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Influence of Application Conditions on Terrain-Aided Navigation*

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***Abstract* - Although the matching rules of terrain-aided navigation (TAN) have always been focused on since TAN was put forward, the application adaptability of matching algorithms are also paid attention to with the application of TAN now because the relationship of application conditions with matching precision is so important for matching algorithms that it will decide TAN's efficiency or feasibility directly. In this paper, underwater TAN is studied and terrain contour matching (TERCOM) is applied as the matching algorithm. The effect of terrain characteristics, map resolution, initial INS error, INS precision, vehicle speed, path length, and sonar precision on matching precision is studied respectively by simulation. Some results, which are helpful for the promotion and application of TERCOM, are concluded finally.**

Index Terms - TERCOM; TAN; Navigation; Map Matching; INS.

I. INTRODUCTION

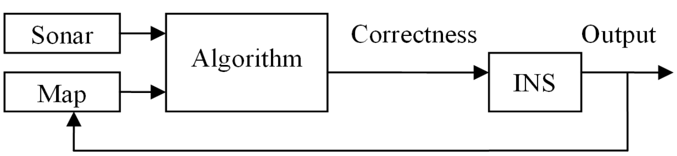

Fig.1. The Scheme of Underwater Terrain-Aided Navigation System

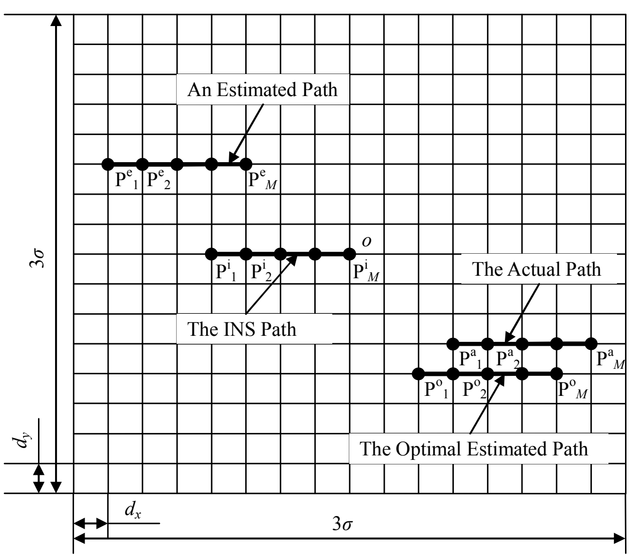

Fig.2 The Sketch of a Matching Process

To follow the designed trajectory accurately, vehicle, such as submarine, needs to acquire its current position continuously or at a frequency. Inertial navigation system (INS), totally autonomous equipment, is a well-known and widely used sensor unit to measure vehicle's real-time velocity and position without additional outside information, but its error is accumulated with running time so that it cannot work alone for a long time. Therefore, other navigation systems, such as global navigation satellite system (GNSS), and/or terrain-aided navigation (TAN), have been put forward to reducing INS accumulated error, but GNSS is easy to be jammed and is difficult to be used for underwater vehicle especially. However, TAN is fully autonomous [1]. Shown in Fig.1 is the scheme of a typical underwater TAN which is composed of INS, sonar, bathymetry map, and matching algorithm. Matching algorithm has always been the focus of TAN since TAN was put forward and many matching algorithms have been proposed and studied. However, the present research on matching algorithms is mainly focused on matching rules while matching algorithm's application adaptability, which will decide matching algorithm's effectiveness or even feasibility, is paid less attention to.

