FRM Part 1

Book 3 - Financial Markets and Products

PROPERTIES OF STOCK OPTIONS

Learning Objectives

After completing this reading you should be able to:

- ✓ Identify the **six factors that affect an option's price** and describe how these six factors affect the price for both European and American options.
- ✓ Identify and compute **upper and lower bounds** for option prices on non-dividend and dividend-paying stocks.
- ✓ Explain **put-call parity** and apply it to the valuation of European and American stock options.
- ✓ Explain the **early exercise features** of American call and put options.

Six Factors Affecting Option Prices

1. **Current stock price (S)**
   - The **value of a call** options **increases** as *S* **increases** (decreases).
   - For put options, the **value of a put decreases** (increases) as *S* **increases** (decreases).
2. **Strike price of the option (K)**
   - For call options, the value **decreases** as the strike price **increases**.
   - For put options, the value **increases** as the strike price **increases**.
3. **Time to expiration (T)**
   - With American style options as the **time to expiration increases**, the value of the option **increases**.
   - With more time, there are **higher chances of the option moving in-the-money**.
     - ✓ However, the same does **not apply to European-style options**, precisely when the underlying has scheduled dividends.

Six Factors Affecting Option Prices

4. **Short-term risk-free rate (r)**
   - As the **risk-free rate increases**, the value of a call (put) **increases** (decreases).
5. **Dividends (D)**
   - As the dividend increases, the value of a call (put) decreases (increases).
   - The reason behind this is that immediately after payment of a dividend, the stock price falls.
   - The call option holder has no access to the dividends because they do not own the underlying stock.

Six Factors Affecting Option Prices

6. Expected volatility of stock prices (σ)

- Volatility is considered the **most significant factor** in the valuation of options.
- As **volatility increases**, the value of **all options increases**.
- In particular, the maximum loss on a call is limited to the premium paid.
  - ✓ Thus, as volatility increases, there are **higher chances of the option expiring in-the-money**.

Pricing Bounds for Options

- For the next slides, let:
  - **c** = value of European call option.
  - **C** = value of an American call option.
  - **p** = value of European put option.
  - **P** = value of an American put option.
  - $\mathbf{S_T}$ = value of the stock at expiration.

Pricing Bounds for Options

- A call option gives the holder the right to buy the stock at a specified price.
  - The value of the call is **always less than the value of the stock**.

$$c \leq S_0 \text{ and } C \leq S_0$$

- If the value of a call **were to be higher than the value of the underlying stock**, arbitrageurs would sell the **call and buy the stock**, earning an instant risk-free profit in the process.

- A put option gives the holder the right to sell the underlying stock at a specified price.
  - The value of a put is **always less than the strike price**.

$$p \leq K \text{ and } P \leq K$$

- If the value of a put **were to be higher than the strike price**, everyone would move swiftly to **sell the option** and then **invest the proceeds at the risk-free rate** throughout the life of the option.

Put-Call Parity

- Put call parity states that the price of a **call option implicitly informs a certain price for the corresponding put option** with the same strike and expiration, and vice versa.
- Expressed mathematically,

$$c + Ke^{-rT} = S_0 + p$$

- Where:
  - ***c*** = value of call option
  - ***K*** = strike price
  - ***p*** = value of put option
  - S_0 = initial stock price

Put-Call Parity

- On the expiration date: $S_T + p = c + K$

- Own the stock trading at \$100, own the put option with EP = \$90: At expiration,

Exp date Price	\$80	\$89	\$110	\$130
Stock	\$80	\$89	\$110	\$130
Put Option	\$10	\$1	\$0	\$0
Portfolio	\$90	\$90	\$110	\$130

- Own the call with EP = \$90, own a bond that matures for \$90. At expiration,

Exp date Price	\$80	\$89	\$110	\$130
Call Option	\$0	\$0	\$20	\$40
Bond	\$90	\$90	\$90	\$90
Portfolio	\$90	\$90	\$110	\$130

- Before the expiration date: $S_0 + p = c + Ke^{-rT}$

Put-Call Parity In European Options

Example

- A **stock** currently sells for **$51**.
  - A 3-month call option on the stock, with a **strike price of $50**, has a **price of $5** .
  - The continuously compounded **risk-free rate is 10%.**
- Determine the price of the **associated put option**.

Solution

- We have the formula:
  - $c + Ke^{-rT} = p + S_0$
- Making *p* the subject,
  - $p = c + Ke^{-rT} - S_0$
  - $= 5 + (50e^{-0.10\times\frac{3}{12}}) - 51$
  - $= 2.77$

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Put-Call Parity In European Options

- If *p* is greater than or less than 2.77, there will be arbitrage opportunities.
- **For example, assume *p* = 3.50**. The following arbitrage opportunities would present themselves:

  I. Buy call for $5;

  II. Short put to realize t $3.50;

  III. Short the stock to realize $51;

  IV. Invest $49.5 (= 51 + 3.5 – 5) for 3 months, making $50.75 (= $49.5e^{10\% \times 3/12}$).

➢ Let S_T be the price of the stock at expiry:

- If $S_T > 50$,
  - Receive $50.75 from the investment
  - Exercise the call to buy the stock for $50.
  - Net profit = $0.75
- If $S_T < 50$,
  - Receive $50.75 from the investment
  - Put exercised by holder: buy the stock for $50.
  - Net profit = $0.75

Put-Call Parity In European Options

- Put call parity is only valid for **European options**.
  - For American options with the **possibility of early exercise**, the relationship turns into an equality:

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

Effect of Dividends

- When a stock pays a dividend, its value must **decrease by the amount of the dividend**.
  - This **increases the value of a put** option and **decreases the value of a call** option.
- The payment of a dividend will **reduce the lower pricing bound** for a **call** and **increase the lower pricing bound** for a **put**.

Lower Bounds of American Options

Option	Minimum Value	Maximum Value
American call	$C \geq \max(0, S_0 - Ke^{-rT})$	S_0
American put	$P \geq \max(0, K - S_0)$	X

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NEXT

TRADING STRATEGIES INVOLVING OPTIONS