Design Basis Report: Phase-Field Simulation of a Double Cantilever Beam Test on Soda-Lime Glass

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Abstract

This report presents a detailed setup for conducting a phase-field simulation of a double cantilever beam (DCB) test using a soda-lime glass specimen. The main objective is to accurately model the propagation of fractures under specified loading conditions. This document thoroughly describes the specimen's geometry, the material properties of soda-lime glass, the applied boundary conditions, and the governing equations for the phase-field fracture model. The results of the simulation will be compared with analytical models (AT) and established reference data to validate the model's predictive accuracy regarding brittle fracture.

Keywords: Double Cantilever Beam, Soda-Lime Glass, Phase-Field Model, Fracture Propagation, Finite Element Method, Brittle Fracture

1. Introduction and Objective

TThis report outlines the complete setup for a computational simulation of a double cantilever beam (DCB) test conducted on a soda-lime glass specimen. The primary objective is to model the stable growth of a pre-existing crack under controlled displacement conditions. The simulation utilizes a phase-field approach to predict the force-displacement response and the crack propagation path. These predictions are then compared with exact analytical solutions to evaluate the accuracy of the model.

2. Geometry and Dimensions

The test specimen is a prismatic bar with a pre-existing edge crack of length A. The arms of the beam are pulled apart via pinholes. The key geometric parameters are summarized in Table 1 and illustrated in Figure 1.

Table 1: Geometric Specifications of the DCB Specimen

Parameter	Symbol	Value	Unit
Length	L	55	mm
Height	H	20	mm
Breadth	B	2.5	mm
Initial Crack	A	25	mm
Pinhole Radius	r	0.3125	mm

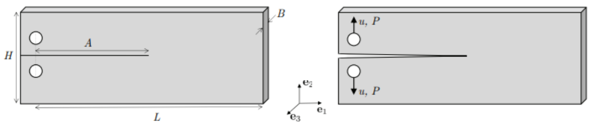

Figure 1: Schematic of the double cantilever beam test specimen with dimensions and loading configuration.

3. Material Properties: Soda-Lime Glass

The specimen is made of soda-lime glass, treated as an isotropic linear elastic brittle material. A complete simulation requires the mechanical and fracture properties detailed in Table 2.

Table 2: Material Properties of Soda-Lime Glass

Property	Symbol	Value	Unit
Young's Modulus	E	72	GPa
Poisson's Ratio	ν	0.22	–
Fracture Toughness	G_c	3.7	J/m^2

4. Boundary and Loading Conditions

The simulation replicates the physical DCB test with the following conditions:

- **Applied Displacement:** A displacement, denoted by u, is applied through the pinholes in the direction e_2, pulling the arms of the beam apart.
- **Symmetry:** To reduce computational cost, symmetry is exploited, and only a quarter of the specimen is simulated.

- **Crack Growth:** As u increases, the beam arms bend until a critical value, u_{cr}, is reached. At this point, the crack propagates straight ahead in the e_1 direction.

5. Simulation and Solver Parameters

The analysis is performed using the Finite Element Method (FEM) coupled with a phase-field model for fracture.

- **Solver:** An unstructured FE mesh is used to solve the governing equations for elasticity and phase-field damage.

- **Element Size:** The mesh is refined to an element size of $h = 0.05\,\text{mm}$ ahead of the crack front to accurately resolve the fracture process.

- **Material Model:** A phase-field model regularizes the sharp crack interface over a small length scale, defined by the regularization length ε. For soda-lime glass, the simulation is conducted with $\varepsilon = 0.16\,\text{mm}$.

- **Governing Equations:** The global force-displacement can be written as:

$$P = \begin{cases} \frac{BH^{3+\alpha_1}\mu E}{32H\alpha_1\mu a^3 + 3\cdot 2^{3+\alpha_1}\alpha_2 H^2\mu a^{1+\alpha_1} + 6\alpha_3 H^{2+\alpha_1} Ea} u & \text{if } 0 \le u < u_{cr} \\ \frac{BH^{3+\alpha_1}\mu E}{32H\alpha_1\mu a^3 + 3\cdot 2^{3+\alpha_1}\alpha_2 H^2\mu a^{1+\alpha_1} + 6\alpha_3 H^{2+\alpha_1} Ea} u & \text{if } u_{cr} \le u \end{cases} \tag{1}$$

where the constants are approximately $\alpha_1 \approx 1.03$, $\alpha_2 \approx 0.324$, and $\alpha_3 \approx 2.277$.

6. Expected and Reference Results

The simulation aims to predict the force-displacement response, crack length evolution, and the final fracture pattern.

6.1. Force-Displacement and Crack Growth (2D Plots)

The relationship between the resultant force P and displacement u, and the evolution of crack length a versus u are analyzed.

- **Expected Result:** The force is expected to increase linearly with displacement until a critical point, after which it will decrease as the crack propagates. The crack length should remain constant at 25 mm before increasing steadily.
- **Reference Result:** As shown in Figure 2, the phase-field model's predictions for both force-displacement and crack evolution show excellent agreement with the exact analytical solutions (labeled "Sharp").

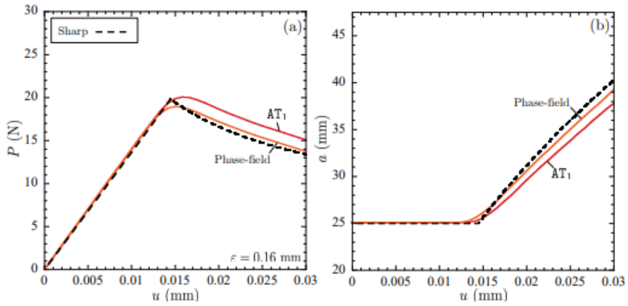

Figure 2: Comparison between exact results (Sharp) and predictions from AT and phase-field models for (a) force-displacement response and (b) crack length evolution.

6.2. Fracture Pattern (3D Plot)

A 3D contour plot of the phase-field variable v (where $v = 1$ indicates a fully cracked region) is generated on the deformed specimen geometry at a displacement of $u = 0.03\,\text{mm}$.

- **Expected Result:** The fracture is expected to be a straight crack propagating along the center-line of the beam, in the e_1 direction.
- **Reference Result:** As seen in Figure 3, the phase-field model accurately predicts a sharp, straight crack path, which is consistent with the physical behavior of brittle fracture in a DCB test.

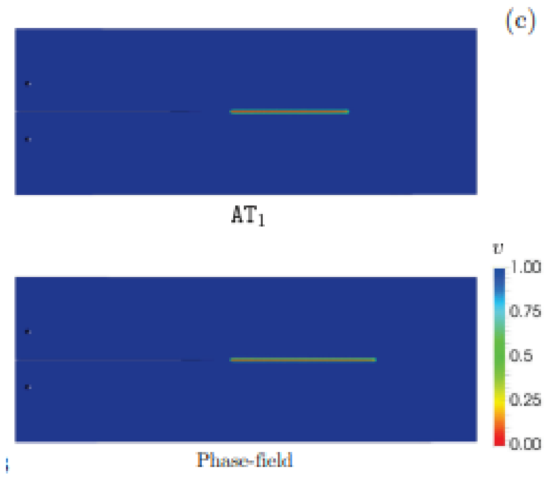

Figure 3: Contour plots of the phase field v over the deformed configuration at $u = 0.03\,\text{mm}$, as predicted by the AT and phase-field models.