Shown in Fig.2 is a typical matching process. When vehicle sails along an actual path $\{\mathrm{P}_i'\}$ ($i = 1, 2, \cdots, M$) (M is matching path length.) which we don't know in sailing, sonar will sample depth of water at a preset frequency and provide a sequence of water depths $\{h_{\mathrm{sonar}}(i)\}$ and INS will provide a path $\{\mathrm{P}_i\}$ with its accumulated error for matching algorithm. Firstly, a matching region with the central of the current INS position is decided according to the estimated INS error σ. Then, $\{h_{\mathrm{map}}(i)\}$, the water depths of all the possible paths with the same shape with the INS path in the region are acquired from the bathymetry map and compared with $\{h_{\mathrm{sonar}}(i)\}$. If

$$\begin{cases} f(\{h_{\mathrm{map}}(i)\}, \{h_{\mathrm{sonar}}(i)\}) < \varepsilon \\ f(\{h_{\mathrm{map}}(i)\}, \{h_{\mathrm{sonar}}(i)\}) \to \min \end{cases},$$

$\{\mathrm{P}_i''\}$ are decided as the optimal estimated path and $\{h_{\mathrm{map}}(i)\}$, the corresponding water depths, is assigned to $\{h_{\mathrm{optimal}}(i)\}$, in which $f(\bullet)$ is a matching rule and ε is a threshold value. Otherwise, the matching region will be enlarged and matching will be performed again until $\{\mathrm{P}_i''\}$ are found. If $\{\mathrm{P}_i''\}$ can not

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be found in the large enough region, $\{P_i\}$ are assigned to $\{P_i''\}$. A matching loop is accomplished and the next loop will begin until vehicle sails out of the map.

Usually, there is difference between $\{P_i'\}$ and $\{P_i''\}$, matching error, which is the function of application conditions, such as terrain characteristics, INS precision, initial error, sonar error, path length, map resolution, and etc. If the relationship of the matching error with the application conditions is acquired qualitatively or quantitatively, it is the exact guideline for matching algorithm's application, such as mapping, sensors selection, route plan, and etc. Therefore, the relationship's importance is self-evidence.

In this paper, underwater TAN with Terrain Contour Matching (TERCOM) is researched. The effect of terrain characteristics, map resolution, initial INS error, INS precision, vehicle speed, path length, and sonar precision on matching error is studied respectively by simulation. The remainder of the paper is organized as follows. The principle of TERCOM is summarized briefly firstly in Section 2. Then, the relationship of matching error with application conditions is studied by simulation in Section 3. Finally, some results are concluded in Section 4.

II. The Principle of TERCOM

A. *Figures and Tables*

TERCOM is one of the most popular matching algorithms and has been applied successful in practice although many efforts are still being made to improve its performance. In the following, the principle of TERCOM [1] is briefly reviewed. The commonly applied matching rules of TERCOM are

$$J_{\mathrm{CC}}(x,y)=\frac{1}{M}\sum_{i=1}^{M}h_{\mathrm{map}}(i)h_{\mathrm{sonar}}(i) \quad (1)$$

$$J_{\mathrm{MAD}}(x,y)=\frac{1}{M}\sum_{i=1}^{M}\left|h_{\mathrm{map}}(i)-h_{\mathrm{sonar}}(i)-(\bar{h}_{\mathrm{map}}-\bar{h}_{\mathrm{sonar}})\right| \quad (2)$$

$$J_{\mathrm{MSD}}(x,y)=\frac{1}{M}\sum_{i=1}^{M}[h_{\mathrm{map}}(i)-h_{\mathrm{sonar}}(i)-(\bar{h}_{\mathrm{map}}-\bar{h}_{\mathrm{sonar}})]^2 \quad (3)$$

in which $J_{\mathrm{CC}}(x,y)$, $J_{\mathrm{MAD}}(x,y)$, and $J_{\mathrm{MSD}}(x,y)$ are the values of cross correlation (CC), mean absolute difference (MAD), and mean square difference (MSD) at the matching point (x,y) respectively, $\bar{h}_{\mathrm{map}}$ and $\bar{h}_{\mathrm{sonar}}$ the mean depths of the estimated and sonar measured paths respectively. It has been proven that the correct matching makes the MAD and MSD values minimum but it unnecessarily makes the CC value maximum at the same time. It is also found that the MAD and MSD rules are equivalent while the CC rule may induce mismatching to some extent [1]. Therefore, the MAD and MSD rules are used in this paper.

