ANNEXURE - I

ANNA UNIVERSITY
CHENNAI - 25

College Code	6	2	0	3								
College Name	BHARATHIYAR INSTITUTE OF ENGINEERING FOR WOMEN											
Register Number	6	2	0	3	1	9	1	0	6	0	1	7
Name of the Candidate	K. DEEPA											
Degree	B.E											
Branch	ELECTRONICS AND COMMUNICATION ENGINEERING				Semester		04					
Question Paper Code	X	1	0	6	6	3						
Subject Code	M	A	8	4	5	1						
Subject Name	PROBABILITY AND RANDOM PROCESSES											
Date	08	07	2021	Session	FN ✓	AN						
No. of Pages used	30	In words	Thirty pages only									
All particulars given above by me are verified and found to be correct												
Signature of the Student with date	K. Deepa 08/7/21											

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Instructions to the Candidate: Put Tick mark (✓) for the questions attended in the tick mark column against each question

PART – A			PART – B & C								Grand Total (in words)
Question No.	✓	Marks	Question No.		(i) ✓	(i) Marks	(ii) ✓	(ii) Marks	(iii) ✓	(iii) Marks	
1	✓		11	a	✓		✓				
2	✓			b							
3	✓		12	a	✓						
4	✓			b							
5	✓		13	a	✓		✓				
6	✓			b							
7	✓		14	a	✓		✓				
8	✓			b							
9	✓		15	a							
10	✓			b	✓		✓				Grand Total
			16	a							
				b							
Total											

Declaration by the Examiner: Verified that all the questions attended by the student are valued and the total is found to be correct

Date	Name of the Examiner	Signature of the Examiner

Register Number 620319106017 | Name of the student K·DEEPA | Subject code : MA8451 Subject Name: probability and Random Processes

Part-A

soln:

1) Let A be the event that Randomly selected person from this community smokes.

Let B be the event that person is made.

$$P(A) = P(AB) + P(AB^c)$$

$$= 0.32 + 0.27$$

$$= 0.59$$

$\therefore$ 59% of the population of this community smoke.

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2) soln:

Let D_1 be the event that we find a defective fuse in first test.

$$P(D_1 \cap D_2) = P(D_1)\,P(D_2/D_1)$$

$$= 2/7 \cdot 1/6 = 1/21$$

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9) soln:

A linear time system is said to be time-invarient if the input $x(t)$ is time shifted by an amount then the corresponding output $y(t)$ should also be time shifted by the same amount.

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K·Dp.

$Y(t) = f[X(t)]$, then $Y(t+h) = F[X(t+h)]$.

3) soln:

$$f(x,y) = \frac{\partial^2 F(x,y)}{\partial x\, \partial y}$$

$$= \partial/\partial x\, [-5e^{-5x} \cdot e^{-5y}(-5)]$$

$$= 5^2 e^{-5y} e^{-5x}(-5)$$

$$= -5^3 e^{-5x} e^{-5y}$$

$$\int_0^\infty \int_0^\infty f(x,y)\,dx\,dy = \int_0^\infty \int_0^\infty -5^3 e^{-5x} e^{-5y}$$

$$= -5^2 \int_0^\infty e^{-5y} [e^{-5x}]_0^\infty dy$$

$$= 5^2 \left[\frac{e^{-5y}}{-5}\right]_0^\infty$$

$$= 5 \neq 1.$$

$\therefore$ F cannot be the joint probability distribution function.

4) soln:

$$f(x,y) = \begin{cases} 6/5\,(x+y^2), & 0 \le x \le 1;\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The marginal p.d.f of X is

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= 6/5 \int_0^1 (x+y^2)\,dy \Rightarrow 6/5\left[xy + y^3/3\right]_0^1$$

$$= 6/5\,(x + 1/3), \quad 0 \le x \le 1.$$

5) $\text{Mean} = E\{X(t)\} = \int_{-\pi/2}^{\pi/2} \alpha(t)+\phi \, d\phi$

$= \int_{-\pi/2}^{\pi/2} \cos(t+\phi) \times 1/\pi \, d\phi$

$= 1/\pi \int_{-\pi/2}^{\pi/2} \cos(t+\phi)\, d\phi$

$= 1/\pi \left(\sin(t+\phi)\right)_{-\pi/2}^{\pi/2}$

$= 1/\pi \left[\sin(t+\pi/2) - \sin(t-\pi/2)\right]$

$= 1/\pi \left[\cos t + \cos t\right]$

$= \dfrac{2\cos t}{\pi}$ which is dependent on $\{X(t)\}$ is not a stationary process.

