

FRM Part 1

Book 3 - Financial Markets and Products

COMMODITY FORWARDS AND FUTURES

Learning Objectives

After completing this reading you should be able to:

- ✓ Apply **commodity concepts** such as storage costs, carry markets, lease rate, and convenience yield.
- ✓ Explain the basic **equilibrium formula** for pricing commodity forwards.
- ✓ Describe an **arbitrage transaction** in commodity forwards, and compute the potential arbitrage profit.
- ✓ Define the **lease rate** and explain how it determines the no-arbitrage values for commodity forwards and futures.
- ✓ Define **carry markets**, and illustrate the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds.
- ✓ **Compute the forward price** of a commodity with storage costs.
- ✓ Compare the lease rate with the convenience yield.
- ✓ Identify **factors** that impact gold, corn, electricity, natural gas, and oil forward prices.
- ✓ Compute a **commodity spread**.
- ✓ Explain how **basis risk** can occur when hedging commodity price exposure.
- ✓ Evaluate the differences between a **strip hedge** and a **stack hedge** and explain how these differences impact risk management.
- ✓ Provide examples of **cross-hedging**, specifically the process of hedging jet fuel with crude oil and using weather derivatives.
- ✓ Explain how to create a **synthetic commodity position**, and use it to explain the relationship between the forward price and the expected future spot price.

Storage Costs, Carry Markets, Lease Rate, and Convenience Yield



Storage cost

- A forward contract to exchange 10,000 tons of corn six months from today would have warehouse costs.
- Others do not have such costs, e.g., a forward contract on goods that are used immediately, such as oil or goods that perish quickly, such as tomatoes
- A commodity for which the forward price compensates a commodity owner for costs of storage is called a **carry market**.



Lease rate

- A trader can also lend out a commodity in the short term if they do not plan to use it.
- The rate of return demanded by the lender is referred to as the lease rate.



Convenience yield

- A good example of a consumption asset that has convenient yield is oil.
- If you hold oil, you'll have the convenience of selling it at a higher price during a shortage.

Equilibrium Formula for Pricing Commodity Forwards

- The **prepaid forward price** is by definition the present value of the commodity on the future date:

$$F_{0,T}^P = e^{-aT} E_0[S_T]$$

- Where:
 - $F_{0,T}^P$ = prepaid forward price; and
 - a = discount rate for the commodity.
- The **forward price** is the future value of the prepaid forward price computed using the risk-free rate:

$$F_{0,T} = F_{0,T}^P e^{rT}$$

- We can combine these two equations to come up with the forward price:

$$F_{0,T} = E_0(S_T) e^{-(a-r)T}$$

- Which implies that the **commodity forward price** is the **discounted expected spot price**.

Arbitrage in Commodity Forwards

Cash-and-carry arbitrage

- Cash-and-carry arbitrage involves **buying a commodity** in cash form, **holding it**, and **delivering it** into a futures contract. The steps involved are as follows:
 - At initiation
 - Borrow money for the term of the contract at market interest rates
 - Buy the underlying commodity at the spot price
 - Sell a futures contract at the current futures price
 - At expiration
 - Deliver the commodity, receiving the futures contract price in the process.
 - Repay the loan plus interest.

Arbitrage in Commodity Forwards

Reverse cash-and-carry arbitrage

- A reverse cash-and-carry arbitrage strategy involves combining a **short position in a commodity** and a **long position in the futures** for that commodity. The steps involved are as follows:
 - Sell commodity short
 - Lend short sale proceeds at market interest rates
 - Buy futures contract at market price
- At the expiration:
 - Collect proceeds of the loan
 - Take delivery of the commodity for the futures price and cover the short sale commitment

Example >>

Arbitrage in Commodity Forwards

Example

- The current spot price of a bushel of corn is **\$10**.
 - There exists an active lending market for corn, where the **annual lease rate is equal to 8%**, the effective annual **risk-free rate is equal to 10%**, and the **1-year forward price for corn is \$10.35 per bag**.
- Does arbitrage exist? What is the risk-free profit?

Answer

- Yes, the risk-free profit is \$0.13.
- *Expected spot price in 1 year* $= S_0 e^{(r - \delta)T}$
 - $= 10e^{(10\% - 8\%)1} = 10.20$
- Since 10.35 is greater than 10.20, arbitrage exists.