The matching process is briefly summarized as follows. Firstly, a matching region with the center of the current INS position is decided according to the estimated INS error σ. Then, the water depths of all possible optimal estimated paths with the same shape with the INS path in the region are acquired from the bathymetry map and the MAD and MSD values are calculated by the equations (2) and (3) respectively. Finally, the minimum MAD and MSD value matching paths are found and if the two paths are the same, the path in map is the optimal estimated. Otherwise, the mean of the two paths is decided as the optimal estimated. Try to position figures and tables at the tops and bottoms of columns and avoid placing them in the middle of columns.

III. Simulations

A. *The Main Simulation Conditions*

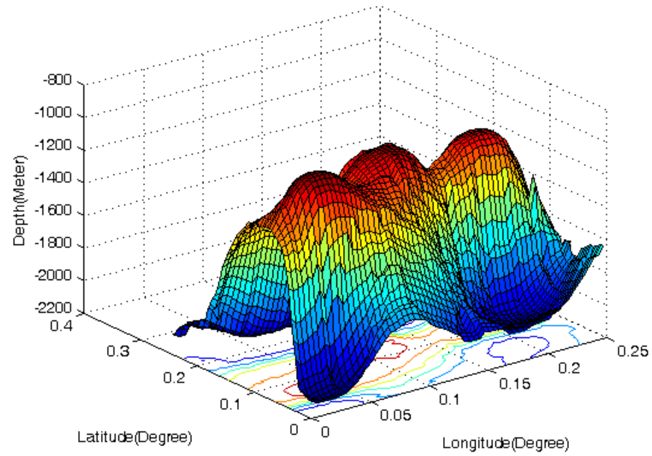


Fig.3 The 3-D Image of the Map (Map1)

If there are no extra comments, the simulation conditions are as follows. The main simulation conditions and sonar error model are listed in Table 1 and Table 2. The 3-D image of a map is shown in Fig.3. The map is a 473×473 square grid and the grid distance (the distance between two adjacent points in the map) is 58m. Its origin is (0°, 0°). Vehicle sails from (0.03030°, 0.17169°) along longitude from west to east. The path length M is 10 points. In Table 1, g is gravity acceleration and 0.02° longitude and latitude errors are about 3.149km. In Table 2, h is water depth measured by sonar. In the following figures, running time refers to vehicle running duration.

B. *Matching in Different Maps*

A major shortcoming of any TAN algorithm is the dependence of its accuracy on the 'information content' of a map. Therefore, a lot of methods with different criterions (such as map's signal-to-noise ratio, entropy, terrain depth variation, correlation length, etc.) have been put forward for judging the information content of a map. Up to now, there are still some new methods being proposed, which also means that there is still a long way to establish an effective criterion for judging the information content. However, the criterion is not studied in this paper. Entropy and the 3-D image of a map are used to judge its information content. The effect of terrain characteristics on matching precision is qualitatively analyzed by simulation in 3 maps shown in Fig.3 (Map1) and Fig.4 (Map2 and Map3).

Table 1 The Main Simulation Conditions

Vehicle	Initial Longitude and Latitude Errors (°)	0.02
	Sample Step (s)	20
	Speed (m/s)	3
Gyroscope	X-axis Bias (°/h)	0.001
	Y-axis Bias (°/h)	0.001
	Z-axis Bias (°/h)	0.001
	X-axis Random Walk (°/h$^{1/2}$)	0.001
	Y-axis Random Walk (°/h$^{1/2}$)	0.001
	Z-axis Random Walk (°/h$^{1/2}$)	0.001
Accelerator	X-axis Bias (g)	1×10^{-6}
	Y-axis Bias (g)	1×10^{-6}
	Z-axis Bias (g)	1×10^{-6}
	X-axis Random Walk (g/s$^{1/2}$)	1×10^{-6}
	Y-axis Random Walk (g/s$^{1/2}$)	1×10^{-6}
	Z-axis Random Walk (g/s$^{1/2}$)	1×10^{-6}

