FRM Part 1

Book 3 - Financial Markets and Products

DETERMINATION OF FORWARD AND FUTURES PRICES

Learning Objectives

After completing this reading you should be able to:

- ✓ Differentiate between **investment** and **consumption** assets.
- ✓ Define **short-selling** and calculate the net profit of a **short sale of a dividend-paying stock**.
- ✓ Describe the differences between forward and futures contracts and explain the relationship between **forward** and **spot prices**.
- ✓ Calculate the forward price given the underlying asset's spot price, and describe an **arbitrage argument** between spot and forward prices.
- ✓ Explain the relationship between forward and futures prices.
- ✓ Calculate a forward foreign exchange rate using the **interest rate parity relationship**.
- ✓ Define **income**, **storage costs**, and **convenience yield**.
- ✓ Calculate the **futures price on commodities** incorporating income/storage costs and/or convenience yields.
- ✓ Calculate, using the **cost-of-carry model**, forward prices where the underlying asset either does or does not have interim cash flows.
- ✓ Describe the various **delivery options** available in the futures markets and how they can influence futures prices.
- ✓ Explain the relationship between current futures prices and **expected future spot prices**, including the impact of systematic and nonsystematic risk.
- ✓ Define and interpret **contango** and **backwardation**, and explain how they relate to the cost-of-carry model.

Investment Asset vs. Consumption Asset

Investment assets

Earning an income or capital gain

Stocks, bonds, etc.

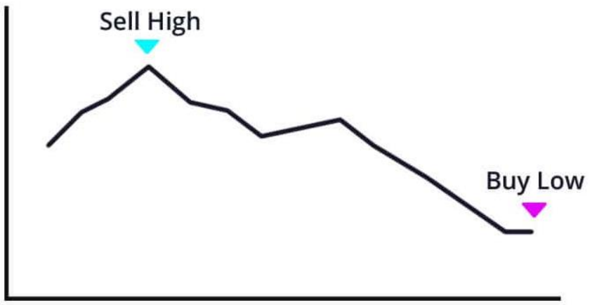
Consumption assets

Purpose of consumption (not for resale)

Oil, coffee, tea, corn, etc.

Short-selling

- Short selling involves the **sale of a security** which the investor **does not own**.
  - The investor borrows the security from the **lender**, usually the broker, with a promise to return it as of a specified date.
  - Their goal, therefore, is to **sell high** and **buy low** and get to keep the difference (profit).



- When the short sale is closed out, the short seller must **return the security to the lender**.
  - The lender may also request to have the asset even **before closeout**, depending on the initial agreement.
- There's always the risk that the security's price will actually rise, forcing the investor to **reacquire it at a higher price** and incur **a loss**.

Short-selling

- Short sales are transacted **through a broker**.
  - The short seller must deposit some **collateral to guarantee** the eventual return of the security to the owner.
  - In addition, the short seller is required to **pay all accrued dividends** to the lender.
- Thus, the net profit is equal to:

$$Net\ profit = Sale\ price\ - Borrowing\ price\ - Dividend\ paid$$

- Stock shorted today at \$100, dividend paid next month of \$4, short position closed out the following month at \$90.
- Net profit = 100 – 90 – 4 = 6
- Return on short sale = 6 ÷ 100 = 0.06

The Forward Price vs. the Spot Price

- Then, the relationship between spot prices and forward prices can be expressed as follows:

$$F_0 = S_0 e^{rT}$$

- Where:
  - F_0 = Forward price today, i.e., at t = 0
  - S_0 = Underlying asset (spot) price today
  - r = Continuously compounded risk-free annual rate
  - T = Time to maturity of the forward contract in years
- The forward price (left side) must equal the right side of the equation, i.e., the **cost of borrowing** funds to buy the underlying asset and **carrying it forward to time T**.
  - If $F_0 < S_0 e^{rt}$, an arbitrageur can make a **risk-free profit** by selling the asset, lending out the proceeds, and buying the forward.
  - If $F_0 > S_0 e^{rt}$, an arbitrageur can make a **risk-free profit** by selling the forward and buying the asset with borrowed funds.

Carrying Costs

- Carrying costs are any **cash flows associated with the underlying asset** over the life of the forward contract.
- The owner of the **forward contract does not receive** any of these cash flows.
  - Therefore, the **present value of these cash flows**, call it P, must be deducted from the spot price when determining the forward price. Thus,

$$F_0 = (S_0 - P)e^{rT}$$

- If the cash flows are in the form of dividends paid at a **continuously compounded rate** q, then:

$$F_0 = S_0 e^{(r - q)T}$$

Value of a Forward Contract

- At initiation, a forward contract has **zero value**.
    - The contract can only gain value once it has already commenced.
- If *K* represents the obligated delivery price, then the value of the contract to the long is given by:

$$value_{\ forward} = S_0 - Ke^{-rT}$$

if the underlying has no carrying costs

$$value_{\ forward} = S_0 - P - Ke^{-rT}$$

if the underlying has cash flows with a present value of *P*

$$value_{\ forward} = S_0 e^{-qt} - Ke^{-rT}$$

if the underyling pays dividends at a continuously compounded rate *q*

Computing Foreign Exchange Rates Using the Concept of Interest Rate Parity

If we let:

- F_0 to be the forward exchange rate;
- S_0 to be spot exchange rate;
- And $(r - r_f)$ to be the **interest rate differential** between the domestic currency and the foreign currency;

- Then,

$$F_0 = S_0 e^{(r - rf)T}$$

Example >>

Computing Foreign Exchange Rates Using the Concept of Interest Rate Parity

Example

- A German trader invests in a **1.5-year** currency futures contract on the U.S. dollar.
  - The risk-free interest rate in the Eurozone is **1.25%**.
  - The U.S. risk-free rate is **1.5%**.
  - The spot exchange rate is **1.098 USD per Euro (USD 1.098/EUR).**
- What is the **1.5-year futures exchange rate**?

Solution

- In this case, the domestic rate is the U.S. and the foreign rate is the Euro.
- $\boldsymbol{F_0 = S_0 e^{(US\ rate - Euro\ rate)T}}$
  - $\boldsymbol{F_0 = 1.098 \times e^{(1.5\% - 1.25\%)1.5}}$ = **1.102 USD per Euro**
- Since the U.S. risk-free rate is **greater** than the Euro risk-free rate, the futures exchange rate must be **greater** than the spot exchange rate.

Storage Costs

- The relationship seen previously between forward prices and spot prices are **only valid for investment assets**.
  - When it comes to **consumption assets**, we have what we call **storage costs**.
  - For example, a forward contract on several tons of corn must have **warehouse costs**.
- If the storage cost is a fixed cost U that's independent of the value of the underlying asset, then:

$$F_0 = (S_0 + U)e^{rT}$$

- If the storage cost *u* is a percentage of the underlying asset (yield), then:

$$F_0 = S_0 e^{(r+u)T}$$

Convenience Yield

- Convenience yield is the **additional value** that comes with **holding the asset** rather than having a long forward or futures contract on the asset.
- A good example of a consumption asset that has convenient yield is **oil**.
  - If you hold oil, you'll have the convenience of selling it at a higher price **during a shortage**.

- If a forward contract has a storage cost *u* expressed as a percentage of the underlying, **as well as a convenient yield *y***, then:

$$F_0 = S_0 e^{(r+u-y)T}$$

Example >>

Storage Costs and Convenience Yield

Example

- The price of a **3-month** crude oil futures contract (CL) is **USD 62.50**.
  - The risk-free rate is **2%**.
  - The storage cost is **10%**.
  - The convenience yield is **1%**.
- What is the **current price** of crude oil?

Solution

- $F_0 = S_0 e^{(r+u-y)T}$
  - $S_0 = \frac{F_0}{e^{(r+u-y)T}} = \frac{62.50}{e^{(2\%+10\%-1\%)(\frac{3}{12})}} = 60.80$

➢ Since the risk-free rate and the storage cost **outweigh the convenience yield**, the spot is lower than the futures price.

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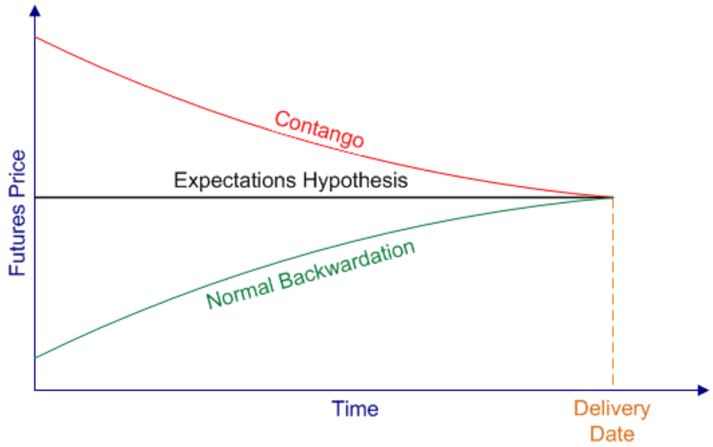
How Backwardation and Contango Relate to the Cost-of-carry Model

- Contango refers to a situation where the futures price is above the spot price.
  - It is likely to occur when there are **little or no benefits associated with holding the asset**, i.e., zero dividends, zero coupons, or zero convenience yield.
- Backwardation refers to a situation where the futures price is below the spot price.
  - It occurs when the **benefits** of holding the asset **outweigh the opportunity cost** of holding the asset as well as any **additional holding costs**.



Book 3 - Financial Markets and Products

Chapter 8

DETERMINATION OF FORWARD AND FUTURES PRICES

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