Momentum Transfer and Mechanical Operations Lab

Stokes Law - Particle Settling & Flow visualization in Hele-Shaw cell

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Team: MTMO 2

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Abstract

The primary objective of this experiment is to determine the Settling Velocities, Reynolds Number (N_{Re}), and Drag coefficient (C_D) for various sized and shaped particles. This was achieved by conducting experiments in different water-glycerol mixtures. These mixtures varied in viscosity, to observe the settling behaviour of particles under different fluid conditions. The experiment mainly aimed to validate Stoke's Law and plot the relationship between the Drag coefficient and the Reynolds Number to illustrate the different settling regimes of particles.

The experiment utilised four different water-glycerol columns. These columns were roughly two metres in length. The set-up of the experiment is described in the later sections along with the specific composition of the liquids used. The settling time for each bead in each column was recorded using two stopwatches, and the descent distance was marked at specific intervals on the columns.

The results were analysed to calculate the Settling Velocities, Reynolds Numbers, and Drag coefficients for each particle type in each fluid mixture. The data was used to plot C_D vs N_{Re}, revealing the different settling regimes and the applicability of Stoke's Law under various conditions. The experiment also included an error analysis to assess the accuracy of the measurements and calculations.

The findings indicated that Stoke's Law was not applicable in cases where $N_{Re} > 1$, and significant deviations from the theoretical predictions were observed. These deviations were attributed to inaccuracies in diameter and time measurements, as well as the non-spherical shape of the particles. Overall, the study confirmed the applicability of Stoke's Law within its limits but underscored the challenges in precise measurement and the influence of non-ideal particle shapes.

A. Particle Settling

1 Aim

- Determine settling velocities, Reynold's Number (N_{Re}), and Drag coefficient (C_D) for the various sized and shaped particles.
- Validate Stokes' Law and plot (C_D) vs (N_{Re}) to show the different regimes under which particles are settling.

2 Background and Motivation

Understanding how particles settle in fluids is vital across many scientific and engineering fields. Stokes' Law, which relates a particle's settling velocity to its size, density, and the fluid's viscosity, is key to comprehending this behavior. The law has broad applications spanning several disciplines, from sedimentation and environmental processes to the design of vehicles and understanding natural phenomena like raindrop formation.

For a Chemical Engineer who would deal with fluid flow characteristics, grasping the practical implications of Stokes' Law is essential, as it directly influences the design and optimization of fluid-related processes. This experiment, focused on particle settling, aims to validate Stokes' Law and deepen our understanding of its relevance in engineering applications.

3 Materials and Methods

Fore-mostly the external experimental conditions include the temperature which is roughly assumed to be 30°C and pressure is standard normal when the experiment was performed.

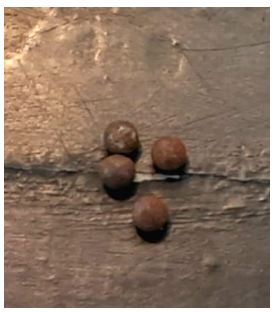
3.1 Apparatus Required

4 Glass columns containing water & glycerol of different composition, markers, Vernier Caliper, Weighing Machine, Stopwatch.

3.2 Materials Used

Various types of closely spherical beads and stones are used.

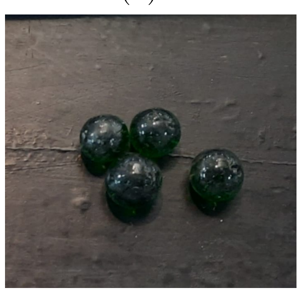
- Round beads made from various materials including iron(brown), plastic(clear), ceramic(white), marble(green), and granite(grey).



(a) Iron Beads

(b) Plastic Peas like Balls

(c) Ceramic White Balls



(d) Green Marbles

(e) Grey Granite Balls

Figure 1: Spherical Beads (almost regular shaped particles)

- Additionally, four irregularly shaped stones were used to study their movement and behavior in the different column solutions (shown in figure 2).

3.3 Experimental Setup Description

- **Water-Glycerol Columns:** Four vertical glass columns (shown in figure 3), each containing different compositions of water and glycerol:
  - Column 1: Filled with 100% water.

