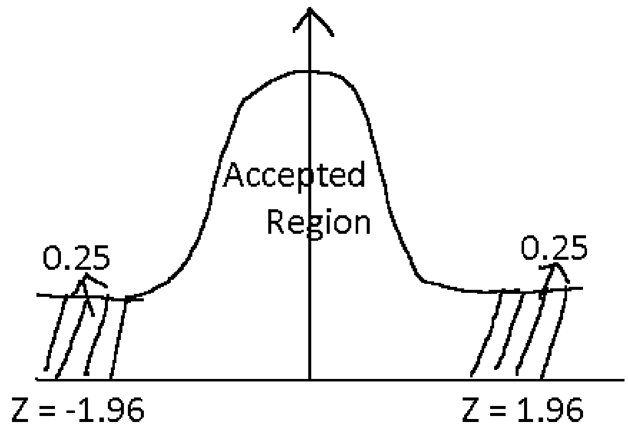
The critical region of a test of statistical hypothesis is that region of the normal curve which corresponds to the rejection of null hypothesis.

The shaded portion in the following figure is the critical region which corresponds to 5% LOS



Critical values (or) significant values

The sample values of the statistic beyond which the null hypothesis will be rejected are called critical values or significant values

	Level of significance		
Types of test	1%	5%	10%
Two tailed test	2.58	1.96	1.645
One tailed test	2.33	1.645	1.28

Two tailed test and one-tailed tests:

When two tails of the sampling distribution of the normal curve are used, the relevant test is called two tailed test.

The alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ is taken in two tailed test for $H_0 : \mu_1 = \mu_2$

When only one tail of the sampling distribution of the normal curve is used, the test is described as one tail test $H_1 : \mu_1 < \mu_2$ (or) $\mu_1 > \mu_2$

$$\left.\begin{array}{l} H_0 = \mu_1 = \mu_2 \\ H_1 = \mu_1 \neq \mu_2 \end{array}\right\} \text{two tailed test}$$

Type I and type II Error

Type I Error : Rejection of null hypothesis when it is correct

UNIT-II

DESIGN OF EXPERIMENTS

Analysis of variance:

The technique of analysis of variance is referred to as ANOVA. A table showing the source of variance, the sum of squares, degrees of freedom, mean squares(variance)and the formula for the "F ratio is known as ANOVA table"

The technique of analysis if variance can be classified as

(i) One way classification(CRD)
(ii) Two way classification(RBD)
(iii) Three way classification(LSD)

One way classification:

In one way classification the data are classified on the basic of one criterion

The following steps are involved in one criterion of classification

(i) The null hypothesis is

$$H_o : \mu_1 = \mu_2 = ... = \mu_k$$

$$H_1 : \mu_1 \neq \mu_2 \neq .... \neq \mu_k$$

(ii) Calculation of total variation

Total sum of squares $V = \sum_i \sum_j {x_{ij}}^2 - \frac{G^2}{N}$

Where $G = \sum_i \sum_j x_{ij}$ (Grand total)

$\frac{G^2}{N}$ =correction formula

(iii) Sum of squares between the variates

$$V_1 = \sum_i \left[\frac{T_i^2}{n_i}\right] - \frac{G^2}{N}$$ With k-1 degree of freedom

(iv) Sum of squares within samples

$$V_2 = V - V_1$$

then the ratio $\dfrac{\frac{V_1}{K-1}}{\frac{V_2}{N-K}}$ follows F-distribution with degrees of freedom. Choosing the ratio which is greater than one, we employ the F-test

If we calculated F < table value $F_{0.05}$, the null hypothesis is accepted.

ANOVA Table for one way classification

Source of variation	Sum of square	Degrees of freedom	Mean square	Variance ratio
Between classes	V_1	K-1	$\dfrac{V_1}{K-1}$	$\dfrac{\frac{V_1}{K-1}}{\frac{V_2}{N-K}}$ (or)
Within classes	V_2	$N-K$	$\dfrac{V_2}{N-K}$	$\dfrac{\frac{V_2}{N-K}}{\frac{V_1}{K-1}}$
	V	$N-1$		

1. To test the significance of the variation of the retail prices of a certain commodity in the four principal plates A,B,C &D, seven shops were chosen at random in each city and the prices observed were as follows (prices in paise)

A	82	79	73	69	69	63	61
B	84	82	80	79	76	68	62
C	88	84	80	68	68	66	66
D	79	77	76	74	72	68	64

Do the data indicate that the prices in the four cities are significantly different?

Solution:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_1 : \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$$

i.e., the prices of commodity in the four cities are same.

we take the origin at $x = 80$ and the calculation are done as follows.

Calculation of ANOVA (use new values)

Cities K=4	Shop(n = 7) 1	2	3	4	5	6	7	T_i	$\frac{T_i^2}{n}$	$\sum x^2$
A	2	-1	-7	-11	-11	-17	-19	-64	585.14	946
B	4	2	0	-1	-4	-12	-18	-29	120.14	505
C	8	4	0	-12	-12	-14	-14	-40	228.57	760
D	-1	-3	-4	-6	-8	-12	-16	-50	357.14	526
	$\frac{G^2}{N}=1196.03$							$G=-183$	$\frac{\sum T_i^2}{n}=1290.9$	$\sum_i\sum_j x_{ij}^2=2737$

Total sum of squares $V=\sum_i\sum_j x_{ij}^2-\frac{G^2}{N}$

$=2737-1196.03$

$V=1540.97$

Sum of squares between cities

$$V_1=\sum\frac{T_i^2}{n}-\frac{G^2}{N}$$

$= 1290.9 - 1196.03$

$$V_1=94.87$$

Sum of squares within cities

$$V_2=V-V_1=1540.97-94.87$$

$$V_2=1446.1$$

ANOVA Table:

Source of variation	Sum of square of deviation	Degrees of f	Mean square	F
Between cities	$V_1=94.87$	K-1=4-1=3	$\frac{V_1}{K-1}=\frac{94.87}{3}$ $=31.62$	$=\frac{60.25}{31.62}$ $=1.90$
Within cities	$V_2=1446.1$	N-K=28-4=24	$\frac{V_2}{N-K}=\frac{1446.1}{24}$ $=60.25$	
Total	V=1540.97	N-1=27		

Number of degrees of freedom = (N - K, K - 1) = (24,3)

Critical value:

The table value of F for (24, 3) degree of freedom at 5% Los is 8.64

Conclusion:

Since F< 8.64, H_o is accepted at 5% Los

$\therefore$ The prices of commodity in the four cities are same

2. Fill up the following Analysis of variance table

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F ratio
Treatments	-	-	117	-
Error	-	704	-	
Total	16	938		

Solution:

From the given table we have,

$$V_2 = 704; V = 938$$

degree of freedom (total) N - 1 = 16 => N = 17

mean squares $\frac{V_1}{K-1} = 117$

We Know that $V_2 = V - V_1$

=> $V_1 = V - V_2$

= 938 – 704

$\boxed{V_1 = 234}$

$$\frac{V_1}{K-1} = 117$$

$$=> \frac{234}{K-1} = 117 => \frac{234}{K-1} = K-1$$

K – 1 = 2

degree of freedom (K-1) = 2

=>K=3

Next, N-K = 17-3 = 4

$$\frac{V_2}{N\text{-}K} = \frac{938}{14} = 50.29$$

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F ratio
Treatments Error	K-1=3-1=2 N-K=17-3=14	$V_1 = 234$ $V_2 = 704$	$\frac{V_1}{K\text{-}1} = 117$ $\frac{V_2}{N\text{-}K} = 50.29$	$\frac{117}{50.29}$ = 2.327
Total	16	V = 938		

3. The following are the number of mistakes made in 5 successive days of 4 technicians working in a photographic laboratory

Technicians I	Technicians II	Technicians III	Technicians IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

Test at the 1% Los whether the difference among the 4 samples means can be attributed to chance

Solution:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

ie., There is no differences among the 4 samples mean

$$H_1 : \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$$

We take the origin at 12 and the calculation are done as follows

Calculation of ANOVA (NEW Values)

Technicians K = 4	Days(5)					T_i	$\frac{T_i^2}{n}$	$\sum x^2$
	1	2	3	4	5			
I	-6	2	-2	-4	-1	-11	24.2	61
II	2	-3	0	-2	2	-1	0.2	21
III	-2	0	-5	3	-1	-5	5	39
IV	-3	0	-4	-2	-1	-10	20	30
Total	$\frac{G^2}{N}=\frac{(-27)^2}{20}=36.45$					G=-27	49.4	151

Total sum of squares:

$$V=\sum_i\sum_j x_{ij}^2-\frac{G^2}{N}$$

$$=151-36.45$$

$$V=114.55$$

Sum of squares b/w cities:

$$V_1=\sum\frac{T_i^2}{n}-\frac{G^2}{N}$$

$$=49.4-36.45$$

$$V_1=12.95$$

Sum of squares within cities:

$$V_2=V\text{-}V_1=114.55-12.95$$

$$V_2=101.6$$

Source of variation	Sum of squares of deviation	Degrees of freedom	Mean squares	F ratio
B/W Technicians Within Technicians	$V_1 = 12.95$ $V_2 = 101.6$	K-1=4-1=3 N-K=20-4=16	$\frac{V_1}{K\text{-}1} = \frac{12.95}{3} = 4.31$ $\frac{V_2}{N\text{-}K} = \frac{101.6}{16} = 6.35$	$= \frac{6.35}{4.31}$
Total	V=114.55	N-1=19		$=1.473$

Degrees of freedom ((N - K, K - 1) = (16,3)

Critical value:

The table value of 'F' for (16,3) degree of freedom at 1% Los is 5.29

Conclusion:

Since F < 5.29, H_0 accepted at 1% level

$\therefore$ There is no difference among the four sample means.

4. The following table shows the lives in hours of four batches of electric lamps.

Batches	Lives in hours							
1	1610	1610	1650	1680	1700	1720	1800	
2	1580	1640	1640	1700	1750			
3	1460	1550	1600	1620	1640	1660	1740	1820
4	1510	1520	1530	1570	1600	1680		

Perform an analysis of the variance on these data and show that a significant test does not reject their homogeneity

Solution:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

I.e., the means of the lives of the four brands are homogeneous.

$$H_1 : \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$$

We take the origin $x_{ij} = \dfrac{\text{old}x_{ij} - 1700}{10}$

Calculation of ANOVA

Brand K=4	Lives								T_i	$\dfrac{T_i^2}{n}$	$\sum_{ij} x_{ij}$
	1	2	3	4	5	6	7	8			
1	-9	-9	-5	-2	0	2	10	-	-13	24.143	295
2	-12	-6	-6	0	5	-	-	-	-19	72.2	241
3	-24	-15	-10	-8	-6	-4	4	12	-51	325.125	1177
4	-19	-18	-17	-13	-10	-2	-	-	-79	1040.167	1247
Total	$\dfrac{G^2}{N} = \dfrac{(-162)^2}{26} = 1009.38$								G=-162	=1461.635	2960

$$N = n_1 + n_2 + n_3 + n_4 = 7 + 5 + 8 + 6 = 26$$

Total sum of squares:

$$V = \sum_i \sum_j (x_{ij})^2 - \frac{G^2}{N}$$

$$= 2960 - 1009.38$$

$$V = 1950.62$$

Total sum of squares b/w brands:

$$V_1 = \sum \frac{T_i^2}{n} - \frac{G^2}{N}$$

$$= 1461.635 - 1009.38$$

$$V_1 = 452.255$$

Sum of squares within brands:

$$V_2 = V - V_1$$

$$= 1950.62 - 452.255$$

$$V_2 = 1498.365$$

ANOVA Table:

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F ratio
B/W Brands Within Brands	$V_1 = 452.255$ $V_2 = 1498.365$	K-1=4-1=3 N-K=26-4=22	$\frac{V_1}{\text{K-1}} = \frac{452.255}{3} = 150.75$ $\frac{V_2}{\text{N-K}} = \frac{1498.365}{22} = 68.11$	$= \frac{150.75}{68.11}$ $= 2.21$
Total	$1950.62 = V$	N-1=25		

Degrees of freedom (3, 22) =3.05

Critical value:

The table value of 'F' for (3,22) d.f at 5% Los is 3.05

Conclusion:

Since F<3.05, H_0 is accepted at 5% level

$\therefore$ The means of the lives of the four brands are homogeneous.

ie., the lives of the four brands of lamps do not differ significantly.

Two way classification:

In two way classification the data are classified on the basis of two criterions

The following steps are involved in two criterion of classification

(i) The null hypothesis

H_{01} and H_{02} framed

We compute the estimates of variance as follows

(ii) $G=\sum_i \sum_j x_{ij} =$ Grand total of $K \times n$ Observations

(iii) S : Total sum of squares $\sum\sum x_{ij}^2 - \frac{G^2}{N}$

(iv) S_1 :Sum of squares b/w rows (class-B) $= \frac{1}{K}\sum_{j=1}^{n} R_j^{\,2} - \frac{G^2}{N}$

(v) S_2 :Sum of squares b/w (classes A) $= \frac{1}{n}\sum_{i=1}^{K} C_i^{\,2} - \frac{G^2}{N}$

S_3 : Sum of squares due to error (or) Residual sum of squares

(vi) Errors (or) Residual $S_3 = S - S_1 - S_2$

(vii) The degrees of freedoms of

$S_1 = n\text{-}1$; $S_2 = k\text{-}1$; $S_3 = (n\text{-}1)(k\text{-}1)$

S=nk-1

ANOVA Table for two way classification

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F ratio
B/W 'B' classes(rows)	S_1	n-1	$\frac{S_1}{n\text{-}1} = Q_B$	$F_1 = \frac{Q_B}{Q_{AB}}$ $d.f = [(n\text{-}1)(k\text{-}1)(n\text{-}1)]$
B/W 'A' classes(column)	S_2	k-1	$\frac{S_2}{k\text{-}1} = Q_A$	$F_2 = \frac{Q_A}{Q_{AB}}$
Residual (or) error	S_3	(n-1)(k-1)	$\frac{S_3}{(n\text{-}1)(k\text{-}1)} = Q_{AB}$	$d.f = [(k\text{-}1),(k\text{-}1)(n\text{-}1)]$
Total	S	nk-1	-	-

Advantages of R.B.D:

The chief advantages of R.B.D are as follows

(i) This design is more efficient or more accurate than CRD. This is because of reduction of experimental error.

(ii) The analysis of the design is simple and even with missing observations, it is not much complicated

(iii) It is Quite flexible, any number of treatments and any number of replication may be used

(iv) It is easily adaptable as in agricultural experiment it can be accommodated well in a rectangular, squares(or)in a field of any shape

(v) It provides a method of eliminating or reducing the long term effects.

(vi) This is the most popular design with experiments in view of its simplicity, flexibility and validity. No other has been used so frequently as the R.B.D

Disadvantages:

(i) The number of treatments is very large, than the side of the blocks will increase and this may introduce heterogeneity within blocks.

(ii) If the interactions are large, the experiments may yield misleading results.

1. The following data represent the number of units of production per day turned out by four randomly chosen operators using three milling machines

Machines.

		M_1	M_2	M_3
Operators	1	150	151	156
	2	147	159	155
	3	141	146	153
	4	154	152	159

Perform analysis of variance and test the hypothesis

(i) That the machines are not significantly different

(ii) That the operators are not significantly different at 5% level

Solution:

H_{01} : There is no significantly difference bet machine and

H_{02} : There is no significantly a difference b/w operator

We take the origin 155 and the calculations are done as follows.

Calculation of ANOVA (using new values)

Operators	Machines			Row total R_j	$\sum_j x_{ij}^2$
	M1	M2	M3		
1	-5	-4	1	-8	42
2	-8	4	0	-4	80
3	-14	-9	-2	-25	281
4	-1	-3	4	0	26
Column total C_i	-28	-12	3	$\boxed{-37}$	429
$\sum_i x_{ij}^2$	286	122	21	429	

Here N=12 ; G=-37

$$\text{Correction factor } \frac{G^2}{N} = \frac{(-37)^2}{12} = 114.08$$

Total sum of squares:

$$S=\sum_i \sum_j x_{ij}{}^2 - \frac{G^2}{N}$$

$$=429-114.08$$

$$=314.92$$

Sum of squares between operators:

$$S_1 = \sum_j \frac{R_j{}^2}{n_j} - \frac{G^2}{N}$$

$$=\frac{1}{3}[(-8)^2+(-4)^2+(-25)^2]-114.08$$

$$=235-114.08$$

$$=120.92$$

Sum of squares between machines:

$$S_2 = \sum_i \left(\frac{C_i{}^2}{n_i}\right) - \frac{G^2}{N}$$

$$=\frac{1}{4}\left[(-28)^2+(-12)^2+(3)^2\right]-114.08$$

$$=234.25-114.08$$

$$S_2 = 120.17$$

Residual sum of squares:

$$S_3 = S\text{-}S_1 - S_2$$

$$=314.92-120.92-120.17$$

$$=73.83$$

AVOVA Table for two way classification

Source of variation	Sum of squares	Degrees of freedom	Mean sum squares	F ratio
B/W operators	120.92	n-1=4-1=3	$Q_B = \frac{S_1}{n-1} = 40.31$	
B/W machines	120.17	k-1=3-1=2	$Q_A = \frac{S_2}{k\text{-}1}$ $= 60.09$	$\frac{40.31}{12.305} = 1.49$ $(3,6)$
Residual	73.83	(n-1)(k-1)=6	$Q_{AB} = \frac{S_3}{(k\text{-}1)(n\text{-}1)}$ $= 12.305$	$\frac{60.09}{12.305} = 4.88$ $(2,6)$
Total	314.92	nk-1=11		

Degrees of freedom $V_1 = 2; V_2 = 6$ (machines)

Degrees of freedom $V_1 = 3; V_2 = 6$ (operators)

Critical value:

(i) Machines

The table value of 'F' for (2,6) d.f at 5% Los is 5.14

(ii) Operators

The table value of 'F' for (3,6) d,f at 5% Los is 4.76

Conclusion:

(i) Operators

Since F< 4.76 , H_{02} is accepted at 5% level

$\therefore$ The operators are not significantly different

(ii) For Machines

Since F< 5.14, H_{01} is accepted at 5% level

$\therefore$ The machines are not significantly different

2. An experiment was designed to study then performance of four different detergents, the following "whiteness" readings were obtained with specially designed equipment for 12 loads of washing distributed over three different models of washing machines.

Machines / Detergents	1	2	3	Total
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

Looking on the detergents as treatment and the machines as blocks, obtain the appropriate analysis of variance table and test at 0.01 level of Significance whether there are differences in the detergents (or) in the washing machines

Solution:

H_{01} : There is no significant different b/w detergent

H_{02} : There is no significant different b/w washing machine

We take the origin is 50 and the calculation are done as follows.

Calculation of ANOVA (using new values)

Detergents	Washing machines			Row total R_j	$\sum_j x_{ij}^2$
	M1	M2	M3		
A	-5	-7	1	-11	75
B	-3	-4	2	-5	29
C	-2	0	5	3	29
D	-8	-13	-1	-22	234
Column total C_i	-18	-24	7	$\boxed{-35}$	367
$\sum_i x_{ij}^2$	102	234	31	367	

Here N=12; G=-35

Correction factor $\frac{G^2}{N} = \frac{(-35)^2}{12} = 102.08$

Total sum of squares: $S = \sum_i \sum_j x_{ij}^2 - \frac{G^2}{N}$

$$= 367 - 102.08$$

$$S = 264.92$$

Sum of squares b/w detergents: $S_1 = \sum_j \frac{R_j^{\,2}}{h_j} - \frac{G^2}{N}$

$$= \frac{1}{3}\left[(-11)^2 + (-5)^2 + (3)^2 + (-22)^2\right] - 102.08$$

$$= 213 - 102.08$$

$$S_1 = 110.92$$

Sum of squares between machines

$$S_2 = \sum_i \left(\frac{C_i^{\,2}}{n_i}\right) - \frac{G^2}{N}$$

$$= \frac{1}{4}\left((-18)^2 + (-24)^2 + (7)^2\right) - 102.08$$

$$= 237.25 - 102.08$$

$$S_2 = 135.17$$

Residual sum of squares $S_3 = S\text{-}S_1 - S_2$

$$= 264.92 - 110.92 - 135.17$$

$$S_3 = 18.83$$

ANOVA table for two way classification:

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F ratio
B/W detergents	$S_1 = 110.92$	n-1=4-1=3	$Q_B = \frac{S_1}{n-1} = \frac{110.92}{3}$ $= 36.97$	$\frac{Q_B}{Q_{AB}} = \frac{36.97}{3.14}$ $= 11.77$
B/W machines	$S_2 = 135.17$	k-1=3-1=2	$Q_A = \frac{S_2}{k-1} = \frac{135.17}{2}$ $= 67.59$	
Residual (or) Error	$S_3 = 18.83$	(n-1)(k-1)=6	$Q_{AB} = \frac{S_3}{(n-1)(k-1)} = \frac{18.83}{6}$ $= 3.14$	$\frac{Q_A}{Q_{AB}} = \frac{67.59}{3.14}$ $= 21.52$
Total	S=264.92	nk-1=11		

Degrees of freedom $V_1 = 2; V_2 = 6$ (machines)

Degrees of freedom $V_1 = 3; V_2 = 6$ (detergents)

Critical value:

(i) Detergents:

The table value of F for (3,6) degree of freedom at 1% Los is 9.78

(ii)Machines

The table value of F for (2,6) degree of freedom at 1% Los is 10.92

Conclusion:

(i) For detergents

Since F>9.78, H_{01} is rejected at 5% level

$\therefore$ The detergents are significantly different

(ii) For machines

Since F>10.92, H_{02} is rejected at 5% level

$\therefore$ The machines are significantly different

3. To study the performance of three detergents and three different water temperatures the following whiteness readings were obtained with specially designed equipment.