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6) Soln:

$\lambda = 3/10 = 0.3/\text{minute} \; ; \; t = 10$

$P(X(10) \leq 3) = \dfrac{e^{-3}3}{0!} + \dfrac{e^{-3}3^1}{1!} + \dfrac{e^{-3}3^2}{2!} + \dfrac{e^{-3}3^3}{3!}$

$= e^{-3}\left[1 + 3 + 9/2 + 27/6\right]$

$= 0.6472$

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7) Let $X(t)$ and $Y(t)$ be two random.

$R_{XY}(t, t+\tau) = E[X(t)\,Y(t+\tau)]$

$= R_{XY}(\tau)$

property -1 :

$$R_{XY}(\tau) = R_{YX}(-\tau)$$

property -2 :

$$|R_{XY}(\tau)| \le \sqrt{R_{XX}(0) \cdot R_{YY}(0)}$$

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8). Soln:

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} d\tau$$

$$= C\int_{-\infty}^{\infty} e^{-\alpha|\tau|} e^{-i\omega\tau} d\tau$$

$$= C\int_{-\infty}^{\infty} e^{-\alpha|\tau|} (\cos\omega\tau - i\sin\omega\tau) d\tau$$

$$= 2C\int_{0}^{\infty} e^{-\alpha\tau} \cos\omega\tau \, d\tau$$

$$= 2C\left[\frac{e^{-a\tau}}{\alpha^2+\omega^2}(-\alpha\cos\omega\tau + \omega\sin\omega\tau\right]_0^{\infty}$$

$$= \frac{2C\alpha}{\alpha^2+\omega^2}$$

Part- B

12) a) Soln:

Given:

$$f_{XY}(x,y) = 1/3\,(x+y), \quad 0 \le x \le 1, \quad 0 \le y \le 2$$

i) The marginal p.d.f of x and y are,

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$$

$$f_X(x) = \int_{0}^{2} f_{XY}(x,y)\,dy$$

$$= 1/3 \int_{0}^{2} (x+y)\,dy$$

$$= 1/3 \left[xy + \frac{y^2}{2}\right]_0^2$$

$$= 1/3\left[2x + 2^2/2\right] - [0]$$

$$= 1/3\,[2x+2]$$

$$f_X(x) = 2/3\,[x+1], \quad 0 \le x \le 1$$

$$f_Y(y) = \int_0^1 f_{XY}(x,y)\,dx$$

$$= 1/3 \int_0^1 (x+y)\,dx$$

$$= 1/3 \left[\frac{x^2}{2} + xy\right]_0^1$$

$$= 1/3\,[1/2 + y]$$

$$f_Y(y) = 1/6\,[1+2y], \quad 0 \le y \le 2$$

$$E[X] = \int_0^1 x f_X(x)\,dx$$

$$= 2/3 \int_0^1 x\,[x+1]\,dx$$

$$= 2/3 \int_0^1 [x^2 + x]\,dx$$

$$= 2/3 \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$$

$$= 2/3\,[1/3 + 1/2]$$

$$1/3 + 1/2 = \frac{2+3}{6} = 5/6$$

$$= 2/3\,(5/6)$$

$$= 10/18$$

$$\boxed{E[X] = 5/9}$$

$$E[y] = \int_0^2 y f_y(y)\,dy$$

$$= 1/6 \int_0^2 y(1+2y)\,dy$$

$$= 1/6 \left[\frac{y^2}{2} + 2\frac{y^3}{3}\right]_0^2$$

$$= 1/6 \left[\left(4/2 + \frac{2(2)^3}{3}\right) - 0\right]$$

$$= 1/6 \left[2 + 16/3\right]$$

$$= 2 + 16/3 = \frac{6+16}{3} = 22/3$$

$$= 1/6 \left[22/3\right]$$

$$= 22/18$$

$$\boxed{E[y] = 11/9}$$

$$E[x2] = \int_0^1 x^2 f_x(x)\,dx$$

$$= \int_0^1 x^2 \cdot 2/3\,(x+1)\,dx$$

$$= 2/3 \int_0^1 x^2(x+1)\,dx$$

$$= 2/3 \int_0^1 (x^3 + x^2)\,dx$$

$$= 2/3 \left[\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1$$

$$= 2/3 \left[1/4 + 1/3\right]$$

$= 2/3 \left[6/8\right]$

$= (2/3)\left(\frac{7}{12}\right)$

$= 7/18$

$$\boxed{E[x^2] = 7/18}$$

$$E[y^2] = \int_0^2 y^2 f_Y(y)\, dy$$

$$= 1/6 \int_0^2 y^2 (1+2y)\, dy$$

$$= 1/6 \left[\frac{y^3}{3} + 2\frac{y^4}{4}\right]_0^2$$

$$= 1/6 \left[\frac{(2)^3}{3} + 2\frac{(2)^4}{4}\right]$$

$$= 1/6 \left[8/3 + 8\right]$$

$$\boxed{E[y^2] = 16/9}$$

$$var[x] = E[x^2] - E[x]]^2$$

$$= 7/18 - \left[5/9\right]^2$$

$$= 7/18 - \frac{25}{81}$$

$$\boxed{var[x] = 13/162}$$

$$var[Y] = E[Y^2] - [E[Y]]^2$$

$$= 16/9 - [11/9]^2$$

$$= 16/9 - \frac{121}{81}$$

$$\boxed{var[Y] = 23/81}$$

$$E[XY] = \int_0^1 \int_0^2 xy\, f_{XY}(x,y)\, dy\, dx$$

$$= 1/3 \int_0^1 \int_0^2 (x^2 y + x y^2)\, dy\, dx$$

$$= 1/3 \int_0^1 \left[\frac{x^2 y^2}{2} + \frac{x y^3}{3}\right]_0^2 dx$$

$$= 1/3 \int_0^1 \left[2x + 8/3\, x\right] dx$$

$$= 1/3 \left[\frac{2x^3}{3} + 8/3\, \frac{x^2}{2}\right]_0^1$$

$$= 1/3 \left[2/3 + 4/3\right]$$

$$= 1/3\, (6/3)$$

$$\boxed{E[XY] = 2/3}$$

$$\operatorname{cov}[x, y] = E[xy] - E[x]E[y]$$

$$= 2/3 - 5/9 \times 11/9$$

$$= 2/3 - \frac{55}{81}$$

$$\operatorname{cov}[x, y] = -1/81$$

$$r(x, y) = \frac{\operatorname{cov}(x, y)}{\sqrt{\operatorname{var}(x)}\ \sqrt{\operatorname{var}(y)}}$$

$$= \frac{\left(-1/81\right)}{\sqrt{13/162} \times \sqrt{23/81}}$$

$$\boxed{r(x, y) = -\sqrt{2/299}}$$

13) a) i)

$$P_n(t) = P\{N(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, \quad n = 0, 1, 2, \ldots$$

proof :

Let λ be the number of occurences of event in unit time

Let $P_n(t) = P(N(t) = n)$ = probability of n occurances in $(0,t)$

$$P_n(t+\Delta t) = P(N(t+\Delta t) = n)$$

$= P$ ($n-1$ occurances in $(0,t)$ and 1 occurances in $(t, t+\Delta t)$ + P (n occurances in $(0,t)$ and 0 occurances in $(t, t+\Delta t)$

$= P_{n-1}(t) \cdot \lambda \Delta t + P_n(t)(1 - \lambda \Delta t)$, neglecting $o(\Delta t)$ terms

$= P_{n-1}(t)\lambda\Delta t + P_n(t) - P_n(t)\lambda\Delta t$

$$\therefore \frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = \lambda(P_{n-1}(t) - P_n(t))$$

Taking limit as $\Delta t \to 0$,

$$P_n'(t) = \frac{d}{dt} P_n(t) = \lambda\left(P_{n-1}(t) - P_n(t)\right) \longrightarrow (1)$$

