FRM Part 1

Book 3 - Financial Markets and Products

PROPERTIES OF INTEREST RATES

Learning Objectives

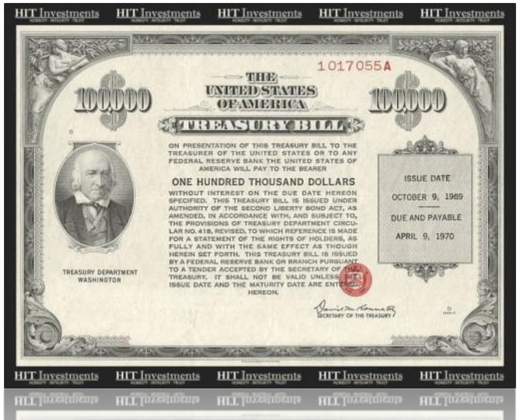
After completing this reading you should be able to:

- ✓ Describe **Treasury rates**, **LIBOR**, and **repo rates**, and explain what is meant by the "risk-free" rate.
- ✓ Calculate the value of an investment using **different compounding frequencies**.
- ✓ **Convert interest rates** based on different compounding frequencies.
- ✓ Calculate the **theoretical price of a bond** using spot rates.
- ✓ Derive **forward interest rates** from a set of spot rates.
- ✓ Derive the value of the cash flows from a **forward rate agreement** (FRA).
- ✓ Calculate the **duration**, **modified duration**, and **dollar duration** of a bond.
- ✓ Evaluate the **limitations of duration** and explain how **convexity** addresses some of them.
- ✓ Calculate the **change in a bond's** price given its duration, its convexity, and a change in interest rates.
- ✓ Compare and contrast the **major theories of the term structure** of interest rates.

Treasury Rates, LIBOR, and Repo rates

Treasury rates

- Treasury rates are the rates earned by investors in instruments used by a **government to borrow in its own currency**.
  - These include Treasury bonds and Treasury bills.
- Treasury rates are **considered “risk-free”** because they have zero risk exposure.
  - That has much to do with the **ability of the government** to use a range of tools at its disposal to avoid default, including **printing of cash** and **increased taxes**.
- T-bill and T-bond rates are used as the **benchmark for nominal risk-free rates**.



Treasury Rates, LIBOR, and Repo rates

LIBOR

- LIBOR, London Interbank Offered Rate, is the rate at which the world's leading banks **lend to each other for the short-term**.
  - It's the **most widely used benchmark** for short-term lending.

Repo rates

- Repo rates are the **implied rates on repurchase (repo) agreements**.
  - A repo agreement is an agreement between two parties – the seller and the buyer – where the seller agrees to sell a security to the buyer with the understanding that they (seller) will **buy it back later at a higher price**.
- The most common repo transactions are **carried out overnight**.
- The **credit risk** in a repo agreement depends on the **term** of the agreement as well as the **creditworthiness** of the seller.

Compounding Frequencies

- Given an initial investment of *A* that earns an annual rate *R* , compounded *m* times a year for a total of *n* years, then we can compute the future value, *FV* , as follows:

$$FV = A\left[1+\frac{R}{m}\right]^{m\text{x}n}$$

- In the presence of **continuous compounding**, then:

$$FV = Ae^{R\text{x}n}$$

➢ **Exam tip:** For any rate *R*, the future value with **continuous compounding** will always be **greater than** the future value with **discrete compounding**.

Compounding Frequencies

- Let R_c be the **continuously compounded rate** that equates the future value under discrete compounding to the future value under continuous compounding:

$$A\left[1+\frac{R}{m}\right]^{m \text{x} n} = Ae^{Rc \text{ x } n}$$

$$R_c = m \text{ x } ln\left[1+\frac{R}{m}\right]$$

- Alternatively, given R_c,

$$R = [e^{\frac{Rc}{m}} - 1]$$

The Theoretical Price of a Bond Using Spot Rates

- The **theoretical price** of a bond is given by the present value of all of the bond's cash flows. Assuming each cash flow is associated with a spot discount factor z_j, then:

$$P = \underbrace{\left[\frac{C}{2} \times \sum_{j=1}^{N} e^{-\frac{z_j}{2} \times j}\right]}_{\text{Coupon payments}} + \underbrace{FV\left(e^{-\frac{z_N}{2} \times j}\right)}_{\text{Principal}}$$

- Where:
  - ***P*** = bond's price
  - $\boldsymbol{z_j}$ = bond equivalent spot rate corresponding to $\frac{j}{2}$ years on a continuously compounded basis
  - ***FV*** = face value of the bond
  - ***N*** = number of semiannual payment periods
- The **yield** of a bond is the single discount rate that equates the **bond's present value to its market price**.
- A bond's **par yield** is the discount rate that equates the **bond's price to its par value**.

