

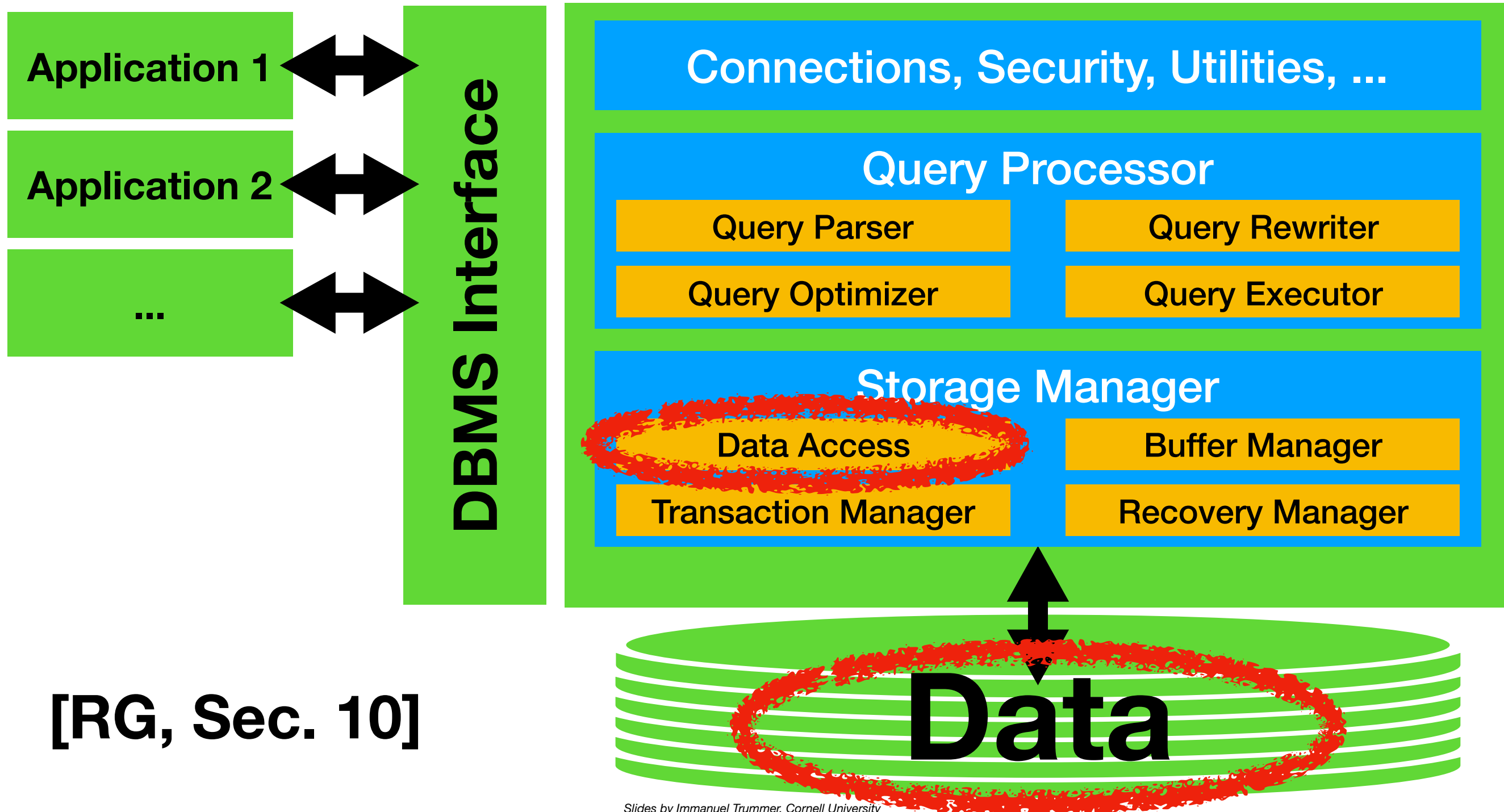
# Hash Indexes

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# Database Management Systems (DBMS)



[RG, Sec. 10]