Table 2 Sonar Error Model

Depth (m)	Error
0~10	0.1m
10~100	0.4%×*h*
100~300	0.6%×*h*
300~1000	0.8%×*h*
1000~8000	1%×*h*
>8000	Not Workable

The simulation origins are (0.03976°, 0.15160°) and (0.03030°, 0.16673°) in Map2 and Map3 respectively and other conditions are shown in Section 3.1. The entropy values of the three maps are 11.7401, 11.7492, and 11.7770 respectively and the entropy value is calculated as follows [9].

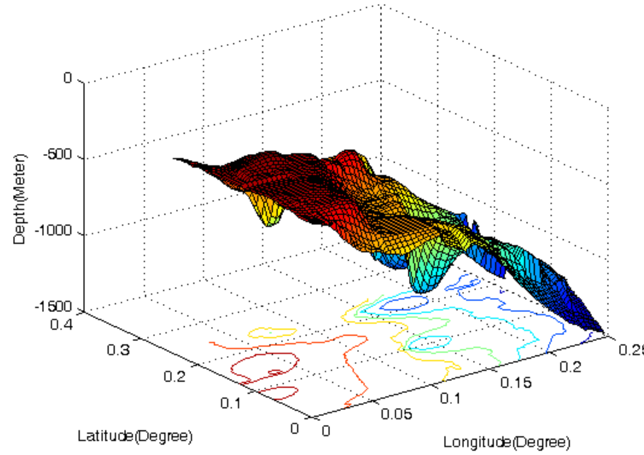
$$H = -K\sum_{i=1}^{N}\sum_{j=1}^{N} P_{ij}\ln P_{ij}\ , \tag{4}$$

in which $K = 1/\ln 2$, $P_{ij} = \dfrac{1\Big/\left(1+x_{ij}\right)}{\sum_{i=1}^{N}\sum_{j=1}^{N} 1\Big/\left(1+x_{ij}\right)}$, $x_{ij} = \dfrac{h_{ij} - \bar{h}}{h_{\max} - h_{\min}}$,

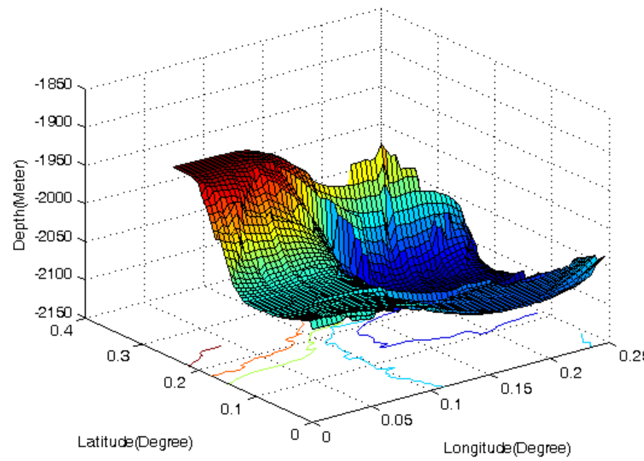
h_{ij} the water depth at a point in map, $\bar{h}$ the mean water depth of map, $h_{\max}$ and $h_{\min}$ the maximal and minimal water depths of map, and N=473.

Shown in Fig.5 are the matching results in the three maps, which shows that the matching error in Map1 is less than in Map2 and the matching in Map3 is divergent. The mean matching errors in Map1 and Map2 are 84.39m and 194.91m respectively. Due to the divergence in Map3, the matching result cannot be used to correct INS and the output of TAN is INS's.