Let the solution of (1) be,

$$P_n(t) = \frac{(\lambda t)^n}{n!} f(t) \longrightarrow (2)$$

Diff (2), with respect to (2) with 't',

$$P_n'(t) = \frac{\lambda^n}{n!}\left[n t^{n-1} f(t) + t^n f'(t)\right] \longrightarrow (3)$$

using (2) and (3) in (1),

$$\frac{\lambda^n}{n!}\left[n t^{n-1} f(t) + t^n f'(t)\right] = \lambda\left[\frac{(\lambda t)^{n-1}}{(n-1)!} f(t) - \frac{(\lambda t)^n}{n!} f(t)\right]$$

$$\frac{\lambda^n t^{n-1}}{(n-1)!} f(t) + \frac{\lambda^n t^n}{n!} f'(t) = \frac{\lambda^n t^{n-1} f(t)}{(n-1)!} - \frac{\lambda(\lambda t)^n}{n!} \cdot f(t)$$

$$\frac{\lambda^n t^n}{n!} f'(t) = -\frac{\lambda(\lambda t)^n}{n!} f(t)$$

$$f'(t) = -\lambda f(t)$$

$$\frac{f'(t)}{f(t)} = -\lambda$$

Integrating we get $\log(f(t)) = -\lambda t + c$

$$f(t) = e^{-\lambda t + c} = ke^{-\lambda t} \longrightarrow (4)$$

where $k = e^{c}$

$$p_n(t) = \frac{(\lambda t)^n}{n!} f(t)$$

$$\therefore\ p_0(t) = f(t)$$

$$\therefore\ p_0(0) = f(0)$$

$$p(X(0) = 0) = ke^{-0} \quad \text{(using 4)}$$

$$\therefore\ 1 = k$$

sub in (4),

$$f(t) = e^{-\lambda t}$$

$\therefore$ By eqn (2),

$$p_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \quad n = 0, 1, 2, 3$$

13) a) ii)

$$p(N(s) = k / N(t) = n).$$

Soln:

$$p(N(s)=k/N(t)=n) = \frac{p(N(s)=k \text{ and } N(t)=n)}{p(N(t)=n}$$

$$= \frac{p(N(s)=k \text{ and } N(t-s)=n-k)}{p(N(t)=n}$$

$$= \frac{p(N(s)=k)\,p(N(t-s)=n-k)}{p(N(t)=n}$$

$$= \frac{e^{-\lambda s}(\lambda s)^k}{k!}\,\frac{e^{-\lambda(t-s)}[\lambda(t-s)]^{n-k}}{(n-k)!} \times \frac{n!}{e^{-\lambda t}(\lambda t)^n}$$

$$= \frac{n!}{k!\,(n-k)!}\,\frac{s^k(t-s)^{n-k}}{t^n}$$

$$= nC_k\,(s/t)^k\left(\frac{t-s}{t}\right)^{n-k}$$

$$= nCk\,(s/t)^k\,(1-s/t)^{n-k}$$

$$\boxed{p(N(s)=k/N(t)=n) = nCk\,(s/t)^k\,(1-s/t)^{n-k}}$$

14) a,

a) i) Soln:

If $x(t)$ and $y(t)$ are uncorrelated random processes, then their cross covariance

$$C_{xy}(t, t+\tau) = 0$$

$$R_{xy}(t, t+\tau) - E[x(t)] \cdot E[y(t)] = 0$$

$$R_{xy}(t, t+\tau) = E[x(t)] \cdot E[y(t)]$$

$$R_{xy}(\tau) = E[x(t)] \cdot E[y(t)] \longrightarrow ①$$

Similarly,

$$R_{yx}(\tau) = E[y(t)] \cdot E[x(t)] = R_{xy}(\tau) \longrightarrow ②$$

By definition we have the autocorelation function of

$$z(t) = x(t) + y(t)$$

$$R_{zz}(t) = E[z(t) \cdot z(t+\tau)]$$

$$= E\{[x(t) + y(t)] \cdot [x(t+\tau) + y(t+\tau)]\}$$

$$= E\{x(t) \cdot x(t+\tau) + x(t) \cdot y(t+\tau) + y(t) \cdot x(t+\tau) + y(t) \cdot y(t+\tau)\} \rightarrow ③$$