Water temp	Detergents A	Detergents B	Detergents C
Cold Water	57	55	67
Worm Water	49	52	68
Hot Water	54	46	58

Solution:

We set the null hypothesis

H_{01} : There is no significant different in the three varieties of detergents

H_{02} : There is no significant different in the water temperatures

We choose the origin at x=50

Water temp	Detergents			Row total R_j	$\sum_j x_{ij}^2$
	A	B	C		
Cold Water	7	5	17	29	363
Worm Water	-1	2	18	19	329
Hot Water	4	-4	8	8	96
Column total C_i	10	3	43	$\boxed{56}$	788
$\sum_i x_{ij}^2$	66	45	677	788	

Total sum of squares:

$$S=\sum_j \sum_i x_{ij}^2 - \frac{G^2}{N}$$

$$= 788 - \frac{(56)^2}{9} = 788 - 348.44$$

$$S=439.56$$

Sum of squares between detergents:

$$S_1 = \sum_i \frac{C_i^2}{n_i} - \frac{G^2}{N}$$

$$= \frac{1}{3}\left[(10)^2 + (3)^2 + (43)^2\right] - 348.44$$

$$= 652.67 - 348.44$$

$$S_1 = 304.23$$

Sum of squares b/w temperatures:

$$S_2 = \sum_j \frac{R_j^2}{n_j} - \frac{G^2}{N}$$

$$= \frac{1}{3}[1266] - 348.44$$

$$= 422 - 348.44$$

$$S_2 = 73.56$$

Error sum of squares:

$$S_3 = S\text{-}S_1 - S_2$$

$$= 439.56 - 304.23 - 73.56$$

$$S_3 = 61.77$$

ANOVA Table:

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F ratio
B/W detergents B/W temperatures	304.23 73.55	2 2	$\frac{304.23}{2}$ $= 152.11$ $\frac{73.56}{2}$ $= 36.78$	$\frac{152.11}{15.445}$ $= 9.848$ $(2, 4)$ $\frac{36.78}{15.445}$ $= 2.381$
Error	61.79	4	15.445	
Total	439.56	8		

Degrees of freedom (2,4) and (2,4)

Critical value:

The table value of F for (2,4) d.f at 5% Los is 6.94

Conclusion:

(i) For detergents:

Since F > 9.85, H_{01} is rejected at 5% Los

$\therefore$ There is a significant different between the three varieties detergents,

(iii) For water temperature

Since F<6.94, H_{02} is accepted at 5% Level

$\therefore$ There is no significant different in the water temperatures.

4. Four experiments determine the moisture content of samples of a powder, each man taking a sample from each of six consignments. These assignments are

Observer	Consignment					
	1	2	3	4	5	6
1	9	10	9	10	11	11
2	12	11	9	11	10	10
3	11	10	10	12	11	10
4	12	13	11	14	12	10

Perform an analysis if variance on these data and discuss whether there is any significant different b/w consignments (or) b/w observers.

Solution:

We formulate the hypothesis

H_{02} : There is no significant different b/w observer

H_{02} : There is no significant different b/w consignment

We take origin at x=11 and the calculations are done are as follows

Calculation ANOVA:

Observer	consignments						Rowtotal	$\sum_j x_{ij}^2$
	1	2	3	4	5	6	R_j	
1	-2	-1	-2	-1	0	0	-6	10
2	1	0	-2	0	-1	-1	-3	7
3	0	-1	-1	1	0	-1	-2	4
4	1	2	0	3	1	-1	6	16
Column total C_i	0	0	-5	3	0	-3	$\boxed{-5}$	37
$\sum_j x_{ij}^2$	6	6	9	11	2	3	37	

Total sum of squares $= \sum_{j}\sum_{i} x_{ij}^{2} - \frac{G^2}{N}$

$$S = 37 - \frac{(-5)^2}{24} = 35.96$$

Sum of squares b/w observers $= \sum \frac{(R_j)^2}{n_j} - \frac{G^2}{N}$

$$S_1 = \frac{1}{6}\left[(-6)^2 + (-3)^2 + (-2)^2 + (6)^2\right] - \frac{25}{24}$$

$$S_1 = 13.13$$

Sum of squares b/w consignments $= \sum\left(\frac{C_i^{\,2}}{n_i}\right) - \frac{G^2}{N}$

$$S_2 = \frac{1}{4}\left[(0+0+25+9+9)\right] - \frac{25}{24}$$

$$S_2 = 9.71$$

Error sum of squares $S_3 = S - S_1 - S_2$

$$= 35.96 - 13.13 - 9.71$$

$$S_3 = 13.12$$

Source of variation	Sum of squares	Degrees of freedom	Mean squares	'F' ratio
B/W Consignments	$S_1 = 9.71$	n-1=5	$\frac{9.71}{5} = 1.94$	$\frac{1.94}{0.87} = 2.23$ (5,15)
B/W observers	$S_2 = 13.13$	k-1=3	$\frac{13.13}{3} = 4.38$	$\frac{4.38}{0.87} = 5.03$
Error	$S_3 = 13.12$	(n-1)(k-1)=15	$\frac{13.12}{15} = 0.87$	(3,15)
Total	$S = 35.96$	nk-1=23		

Critical value:

(i) For consignments ,
The table value of 'F' for (5, 15) d.f at 5% Los is 2.90

(ii) For observers:
The table value of F for (3, 15) d,f at Los 3.29

Conclusion:

(i) For observers
Since F>3.29, H_{01} is rejected
Hence these is a difference between observers is significant

(ii) For consignment:
Since F<2.33, H_{02} is accepted
$\therefore\therefore$ There is no significant different b/w the consignments

LATIN SQUARES DESIGN:

A Latin squares is a squares arrangement of m-rows and m-columns such that each symbol appearly once and only once in each row and column.

In randomized block design the randomization is done within blocks the units in each block being relatively similar in L.S.D there are two restrictions

(i) The number of rows and columns are equal
(ii) Each treatment occurs once and only once in each row and column.