The results prove that the matching precision in different maps is different. In the three maps, the entropy value of Map3 is the highest and the one of Map1 is the lowest. According to information theory, the higher entropy value means the less difference in the map and the less information content and verse vice. It can also be known from the 3-D images of the three maps that it is much flatter in Map3 than in Map1 and Map2. In Map2, there are some small local plat regions where TERCOM is easy to mismatch. Therefore, TERCOM is more suitable to be applied in the maps with lower entropy values. However, there are more than 20 criterions to judge the information content of a map and entropy is just one of them. Entropy alone is very difficult to judge maps comprehensively for all matching algorithms or even TERCOM alone. It is necessary to establish more effective criterion for a matching algorithm to judge maps accurately, which is not studied further in this paper.



(a)



(b)

Fig.4 The 3-D Image of the Map
(a) Map2; (b) Map3

C. *Matching with Different Initial INS Errors*

Shown in Fig.6 are the matching results with the initial longitude and latitude errors of 0.03°, 0.04°, and 0.05°. The simulation origin is changed to the point (0.06821°, 0.12117°) to ensure that the first matching region is located in Map1. The results show that with the enlarging of initial INS error, the matching error tends to increase too. When the initial errors are 0.03°, 0.04°, and 0.05°, the mean matching errors are 204.79m, 144.37m, and 2047.50m respectively. If the initial INS error is too large, the duration to converge increases and there are some times of serious mismatching in the matching process. The main reason is that with the increase of initial INS error, the matching region is enlarged accordingly, which induces the increased areas with similar terrain characteristics and the increased probability of mismatching correspondingly and is no good for the improvement of matching precision. However, even though the initial error is as large as 0.04°, TERCOM is convergent and the mean matching error is no more than 200m, which proves that TERCOM is capable of matching and converging rapidly with large initial INS error and high precision in a high-information-content-level map.