$$= E[x(t) \cdot x(t+\tau)] + E[x(t) \cdot y(t+\tau)] + E[y(t) \cdot x(t+\tau)] + E[y(t) \cdot y(t+\tau)]$$

$$= R_{xx}(\tau) + R_{xy}(\tau) + R_{yx}(\tau) + R_{yy}(\tau)$$

$$R_{zz}(\tau) = R_{xx}(\tau) + 2R_{xy}(\tau) + R_{yy}(\tau) \longrightarrow ④$$

The power spectral density of $\{Z(t)\}$ is,

$$F[R_{ZZ}(\tau)] = F[R_{XX}(\tau)] + 2F[R_{XY}(\tau)] + F[R_{YY}(\tau)]$$

$$S_{ZZ}(\omega) = S_{XX}(\omega) + 2S_{XX}(\omega) + S_{YY}(\omega) \longrightarrow (5)$$

∴ by definition of power density and cross spectral density]

Cross - correlation function is,

$$R_{XZ}(\tau) = E[X(t) \cdot Z(t+\tau)]$$

$$= E\{X(t) \cdot [X(t+\tau) + Y(t+\tau)]$$

$$= E[X(t) \cdot X(t+\tau) + X(t) \cdot Y(t+\tau)]$$

$$= E[X(t) \cdot X(t+\tau)] + E[X(t) \cdot Y(t+\tau)]$$

$$R_{XZ}(\tau) = R_{XX}(\tau) + R_{XY}(\tau) \longrightarrow (6)$$

$$F[R_{XY}(\tau)] = F[R_{XX}(\tau)] + F[R_{XY}(\tau)]$$

$$S_{XZ}(\omega) = S_{XX}(\omega) + S_{XY}(\omega)$$

Similarly we get,

$$S_{YZ}(\omega) = S_{XY}(\omega) + S_{YY}(\omega)$$

14) a)

ii) Soln:

Given:

$$S(\omega) = \frac{\omega^2+9}{\omega^4+5\omega^2+4}$$

$$= \frac{\omega^2+9}{(\omega^2+4)(\omega^2+1)} \longrightarrow ①$$

put $\omega^2 = u$

$$① \Rightarrow \frac{u+9}{(u+4)(u+1)}$$

$$= \frac{A}{u+4} + \frac{B}{u+1} \longrightarrow ②$$

$$u+9 = A(u+1) + B(u+4)$$

put $u = -1 \Rightarrow$

$$8 = 3B$$

$$\boxed{B = 8/3}$$

put $u = -4 \Rightarrow$

$$5 = -3A$$

$$\boxed{A = -5/3}$$

$$② \Rightarrow \frac{-5/3}{u+4} + \frac{8/3}{u+1}$$

$$① \Rightarrow S(\omega) = \frac{\omega^2+9}{(\omega^2+4)(\omega^2+1)}$$

$$= -5/3 \frac{1}{\omega^2+4} + 8/3 \frac{1}{\omega^2+1}$$

$$R_{XX}(\tau) = F^{-1}[S_{XX}(\omega)]$$

$$= -\frac{5}{3} F^{-1}\left[\frac{1}{\omega^2+4}\right] + \frac{8}{3} F^{-1}\left[\frac{1}{\omega^2+1}\right]$$

$$= -\frac{5}{3}\left[\frac{1}{4} e^{-2|\tau|}\right] + 8/6\left[e^{-|\tau|}\right]$$

$$= -\frac{5}{12} e^{-2|\tau|} + 8/6\, e^{-|\tau|}$$

The mean square value $= R_{XX}(0)$

$$= -\frac{5}{12} + 8/6$$

$$\boxed{= 11/12}$$

15)

b) i) Soln:

$$S_{yy}(\omega) = S_{XX}(\omega)\,|H(\omega)|^2 \longrightarrow ①$$

$$H(\omega) = F[h(t)]$$

$$= \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\, dt$$

$$= \int_{0}^{T} {}^{1}/_{T}\, e^{-j\omega t}\, dt$$

$$= {}^{1}/_{T} \left(\frac{e^{-i\omega t}}{-i\omega} \right)_0^T$$

$$= {}^{1}/_{T} \left(\frac{e^{i\omega t} - 0}{-i\omega} \right)$$

$$= \frac{1}{Ti\omega} [1 - (\cos \omega T - i \sin \omega T)]$$

$$= \frac{1}{Ti\omega} [(1 - \cos \omega T) + i \sin \omega T]$$

$$= \frac{1}{T\omega} [-i(1 - \cos \omega T) - i^2 \sin \omega T]$$

$$H(\omega) = \frac{1}{T\omega} [\sin \omega T - i(1 - \cos \omega T)]$$

$$\overline{H(w)} = \frac{1}{Tw}\left[\sin wT + i(1-\cos wT)\right]$$

$$|H(w)|^2 = H(w)\,\overline{H(w)}$$

$$= \frac{1}{T^2w^2}\left[\sin^2 wT - i^2(1-\cos wT)^2\right]$$

$$= \frac{1}{T^2w^2}\left[\sin^2 wT + 1 + \cos^2 wT - 2\cos wT\right]$$

$$= \frac{1}{T^2w^2}\left[1+1-2\cos wT\right]$$

$$= \frac{1}{T^2w^2}\left[2-2\cos wT\right]$$

$$= \frac{2}{T^2w^2}\left[1-\cos wT\right]$$

$$= \frac{2}{T^2w^2}\left[2\sin^2(wT/2)\right]$$

$$|H(w)|^2 = \frac{4}{T^2w^2}\sin^2(wT/2)$$

$$① \Rightarrow S_{yy}(w) = \frac{4}{T^2w^2}\sin^2(wT/2)\cdot S_{xx}(w)$$

15) b)

ii) soln:

To find the spectral density of $y(t)$ first we will find the autocorrelation function of $y(t)$.

$$y(t_1)y(t_2) = \{A\cos(w_0t_1+\theta)+N(t_1)\}\{A\cos(w_0t_2+\theta)+N(t_2)\}$$

$$= A^2\cos(w_0t_1+\theta)\cos(w_0t_2+\theta) + A[N(t_1)\cos(w_0t_2+\theta)+N(t_2)\cos(w_0t_1+\theta)] + N(t_1)N(t_2)$$

$$\therefore R_{yy}(t_1,t_2) = E(y(t_1)y(t_2))$$

$$= E[A^2\cos(w_0t_1+\theta)\cos(w_0t_2+\theta) + AN(t_1)\cos(w_0t_2+\theta) + AN(t_2)\cos(w_0t_1+\theta)+N(t_1)N(t_2)]$$

$$= A^2E[\cos(w_0t_1+\theta)\cos(w_0t_2+\theta)] + AE[N(t_1)\cos(w_0t_2+\theta)] + AE[N(t_2)\cos(w_0t_1+\theta)] + E[N(t_1)N(t_2)]$$

$$= \frac{A^2}{2}E[\cos\{w_0t_1+\theta-(w_0t_2+\theta)\} + \cos(w_0t_1+\theta+w_0t_2+\theta)] + AE[N(t_1)]E[\cos(w_0t_2+\theta)] + AE[N(t_2)]E[\cos(w_0t_1+\theta)] + E[N(t_1)N(t_2)]$$

$$\left[\because \cos A \cos B = 1/2 \cos(A-B) + \cos(A+B)\right]$$

$$= \frac{A^2}{2}\left[E\{\cos w_0(t_1-t_2)\} + E\{\cos(w_0(t_1+t_2)+2\theta)\}\right] +$$

$$AE[N(t_1)] E[\cos(w_0 t_2+\theta)] +$$

$$AE[N(t_2) E[\cos(w_0 t_1+\theta)] + E[N(t_1)N(t_2)]$$

$$= \frac{A^2}{2} E\{\cos w_0(t_1-t_2)\} + \frac{A^2}{2} E[\cos(w_0(t_1+t_2)+2\theta)]$$

$$+ AE[N(t_1) E[\cos(w_0 t_2+\theta)] +$$

$$AE[N(t_2) E[\cos(w_0 t_1+\theta) + E[N(t_1) N(t_2)] \rightarrow (1)$$

consider,

$$E[\cos\{w_0(t_1+t_2)+2\theta\}] = \int_{-\infty}^{\infty} \cos\{w_0(t_1+t_2)+2\theta\} f(\theta)\, d\theta$$

