TERRAIN AIDED NAVIGATION USING MAXIMUM A POSTERIORI ESTIMATION

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INTRODUCTION

Terrain aided navigation (TAN) is a technique to estimate the position of a moving vehicle by comparing the measured terrain profile under the vehicle to a stored elevation map. TAN has been operational for unmanned vehicles for some time. Although this operational system has proven to be reliable and cost effective, it is desirable to develop enhancements which can either reduce the pre-planning effort or increase the operational envelope, i.e., reliable operation in terrain with less relief or with stored elevation data with larger errors. It is anticipated that terrain aided navigation will be in use for many years to come due to the long term stability of the terrain profile of earth, the relative ease of mapping and maintaining maps of large operational areas, the ease and reliability with which on-board measurements can be made and the relatively low computational burden of computing navigation updates in an embedded vehicle processor.

In this paper the theoretical development of the maximum a posteriori (MAP) estimation technique is presented along with the application of the algorithm to TAN. Simulation results of the MAP algorithm are presented. In addition, the performance of the MAP algorithm is quantitatively compared to that of a mean absolute difference (MAD) batch algorithm and an extended Kalman filter (EKF) recursive algorithm.

BACKGROUND

A number of TAN techniques have been developed and tested. These fall into two general algorithmic categories: batch and recursive. In addition, there are two general map storage techniques: small, high fidelity maps which are used at specific points along the intended route of the vehicle; and a single, large, low fidelity map which encompasses the entire operating area of the vehicle. These techniques are shown in Figure 1 and are associated with the two most widely understood TAN implementations: Terrain Contour Matching (TERCOM) (Reference 1) and Sandia Inertial Terrain Aided Navigation (SITAN) (Reference 2).

The most widely known form of TAN is TERCOM. With TERCOM a strip of terrain elevation measurements are collected while the vehicle flies along the intended route and the measurements are post processed by a batch algorithm to provide a

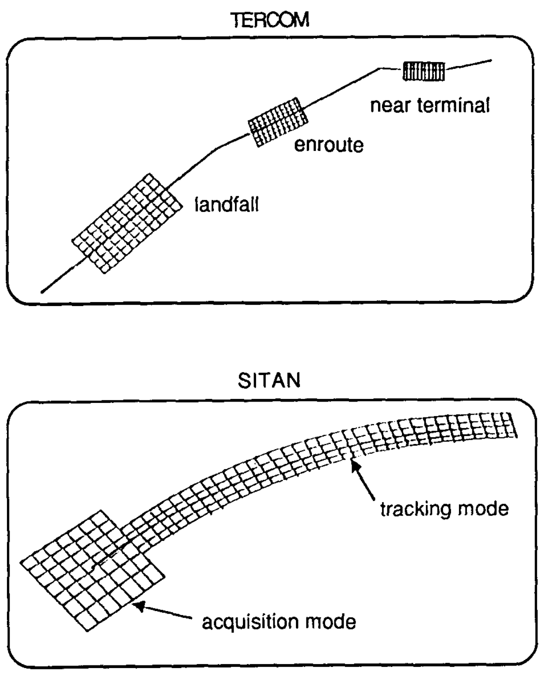

Figure 1. Terrain Aided Navigation Concepts

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correlation with a high fidelity map. In the operational missile systems employing TERCOM the stored map preparation and validation process includes extensive analysis to evaluate the probability of obtaining a strong and unambiguous correlation with candidate maps. The map size in the crosstrack direction is determined by the "worst case" navigation uncertainty and in the downtrack direction by the larger of "worst case" navigation uncertainty or the map length necessary to provide an unambiguous update opportunity. A sequence of maps are then developed to provide navigation update opportunities from the launch point to the target. The operational TERCOM applications use a mean absolute difference (MAD) algorithm which is only a modest computational requirement in an embedded flight processor. In addition, the map storage requirements are minimized by carefully selecting the minimum number and size of maps required for each mission.

In the late 1970's TAN in the form of SITAN was proposed. SITAN uses an extended Kalman filter (EKF) and a local terrain linearization technique to implement a recursive algorithm. This algorithm operates on individual terrain elevation measurements as they become available and for the entire duration of the mission. This requires a map for the entire mission. For missile applications the map could be for the length of the mission with the width determined by navigation uncertainty and terrain uniqueness or suitability. However, for manned aircraft applications map data for the entire operating area must be stored because the pilot may deviate from the preplanned route at any time. SITAN has been developed and evaluated for the manned aircraft application using Digital Terrain Elevation Data (DTED) which is a low fidelity Defense Mapping Agency (DMA) product readily available in most operational areas.

Other TAN techniques including TERPROM and SPARTAN have been developed and evaluated; however, only the fundamental concepts of the TERCOM and SITAN techniques will be discussed and compared in this paper.