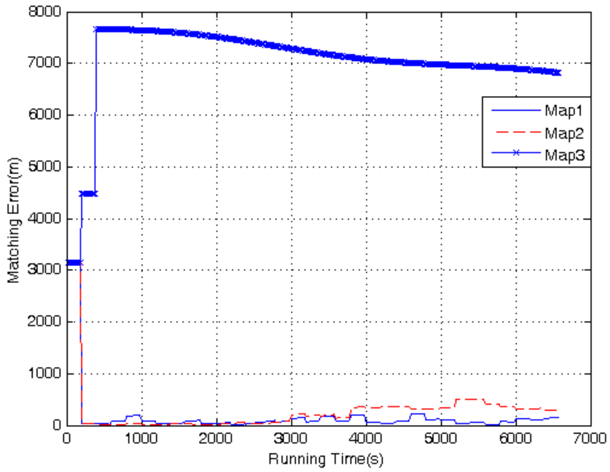


Fig.5 The Matching Results in the Three Maps

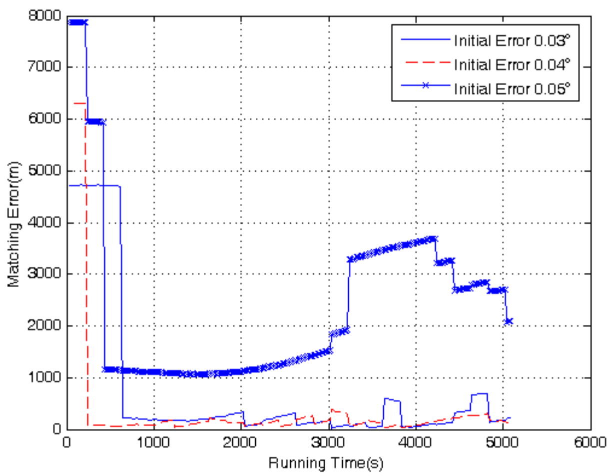


Fig.6 The Matching Results with Different Initial Errors

D. Matching with Different Map Resolutions

The matching results in Map1 with the resolutions of 116m, 223m, and 464m are shown in Fig.7 and the mean matching errors are 798.58m, 819.79m, and 1786.10m respectively. It is shown that with the decrease of map's resolution, the matching error increases sharply and the matching process is unstable heavily and the probability of mismatching increases too. The main reason is that with the decrease of map's resolution, the effective sample points of matching path decrease accordingly because only the water depth at point is exactly known in map and a sample point will be approximated to the nearer point if it is between two adjacent points in map. For example, the effective path length reduces from 10 points to 2 points when map's resolution reduces from 58m to 464m. In the simulation, sample step is 20s and vehicle speed is 3m/s i.e. sample length is 60m. Therefore, the effective path length is decreased with the decreasing of map's resolution and the uniqueness of matching path is reduced so that it is easier for TERCOM to mismatch. It is necessary to increase map's resolution for improving matching precision.

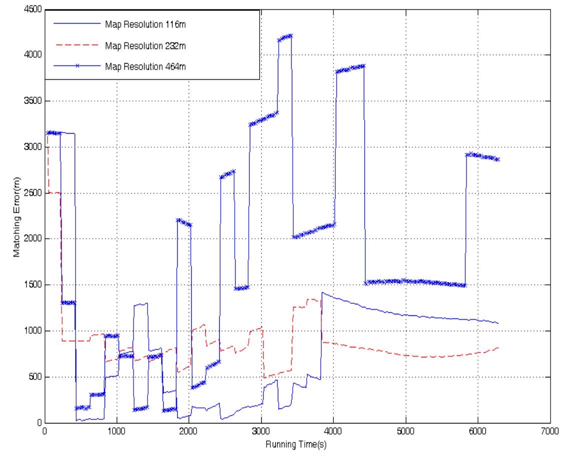


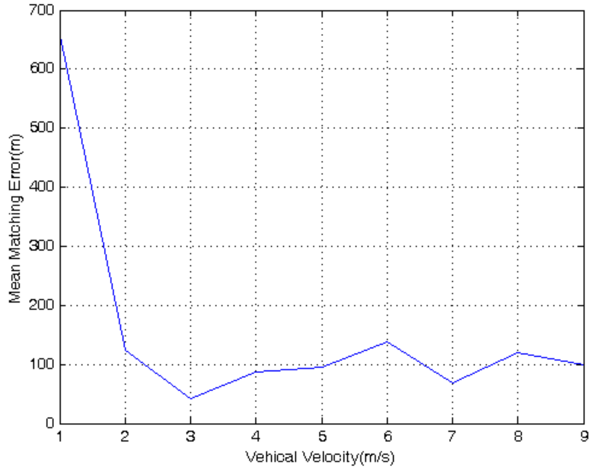
Fig.7 The Matching Results in Map1 with Different Resolutions

E. Matching with Different Vehicle Speeds

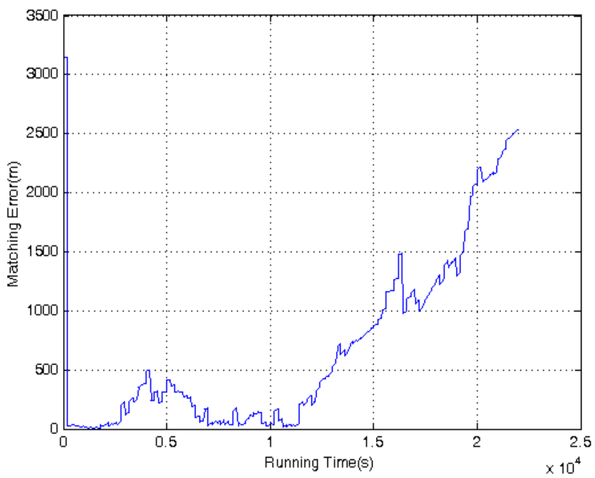
Shown in Fig.8 (a) are the matching results in Map1 with different vehicle speeds. If sample step is fixed, the slower the vehicle speed is, the shorter the sample length is. When sample length is shorter than grid distance, the effective path length will be reduced, which will reduce the uniqueness of matching path and TERCOM is easy to mismatch. Shown in Fig.8 (b) is the matching result in Map1 with the vehicle speed of 1m/s, which shows that TERCOM is divergent at the latter half of matching process. When sample length is close to or larger than grid distance, the effect of vehicle speed on matching error is not so remarkable. Especially, when sample length is close to grid distance, matching error is minimal. In the simulation, grid distance is 58m and matching error is minimal when sample length is 60m if vehicle speed is 3m/s. Therefore, it is necessary to set vehicle speed properly according to map's resolution.