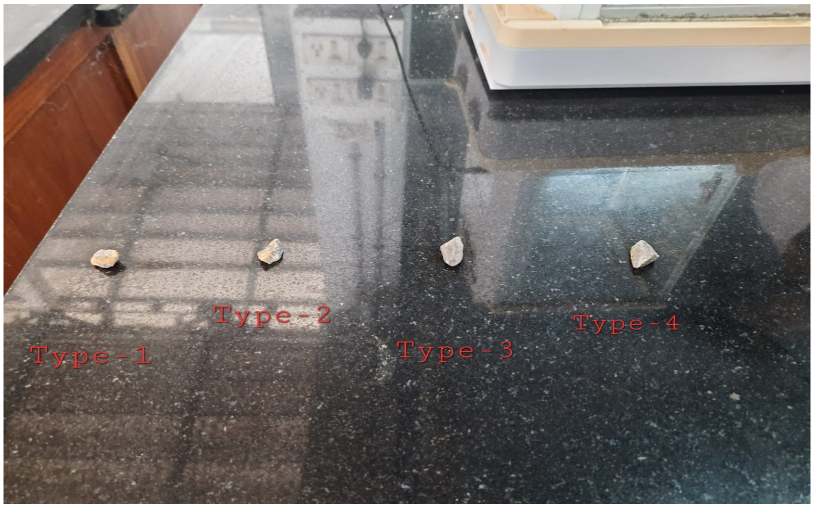


Figure 2: Irregular Shaped Stones (Type markings are corresponding to the specific liquids used for irregular particles

  - Column 2: A mixture of 50% water and 50% glycerol.
  - Column 3: A mixture of 25% water and 75% glycerol.
  - Column 4: Filled with 100% glycerol.

- **Markers:** Each column is equipped with markers to facilitate measurement during the experiment.
- **Vernier Caliper:** A precision instrument used to measure the diameter of the beads and the dimensions of the irregular stones to ensure accurate data collection.
- **Weighing Machine:** Used to measure the mass of the beads and stones before they are introduced into the columns.
- **Stopwatch:** Essential for timing the descent of the beads and stones through the liquid columns to calculate their terminal velocity and other relevant parameters.

3.4 Procedure

- The experiment involved selecting four particles of six different materials / sizes. We label them based on their appearance as - 'Brown', 'Clear', 'White', 'Green', 'Grey' and 'Irregular'.
- The diameters of each of the five almost-spherical particles were measured using a vernier caliper, and the results were averaged accordingly.

Figure 3: Set-up for particle-settling experiment

- Each of the irregular particle was considered to have a distinct D_p. We assumed the D_p to be equal to the average of Ferret's diameters from two different orientations. Volume of each particle was also calculated using water displacement to verify value of D_p.
- The weights of the particles were then determined using a weighing machine, with careful taring conducted after each measurement.
- Four long cylindrical columns were prepared with glycerol-water mixture with different percentage of glycerol in each (0%, 50%, 75% and 100%).
- Each particle was dropped into the four columns, and the time taken to traverse a predetermined distance was recorded using by two stopwatches and a video recording.
- This procedure was repeated for each bead material across all columns, resulting in 72 tabulations.

4 Calculations & Tabulations

For calculations, the following theoretical context is being used and all the experimental observations are tabulated along wtih some more columns of calculated features.

4.1 Stoke's Law & Particle Settling

Stoke's Law is a fundamental equation in fluid dynamics that governs the settling of small particles through a viscous fluid under the influence of gravity. It mathematically relates the settling velocity (u_t) of a spherical particle to the particle's diameter (D_p), the density of the particle (ρ_p), the density of the fluid (ρ), the acceleration due to gravity (g), and the dynamic viscosity of the fluid (μ):

$$u_t = \frac{1}{18\mu} g D_p^2 (\rho_p - \rho) = \frac{h}{t} \tag{1}$$

The above equation is in practice related to the formula ($u_t = h/t$), where h is the distance (descent distance) travelled by the particle under consideration in time t (descent time).

