**Indian Institute of Technology Roorkee**
**Spring Semester 2023-24**
**MAI-102 (Mathematics II)**
**Assignment 8**

*Topics:* ***Testing of Hypothesis:*** *Simple and composite hypothesis, Type I and Type II errors, power of a test. Hypothesis testing for mean, variance and proportion for normal populations.*

---

(1) A brochure inviting subscriptions for a new diet program states that the participants are expected to lose over 22 pounds in five weeks. Suppose that, from the data of the five-week weight losses of 56 participants, the sample mean and sample standard deviation are found to be 23.5 and 10.2, respectively. Could the statement in the brochure be substantiated on the basis of these findings? Test at the $\alpha = .05$ level.

(2) Test the null hypothesis $H_0 : \mu = 0.340$ against the alternative hypothesis $H_1 : \mu \neq 0.340$ with standard deviation 0.010 at 0.05 level of significance when a sample of size 35 is tested, giving mean 0.343.

(3) It is known from experience that the standard deviation of the weight of 8-ounce packages of cookies made by a certain bakery is 0.16 ounce. To check whether its production is under control on a given day, employees select random samples of 25 packages and find that their mean weight is 8.091 ounce. Since the bakery stands to lose money when $\mu > 8$ and the customers lose out when $\mu < 8$, test the null hypothesis $H_0 : \mu = 8$ against the alternative hypothesis $H_1 : \mu \neq 8$ at 0.01 level of significance.

(4) Test the null hypothesis $H_0 : \mu \geq 22000$ miles against the alternative hypothesis $H_1 : \mu < 22000$ miles at 0.05 level of significance, when the mean of 100 tyres made by a certain manufacturer lasted on an average 21819 miles with a standard deviation of 1295.

(5) A trucking firm suspects the claim that the average lifetime of certain tyres is at least $28,000$ miles. To check the claim, the firm puts 40 of these tyres on its trucks and gets a mean lifetime of $27,463$ miles with a standard deviation of $1,343$ miles. What can it conclude if the probability of type-I error is to be at most 0.01?

(6) The specifications for a certain kind of ribbon call for a mean breaking strength of 180 pounds. If five pieces of the ribbon have a mean breaking strength of 169.5 pounds with a standard deviation of 5.7 pounds, test the null hypothesis $H_0 : \mu = 180$ against the alternative hypothesis $H_1 : \mu < 180$ at the 0.01 level of significance.

(7) In 64 randomly selected hours of production, the mean and the standard deviation of the number of acceptable pieces produced by an automatic stamping machine are $1,038$ and 146 respectively. At the 0.05 level of significance, does this enable us to reject the null hypothesis $H_0 : \mu = 1,000$ against the alternative hypothesis $H_1 : \mu > 1,000$?

(8) Given the probability density function $f(x, \theta) = \theta e^{-\theta x},\ 0 \leq x < \infty,\ \theta > 0$. The null hypothesis $H_0 : \theta = 2$ against the alternative hypothesis $H_1 : \theta > 2$ will be tested on the following procedure. $H_0$ should be rejected if a sample $x$ drawn from the population is greater than or equal to 6. Find
 (i) the probability of Type-I error,
 (ii) the probability of Type-II error,
 (iii) the power of the test.

(9) Let $\{X_1, X_2\}$ be a random sample from a normal population with $\sigma^2 = 1$. If the null hypothesis $\mu = \mu_0$ is to be rejected in favour of the alternative hypothesis $\mu = \mu_1 > \mu_0$ when $\overline{x} > \mu_0 + 1$, what is the size of the critical region?

(10) A study of the number of business lunches that executives in the insurance and banking industries claim as deductible expenses per month was based on random samples and yielded the following results: $n_1 = 40, \overline{x}_1 = 9.1, s_1 = 1.9$ and $n_2 = 50, \overline{x}_2 = 8.0, s_2 = 2.1$.

Test the null hypothesis $\mu_1 - \mu_2 = 0$ against the alternative $\mu_1 - \mu_2 \neq 0$ at 0.05 level of significance.

(11) Following are the are the average weekly losses of work hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation: 45 and 36, 73 and 60, 46 and 44, 124 and 119, 33 and 35, 57 and 51, 83 and 77, 34 and 29, 26 and 24, 17 and 11. Test at 0.05 level of significance whether the safety program is effective.