F. Matching with Different Precision INSs

Shown in Fig.9 are the matching results in Map1 with different precision INSs which are listed in Table 3 where gyroscope and accelerator bias and random walk values of each INS are listed (i.e., each value of gyroscope and accelerator bias and random walk in Table 1 is substituted for the one in Table 3) . It is shown that with the decrease of INS precision, matching error increases and there are some times of mismatching. The effect of gyroscope precision on matching error is more significant because the shape of matching path copies the one of INS path which is mainly decided by gyroscope. If the difference between the shapes of INS and actual paths is too large, there will be significant mismatching. Therefore, it is necessary to improve INS precision, especially gyroscope precision, for improving matching precision.



(a)



(b)

Fig.8 The Matching Results in Map1
(a) Matching with Different Vehicle Speeds;
(b) Matching with the Vehicle Speed of 1m/s

G. *Matching with Different Path Lengths*

Shown in Fig.10 are the matching results in Map1 with different path lengths. It is shown that path length influences matching precision to some extent. The mean matching error is the lowest when path length is 2 or 3 points while it is the highest when path length is 12 to 15 points and it is much stable when path length is 6 to 10 points. The main reason is that the uniqueness of matching path in a map with a high information-content level is still remarkable even though path length is much short or moderate. However, the shape of INS path is much different from the one of actual path in the long process of accumulating matching points when path length is too long, which increases the probability of mismatching and decreases matching precision. Therefore, path length can be reduced in a map with a high information-content level to improve matching speed with much high matching precision. However, it is easy for TERCOM to mismatch with a short path length due to too little terrain information content of matching path, especially in a map with a low or moderate information-content level. Therefore, it is advisable to set a moderate path length.