In the context of particle settling, the drag force (F_d) acting on the particle, which opposes its motion, is obtained from the force balance equation:

$$(\frac{m_p}{\rho_p})\rho_p g - F_d - (\frac{m_p}{\rho_p})\rho g = m_p \frac{du}{dt}\Big|_{u=u_t} = 0 \tag{2}$$

where m_p is the mass of the particle. When the particle reaches its terminal velocity ($u = u_t$), then the net force on the becomes zero and the following expression for drag force is obtained:

$$F_d = \frac{1}{6}\pi D_p^3 (\rho_p - \rho) g \tag{3}$$

Theoretically the expression for drag force is as follows:

$$F_d = C_d (\frac{u_t^2}{2}) \rho A_p \tag{4}$$

where C_d is the drag coefficient, A_p is the projected area of the particle and u_t is the particle velocity. In this context u_t is nothing but the terminal velocity of the particle.

The Reynolds Number ($N_{Re,p}$) is a dimensionless parameter that characterizes the relative importance of inertial forces to viscous forces in fluid flow. For particle settling, it is defined as:

$$N_{Re,p} = \frac{\rho u_t D_p}{\mu} \tag{5}$$

The drag coefficient (C_d) quantifies the drag force experienced by a particle relative to its effective cross-sectional area. It is expressed as (from eq 3 & eq 4):

$$C_d = \frac{F_d}{\frac{1}{2}\rho u_t^2 A_p} = \frac{2 m_p g (\rho_p - \rho)}{\rho \rho_p A_p u_t^2} \tag{6}$$

Note: Here for calculation purpose we are taking average values of the human observed and video analyzed velocity (u_t) & descent time (t) values.

4.2 Settling Velocity in Newton's Regime

In case of turbulent flow, the settling velocity of particles (u_t) follows an expression different from eq 1 which is as follows:

$$u_t = 1.75\Big[\frac{gD_p(\rho_p - \rho)}{\rho}\Big]^{1/2} = \frac{h}{t} \tag{7}$$

4.3 Observation Tables

The fluid properties are calculated using an online calculator by providing the volume fraction of glycerol and water and the temperature the experiment was conducted. The temperature is roughly approximated to be at 30deg C and the liquid mixtures are considered to be in their weight %.

Glycerol %	Viscosity(Pa.s)	Height(m)	Density(kg/m^3)
0	0.0007972	1.4460	995.67
50	0.0042029	0.7540	1120.8
75	0.021471	0.9060	1188.2
100	0.6484	0.8720	1260.0

Table 1: Characteristics of Fluid mixtures (specified using glycerol %)

The following observation tables contain 3 different stopwatch times. The first T_1 & T_2 columns are taken during experiment and data for T_3 column are tabulated after analyzing the videos of the corresponding liquid-particle settling observation. And T_{avg} is the average of all these time data.

Particle Type	m_p(g)	D_p(mm)	$\rho_p(kg/m^3)$	T_1(s)	T_2(s)	T_3(s)	T_{avg}(s)
Brown	0.9850	6.280	7595.540	1.45	1.50	1.16	1.37
Clear	0.6025	7.590	2631.680	3.91	3.80	3.35	3.69
White	2.1125	10.275	3719.226	1.71	1.71	1.43	1.62
Green	5.1150	15.490	2628.408	2.07	2.77	2.31	2.38
Grey	11.4250	20.120	2679.006	1.85	2.14	1.86	2.04
Irregular	2.0420	14.400	1146.078	4.24	4.01	4.20	4.15

Table 2: Observations in 0% GL mixture

Particle Type	m_p(g)	D_p(mm)	$\rho_p(kg/m^3)$	T_1(s)	T_2(s)	T_3(s)	T_{avg}(s)
Brown	0.9850	6.280	7595.540	0.94	0.92	0.78	0.88
Clear	0.6025	7.590	2631.680	1.66	1.71	1.78	1.72
White	2.1125	10.275	3719.226	1.12	1.13	0.99	1.08
Green	5.1150	15.490	2628.408	1.27	1.46	1.29	1.34
Grey	11.4250	20.120	2679.006	3.18	3.25	1.14	2.52
Irregular	3.5110	18.250	1135.761	1.98	2.00	2.01	2.00

Table 3: Observations in 50% GL mixture

Particle Type	m_p(g)	D_p(mm)	$\rho_p(kg/m^3)$	T_1(s)	T_2(s)	T_3(s)	T_{avg}(s)
Brown	0.9850	6.280	7595.540	1.79	1.72	1.10	1.54
Clear	0.6025	7.590	2631.680	2.31	2.29	2.29	2.30
White	2.1125	10.275	3719.226	1.47	1.51	1.27	1.42
Green	5.1150	15.490	2628.408	2.04	1.87	1.56	1.82
Grey	11.4250	20.120	2679.006	1.26	1.41	1.31	1.33
Irregular	1.9460	15.900	1253.584	3.43	3.21	3.03	3.22