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Arbitrage in Commodity Forwards

To take advantage of this opportunity, an arbitrageur can make the following moves:

- At initiation
 - Borrow \$10 at the rate of 10%
 - Buy a bushel of corn at \$10
 - Go short on a corn futures contract
 - Lend the bushel of corn at 8%
 - At maturity
 - Take back the bushel of corn plus proceeds from the lease amounting to **\$0.83** (= $\$10e^{8\% \times 1} - \10)
 - Deliver the bushel of corn; receive **\$10.35**
 - Repay borrowed funds amounting to **\$11.05** (= $10 e^{10\% \times 1}$)
- Net profit = $10.35 + 0.83 - 11.05 = \$0.13$

Factors that Impact Gold, Corn, Electricity, Natural Gas, and Oil Forward Prices



Gold

- Durable, non-perishable
- When the lease rate is positive, investors prefer to hold synthetic gold rather than physical gold



Corn

- Seasonality in production, plus storage costs
- The forward price curve of corn rises to reward storage between harvests, and it falls at harvest



Electricity

- Storage is often not possible
- Most highly volatile commodity on a daily basis because of day-to-day market forces (demand and supply)



Natural gas

- Seasonality, transportation costs, and storage costs are important.
- These factors influence the forward curve.



Oil

- Transportation costs are important

Commodity Spread

- Commodity spread refers to the **price differential** between a **raw material** commodity and the **finished product** created from that commodity.
 - For example, crude oil can be refined to produce heating oil, gasoline, and kerosene.
- **To trade on the spread**, an investor typically combines a long position in raw materials with a short position in a finished product.

Common commodity spreads include:

1. Crack spread

- Crack spread describes the price differential between crude oil and the **various petroleum products extracted from it**, particularly gasoline, heating oil, and kerosene.
- The refining process is called cracking and results in a **ratio between the raw material and the final products made**.

Example >>

Commodity Spread

1. Crack spread

Example

- A trader plans to **buy crude oil in one month** to produce **heating oil and gasoline in two months**.
 - The 1-month futures price for crude oil is currently \$30 / barrel .
 - The 2-month futures prices for heating oil and gasoline are \$43 / barrel and \$33.5 / barrel, respectively.
- Calculate the 7-4-3 crack (commodity) spread.

Solution

- The 7-4-3 spread implies that when refined, 7 barrels of crude oil produce 4 barrels of heating oil and 3 barrels of gasoline.
- By going long on crude oil and short in both heating oil and gasoline, the trader can lock in a profit:
 - Profit for a 7-4-3 spread = $(4 \times \$43 + 3 \times \$33.5 - 7 \times \$30) = \62.5
 - Thus, the profit per barrel of crude oil = $\$62.5 / 7 = \8.93

Commodity Spread

2. Crush spread

- A crush spread describes the price differential between **soybeans** and its two main products: **soybean meal** and **soybean oil**.
 - A trader takes a **long position in soybeans** and **short positions** in soybean meal and soybean oil.

3. Spark spread

- The spark spread uses **natural gas** as the raw material and **electricity** as the finished product.
 - The heat rate can be calculated in terms of Btu/kWh.

Basis Risk in Commodity Hedging

- Basis risk in commodity futures and forwards occurs when **spot price** and the **futures price do not converge**.
- Basis risk manifests in two situations:
 - When the risk attached to a commodity's price cannot be entirely hedged using the commodity futures contracts available in the market due to a **quantity mismatch**.
 - In cross-hedging, when futures contracts on a particular commodity are unavailable, hedgers turn to **closely related commodities** that exhibit similar price movement.
 - I. A good example would be the use of **gasoline futures to hedge exposure to jet fuel** if jet fuel futures do not exist in the market. Cross-hedging often results in **imperfect hedges**.
 - II. Another example would be an **energy-related company** hedging using **weather derivatives**.

Strip Hedging vs. Stack-and-roll hedging

Strip Hedging

- Happens when a series of futures contracts over many maturities ranges are purchased to hedge the underlying cash positions.
- **Example**
 - Suppose a farmer has entered into a contract to supply a **fixed number of bushels of corn per month at a fixed price**.
 - The farmer could set up a strip hedge by **buying futures** contracts that **match the maturity and quantity** for every month of the obligation.
- A strip hedge strategy has **no basis risk** because the basis becomes locked.

Strip Hedging vs. Stack-and-roll hedging

Stack-and-roll hedging

- Involves purchasing futures contracts for a **nearby delivery date** and on that date **rolling the position forward** by purchasing a fewer number of contracts.
 - The **process continues** for future delivery dates until the exposure at each maturity date is hedged.
 - **Example**
 - Using the example from the farmer, a stack hedge would involve entering into a **one-month futures contract** equaling the total value of the year's promised deliveries.
 - At the end of the first month, the farmer **rolls into the next one-month contract**, and so forth.
 - In normal circumstances, short-term contracts require **fewer transaction costs**. Thus, stack-and-roll hedges are **cheaper than strip hedges**.
 - However, **stack-and-roll hedging exposes the hedger to basis risk.**
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