This design is a three way classification model analysis of variance

The following steps are involved in Latin square design

Correction factor $=\frac{G^2}{N}$; G -> Grand total

$$\text{S.S b/w rows}=S_a=\sum_{i=1}^{m}\frac{S_i^{\,2}}{m}-\text{C.F} \qquad \text{(S.S means Sum of Squares)}$$

$$\text{S.S b/w Columns}=S_b=\sum_{j=1}^{m}\frac{S_j^{\,2}}{m}-\frac{G^2}{N}|\text{C.F}$$

$$\text{S.S b/w Varieties}=S_c=\sum_{i=1}^{m}\frac{V_i^{\,2}}{m}-\text{C.F}$$

$$\left.\begin{matrix}\textit{Total sum of}\\ \textit{squares}\end{matrix}\right\} S=\sum_j\sum_i x_{ij}^{\;2}-C.F$$

$$\text{and } S_d = S - S_a - S_b - S_c$$

Here S_i=sum of i^{th} row

S_j=sum of j^{th} column

V_i=sum of i^{th} variety

ANOVA Table:

Source of variation	Sum of squares	Degrees of freedom	Mean squares	'F' ratio
B/W Rows	S_a	m-1	$\frac{S_a}{m-1} = R$	$\frac{R}{E}$ $[(m-1),(m-1)(m-2)]$
B/W Columns	S_b	m-1	$\frac{S_b}{m-1} = C$	$\frac{C}{E}$ $[(m-1),(m-1)(m-2)]$
B/W varieties	S_c	m-1	$\frac{S_c}{m-1} = V$	$\frac{V}{E}$ $[(m-1),(m-1)(m-2)]$
Error	S_d	(m-1)(m-2)	$\frac{S_d}{(m-1)(m-2)} = E$	
Total	S	$m^2 - 1$		

Comparison of LSD and RBD

(i) In LSD, the number of rows and number of columns are equal and hence the number of replication is equal to the number of treatments there is no such restriction in RBD

(ii) L.S.D is suitable for the case when the number of treatments is b/w 5 and 12 where as R.B.D can be used for any number of treatments and replications

(iii) The main advantage of L.S.D is that it removes the variations b/w rows and columns from that within the rows resulting in the reduction of experiment error to a large extent

(iv) The RBD can be performed equally on rectangular of square plots but for LSD, a mose (or) less a squares field is required due to (iii) LSD is preferred over RBD

Note: A 2×2 Latin Square Design is not possible. The degree of freedom for error in a $m\times m$ Latin squares design is (m-1)(m-2)

For m=2 the degree of freedom is 'o' and hence comparisons are not possible.

Hence a 2×2 LSD is not possible.

1. The following is the LSD layout of a design when 4 varieties of seeds are being tested set up the analysis of variance table and state four conclusion

A 105	B 95	C 125	D 115
C 115	D 125	A 105	B 105
D 115	C 95	B 105	A 115
B 95	A 135	D 95	C 115

Solution:

H: There is no significant difference

we take the origin as $u_{ij} = \dfrac{x_{ij} - 100}{5}$ and the calculations are done as follows

Varieties	Values				V_i
A	1	1	3	7	12
B	-1	1	-1	-1	0
C	5	3	-1	3	10
D	3	5	3	-1	10

Columns / Rows	C_1	C_2	C_3	C_4	Row total R_j	$\sum_i x_{ij}^2$
R_1	1	-1	5	3	8	36
R_2	3	5	1	1	10	36
R_3	3	-1	1	3	6	20
R_4	-1	7	-1	3	8	60
Columns total C_i	6	10	6	10	G=32	152
$\sum_j x_{ij}^2$	20	76	28	28	152	

G=32 N=16; $\sum_j \sum_i x_{ij}^2 = 152$

$$\text{C.F} = \frac{G^2}{N} = \frac{(+32)^2}{16} = 64$$

Total sum of squares $= \sum_{j}\sum_{i} x_{ij}^{2} - \frac{G^2}{N}$

$$= 152 - \frac{(32)^2}{16}$$

$$= 152 - 64$$

$$S = 88$$

Sum of squares b/w rows $= \frac{1}{4}\left[8^2 + 10^2 + 6^2 + 8^2\right] - 64$

$$= 66 - 64$$

$$S_a = 2$$

Sum of squares b/w columns $= \frac{1}{4}\left[6^2 + 10^2 + 6^2 + 10^2\right] - 64$

$$S_b = 68 - 64$$

$$S_b = 4$$

Sum of squares b/w Varieties $= \frac{1}{4}\left[12^2 + 0^2 + 10^2 + 10^2\right] - 64$

$$= 86 - 64$$

$$S_c = 22$$

Error sum of squares $S_d = S - S_a - S_b - S_c$

$$= 88 - 2 - 4 - 22$$

$$S_d = 60$$

ANOVA Table:

Source of variation	Sum of squares	Degrees of freedom	Mean sum of squares	'F' ratio
B/W rows	$S_a = 2$	m-1=4-1=3	$\frac{S_a}{\text{m-1}} = \frac{2}{3} = 0.67$	$\frac{0.67}{10} = 0.067$
B/W columns	$S_b = 4$	m-1=4-1=3	$\frac{S_b}{\text{m-1}} = \frac{4}{3} = 1.33$	$\frac{1.33}{10} = 0.133$
B/W varieties	$S_c = 22$	m-1=3	$\frac{S_c}{\text{m-1}} = \frac{22}{3} = 7.33$	$\frac{7.33}{10} = 0.733$
Error	$S_d = 60$	(m-1)(m-2) $=3\times 2=6$	$\frac{S_d}{\text{(m-1)(m-2)}} = 10$	-
Total	$S = 88$	$m^2 - 1 = 15$	-	-

Number of degrees of freedom $V_1 = 3$; $V_2 = 6$

Critical value:

The table value of F for (3, 6) d.f at 5% Los is 4.76

Conclusion:

Since F<4.76, for all the case.