# Hash vs. Tree Indexes

- **Tree index:** traverse search tree to find interesting leafs
- **Hash index:** evaluate hash function to find buckets

# Supported Predicates: Tree Indexes

- Tree indexes are based on a **sort order** between keys
  - Can handle **equality and inequality** conditions
    - **Consecutive** keys stored close together
- Composite keys: useful for conditions on key **prefix**
  - Keys with **same prefix** value stored close together

# Supported Predicates: Hash Indexes

- Hash indexes are based on key **hash values**
- Only useful for **equality** conditions
  - **Consecutive** keys may be stored far apart
    - Similar hash value  $\nRightarrow$  similar key value
- Condition must constrain **all** components
  - Keys with **same prefix** may be stored far apart

# Hash Index Variants

- **Static** hashing
  - Bad for dynamic data
- **Extendible** hashing
  - Expands with few high-overhead operations
- **Linear** hashing
  - Expands more "smoothly"

# Static Hashing

- Hash **bucket** pages contain references to data
  - Alternatively, may contain **data** directly
- Hash buckets are associated with **hash value ranges**
- Can use hash index to find **entries with key V**
  - Calculate **hash value h** for V as  $h(V)$
  - Look up **bucket page** associated with h

# Static Hashing Example

## Hash Function (Not Stored)

Key	Hash
Alan	1
Bob	0
Chan	2
Dora	5
David	1
Ester	7
Felix	4
Gert	2
Holy	7
Ida	1
Jana	0
Kyle	6
Lana	6
Levi	5
Olivia	3
Philip	7
Rosa	3
Tia	6
Victor	5
Zemin	4

$$\text{PageID} = \text{Hash} \% \text{NrBuckets}$$

P0		P1		P2		P3	
Bob	P23,1	Alan	P24,2	Chan	P21,3	Olivia	P47,2
Jana	P42,1	David	P36,1	Gert	P91,1	Rosa	P62,1
Zemin	P56,3	Ida	P62,3	Lana	P74,2		



# Updates and Static Hashing

- **Deletions** are easy - just remove associated entries
- **Insertions** are more difficult - what if bucket is full?
  - Can add "**overflow**" pages (like ISAM index!)
  - **Initial bucket** page stores pointer to first overflow page
  - Overflow pages form **linked list** if more than one
- Can **rehash** if number of overflow pages increases

# Overflow Pages Example

## Hash Function (Not Stored)

Key	Hash
Alan	1
Bob	0
Chan	2
Dora	5
David	1
Ester	7
Felix	4
Gert	2
Holy	7
Ida	1
Jana	0
Kyle	6
Lana	6
Levi	5
Olivia	3
Philip	7
Rosa	3
Tia	6
Victor	5
Zemin	4

$$\text{PageID} = \text{Hash} \% \text{NrBuckets}$$

P0	P1	P2	P3
Bob P23,1	Alan P24,2	Chan P21,3	Olivia P47,2
Jana P42,1	David P36,1	Gert P91,1	Rosa P62,1
Zemin P56,3	Ida P62,3	Lana P74,2	

P101	P105	P107
Felix P76,1	Levi P54,2	Kyle P44,1
	Victor P38,2	Tia P35,2

## Overflow Pages

# Static Hashing: Pros/Cons

- Can get data with **one read**
- May need multiple reads in case of **overflow** pages
- Will waste space if too many **deletions** (empty pages)
- Can use **rehashing** but creates significant overheads

# Extendible Hashing

- Idea: use **directory** to map hash buckets to pages
  - More **flexible** than using page IDs directly
- Redistribute **overflowing** buckets to multiple pages
  - More **efficient** than having to rehash all data
- Need to **increase directory size** if too many splits

# Extendible Hashing Example

## *Hash Function (Not Stored)*

Key	Hash
Alan	1
Bob	0
Chan	2
Dora	5
David	1
Ester	7
Felix	4
Gert	2
Holy	7
Ida	1
Jana	0
Kyle	6
Lana	6
Levi	5
Olivia	3
Philip	7
Rosa	3
Tia	6
Victor	5
Zemin	4