Table 4: Observations in 75% GL mixture

Particle Type	m_p(g)	D_p(mm)	$\rho_p(kg/m^3)$	T_1(s)	T_2(s)	T_3(s)	T_{avg}(s)
Brown	0.9850	6.280	7595.540	5.73	5.76	5.69	5.73
Clear	0.6025	7.590	2631.680	18.80	18.61	19.17	18.86
White	2.1125	10.275	3719.226	6.47	6.18	6.53	6.39
Green	5.1150	15.490	2628.408	8.12	8.07	8.38	8.19
Grey	11.4250	20.120	2679.006	5.77	6.33	6.20	6.10
Irregular	3.9440	18.200	1281.237	8.53	8.56	8.78	8.62

Table 5: Observations in 100% GL mixture

5 Results

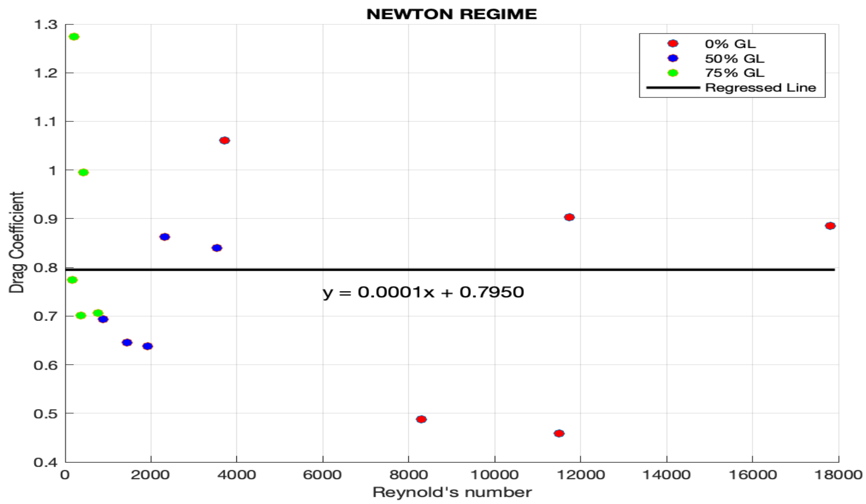


Figure 4: Results with 0%, 50%, 75% glycerol are coming under Newton's Regime

Note: The symbol ($\hat{x}$) signifies the theoretical value of that quantity in the following tables (table 6, 7, 8, 9).

Particle Type	u_t	$\hat{u}_t$	$N_{Re,p}$	$\hat{N}_{Re,p}$	C_d	$\hat{C}_d$	F_d
Brown	1.0564	1.1183	8286	8771	0.4879	0.4354	0.0084
Clear	0.3922	0.6121	3718	5803	1.0605	0.4354	0.0037
White	0.8953	0.9189	11490	11792	0.4586	0.4354	0.0152
Green	0.6065	0.8736	11734	16900	0.9032	0.4354	0.0312
Grey	0.7089	1.0109	17814	25403	0.8854	0.4354	0.0704
Irregular	0.3483	-	6264	-	0.2673	-	0.0048

Table 6: Practical & Theoretical values of quantities in 0% GL

6 Conclusion and Remarks

- Stoke's Law loses its validity when dealing with situations characterized by Reynolds numbers (Re) that are greater than 1.