Deriving Forward Rates from a Set of Spot Rates

- Given a set of spot rates:

Year	1	2	3	4
Spot rate	1.2%	1.5%	1.9%	2.4%

- y_n, the *n*-year spot rate, is a measure of the **average interest rate** over the period from **now until *n* years' time**.
- The forward rate, $f_{t,r}$, is a measure of the **average interest rate** between **times *t* and *t* + *r***.
  - It's the interest rate agreed today (time 0) an investment made at time t>0 a period of *r* years.
- The **one-year forward rate**, $f_{t,1}$, is therefore the rate of interest from **time t to time t + 1**.
  - It can be expressed **in terms of spot rates** as follows:

$$1 + f_{t,1} = \frac{(1 + y_{t+1})^{t+1}}{(1 + y_t)^t} = \frac{(1.015)^2}{(1.012)^1} = 1.018$$

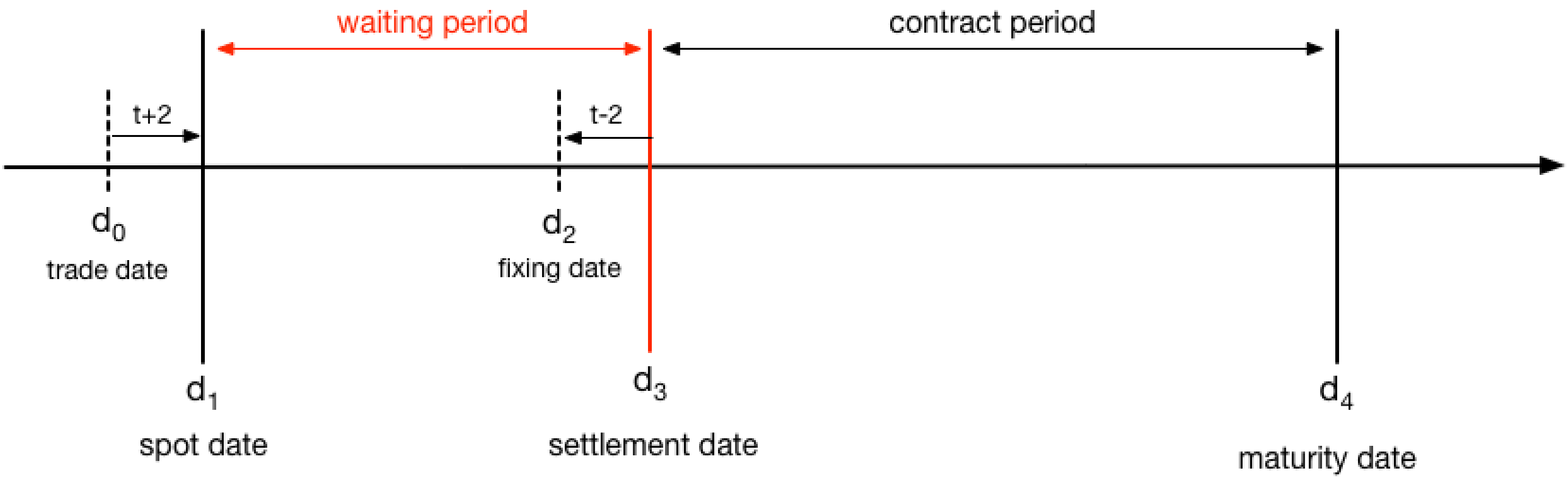
Forward Rate Agreements

- A forward rate agreement is an agreement between two parties **to lock in an interest rate for a specified period** of time **starting on a future settlement date**, based on a notional amount.
- If Firm A and Firm B enter into a forward rate agreement by agreeing on an interest rate R_K, the cash flows will be:
  - $\boldsymbol{Firm\ A}$: $\boldsymbol{L(R_K - R_M)(T_2 - T_1)}$
  - $\boldsymbol{Firm\ B}$: $\boldsymbol{L(R_M - R_K)(T_2 - T_1)}$
- Where
  - $\boldsymbol{L}$ = principal amount
  - $\boldsymbol{R_K}$ = interest rate agreed to in the FRA
  - $\boldsymbol{R_M}$ = actual interest rate observed between T_1 and T_2

Illustration >>

Forward Rate Agreements

- FRAs are **cash-settled** on the settlement date – the start date of the notional loan or deposit.
    - The **interest rate differential** between the market rate and the FRA contract rate determines the exposure to each party.
- It’s important to note that as the principal is a notional amount, there are **no principal cash flows**.