## *Directory Pages*

P116	
***	P131

## *Bucket Pages*

P131	
Alan	P41,2

***Insert Student "Alan"***

# Extendible Hashing

## Example

### *Hash Function (Not Stored)*

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

### *Directory Pages*

P116	
***	P131

### *Bucket Pages*

P131	
Alan	P41,2

***Insert Student "Alan"***

# Extendible Hashing

## Example

### *Hash Function (Not Stored)*

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

### *Directory Pages*

P116	
***	P131

### *Bucket Pages*

P131	
Alan	P41,2
Bob	P24,1

***Insert Student "Bob"***

# Extendible Hashing Example

## *Hash Function (Not Stored)*

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

## *Directory Pages*

P116	
***	P131

## *Bucket Pages*

P131	
Alan	P41,2
Bob	P24,1
Chan	P43,3

***Insert Student "Chan"***



# Extendible Hashing Example

## Hash Function (Not Stored)

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

## Directory Pages

P116	
**0	P131
**1	P133

## Bucket Pages

P131	
Bob	P24,1
Chan	P43,3

P133	
Alan	P41,2
Dora	P49,3

**Insert Student "Dora"**

# Extendible Hashing Example

## Hash Function (Not Stored)

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

## Directory Pages

P116	
**0	P131
**1	P133

## Bucket Pages

P131	
Bob	P24,1
Chan	P43,3

P133	
Alan	P41,2
Dora	P49,3
David	P44,2

**Insert Student "David"**

# Extendible Hashing Example

## Hash Function (Not Stored)

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

## Directory Pages

P116		P117	
*00	P131	*10	P131
*01	P133	*11	P139

## Bucket Pages

P131		P133		P139	
Bob	P24,1	Alan	P41,2	Ester	P52,2
Chan	P43,3	Dora	P49,3		
		David	P44,2		

**Insert Student "Ester"**

# Extendible Hashing Example

## Hash Function (Not Stored)

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

## Directory Pages

P116		P117	
*00	P131	*10	P131
*01	P133	*11	P139

## Bucket Pages

P131		P133		P139	
Bob	P24,1	Alan	P41,2	Ester	P52,2
Chan	P43,3	Dora	P49,3		
Felix	P68,1	David	P44,2		

**Insert Student "Felix"**

# Extendible Hashing Example

## Hash Function (Not Stored)

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

## Directory Pages

P116		P117	
*00	P131	*10	P126
*01	P133	*11	P139

## Bucket Pages

P131		P133		P139		P126	
Bob	P24,1	Alan	P41,2	Ester	P52,2	Gert	P33,1
Felix	P68,1	Dora	P49,3			Chan	P43,3
		David	P44,2				

**Insert Student "Gert"**

# Extendible Hashing Example

## Hash Function (Not Stored)

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

## Directory Pages

P116		P117	
*00	P131	*10	P126
*01	P133	*11	P139

## Bucket Pages

P131		P133		P139		P126	
Bob	P24,1	Alan	P41,2	Ester	P52,2	Gert	P33,1
Felix	P68,1	Dora	P49,3	Holy	P71,1	Chan	P43,3
		David	P44,2				

**Insert Student "Holy"**

# Extendible Hashing

## Example

### Hash Function (Not Stored)

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

### Directory Pages

P116	P117	P118	P119
000 P131	010 P126	100 P131	110 P126
001 P133	011 P139	101 P128	111 P139

### Bucket Pages

P131	P133	P139	P126
Bob P24,1	Alan P41,2	Ester P52,2	Gert P33,1
Felix P68,1	David P44,2	Holy P71,1	Chan P43,3
	Ida P42,2		

P128
Dora P49,3

**Insert Student "Ida"**

# Insertions Summary

- Calculate **hash value** for key of new entry
- Consult **directory** to identify current bucket
- Current bucket has **space**? Simply insert.
- Current bucket is **overflowing**?
  - Add new bucket page, **rehash** existing and new entry
    - For rehashing: consider **one more bit** of hash value
    - **Expand directory** if it does not consider enough bits