Particle Type	u_t	$\hat{u}_t$	$N_{Re,p}$	$\hat{N}_{Re,p}$	C_d	$\hat{C}_d$	F_d
Brown	0.8572	1.0440	1436	1748	0.6458	0.4354	0.0082
Clear	0.4394	0.5544	889	1122	0.6931	0.4354	0.0034
White	0.6986	0.8460	1914	2318	0.6384	0.4354	0.0145
Green	0.5622	0.7912	2322	3268	0.8622	0.4354	0.0288
Grey	0.6602	0.9167	3542	4919	0.8393	0.4354	0.0652
Irregular	0.3775	-	1837	-	0.0217	-	0.0222

Table 7: Practical & Theoretical values of quantities in 50% GL

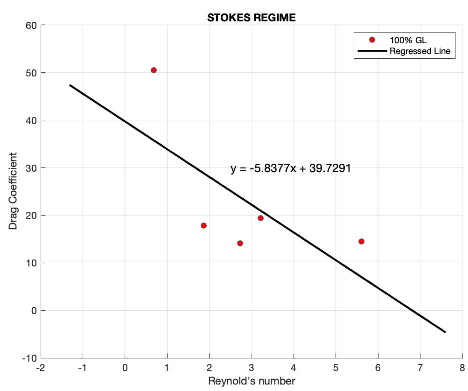
Particle Type	u_t	$\hat{u}_t$	$N_{Re,p}$	$\hat{N}_{Re,p}$	C_d	$\hat{C}_d$	F_d
Brown	0.5896	1.0087	204.9	350.5	1.2741	0.4354	0.0082
Clear	0.3946	0.5263	165.7	221.1	0.7748	0.4354	0.0032
White	0.6391	0.8109	363.4	461.1	0.7008	0.4354	0.0141
Green	0.4967	0.7510	425.8	643.8	0.9953	0.4354	0.0275
Grey	0.6837	0.8709	761.3	969.7	0.7064	0.4354	0.0624
Irregular	0.2810	-	247.2	-	0.1069	-	0.0043

Table 8: Practical & Theoretical values of quantities in 75% GL

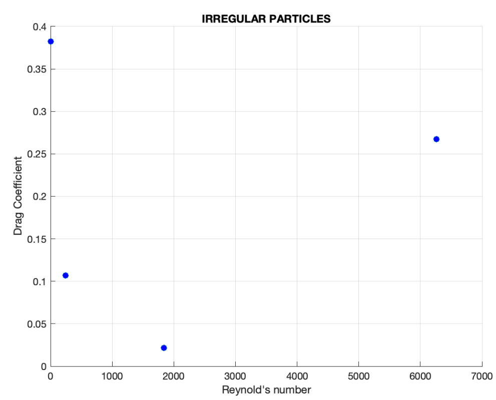
- In such cases, significant deviations from the expected outcomes of Stoke's Law can be observed.
- These deviations primarily arise due to the fact that these points actually lie in the Newton's regime instead of the Stoke's regime.
- As such, the theoretical values of liquids (0%, 50% and 75% Glycerol) are calculated using Newton's formula, whereas the theoretical values of liquid (100% Glycerol) is calculated using Stoke's formula.
- It is also concluded that the errors shown by irregular particles are too high due to their unsymmetrical geometry and shape.

Particle Type	u_t	$\hat{u}_t$	$N_{Re,p}$	$\hat{N}_{Re,p}$	C_d	$\hat{C}_d$	F_d
Brown	0.1523	0.2105	1.863	2.576	17.8102	9.3180	0.0081
Clear	0.0462	0.0666	0.684	0.985	50.5521	24.3784	0.0031
White	0.1364	0.2188	2.730	4.379	14.1021	5.4808	0.0137
Green	0.1065	0.2767	3.213	8.348	19.4124	2.8748	0.0261
Grey	0.1429	0.4840	5.603	18.971	14.5054	1.2651	0.0594
Irregular	0.1011	-	3.585	-	0.3826	-	0.0015

Table 9: Practical & Theoretical values of quantities in 100% GL



(a) Stoke's Regime Results



(b) Results for irregular particles

Figure 5: Results with 100% glycerol mixture are coming close enough to stoke's regime

7 Error Analysis

Least Count of wooden length scale $\equiv \Delta h = 1$ mm
Least Count of stopwatch / video duration $\equiv \Delta t = 0.01$ sec
Least Count of vernier caliper $\equiv \Delta D_p = 0.02$ mm
Least Count of weighting scale $\equiv \Delta m_p = 0.0001$ g