$$f(\theta) = \begin{cases} \frac{1}{2\pi - 0}, & 0<\theta<2\pi \end{cases}$$

$$= \begin{cases} 1/2\pi, & 0<\theta<2\pi \\ 0, & \text{otherwise} \end{cases}$$

Hence, $E[\cos\{w_0(t_1+t_2)+2\theta\}]$

$$= \int_0^{2\pi} \cos\{w_0(t_1+t_2)+2\theta\}\, 1/2\pi \, d\theta$$

$$= 1/2\pi \left[\frac{\sin\{w_0(t_1+t_2)+2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\sin\{w_0(t_1+t_2)\} - \sin\{w_0(t_1+t_2)\}\right]$$

$$= 0$$

similarly,

$$E[\cos(w_0t_1+\theta)] = E[\cos(w_0t_2+\theta)] = 0 \longrightarrow (3)$$

Hence (2) becomes,

$$R_{YY}(t_1,t_2) = \frac{A^2}{2}\cos[w_0(t_1-t_2)] + 0 + 0 + 0 + R_{NN}(t_1,t_2)$$

$$\left[\because E[\cos(w_0(t_1-t_2))] = \cos\{w_0(t_1-t_2)\}\right]$$

$$= \frac{A^2}{2}\cos[w_0(t_1-t_2)] + R_{NN}(t_1,t_2)$$

$$R_{YY}(t_1,t_2) = \frac{A^2}{2}\cos(w_0\tau) + R_{NN}(\tau), \quad [t_1-t_2=\tau]$$

$$\therefore S_{YY}(\omega) = \int_{-\infty}^{\infty} R_{YY}(\tau)e^{-i\omega\tau}\,d\tau + \int_{-\infty}^{\infty} R_{NN}(\tau)e^{-i\omega\tau}\,d\tau$$

$$= \frac{A^2}{2}\int_{-\infty}^{\infty}\cos(w_0\tau)e^{-i\omega\tau}\,d\tau + S_{NN}(\omega)$$

$$= \frac{A^2}{2}\left[\frac{1}{2}\int_{-\infty}^{\infty}[e^{-i(\omega-\omega_0)\tau} + e^{-i(\omega+\omega_0)\tau} d\tau\right] + S_{NN}(\omega)$$

$$= \frac{A^2}{2}\left[\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0) + S_{NN}(\omega)\right]$$

$$\left[\because \int_{-\infty}^{\infty} e^{i\omega\tau} d\tau = 2\pi\delta(\omega)\right]$$

where $S_{NN}(\omega)$ is the spectral density function corresponding to $R_{NN}(\tau)$

11) a)

i) Soln:

Given:

Gamma distribution

$$\boxed{P(x) = \frac{e^{-x}.x^{\lambda-1}}{\lambda!}} \longrightarrow (1)$$

i) To find moment generating function

$$M_X(t) = E(e^{tx})$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\,dx \quad \text{(by (1))}$$

$$= \int_{0}^{\infty} e^{tx}\left[\frac{e^{-x}.x^{\lambda-1}}{\lambda!}\right] dx$$

$$M_X(t) = \frac{1}{\lambda!}\int_{0}^{\infty} (e^{tx}.e^{-x}.x^{\lambda-1})\,dx$$

$$\boxed{M_X(t) = \frac{1}{\lambda!}\int_{0}^{\infty} (e^{-x(1-t)}.x^{\lambda-1}).\,dx} \longrightarrow (2)$$

Now,

$$\boxed{u = x(1-t)}$$

Diff. w.r to x,

$$\frac{du}{dx} = 1(1-t)$$

$$\boxed{\frac{du}{1-t} = dx}$$

$u = x(1-t)$

$$\boxed{\frac{u}{1-t} = x}$$

using this in ②,

$$m_X(t) = \frac{1}{\lambda!}\int_0^\infty (e^{-x(1-t)} \cdot x^{\lambda-1})\,dx$$

$$= \frac{1}{\lambda!}\int_0^\infty e^{-u}\left(\frac{u}{1-t}\right)^{\lambda-1} \cdot \left(\frac{du}{1-t}\right)$$