# Terminology

- **Global depth**: how many hash bits directory considers
- **Local depth**: how many hash bits for specific bucket

# Local vs. Global Depth

Hash Function  
(Not Stored)

Key	Hash
Alan	001
Bob	000
Chan	010
Dora	101
David	001
Ester	111
Felix	100
Gert	010
Holy	111
Ida	001
Jana	000
Kyle	110
Lana	110
Levi	101
Olivia	011
Philip	111
Rosa	011
Tia	110
Victor	101
Zemin	100

Directory Pages **Global Depth: 3**

P116	P117	P118	P119
000 P131	010 P126	100 P131	110 P126
001 P133	011 P139	101 P128	111 P139

Bucket Pages **Local Depth**

P131	P133	P139	P126
Bob P24,1	Alan P41,2	Ester P52,2	Gert P33,1
Felix P68,1	David P44,2	Holy P71,1	Chan P43,3
	Ida P42,2		

P128
Dora P49,3

# Deletions

- Can **merge bucket** pages if they become empty
- Can **half directory** size if number of buckets shrinks
- Often **no compaction** in practice
  - Assumption: **inserts** are more common than deletes

# Extendible Hashing: Pros/Cons

- Avoids **overflow pages**
- No need for **expensive rehashing**
  - Only rehash **one bucket** at a time
- Need additional **directory access**
- Need to **double directory** occasionally
  - This may take up some **time**

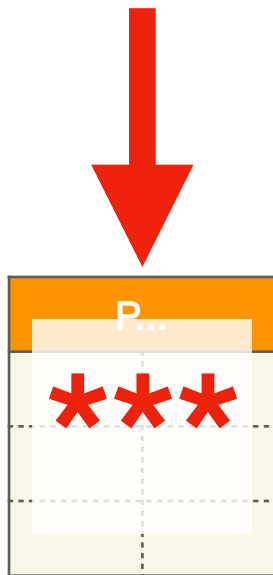
# *Can We Always Avoid Overflow Pages?*

# Linear Hashing

- Idea: avoid directory by **fixing next bucket** to split
  - Means we do not always split **overflowing** bucket!
  - I.e., we may have **temporary** overflow pages
  - Buckets to split are selected in **round robin** fashion
  - Means overflowing bucket will be split **eventually**