In the late 1950's and throughout the 1960's when TAN concepts were originally developed and in the 1970's when TAN concepts were applied to missile applications digital computer capabilities were limited. In particular, the memory required for the storage of large maps, the throughput required to implement computationally intensive algorithms and the data links for embedded flight computers were constraints that strongly influenced the development of TAN. Within the past few years the computational, data storage and memory access capabilities of embedded vehicle computers have improved dramatically. Thus, the previously assumed computational constraints do not apply as techniques are developed to enhance the performance and to expand the operational envelope of TAN techniques.

TECHNICAL APPROACH

The TERCOM and SITAN approaches both have attributes that are of interest. Although enhancements can be envisioned in a number of areas the approach here is to investigate algorithm techniques which would make more complete use of the information content of the stored elevation data, the a priori knowledge of the errors in the stored elevation data and the elevation measurements. A batch processing algorithm was selected because: no linearization of the terrain profile is necessary; it does not require an acquisition process; and the algorithm techniques are applicable to both small discrete maps and large maps which can support continuous navigation updating.

The following inputs are assumed to be available to the TAN algorithm:

- o Vehicle position (pn_i, pe_i, pv_i) from an inertial navigation system (INS),
- o Ground clearance measured by a radar altimeter (h_i),
- o Stored elevation map data near the flight path (elevation map).

The objective of the proposed TAN algorithm is to compute the best estimate of the INS position error based on the maximum a posteriori (MAP) estimation technique (Reference 3).* The maximum a posteriori estimate of the INS position error, x_{MAP}, is defined as:

$$\left. \frac{\partial P(x|z)}{\partial x} \right|_{x = x_{MAP}} = 0$$

where x is the INS position error, z represents all the input measurements including pn_i, pe_i, pv_i, h_i, and elevation map, and $P(x|z)$ is the conditional probability density function. x_{MAP} can also be obtained by maximizing the natural logarithm of $P(x|z)$ with respect to x:

* With gaussian a priori statistics, the MAP estimate is equivalent to a least squares fit when the information matrix is used as the weighting matrix.

$$\left.\frac{\partial \ln P(x|z)}{\partial x}\right|_{x = x_{MAP}} = 0$$

Using Bayes' rule it follows that

$$P(x|z) = \frac{P(z|x)P(x)}{P(z)}$$

and

$$\ln P(x|z) = \ln P(z|x) + \ln P(x) - \ln P(z)$$

Observing that P(z) is not a function of x and if the initial position uncertainty is unknown, then the resulting estimate is the maximum likelihood (ML) estimate:*

$$\left.\frac{\partial \ln P(z|x)}{\partial x}\right|_{x = x_{ML}} = 0$$

In other words, $x_{MAP} = x_{ML}$ if the INS initial position uncertainty is unknown. If the INS a priori position error is x_0 with covariance P_0 and the ML estimate of the INS position error is x_{ML} with covariance R, then MAP estimate can be computed as:

$$x_{MAP} = x_0 + P_0H^T(HP_0H^T + R)^{-1}(x_{ML} - x_0)$$

where H is the identity matrix.

ALGORITHM DEVELOPMENT

The application of MAP estimation to the TAN function is presented in Figure 2. After initialization, the first task if to determine whether the likelihood computation is such that it covers at least three times the predicted INS error. When the necessary sequence of measurements has been collected, the likelihood computation is performed. The resulting likelihoods are fitted with a quadratic function around the located maximum. Based on the estimated quadratic function, the position error and its uncertainty are calculated. If the uncertainty exceeds a threshold, then a second sequence of measurements are collected and processed. The likelihood results of the second sequence are added to the likelihoods of the first sequence. This process is repeated until the uncertainty is lower than the threshold, then the position error and uncertainty results are processed by the Kalman filter to obtain the MAP estimate.

* P(z|x) is commonly referred as the likelihood function.

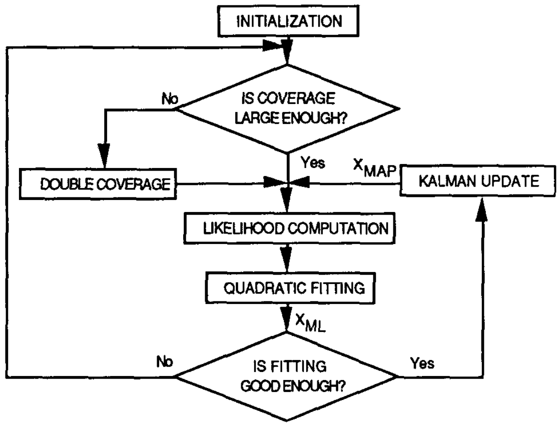

Figure 2. Terrain Aided Navigation Algorithm

SIMULATION DESCRIPTION

Terrain correlation is fundamentally a nonlinear problem. The use of conventional covariance analysis techniques would not necessarily reveal problems peculiar to a nonlinear system such as divergence. Therefore, a direct simulation was developed to evaluate candidate TAN algorithms as shown in Figure 3. This simulation was then exercised in a Monte Carlo manner to obtain statistical performance results. Although the simulation of the altimeter and the INS is relatively straightforward, the generation of the truth terrain and the error characteristics of the on-board stored map are not trivial. The roughness of the truth terrain and the error characteristics of the on-board stored map must reflect realistic operational data. The extent to which the following models are realistic are unknown to the authors. It is believed