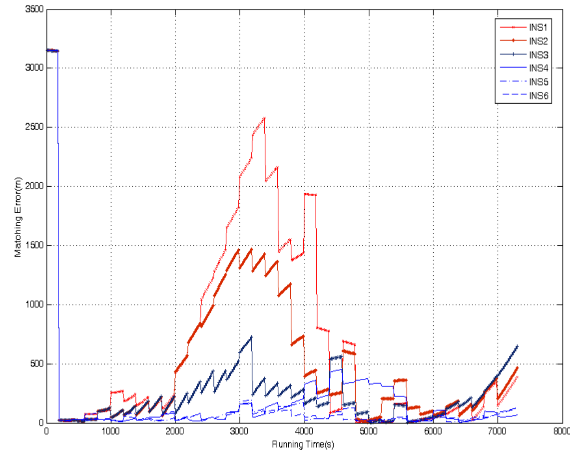


Fig.9 The Matching Results in Map1 with Different Precision INSs

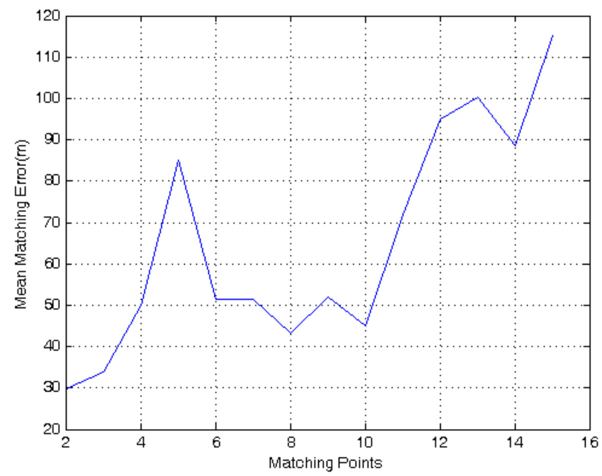


Fig.10 The Matching Results in Map1 with Different Path Lengths

H. *Matching with Different Sonar Errors*

Shown in Fig.11 are the matching results in Map1 with different precision sonars which are listed in Table 4. The mean matching errors with Sonar1, Sonar2, and Sonar3 are 91.74m, 88.30m, and 446.26m respectively. It is shown that sonar's error influences matching precision to some extent and too large sonar error leads to much large mismatching and low matching precision. In a matching algorithm, the sonar measured path is treated as the actual path of vehicle and compared with the estimated paths to decide the optimal estimated path by matching rules. If there is much large difference between the sonar measured and actual paths, the optimal estimated path will be much far from the actual one, although it may be much close to the sonar measured one, which will lead to much large matching error. However, if sonar's error is small enough and the difference between the sonar measured and actual paths is not so obvious, matching error will not reduce remarkably by improving sonar's precision further. Therefore, it is the basis for TAN to navigate accurately by assembling high-precision sonar, but it is no help and unnecessary to improve sonar's precision further if it is high enough (such as Sonar2).

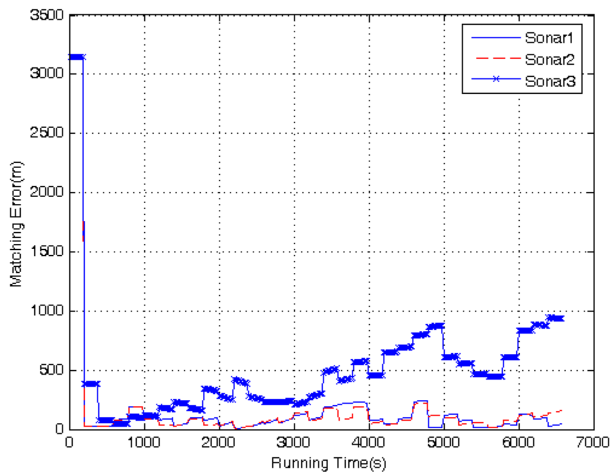


Fig.11 Matching Results in Map1 with Different Sonar Errors

IV. CONCLUSIONS

In this paper, TERCOM is studied thoroughly for underwater TAN by simulation and the effect of different application conditions on matching precision is focused on. The simulation results prove the following points.

(1) The map's information-content level decides matching precision directly. TERCOM's matching error is as low as grid distance in a map with a high information-content level while it is easy to diverge in a map with a low information-content level.

(2) Initial INS error affects matching precision to some extent. Especially, when initial INS error is too large, not only will the first matching region be enlarged, which decreases matching speed, but also TERCOM is easy to mismatch due to the appearance of too many areas with similar terrain characteristics in the region, which decreases matching precision. However, the simulation results also show that TERCOM is capable of matching with much large initial INS error.

(3) With the decreasing of map's resolution or vehicle speed, matching precision is reduced due to the shortened effective path length and the increased probability of mismatching. When sample length is close to grid distance, matching precision is the highest.

(4) Due to the decrease of INS precision, especially gyroscope precision, the shape of matching path is much different from the one of actual path, which increases the probability of mismatching and reduces matching precision. Although path length influences matching precision slightly in a map with a high information-content level, too long path length is no good for improving matching precision due to much difference between the shapes of matching and actual paths. Usually, path length should not be too long or short to ensure the stability and precision of matching.

(5) High-precision sonar is the basis of TAN, but if its precision is high enough, it is unnecessary to improve it further.

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