Forward Rate Agreements

- As time passed, the agreed fixed rate (R_K) **remains the same** but the forward LIBOR rate (R_F) is **likely to move** in either direction.
- Therefore, the value of the forward contract to both parties will be:
  - $\boldsymbol{Firm\ A : V_{FRA} = L(R_K - R_F)(T_2 - T_1)e^{-R_2 T_2}}$
  - $\boldsymbol{Firm\ B : V_{FRA} = L(R_F - R_K)(T_2 - T_1)e^{-R_2 T_2}}$

- Where:
  - $\mathbf{V_{FRA}}$ = value of the forward contract; and
  - $\mathbf{R_2}$ = the continuously compounded risk-free rate for a maturity T_2

Example >>

Forward Rate Agreements

Example

- A German bank and a French bank entered into a **semiannual** forward rate agreement contract where the German bank **will pay a fixed rate of 4.2%** and **receive the floating** rate on the **principal of €700 million**.
  - The forward rate between 0.5 years and 1 year is **5.1%**.
  - If the risk-free rate at the **1-year mark is 6%.**
- What is the **value of the FRA contract** between the two banks?

Solution

- V_{FRA} = Principal × (Floating rate – Fixed rate) × $0.5e^{-R_2 \times T_2}$
- V_{FRA} = €700 million × (5.1% – 4.2%) × $0.5e^{-0.06 \times 1}$ = €2,966,558

Duration

- Duration, sometimes referred to as **Macaulay duration**, is an approximate measure of a **bond's price sensitivity to changes in interest rates**.
    - Bond prices have an **inverse relationship** with interest rates.
    - When interest rates rise, bond prices fall; when interest rates fall, bond prices rise.
- Duration is **expressed in years**.

Example

- An investor buys a **6% annual payment** bond with **three years to maturity**.
- The bond has a **yield-to-maturity of 8%** and is currently priced at **95 per 100 of par**.
- The bond's Macaulay duration is equal to:

$$\mathbf{MacDur} = \frac{\frac{6}{1.08^1} \times 1 + \frac{6}{1.08^2} \times 2 + \frac{106}{1.08^3} \times 3}{95} = 2.82$$

- For a **zero-coupon bond**, its duration is simply its **time to maturity**.
- For a **coupon bond**, its duration is **shorter than maturity** because the cash flows have different weights.

Modified Duration

- Modified duration is used **in the absence of continuous compounding** of the yield to maturity. If the yield, y, is expressed as a rate compounded *n* times a year, then:

$$\boldsymbol{Modified\ duration = \left[\frac{Macaulay\ duration}{(1+\frac{y}{n})}\right]}$$

➢ **Exam tip**: Unless given, you must calculate the **Macaulay duration** to determine the **modified duration**.

Dollar Duration and DV01

Dollar Duration:

- The dollar duration, DD, of a bond is a **product** of its **modified duration** and its **market price**. If we use D^* to denote the modified duration, and P_0 to denote the bond's market price, then:

$$DD = D^* \times P_0$$

DV01 of a Bond:

- The **dollar value of a basis point**, DVBP, also represented as DV01, is the dollar exposure of a bond price for a **change in yield of 0.01%** (1 basis point).
- It is also the duration times the value of the bond and is **additive across the entire portfolio**.