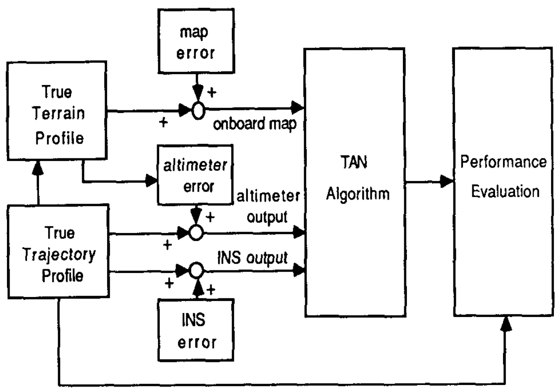

Figure 3. TAN Direct Simulation

that the truth terrain profile is realistic; however, the errors in the creation of map data such as DTED are known in magnitude but unknown in statistical distribution and spacial correlation characteristics. Thus, the following models are presented only for the purposes of evaluating candidate algorithms.

A frequency domain technique used to generate the synthetic terrain map and its associated error map is described in Figure 4. It is assumed that the sample terrain is represented by a two-dimensional third-order markov model (Reference 4) along with the corresponding power spectral density function:

$$S(i,j) = \frac{16\beta^5\sigma^2}{3\left(\omega_i^2 + \omega_j^2 + \beta^2\right)^3}$$

where σ^2 is the variance and β is 2.903/D where D is the correlation distance. Accordingly, the terrain map error is represented by a second-order markov model. The simulated terrain profile and the mapping errors are shown in Figure 5 and Figure 6 respectively.

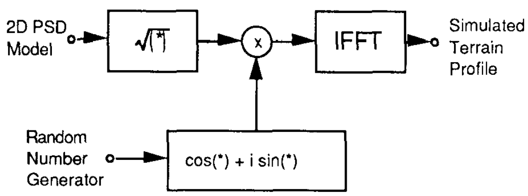

Figure 4. Synthetic Terrain Map Generation

SIMULATION RESULTS

The simulation described above was run in a Monte Carlo manner to evaluate three different TAN algorithms: the MAP estimation algorithm; a MAD algorithm representative of the current operational TERCOM systems; and a recursive EKF algorithm representative of a SITAN approach. 225 trials each consisting of 64 measurement points were used for each experiment. For MAP and MAD algorithm evaluation, the initial INS north and east position errors for each trial were varied between -700m to 700m uniformly. For EKF algorithm evaluation, the initial INS position errors were assumed to be zero due to the divergence problem. The simulated sensor error models were:

Radar altimeter noise:	2m	
INS velocity error:	0.5m/s	level
	0.1m/s	vertical
Baro altimeter bias:	50m	

The radar altimeter noise was varied from measurement to measurement while the INS velocity error and the baro altimeter bias were varied from trial to trial. The terrain profile is 100m one-sigma with a 2.5km correlation distance. The mapping error shown is 15m one-sigma with a 2.5km correlation distance. The terrain profile was held constant and the mapping error was varied in magnitude as shown in the following results.

