Design Basis Report: Phase-Field Simulation of a Torsion Test on a Soda-Lime Glass Tube

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Abstract

This report provides a comprehensive Design Basis Report (DBR) for the numerical simulation of a torsion test on a thin-walled soda-lime glass tube. The objective is to replicate the fracture behavior using a phase-field model. The report outlines the specimen's geometry, the material properties of soda-lime glass, boundary and loading conditions, and the governing equations for stress and strain. Additionally, it specifies the solver parameters, meshing strategy, fracture model parameters, and the expected outputs for both 2D and 3D plots. The tube has an initial length of 5 mm, an inner radius of 2.85 mm, and an outer radius of 3 mm. The simulation aims to predict the stress-strain response and the crack nucleation pattern, with results benchmarked against provided reference data.

Keywords: Torsion Test, Soda-Lime Glass, Phase-Field Model, Fracture Mechanics, Finite Element Analysis, Thin-Walled Tube

1. Objective

The primary objective of this report is to provide a clear and comprehensive framework for simulating a torsion test on a thin-walled soda-lime glass tube. It details all the necessary parameters needed to accurately replicate the simulation, which aims to predict the stress-strain response and fracture behavior of the tube using a phase-field modeling approach..

2. Geometry and Dimensions

The test specimen is a thin-walled circular tube. The geometric parameters are specified in Table 1 and visualized in Figure 1. The wall thickness is $t = B - A = 0.15$ mm.

Table 1: Geometric Specifications of the Circular Tube

Parameter	Symbol	Value	Unit
Initial Length	L	5.00	mm
Inner Radius	A	2.85	mm
Outer Radius	B	3.00	mm

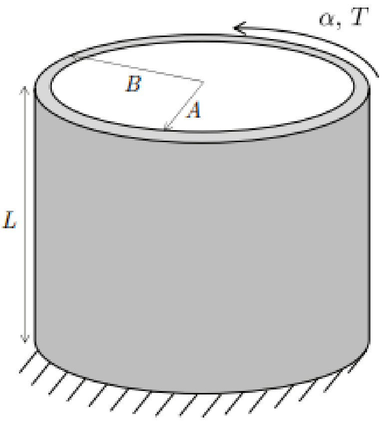

Figure 1: Schematic of the torsion test setup, showing the tube dimensions and applied twist α.

3. Material Properties: Soda-Lime Glass

The tube is fabricated from soda-lime glass. The mechanical properties used for the linear elastic and fracture simulation are detailed in Table 2. The Shear Modulus (G) is derived from Young's Modulus (E) and Poisson's Ratio (ν) via the relation $G = E/(2(1 + \nu))$.

Table 2: Material Properties of Soda-Lime Glass

Property	Symbol	Value	Unit
Young's Modulus	E	70	GPa
Poisson's Ratio	ν	0.22	
Shear Modulus	G	28.69	GPa
Shear Strength	s_{ss}	58	MPa

4. Boundary and Loading Conditions

The simulation replicates a standard torsion test setup:

- **Fixed End:** One end of the tube is fully constrained, preventing all translations and rotations.

- **Twisted End:** The opposite end is subjected to a prescribed small angle of twist, α, around the tube's central axis (z-axis). This loading induces a state of nearly uniform shear stress throughout the thin wall.

5. Formulation and Governing Equations

Due to the tube's thin wall, the stress and strain are considered approximately uniform.

5.1. Strain and Stress Tensors

The Green-Lagrange strain tensor $\mathbf{E}$ is given by:

$$\mathbf{E} = \gamma(\mathbf{e}_\theta \otimes \mathbf{e}_z + \mathbf{e}_z \otimes \mathbf{e}_\theta) \tag{1}$$

where the average shear strain γ is defined as:

$$\gamma = \frac{\alpha(A+B)}{2L} \tag{2}$$

The corresponding Cauchy stress tensor $\mathbf{S}$ is:

$$\mathbf{S} = \tau(\mathbf{e}_\theta \otimes \mathbf{e}_z + \mathbf{e}_z \otimes \mathbf{e}_\theta) \tag{3}$$

where τ is the shear stress.