Important fractional errors are as follows:

$$\frac{\Delta u_t}{u_t} = \frac{\Delta h}{h} + \frac{\Delta t}{t} \tag{8}$$

$$\frac{\Delta\rho_p}{\rho} = \frac{\Delta m_p}{m_p} + 3 \cdot \frac{\Delta D_p}{D_p} \tag{9}$$

$$\frac{\Delta N_{Re,p}}{N_{Re,p}} = \frac{\Delta D_p}{D_p} + \frac{\Delta u_t}{u_t} \tag{10}$$

$$\frac{\Delta C_d}{C_d} = \frac{\Delta m_p}{m_p} + 2 \cdot \frac{\Delta D_p}{D_p} + 2 \cdot \frac{\Delta u_t}{u_t} + \frac{\rho\ \Delta\rho_p}{\rho_p(\rho_p - \rho)} \tag{11}$$

$$\frac{\Delta F_d}{F_d} = \frac{\Delta C_d}{C_d} + 2 \cdot \frac{\Delta u_t}{u_t} + 2 \cdot \frac{\Delta D_p}{D_p} \tag{12}$$

Particle Type	0% GL	50% GL	75% GL	100% GL
Brown	0.0553	0.1789	0.4154	0.2767
Clear	0.3593	0.2074	0.2504	0.3056
White	0.0257	0.1742	0.2118	0.3766
Green	0.3057	0.2894	0.3386	0.6152
Grey	0.2988	0.2798	0.2149	0.7047

Table 10: Fractional Error in theoretical and practical u_t values

Particle Type	0% GL	50% GL	75% GL	100% GL
Brown	0.1206	0.4833	1.9265	0.9114
Clear	1.4358	0.5919	0.7795	1.0736
White	0.0534	0.4664	0.6097	1.5730
Green	1.0746	0.9804	1.2860	5.7525
Grey	1.0336	0.9278	0.6224	10.4661

Table 11: Fractional Error in theoretical and practical C_d values

B. Hele-Shaw Apparatus

1 Aim

- To observe and record (pics/videos) flow profiles around objects of varying shapes and size.
- Compare streamlines that have been observed via flow visualizations using the Hele-Shaw apparatus with theoretical predictions for potential flow.

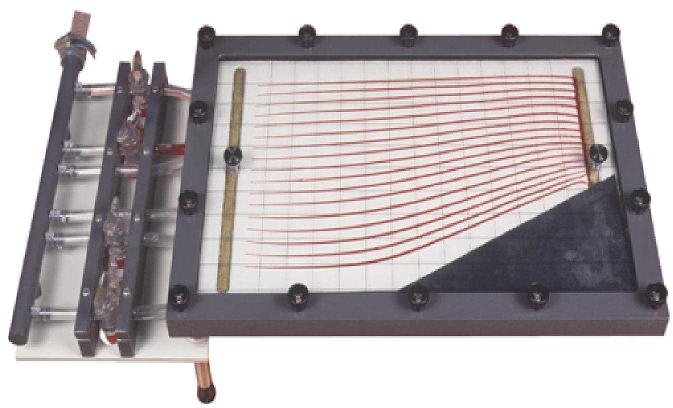


Figure 6: Hele-Shaw Apparatus

2 Background and Motivation

Visualizing fluid flow around objects and inside microreactors are increasingly becoming important in order to understand hydrodynamic parameters in such systems.

Here, we will employ the Hele-Shaw Apparatus to study flow characteristics around objects of varying shapes and sizes.

3 Materials and Methods

3.1 Apparatus Required

Hele-Shaw Apparatus, 2 five liter containers, 3 connecting pipes of appropriate size, plastic cutouts of different shapes, A4 size plastic board (white).

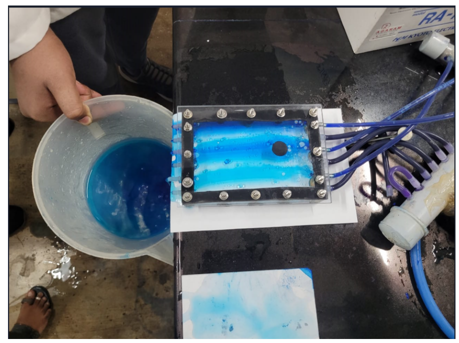
3.2 Materials Used

Water (5 liters), Methylene blue dye (3 liters).

