FRM Part 1

Book 3 - Financial Markets and Products

INTEREST RATE FUTURES

Learning Objectives

After completing this reading you should be able to:

- ✓ Identify the most commonly used **day count conventions**, describe the markets that each one is typically used in, and apply each to an interest calculation.
- ✓ Calculate the **conversion of a discount rate** to a price for a US Treasury bill.
- ✓ Differentiate between the **clean** and **dirty price** for a US Treasury bond; calculate the **accrued interest** and dirty price on a US Treasury bond.
- ✓ Explain and calculate a US Treasury bond futures contract **conversion factor**.
- ✓ Calculate the **cost of delivering** a bond into a Treasury bond futures contract.
- ✓ Describe the impact of the **level and shape of the yield curve** on the cheapest-to-deliver Treasury bond decision.
- ✓ Calculate the **theoretical futures price** for a Treasury bond futures contract.
- ✓ Calculate the final contract price on a **Eurodollar futures contract**.
- ✓ Describe and compute the Eurodollar futures contract **convexity adjustment**.
- ✓ Explain how Eurodollar futures can be used to extend the **LIBOR zero curve**.
- ✓ Calculate the **duration-based hedge ratio** and create a duration-based hedging strategy using interest rate futures.
- ✓ Explain the **limitations** of using a duration-based hedging strategy.

Day Count Conventions

- A day count convention dictates **how interest accrues over time** in a variety of financial instruments, including bonds, swaps, and loans.
- It's usually expressed as a **fraction A/B**.

$$\text{Accrued interest} = \text{Coupon} \times \frac{\text{Number of days between dates}}{\text{Number of days in reference period}}$$

- **Common day count conversions**:
  - Actual/actual (Treasury bonds)
  - 30/360 (Corporate and municipal bonds)
  - Actual/360 (Market instruments, e.g., T-bills, commercial paper, and certificates of deposit)

Example >>

Day Count Conventions

Example

- Suppose we have a **Treasury bond** paying a coupon rate of **10% per annum** on a principal of **$100** on *March 1st* and *September 1st*.
- Compute the accrued interest as of *May 31st.*

Solution

- This implies a **coupon of $5** on *March 1st* and *September 1st*.
- We will have to determine the actual number of days between *May 31st* and the last coupon date, i.e., *March 1st*.
    - That's 30+30+31 = 91 *days*
- $\boldsymbol{Accrued\ interest} = \frac{\mathbf{91}}{\mathbf{184}} \times \$\mathbf{5}$ = *$2.47*

Clean Price vs. Dirty Price

- When a bond is between coupon payment dates, the price has **two components**:
  1. The clean (or flat) price; and
  2. The accrued interest (AI).

- The sum of these two is the dirty (or full) price:

$$PV_{Dirty} = PV_{clean} + Accrued\ interest$$

- The **clean price** is simply the dirty price minus the accrued interest:

$$PV_{Clean} = PV_{Dirty} - Accrued\ interest$$

- Bond prices can be quoted either way.

T-Bill Prices

- Like other money market instruments, Treasury Bills are **issued at a discount to par value**, on an **actual/360** day count basis.
  - The quoted price is also the discount rate:

$$P = \frac{360}{n}(100 - Y)$$

- Where:
  - ***n*** = number of days to maturity; ***Y*** = the bill's cash price

- Alternatively, you might be asked to compute a T-bill's cash price, in which case you should just make Y the subject of the formula:

$$Y = 100 - \frac{Pn}{360}$$

- A 120-day treasury bill with a cash price of 99 would have a quoted price of:

$$P = \frac{360}{120}(100 - 99) = 3$$

U.S. Treasury Bonds Futures Contracts

- Deliverable securities for T-Bond futures contracts are bonds with remaining terms to maturity of **15 years or more**.
- The **cash received by the short position** in a T-bond futures contract is given by:

$$\boldsymbol{Cash\ recieved = \frac{360}{n}(QFP \times CF) + AI}$$

- Where:
  - ***QFP*** = quoted futures price/settlement price
  - ***CF*** = conversation factor
  - ***AI*** = accrued interest since the last coupon date on the bond delivered

$$\boldsymbol{CF = \frac{Discounted\ bond\ price\ - Accured\ interest}{Face\ value}}$$

U.S. Treasury Bonds Futures Contracts

- The **conversion factor** is given by:

$$CF = \frac{Discounted\ bond\ price\ - Accured\ interest}{Face\ value}$$

- For instance, if a bond has a **present value of $125**, **accrued interest of $5**, and a **face value of $100**, then:

$$CF = \frac{125-5}{100} = 1.2$$

- Conversion factors for different bonds are issued on a **daily basis by the Chicago Board of Trade**.

A conversion factor is actually the **approximate decimal price** at which $1 par of a bond would trade **if it had a 6% yield to maturity** (YTM).