5.2. Stress-Strain Relationship

For soda-lime glass, the shear stress S is linearly proportional to the applied twist α until fracture:

$$S = \begin{cases} \mu\left(\frac{\alpha(A+B)}{2L}\right) & \text{if } \alpha < \alpha_{ss} \\ 0 & \text{if } \alpha \geq \alpha_{ss} \end{cases} \tag{4}$$

where μ is the shear modulus (G) and α_{ss} is the angle of twist at which S reaches the material's shear strength s_{ss}.

6. Numerical Simulation Setup

6.1. Solver and Material Model

- **Analysis Type:** Phase-field fracture simulation.

- **Material Model:** Linear elastic behavior coupled with a phase-field model to govern fracture initiation and propagation.

6.2. Mesh and Element Size

- **Mesh Type:** Unstructured Finite Element (FE) mesh.

- **Element Size (h):** 0.015 mm. A fine mesh is crucial for resolving the length scale of the phase-field.

6.3. Fracture Parameters

- **Fracture Model:** Phase-field approach.

- **Regularization Length (ε):** This parameter regularizes the sharp crack topology. The simulation is evaluated for three values: 0.16 mm, 0.08 mm, and 0.016 mm.

- **Critical Shear Stress (AT_1 Model):** For comparison, the AT_1 model predicts fracture at a critical stress $s_{ss}^{\mathrm{AT}_1}$, which is dependent on ε:

$$s_{ss}^{\mathrm{AT}_1} = \sqrt{\frac{3GE}{16(1+\nu)\varepsilon}} \tag{5}$$

This model is noted to be inconsistent with experimental observations for larger ε.

7. Expected Results and Outputs

7.1. 2D Plots: Stress-Strain Response

A 2D plot of shear stress S (MPa) versus the normalized angle of twist $\alpha(A+B)/(4L)$ is required. The plot should show a linear response terminating at fracture. The results are to be compared with the analytical solution as shown in figure and the reference plots in the source document.

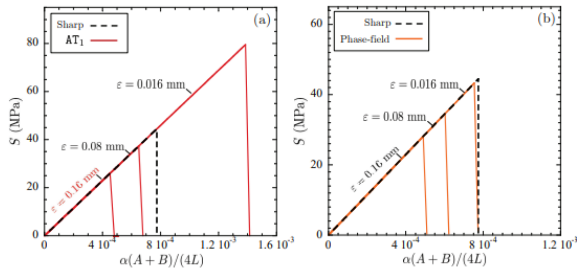

Figure 2: Torsion test of a soda-lime glass thin-walled circular tube. Comparisons between the exact result (10) for the stress-strain response of the tube and the predictions by (a) the AT1 and (b) the phase-field models for three different values of the regularization length .

7.2. 3D Plots: Phase-Field Contour

A 3D contour plot of the phase-field variable v (where $v = 0$ is undamaged and $v = 1$ is fully cracked) on the undeformed geometry .

- **Expected Outcome:** For a sufficiently small ε (e.g., 0.016 mm), the model should predict crack nucleation at a 45° angle to the tube's axis, corresponding to the plane of maximum principal tensile stress. This contrasts with the AT_1 model, which incorrectly predicts a crack perpendicular to the twist direction.

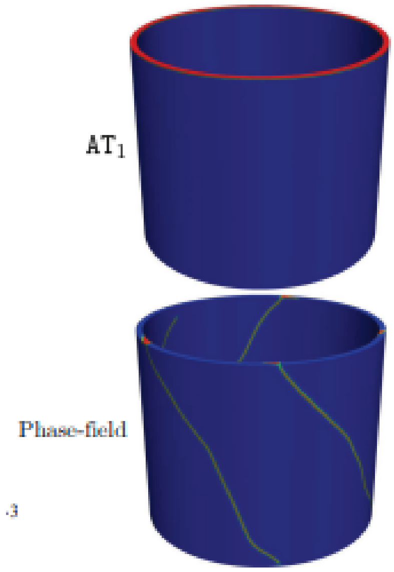

Figure 3: Contour plots of the phase field v over the undeformed configuration of the tube right after fracture nucleation, as predicted by the AT1 and phase-field models for = 0.016 mm.

7.3. Reference Results

All simulation outputs should be validated against the results presented in Figure 8 of the source document, which compares the AT_1 and phase-field models against the exact solution and shows the final crack pattern.