3.3 Experimental Setup Description

The two tanks of water and methylene-blue are connected to the hele-shaw apparatus as shown in the figure 7:

(a) Two fluid tanks



(b) Hele-Shaw Apparatus

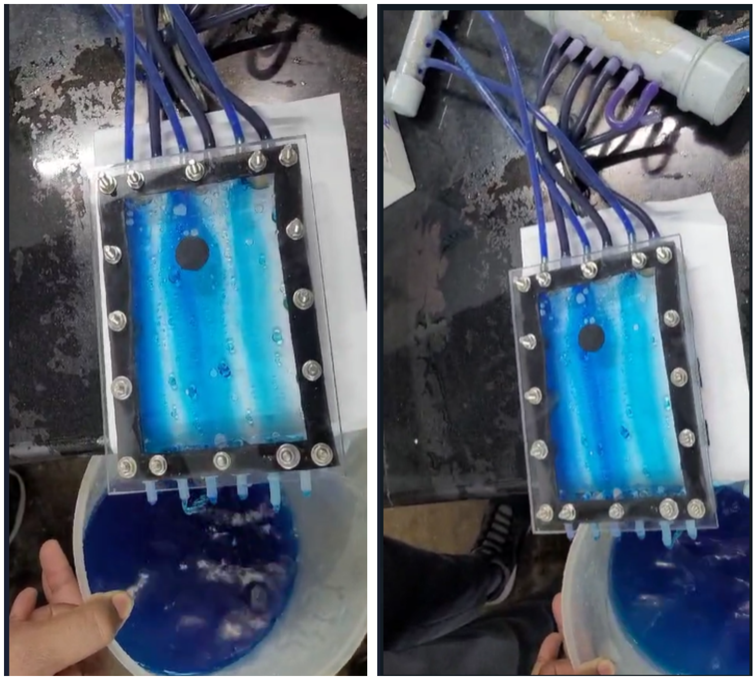
Figure 7: Inlet of Hele-Shaw appratus are connected to the two fuild tanks

3.4 Procedure

- We take the two tanks and fill them up with water and methylene blue dye.
- We then connect the tank to the Hele-Shaw apparatus using appropriate pipes. One pipe is used to drain the waste flow. The ball type valves are initially closed.
- The main box is unscrewed, and a plastic cutoff of circular shape is placed inside. The box is then screwed tightly.
- The valves for water and dye flow are turned on. Videos of the dye patterns are taken for different speeds of water flow.
- Valves are turned off and the whole process is repeated for a plastic cutout of diamond shape.

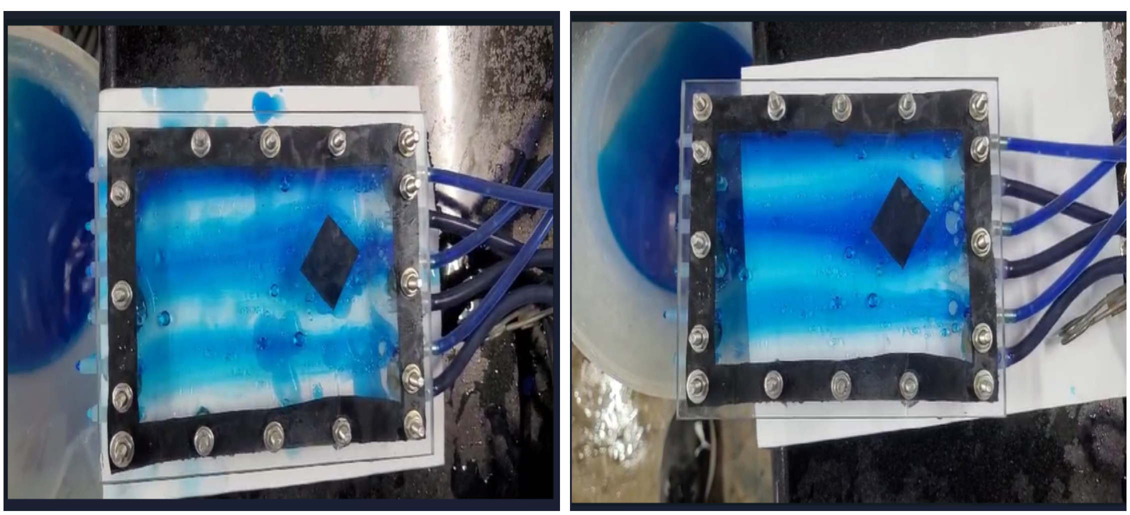
4 Observations

We have used two different shaped particle while observing the streamlines around then in this apparatus. One was circular cross-sectional and the other one was diamond-shaped. Both of them are shown below in the figure 8 & figure 9:



(a) Dye flow speed is slower (b) Dye flow speed is larger

Figure 8: Flow characteristics around circle shaped object

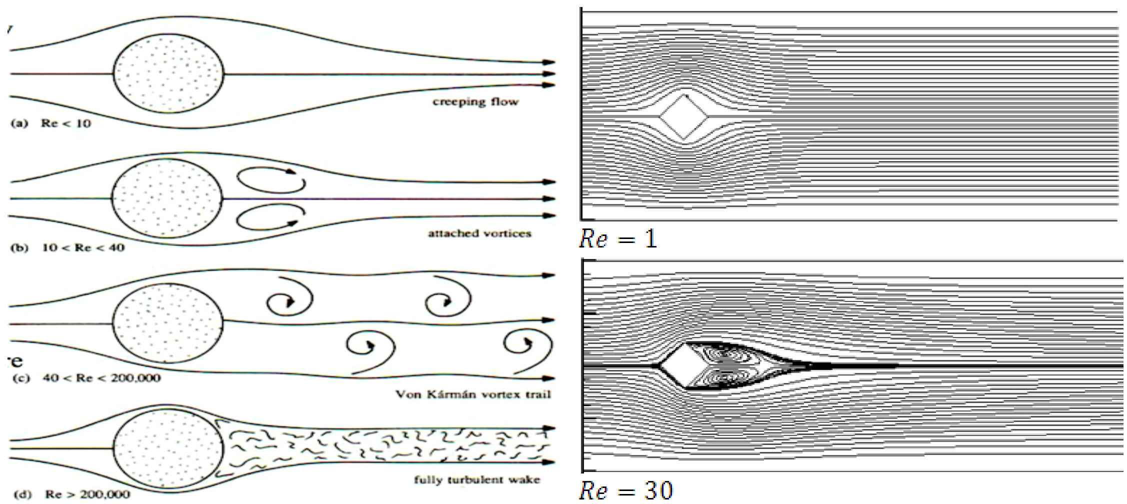


(a) Dye flow speed is slower (b) Dye flow speed is larger

Figure 9: Flow characteristics around diamond shaped object

5 Results and Discussions

The observed flow characteristics are close to the theoretical characteristics followed under the same types of objects used. The theoretically expected potential flow lines are shown in the below figure 10:



(a) Flow around a spherical object (b) Flow around a diamond shaped object

Figure 10: Flow characteristics around diamond shaped object (theorical expectations)

6 Precautions

- Confirm that the main box is properly screwed, else air bubbles may enter the flow.
- The pipes should be checked for any leakages.
- The water flow rate should not be set too high, otherwise it will become harder to see the streamlines.

Thought Question / Open-Ended

Q. Can you explain how Stokes' Law applies to horizontal movement of debris in water. What are the velocities that the debris flow reached while this tragedy unfolded? Use relevant equations to show how this relates to your experiment performed in the lab?

A. We can modify the Stokes' Law appropriately to accomodate the fact that the motion of particle is in horizontal direction instead of vertical. To do this, we start with the force balance equation :

$$F_d = F_e \tag{13}$$

Here, F_d is the drag force on the particle (or debris), and F_e is the external force driving the flood water. Now, putting the value of F_d in the above equation gives us :

$${u_t}^2 = \frac{1}{12}\frac{N_{Re,p} \cdot F_e}{\rho\ A_p} \tag{14}$$

Substituting the value of $N_{Re,p}$ in the above equation gives us:

$$u_t = \frac{1}{3\pi}\frac{F_e}{\mu D_p} \tag{15}$$

This equation can be used to calculate the speed of the debris flowing inside the flood water.

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Rapolu Paranay Reddy: Apparatus & Materials; Experimental Setup Description for part A of the report.
Atharva Sunilkumar Ghodke: Procedure of part A of the report.
Anomol Upadhyay: Aim (Objective); Background & Motivation of the part A of report.
Lakkireddy Vishnu Vardhan Reddy: Abstract
Rest of the parts along with inclusion of Hele-Shaw visualization are done by Deepanjhan Das (general editor) & Aayush Bhakna (proof reader).

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