$$DVBP = DD \times \Delta y = (D^* \times P_0) \times 0.0001$$

Effective Duration

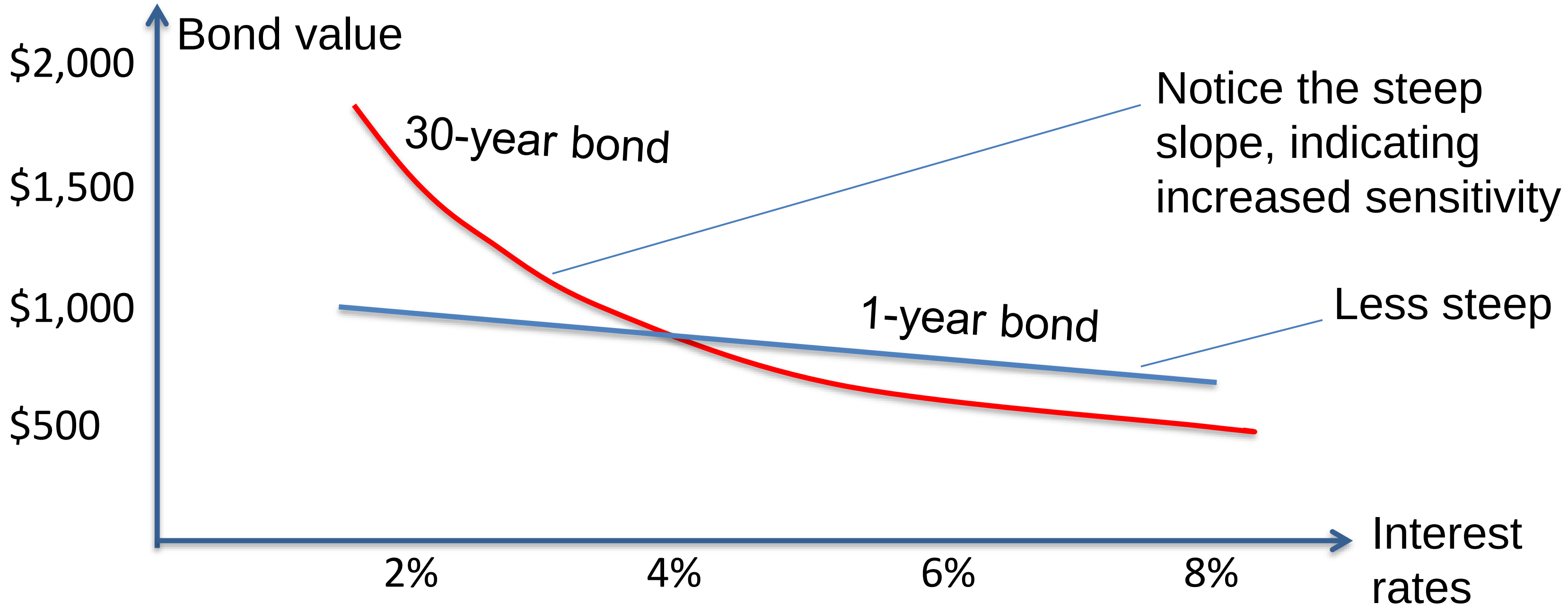
- For bonds with embedded options, **neither the Macaulay duration nor the modified duration** is appropriate as a measure of interest rate sensitivity.
  - This is because such bonds do not have **well-defined internal rate of return** (yield-to-maturity).
- For callable and putable bonds, you ought to compute the **effective duration**.

$$Effective\ duration = \frac{\mathrm{B}V_{-\Delta y} - \mathrm{B}V_{+\Delta y}}{2 \times \mathrm{B}V_0 \times \Delta y}$$

- Where:
  - $BV_{-\Delta y}$ = price estimate if yield decreases by a given amount Δy
  - $\mathrm{B}V_{+\Delta y}$ = price estimate if yield increases by a given amount Δy
  - BV_0= initially observed bond price
  - Δy = change in yield, expressed in decimal form

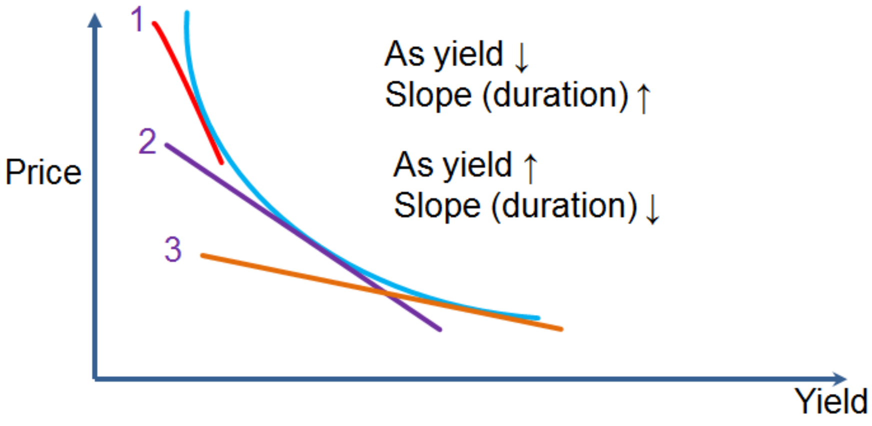
Convexity

- **Duration** usually **increases** for a longer time-to-maturity.
  - Longer maturity bond prices are **more sensitive** to changes in yields than shorter maturity bonds.
  - Interest rate risk is **larger** for longer maturity bonds.