$$m_X(t) = \frac{1}{\lambda!}\int_0^\infty e^{-u} \cdot \frac{u^{\lambda-1}}{(1-t)^{\lambda-1}} \cdot \frac{du}{(1-t)^1}$$

$$= \frac{1}{\lambda!}\int_0^\infty \frac{e^{-u} \cdot u^{\lambda-1} \cdot du}{(1-t)^\lambda}$$

$$= \frac{1}{\lambda!(1-t)^\lambda}\int_0^\infty e^{-u} \cdot u^{\lambda-1}\,du$$

$$\left[\int_0^\infty e^{-u} \cdot u^{\lambda-1}\,du = \lambda!\right]$$

$$= \frac{1}{\lambda!\,(1-t)^\lambda}[\lambda!]$$

$$= \frac{1}{(1-t)^\lambda} = (1-t)^{-\lambda}$$

$$\boxed{M.G.F = (1-t)^{-\lambda}}$$

ii) To find mean (or) E(X)

$$\boxed{M_X(t) = (1-t)^{-\lambda}}$$

D.w.r to 't'

$$M_X'(t) = -\lambda(1-t)^{-\lambda-1}(0-1)$$

$$\boxed{M_X'(t) = \lambda(1-t)^{-\lambda-1}} \longrightarrow (3)$$

put $t=0$,

$$M_X'(0) = \lambda(1-0)^{-\lambda-1} = \lambda(1)$$

$$M_X'(0) = \lambda$$

$$\boxed{\text{Mean} = E(X) = \lambda}$$

Now,

From eqn (3),

$$\boxed{M_X'(t) = \lambda(1-t)^{-\lambda-1}}$$

Again D.w.r to t

$$M_X''(t) = \lambda(-\lambda-1)(1-t)^{-\lambda-1-1}\cdot(0-1)$$

$$M_X''(t) = \lambda(\lambda+1)(1-t)^{-\lambda-2}$$

Put $t = 0$,

$$M_X''(0) = \lambda(\lambda+1)(1-0)^{-\lambda-2}$$

$$M_X''(0) = \lambda(\lambda+1)(1)$$

$$\boxed{E(X^2) = \lambda(\lambda+1)}$$

iii) To find Variance (or) σ^2.

$$\boxed{Var(X) = E(X^2) - [E(x)]^2}$$

$$Var(X) = \lambda(\lambda+1) - (\lambda)^2$$

$$= \lambda(\lambda+1) - \lambda^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$\boxed{Var(X) = \lambda}$$

Result:

Gamma distribution

M·G·F	Mean	Variance
$(1-t)^{-\lambda}$	$E(x) = \lambda$	$\sigma^2 = \lambda$ (or) $Var(X) = \lambda$

11) a)
ii)

$$f(R) = \begin{cases} \frac{1}{4-2} = 1/2 & \text{if } 2 < R < 4 \\ 0 & , \text{ otherwise} \end{cases}$$

$$P(2<R<4) = \int_2^4 f(x)\,dx$$

since we have $2<R<4$,

we substitute $f(x) = 1/2$, Thus

$$P(2<R<4) = \int_2^4 1/2 \, dx$$

$$= 1/2 \int_2^4 dx$$

$$= 1/2 \, [x]_2^4$$

$$= 1/2 \, (4-2)$$

$$= 1/2 \, (2)$$

$$P(2<R<4) = 1$$

~~Part-A~~

Part -A

10). Soln:

convolution $y(t) = h(t) * x(t)$

$$= \int_{-\infty}^{\infty} h(u)\,(t-u)\,du$$

$$E[y(t)] = E\left[\int_{-\infty}^{\infty} h(u)\,x(t-u)\,du\right]$$

$$= \int_{-\infty}^{\infty} h(u)\,E[x(t-u)]\,du$$

$$E[x(t-u)] = \mu x$$

$$\therefore E[y(t)] = \int_{-\infty}^{\infty} h(u)\,\mu x\,du$$

$$= \mu x \int_{-\infty}^{\infty} h(u)\,du$$

$$H(\mu) = \int_{-\infty}^{\infty} h(t)\,dt$$

$$H(0) = \int_{-\infty}^{\infty} h(t)\,dt = \int_{-\infty}^{\infty} h(u)\,du$$

$$\therefore E[y(t)] = \mu x\; H(0).$$