# Linear Hashing Example

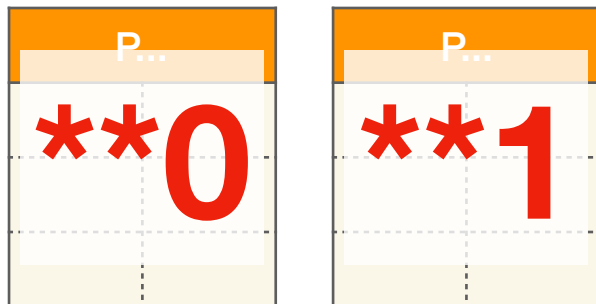
**Split Next**



***Round 1***

# Linear Hashing Example

**Split Next**

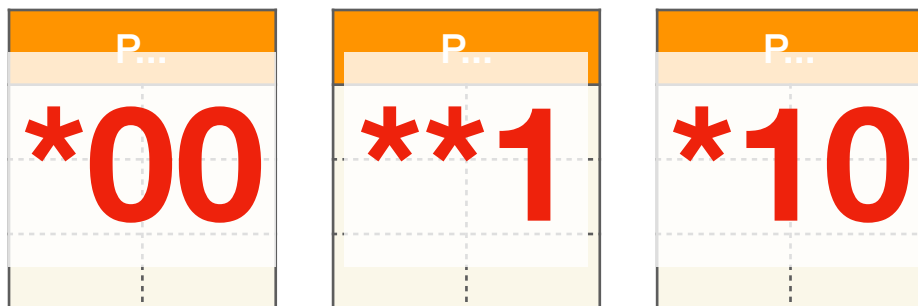


***Round 2***



# Linear Hashing Example

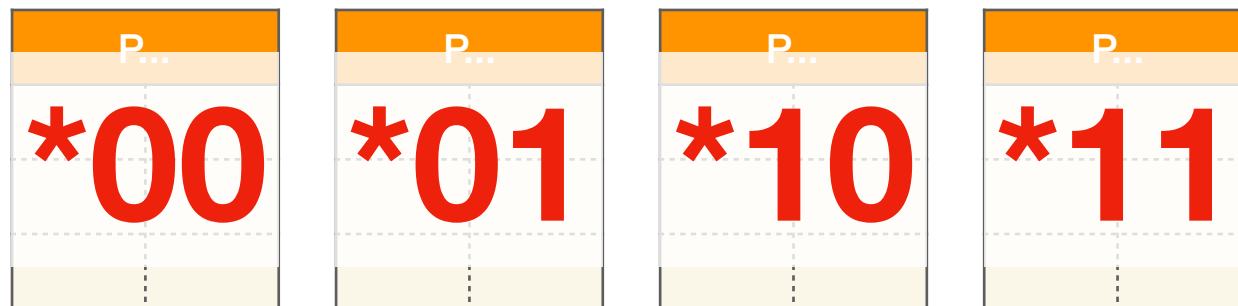
**Split Next**



***Round 2***

# Linear Hashing Example

**Split Next**



***Round 3***

# Linear Hashing Example

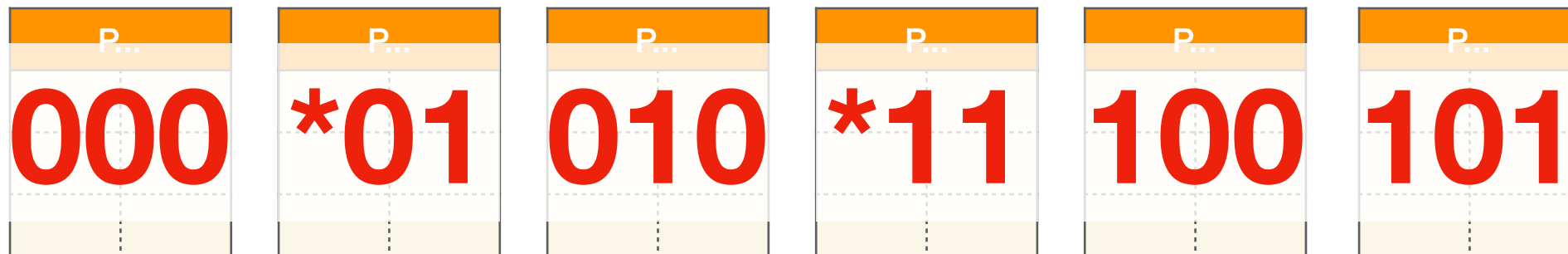
**Split Next**



***Round 3***

# Linear Hashing Example

**Split Next**



***Round 3***

# Insertions Summary

- Calculate **hash value** for new entry to insert
- **Add entry** on page or - if necessary - on overflow page
- **Split next bucket** if trigger condition is satisfied
  - May **eliminate** previously generated overflow pages
  - Some flexibility in choice of **trigger condition**

# Splitting Summary

- Splitting proceeds in **rounds**
  - All buckets present at round start split → round **ends**
  - "Next Split" pointer is **reset** to first page at round end
- We always split the bucket pointed to by **"Next Split"**
  - Add one new page, **redistribute** split bucket entries
    - Consider **one more bit** when redistributing

# Linear Hashing: Pros/Cons

- **Avoids a directory** - no expensive directory **doubling**
- May temporarily admit **overflow** pages
- May split empty pages - inefficient **space** utilization

# Optimizations

- Can apply **same optimizations** as for tree indexes
- Have **many entries** for same search key value?
  - Store key value, followed by **list** of references
- Want to get rid of one level of **indirection**?
  - Can **store data** directly instead of references
    - Leads to "**clustered index**", only one per table!
    - "Clustered index" in general: **data sorted** by index key



# Choose Index Type in Postgres

- CREATE INDEX **<index-name>** ON **<table>**  
USING **<method>** (**<column-list>**)
- Can choose **btree** or **hash** for method
  - Other choices as well, not covered here