$\therefore$ There is no significant difference for the varieties

2. Analyse the variance in the following Latin squares of fields (in keys) of paddy where A,B,C,D denote the difference methods of calculation

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C122	D123	B121	A122

Examine whether the different methods of cultivation have given significantly different fields.

Solution:

Re arrange the table in order

A121	A122	A120	A122
B122	B124	B119	B121
C123	C123	C121	C122
D122	D125	D120	D123

We take the origin 122 and the table is

Letter	Values				V_i total
A	-1	0	-2	0	-3
B	0	2	-3	-1	-2
C	1	1	-1	0	1
D	0	3	-2	1	2

Calculation of LSD:

Columns / Rows	1	2	3	4	Row total	$\sum_j x_{ij}^2$
1	0	-1	1	0	0	2
2	2	1	0	3	6	14
3	-2	-3	-2	-1	-8	18
4	0	1	-1	0	0	2
Columns total	0	-2	-2	2	$\boxed{-2}$	36
$\sum_i x_{ij}^2$	8	12	6	10	36	

Here N=16; G=-2

Correction factor = $\frac{G^2}{N} = \frac{4}{16} = 0.25$

Total sum of squares $S = \sum_i \sum_j x_{ij}^2 - \frac{G^2}{N}$

$$= 36 - 0.25$$

$$S = 35.75$$

Sum of squares b/w rows $S_a = \sum_{i=1}^{m} \frac{S_i^2}{m} - \frac{G^2}{N}$

$$= \frac{1}{4}\left[(6)^2 + (-8)^2\right] - 0.25$$

$$= 25 - 0.25$$

$$S_a = 24.75$$

Sum of squares b/w columns $S_b = \sum_{j=1}^{m} \frac{S_j^2}{m} - \frac{G^2}{N}$

$$= \frac{1}{4}\left[(0)^2 + (-2)^2 + (-2)^2 + (2)^2\right] - 0.25$$

$$S_b = 2.75$$

Sum of squares b/w varieties $S_c = \sum_{i=1}^{m} \frac{V_i^2}{m} - \frac{G^2}{N}$

$$= \frac{1}{4}\left[(-3)^2 + (-2)^2 + (1)^2 + (2)^2\right] - 0.25$$

$$= 4.5 - 0.25$$

$$S_c = 4.25$$

Error (or) Residual $S_d = S - S_a - S_b - S_c$

$$= 35.75 - 24.75 - 2.75 - 4.25$$

$$S_d = 4$$

LSD Table:

Source of variation	Sum of squares	Degrees of freedom	Mean sum of squares	'F' ratio
B/W rows	$S_a = 24.75$	m-1=3	$\frac{S_a}{m\text{-}1} = \frac{24.75}{3} = 8.25$	$\frac{8.25}{0.67} = 12.31$
B/W columns	$S_b = 2.75$	3	$\frac{S_b}{m\text{-}1} = \frac{2.75}{3} = 0.92$	$\frac{0.92}{0.67} = 1.37$
B/W varieties	$S_c = 4.25$	3	$\frac{S_c}{m\text{-}1} = \frac{4.25}{3} = 1.42$	$\frac{1.42}{0.67} = 2.12$
Error (or) Residual	$S_d = 4.0$	6=(m-1)(m-2)	$\frac{S_d}{(m\text{-}1)(m\text{-}2)} = 0.67$	
Total	$S = 35.75$	$m^2 - 1 = 8$		

Critical value:

The value of 'F' for (3,6) d.f at 5% Los is 4.76

Conclusion:

Since F<4.76, we accept the null hypothesis

∴ The difference between the methods of cultivation is not significant.

3. The following data resulted from an experiment to compare three burners A,B, and C,A Latin squares design was used as the tests were made on 3 engines and were spread over 3 days.

	Engine 1	Engine 2	Engine 3
Day 1	A 16	B 17	C 20
Day 2	B16	C 21	A 15
Day 3	C15	A 12	B 13

Test the hypothesis that there is no diff between the burners

Solution:

We take the origin x=15 and the calculation are done as follows

Re arrangement of given table is

A 16	B 17	C 20
A 15	B 16	C 21
A 12	B 13	C 15

Varieties	Values			V_i
A	1	0	-3	-2
B	2	1	-2	1
C	5	6	0	11

Calculation of LSD

Columns/ Rows	C_1	C_2	C_3	Row total	$\sum_j x_{ij}^2$
R_1	1	2	5	8	30
R_2	1	6	0	7	37
R_3	0	-3	-2	-5	13
Column total	2	5	3	10	80
$\sum_i x_{ij}^2$	2	49	29	80	

