Physics Olympiad Problems on Simple Harmonic Motion (1-30)

Physcraft

Problems

1. Earthquake Dynamics and Rock Contact

A rock rests on a concrete sidewalk during an earthquake. The ground moves vertically in simple harmonic motion with a frequency of 2.40 Hz. Assume the rock and ground remain in contact until the acceleration of the ground matches gravitational acceleration g.

(a) Derive an expression for the critical amplitude of vibration when the rock just begins to lose contact. Explain the physical reasoning behind this condition.

(b) Calculate the critical amplitude numerically.

(c) If the earthquake frequency is halved, determine the change in the critical amplitude and provide a conceptual explanation.

(d) A second rock, submerged in water during the same vibration, experiences a buoyant force. Analyze how the buoyant force changes the critical amplitude condition.

(e) Suppose the ground's motion is damped due to energy dissipation. Discuss qualitatively how damping affects the likelihood of the rock losing contact.

2. Car Suspension and Mass Dynamics During an Earthquake

A car with a mass of 1130 kg, carrying four passengers (72.4 kg each), undergoes vertical oscillations during an earthquake. After the earthquake ends, the passengers quickly leave the car. Assume the car's suspension behaves as an ideal spring system with natural frequency $f = 1.80\,\text{Hz}$.

(a) Derive the expression for the spring constant k in terms of the car's mass and oscillation frequency.

(b) Calculate the spring constant numerically.

(c) Determine the upward displacement of the car's suspension when all passengers leave.

(d) If one passenger remains in the car, calculate the new displacement and discuss the proportionality between displacement and mass.

(e) Conceptually analyze how suspension stiffness (value of k) affects the car's ability to damp oscillations caused by earthquakes.

3. Walking Modeled as a Physical Pendulum

A human leg is modeled as a uniform rod of length $L = 0.85\,\text{m}$, swinging as a physical pendulum through half a cycle. The leg swings with an amplitude of 28°. Assume small oscillations to approximate the motion as simple harmonic.

(a) Derive the walking speed equation $v = \sqrt{\frac{6gL}{\pi}}\sin\theta_{\text{max}}$.

(b) Using the derived expression, calculate the walking speed for the given data.

(c) If the leg length is doubled, determine the resulting walking speed and explain the effect of leg length on speed.

(d) Extend this model to predict how walking speeds vary between different animals based on their leg lengths and swing amplitudes.

(e) Critically discuss the limitations of assuming simple harmonic motion for the leg swing, considering real-world factors such as air resistance and muscle dynamics.

Solutions

1. Earthquake Dynamics and Rock Contact

(a) The condition for the rock to lose contact is when the upward ground acceleration equals gravitational acceleration g:

$$a = \frac{g}{(2\pi f)^2}.$$

(b) For $f = 2.40\,\text{Hz}$, the critical amplitude is:

$$A = \frac{9.8}{(2\pi \times 2.40)^2} \approx 0.0163\,\text{m}\,(1.63\text{ cm}).$$

(c) If the frequency is halved ($f = 1.20\,\text{Hz}$), the amplitude increases by a factor of 4:

$$A = 4 \times 0.0163 = 0.065\,\text{m}.$$

(d) Buoyant force reduces the effective weight of the rock, so the critical amplitude decreases proportionally.

(e) Damping reduces oscillation amplitude over time, making it less likely for the rock to lose contact.

2. Car Suspension and Mass Dynamics During an Earthquake

(a) From $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, we get:

$$k = (2\pi f)^2 m.$$

(b) For $f = 1.80\,\text{Hz}, m = 1130 + 4 \times 72.4$, we find:

$$k = (2\pi \times 1.80)^2(1420.6) = 128.3\,\text{kN/m}.$$

(c) Removing 4 passengers:

$$\Delta x = \frac{4mg}{k}, \quad \Delta x \approx 8.87\,\text{cm}.$$

(d) With 1 passenger remaining:

$$\Delta x \approx 6.65\,\text{cm}.$$

(e) A stiffer suspension (higher k) reduces displacement and increases ride discomfort.