Convexity

- Bonds with higher coupon rates are **less sensitive** to changes in yields. Interest rate risk is **inversely** related to the coupon rate.
  - Duration assumes that interest rates and bond prices have a **linear relationship**.
  - It's, therefore, a fairly good measure of exactly how bond prices are affected by **small changes in interest rate**.



- However, the relationship between bond prices and interest rates is **actually non-linear, i.e., convex**.

Convexity

- This makes **convexity a better measure of risk**, especially in the presence of **large and frequent fluctuations in interest rates**.
- For the purpose of the percentage change in price triggered by convexity, i.e., the **price change not explained by duration**, we must calculate the **convexity effect**.

$$\boldsymbol{Convexity\ effect = \frac{1}{2} \times Convexity \times \Delta y^2}$$

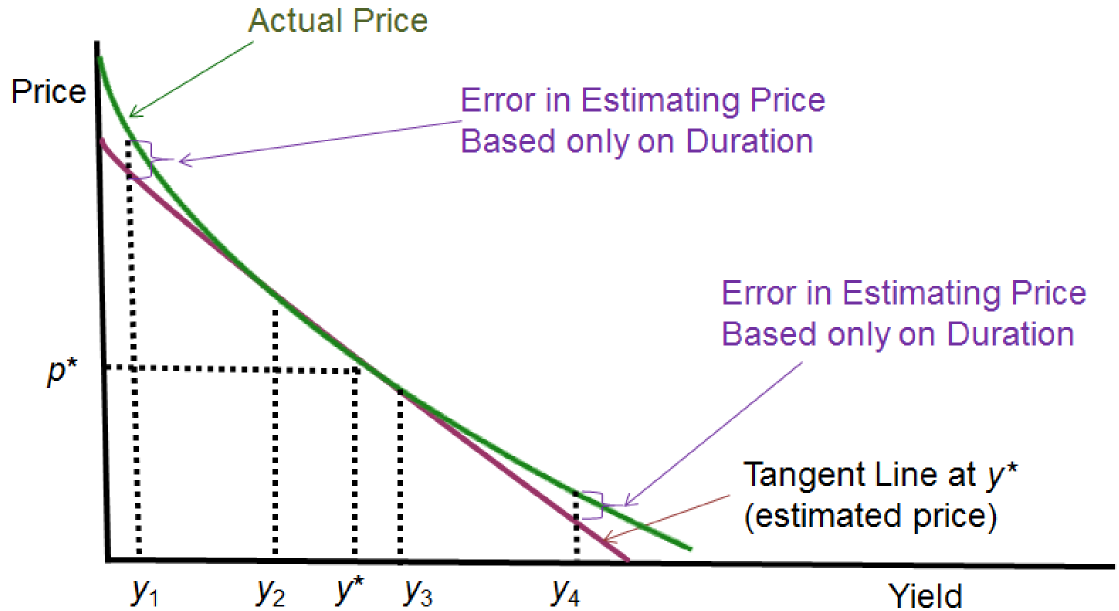
- **Exam tip:** Convexity is always positive for regular coupon-paying bonds

Convexity

- Combining duration and convexity results in a far more **accurate estimate** of the change in the price of a bond given a change in yield.

$$Change\ in\ price = Duration\ effect + Convexity\ effect$$

$$= [-Duration \times Price \times \Delta y\,]\left[\frac{1}{2} \times Convexity \times Price \times \Delta y^2\right]$$



Example >>

Convexity

Example

- A portfolio manager has a bond position worth **CAD 200 million**.
  - The position has a **modified duration of 6 years**; and
  - a **convexity of 120**.
- Assuming that the term structure is flat, by how much does the value of the position change if **interest rates increase by 50 basic points**?

Solution

- $\boldsymbol{Change\ in\ price = Duration\ effect + Convexity\ effect}$
  - $\boldsymbol{= \ [-Duration \times Price \times \Delta y\,]\left[\frac{1}{2} \times Convexity \times Price \times \Delta y^2\,\right]}$
  - $\boldsymbol{= -6 \times 200\ million \times 0.005 + \left[0.5 \times 120 \times 200\ million \times 0.005^2\right]}$
  - $\boldsymbol{= -6\ million + 300,000 = -5.7\ million}$

➢ With every increase in interest rates of 50 basis point, the bond's price will **decrease by $5.7 million**.

INTEREST RATES

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