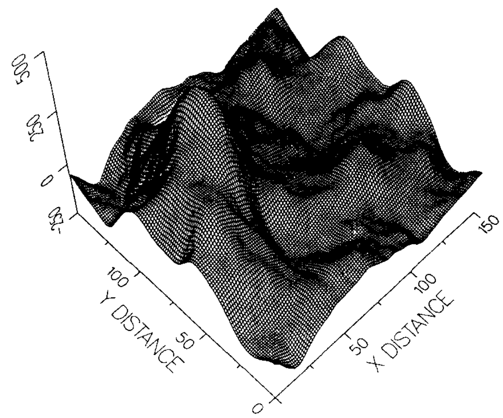

Figure 5. Synthetic Terrain Profile

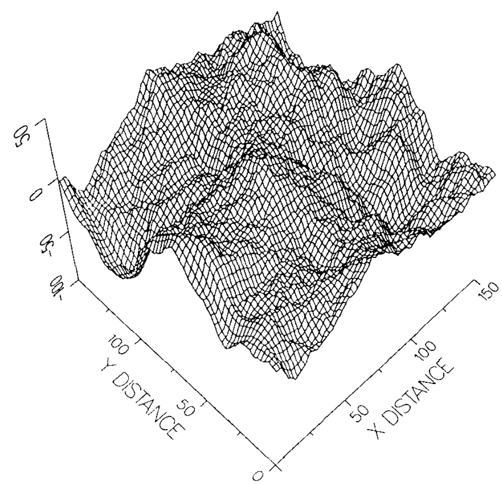

Figure 6. Mapping Error

The radial position error performance for 50% and 100% map error cases are presented in Figure 7 and Figure 8, respectively. The categories 1 through 9 represent radial position error of 0-25m, 25-50m, 50-75m, 75-100m, 100-125m, 125-150m, 150-175m, 175-200m, and over 200m, respectively. These data are presented as histograms show that the MAP estimator algorithm is significantly better than the other two algorithms. For example, for the 50% *map error case* the MAP algorithm has 174 of the 225 trials with a radial position error less than 50m whereas the recursive algorithm has 78 and the MAD algorithm has 65. Although the recursive algorithm seems to perform better than the MAD algorithm, it is worth noting that the initial position error for the recursive algorithm was assumed to be zero making this comparison unrealistic. As the map error is increased the MAD algorithm tends to produce significantly more trials with over 200m radial position error.

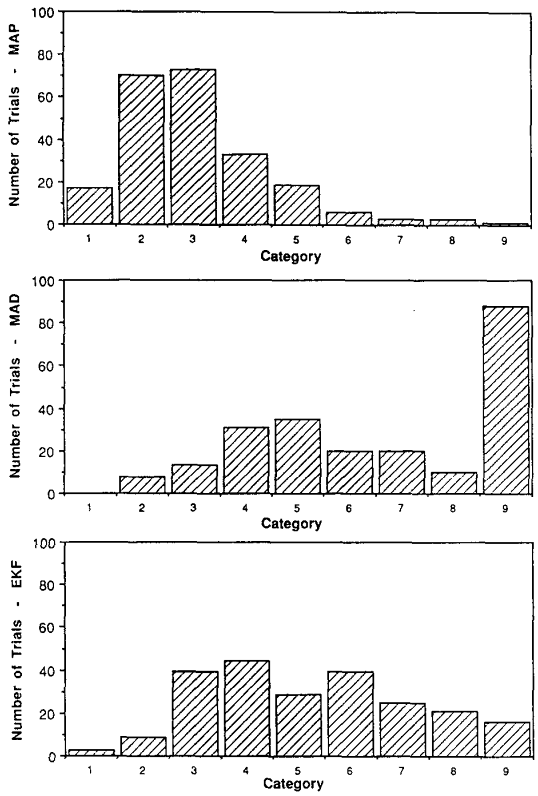

Figure 7. Radial Position Accuracy with 100% Map Error

CONCLUSIONS

In this paper the theoretical development of the maximum a posteriori estimation technique and its application to a terrain aided navigation algorithm were presented. This technique makes more complete use of the information content of the stored elevation data, the a priori knowledge of the errors in the stored elevation data and the elevation measurements. The simulation results presented

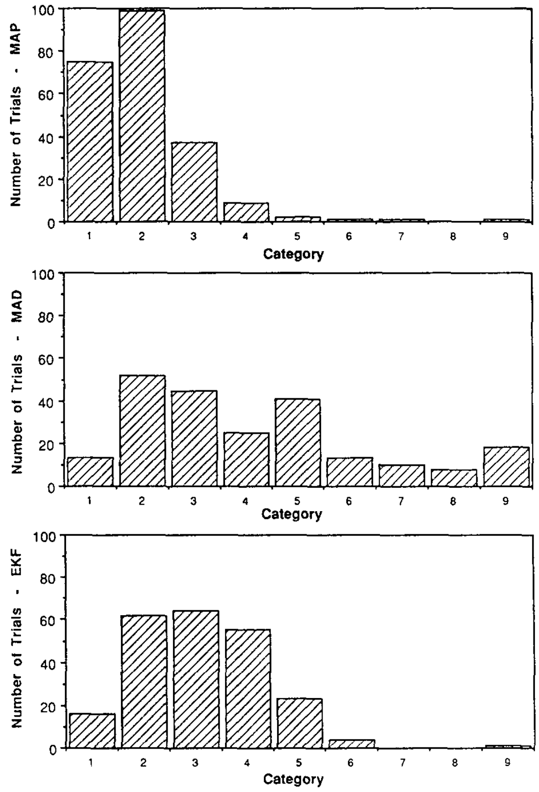

Figure 8. Radial Position Accuracy with 50% Map Error

show that this technique provides a significant performance improvement relative to both a mean absolute difference batch algorithm and an extended Kalman filter recursive algorithm. Although these results are gratifying, further research is needed to study the sensitivitites of mapping errors of operational systems. In addition, this algorithm has been flight tested in a 1750A embedded processor to demonstrate the throughput compatibility.

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