Cheapest to Deliver Bond

- The cheapest to deliver (CTD) the bond refers to the **cheapest bond** that could be **delivered to the long position** in line with contractual specifications.
  - Determining the CTB bond is necessitated by a **discrepancy between the market price of a security** and the **conversion factor** used to determine the value of the security being delivered.
- Thus, picking one bond for delivery over another can be **advantageous to the short position**.

$$CTD = Current\ Bond\ Price - Settlement\ Price \times Conversion\ Factor$$

Cheapest to Deliver Bond

- CTB calculations are relevant in all cases where **multiple financial instruments can satisfy the contract**.
- The theoretical Futures Price for a Treasury bond futures, F_0, is given by:

$$F_0 = (S_0 - I)e^{rT}$$

- Where:
  - S_0 = Spot price of the bond
  - I = Present value of cash flows, i.e., coupons
  - r = Risk-free rate of interest
  - T = Time of maturity

The Final Price of Eurodollar Futures Contracts

- Eurodollars are U.S. dollars deposited in banks **outside the United States**.
  - Eurodollar futures provide a valuable tool for **hedging fluctuations** in short-term U.S. dollar interest rates.
- Eurodollar futures have a maturity term of **3 months.**
- The final price of a Eurodollar futures contract is **determined by LIBOR** on the last trading day.
- Eurodollar futures contract settle in cash and are based on a Eurodollar **deposit of $1million**.

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The Final Price of Eurodollar Futures Contracts

- The minimum price change is **one "tick,"** which is equivalent to one interest rate basis point = 0.01 price points = $25 per contract.

$$\boldsymbol{Eurodollar\ futures\ price = \$10,000[100 - (0.25)(100 - Z)]}$$

- Where:
  - ***Z*** = quoted price for a Eurodollar futures contract

Example: If the quoted price Z is 98.5, then we have the following futures price:

$$\boldsymbol{Eurodollar\ futures\ price = \$10,000[100 - (0.25)(100 - 98.50)] = \$996,250}$$

➢ The three-month forward LIBOR for each contract is 100 – Z .

The Final Price of Eurodollar Futures Contracts

- In practice, however, daily marking-to-market can result in **differences** between **actual forward rates** and those implied by **futures contracts**.
  - To reduce this difference, we use a convexity adjustment:

$$\mathit{Actual\ forward\ rate} = \mathit{Forward\ rate\ implied\ by\ futures} - \left(\frac{1}{2} \times \sigma^2 \times T_1 \times T_2\right)$$

- Where:
  - T_1 = maturity on the futures contract
  - T_2 = time to the maturity of the rate underlying the contract (90 days)
  - σ = annual standard deviation of the change in the rate underlying the futures contract, or 90-day LIBOR

Duration-based Hedge Ratio

- A duration-based hedge ratio is a hedge ratio constructed when interest rate futures contracts are used to hedge positions in an **interest-dependent asset**, usually bonds money market securities.
- The number of futures contracts (N) required to hedge against a given change in yield, (Δy) is:

$$N = -\frac{P \times DP}{FC \times DF}$$

- Where:
  - ***P*** = forward value of the fixed-income portfolio being hedged
  - ***DP*** = duration of the portfolio at the maturity date of the hedge
  - ***FC*** = futures contract price
  - ***DF*** = duration of the asset underlying the futures

The negative sign implies that the number of contracts taken up must be the **opposite of the original position**.

Duration-based Hedge Ratio

- A pension fund has **$25 million portfolio** of Treasury bonds with portfolio **duration of 6.1**. The cheapest to deliver bond has a **duration of 4.7**. The six month treasury bond futures price is 127. The number of futures contracts to fully hedge the portfolio is:

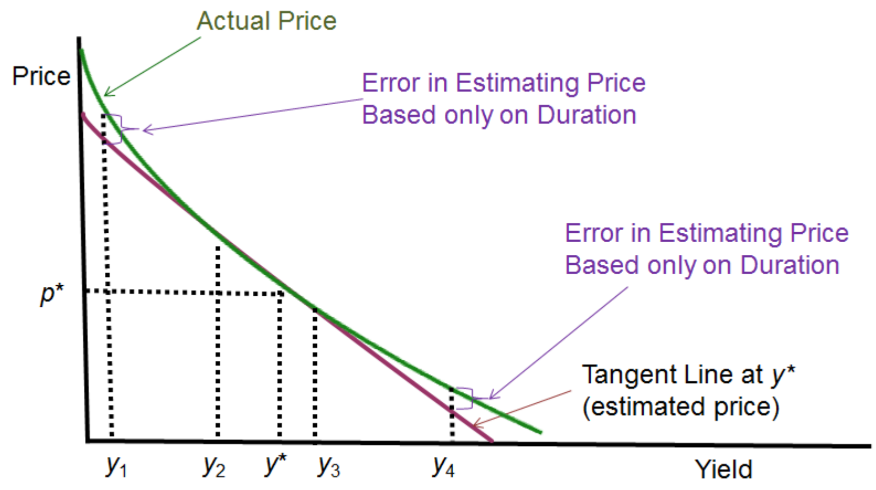
$$N = -\frac{P \times DP}{FC \times DF}$$

$$N = -\frac{\$25,000,000 \times 6.1}{\$127,000 \times 4.7} = -255$$

- If yields rise over the next 6 months, it's **bad news for the portfolio** as it will lose value. Suppose the new value is $23.5 million:
    - Then it's **good news in the futures market** as the gains can be used to offset the spot market losses.
    - In fact, if the **hedge is executed properly** and the yield curve changes are parallel, then it is possible to gain $1.5 million in the derivatives market.
- Total portfolio value after the hedge is $25 million.

Limitations of a Duration-based Hedging Strategy

- The major limitation of employing a duration-based hedging strategy has much to do with the fact that **duration measures are only accurate for small changes in yield**.
  - For large changes in yield, the price/yield relationship is not linear but is actually **convex**.
- Thus, using the strategy in the face of large moves in yield will result in "**underhedging**."



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NEXT

SWAPS