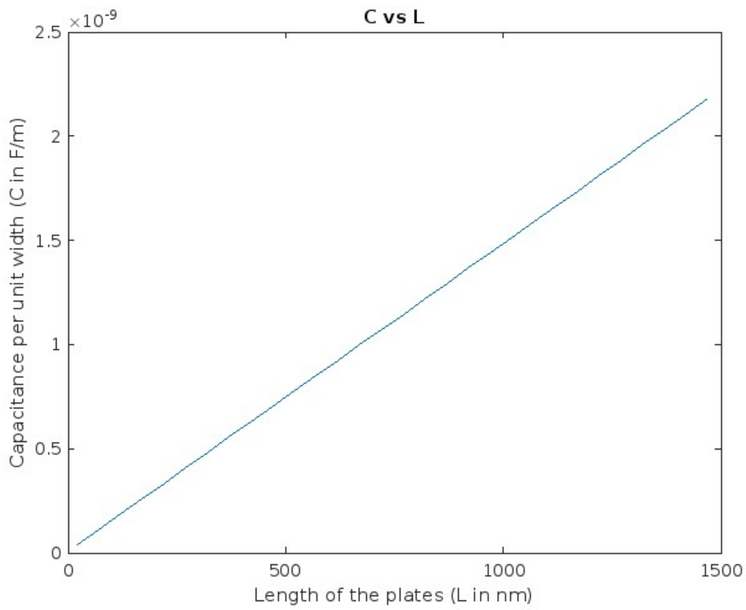
A. Vary L from 20 nm to 1500 nm in steps of 50 nm. Plot C as a function of L.

B. Calculate the parasitic capacitance $C_p(L) = C(L) - C_{th}(L)$ and plot it as a function of L. Qualitatively explain the nature of the plot.

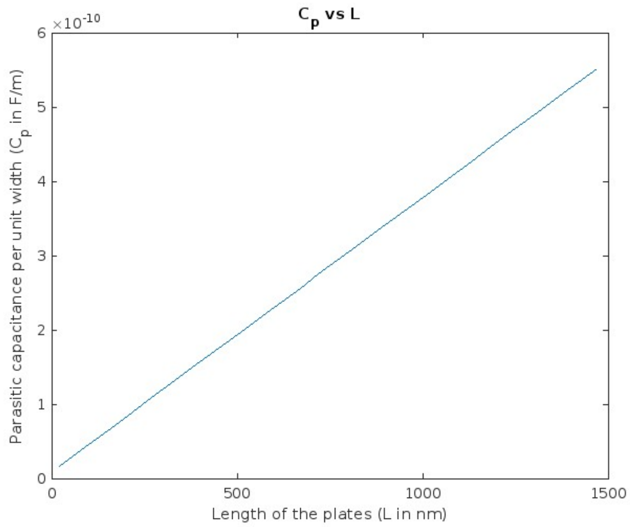


RESULTS

A. The plot of C as a function of L is almost linear as given below.



B. The value of parasitic capacitance increases linearly as length of the plate increases as shown below.



The ratio of parasitic capacitance to capacitance decreases as the length of the plates is increased. So, the fringing effect decreases with increase in length of plates.