(12) A food processor wants to know wether the probability $p$ is really 0.60 that a customer will prefer a new kind of packaging to the old kind. If, in a random sample, 7 of 18 customers prefer the new kind of packaging to the old kind, test the null hypothesis $p = 0.60$ against the alternative hypothesis $p \neq 0.60$ at 0.05 level of significance.

(13) The manufacturer of a spot remover claims that his product removes 90 percent of all spots. If, in a random sample, only 174 of 200 spots were removed with the manufacturer's product, test the null hypothesis $p = 0.90$ against the alternative hypothesis $p < 0.90$ at 0.05 level of significance, where $p$ represents the population proportion.

(14) A random sample of size 20 from a normal population gives a sample mean of 42 and a sample standard deviation of 6. Test the hypothesis that the population standard deviation is 9 at 5% significant level.

(15) Weights (in kgs) of 10 students are given below:

$$38, \ 40, \ 45, \ 53, \ 47, \ 43, \ 55, \ 48, \ 52, \ 49.$$

Can we say that the variance of distribution of weights of all students from which the above sample of 10 students was drawn, is greater than 20? (Test at 5% significance level).

(16) An engineer measured the Brinell hardness of 25 pieces of ductile iron that were subcritically annealed. The resulting data were:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 170 | 167 | 174 | 179 | 179 | 187 | 179 | 183 | 179 |
| 156 | 163 | 156 | 187 | 156 | 167 | 156 | 174 | 170 |
| 183 | 179 | 174 | 179 | 170 | 159 | 187 | | |

Is the engineer's claim that the mean Brinell hardness of all such ductile iron pieces is greater than 170, true? Test at 5% significance level assuming the population to be normal.

(17) A biologist was interested in determining whether sunflower seedlings treated with an extract from Vinca minor roots resulted in a lower average height of sunflower seedlings than the standard height of 15.7 cm. The biologist treated a random sample of $n = 30$ seedlings with the extract and subsequently obtained the following heights:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 11.5 | 11.8 | 15.7 | 16.1 | 14.1 | 10.5 | 09.3 | 15.0 |
| 15.2 | 15.1 | 12.8 | 12.4 | 19.2 | 13.5 | 12.2 | 13.3 |
| | 13.5 | 14.4 | 16.7 | 10.9 | 13.0 | 10.3 | 15.8 |
| | 17.1 | 13.3 | 12.4 | 08.5 | 14.3 | 12.9 | 13.5 |

Is the Biologist's claim that the mean height of the sunflower seedlings will be less than the standard height of 15.7, true? Test at 2%significance level assuming the population to be normal.

(18) A manufacturer claims that the thickness of the spearmint gum it produces is 7.5 one-hundredths of an inch. A quality control specialist regularly checks this claim. On one production run, he took a random sample of $n = 10$ pieces of gum and measured their thickness. He obtained:

| | | | | |
|---|---|---|---|---|
| 7.65 | 7.60 | 7.65 | 7.70 | 7.55 |
| 7.55 | 7.40 | 7.40 | 7.50 | 7.50 |

Is the manufacturer claims that the thickness of the spearmint gum it produces is 7.5 one-hundredths of an inch, correct? Test at 5% significance level assuming the population to be normal.

(19) The marks of 10-year old children in some test is known to have a standard deviation 5.2. If a random sample of size 20 shows a standard deviation of 5.8, test at 5% significance level, if the claim that the standard deviation of the marks is 5.2, true (assuming the population to be normal)?

(20) 11 measured values of a physical quantity have a standard deviation 0.14. Is the claim that the standard deviation of the population is greater than 0.1, true? Test at 10% significance level, assuming the population to be normal.

---

## ANSWERS

---

(1) We fail to reject the null hypothesis and cannot substantiate the brochure's claim based on these results.
(2) $H_0$ is accepted.
(3) $H_0$ is rejected.
(4) $H_0$ is accepted.
(5) $H_0$ is rejected.
(6) $H_0$ is rejected.
(7) $H_0$ is rejected.
(8) (i) $e^{-12}$, (ii) $1 - e^{-6\theta}(\theta > 2)$, (iii) $e^{-6\theta}(\theta > 2)$.
(9) 0.08.
(10) $H_0$ is rejected.
(11) $H_0$ is rejected (the safety program appears to be effective).
(12) $H_0$ accepted.
(13) $H_0$ is accepted.
(14) $H_0$ is rejected.
(15) $H_0$ is accepted.
(16) $H_0$ is accepted.
(17) $H_0$ is accepted.
(18) $H_0$ is accepted.
(19) $H_0$ is accepted.
(20) $H_0$ is accepted.