Here N=9; G=10

Correction Factor $=\frac{G^2}{N}=\frac{(10)^2}{9}=11.11$

Total sum of squares $S=\sum_{j}\sum_{i}x_{ij}^{2}-C.F$

$$=80-11.11$$

$$S=68.89$$

Sum of squares b/w Rows $S_a=\sum_{i=1}^{m}\frac{S_i^{2}}{m}-C.F$

$$=\frac{1}{3}[8^2+7^2+(-5)^2]-11.11$$

$$=46-11.11$$

$$S_a=34.89$$

Sum of squares b/w columns $S_b=\sum_{j=1}^{m}\frac{S_j^{2}}{m}-C.F$

$$=\frac{1}{3}[(2)^2+(5)^2+(3)^2]-11.11$$

$$=1.56$$

Sum of squares b/w varieties $S_c=\sum_{i=1}^{m}\frac{V_i^{2}}{m}-C.F$

$$=\frac{1}{3}[(-2)^2+1^2+11^2]-11.11$$

$$S_c=30.89$$

Error (or) Residual $S_d=S\text{-}S_a-S_b-S_c$

$$=68.89-34.89-1.56-30.89$$

$$S_d=1.55$$

Source of variation	Sum of squares	Degrees of freedom	Mean sum of squares	'F' ratio
B/W rows	$S_a = 34.89$	m-1=2	$\frac{S_a}{m-1} = \frac{34.89}{2}$ $= 17.445$	$\frac{17.445}{0.775} = 22.5$
B/W columns	$S_b = 1.56$	m-1=2	$\frac{S_b}{m-1} = \frac{1.56}{2}$ $= 0.78$	$\frac{0.78}{0.775} = 1.01$
B/W varieties	$S_c = 30.89$	m-1=2	$\frac{S_c}{m-1} = \frac{30.89}{2}$ $= 15.445$	$\frac{15.445}{0.775} = 19.93$
Error (or) Residual	$S_d = 1.55$	(m-1)(m-2)	S_d(m-1)(m-2) $= \frac{1.55}{2}$ $= 0.775$	
Total	$S = 68.89$	$m^2 - 1 = 8$		

Critical value:

The value of 'F' for (2,8) d.f at 5% Los is 4.46

Conclusion:

Since F> the table value for the burners

$\therefore$ There is a significant difference between the burners

and also F> tabulated F for columns the difference b/w the engines is not significant.

Homework:

1. Analyse the variance in the following LS:

B 20	C 17	D 25	A 34
A 23	D 21	C 15	B 24
D 24	A 26	B 21	C 19
C 26	B 23	A 27	D 22

\

2. Analyse the variance in the following LS:

A 8	C 18	B 9
C 9	B 18	A 16
B 11	A 10	C 20

Factorial Experiments

Definition 1:

A factorial experiment in which each of m factors at 'S' is called a symmetrical factorial experiment and is often known as S^m factorial design

Definition 2:

2^m - Factorial experiments means a symmetrical factorial experiments where each of the m-factors is at two levels

2^2 -a factorial experiment means a symmetrical experiment where each of the factors is at two levels

Note:

If the numbers of level of the different factors are equal the experiments is called as a symmetrical factorial experiment.

Uses advantages of factorial experiments:

(i) Factorial designs are widely used in experiments involving several factors where it is necessary
(ii) F.D allow effects of a factor to be estimated at several levels of the others, giving conclusions that are valid over a range of experimental conditions
(iii) The F.D are more efficient than one factor at a time experiments.
(iv) In F.D individual factorial effect is estimated with precision, as whole of the experiment is devoted to it.
(v) Factorial designs from the basis of other designs of considerable practical value.
(vi) F.D are widely used in research work. These design are used to apply the results over a wide range of conditions

2^2 -Factorial experiment:

A factorial design with two factors, each at two levels is called a 2^2 factorial design

Yates's notation:

The two factors are denoted by the letters A and B the letters 'a' and 'b' denote one of the two levels of each of the corresponding factors and this will be called the second level.

The first level of A and B is generally expressed by the absence of the corresponding letter in the treatment combinations. The four treatment combinations can be enumerated as follows.

Symbols used:

a_0b_0(or)1:Factors A and B both at first level

a_1a_0(or)a:A at second level and B at first level

a_0a_1 (or) b :A at first level and B at second level

a_1a_1(or) ab : A and B both second levels.

Yates's method of computing factorial effect totals

For the calculation of various factorial effect total for 2^2-factorial experiments the following table is need

Treatment combination	Total yield from all replicates	(3)	(4)	Effect Totals
'1'	[1]	[1]+[a]	[1]+[a]+[b]+[ab]	Grand total
a	[a]	[b]+[ab]	[ab]-[b]+[a]-[1]	[A]
b	[b]	[a]-[1]	[ab]+[b]-[a]-[1]	[B]
ab	[ab]	[ab]-[b]	[ab]-[b]-[a]+[1]	[AB]

2^2-factorial experiment conducted in a CRD

Let $x_{ij} = j^{th}$ observation of i^{th} treatment combinations i=1, 2, 3, 4; j=1,2….(say)

i.e., $x_1 = [1]; x_2 = [a]; x_3 = [b]; x_4 = [ab]$

Where

x_i=total of i^{th}treatment combination .

$G=\sum_i \sum_f x_{ij}$ grand total

n=4r=Total number of observations

$$TSS=\sum_j \sum_i x_{ij}^2 - \frac{G^2}{4r}$$

1. The follwoing table gives the plan and yields of a 2^2 – factorial experiment conducted in CRD

Analyse the design and give your comments

(1) 20	a 28	a 24	b 10
ab 23	b 11	ab 22	(1) 17
a 24	b 15	ab 21	(1) 19

Solution:

Arrange the observation as in one-way classification, we proceed as follows

Treatment Combination				Total
(1)	20	17	19	56
a	28	24	24	76
b	10	11	15	36
ab	23	22	21	66
Total			G=	234

$$\text{Correction Formula} = \frac{G^2}{2^2 \times r} = \frac{234^2}{4 \times 3} = 4563$$

$$\sum_j \sum_i x_{ij}^2 = 20^2 + 17^2 + 19^2 + 28^2 + 24^2 + 24^2 + 10^2 + 11^2 + 15^2 + 23^2 + 22^2 + 21^2$$

$$\sum_j \sum_i x_{ij}^2 = 4886$$

$$TSS = \sum_j \sum_i x_{ij}^2 - \frac{G^2}{4r} = 4886 - 4563 = 323$$

The values of SSA, SSB and SSAB are obtained by yate's method

Treatment combination	Total (2)	(3)	(4)	Divisor (5)	Sum of squares (6)
1	[1]	[1]+[a]	[1]+[a]+[b]+[ab]=[M]	-	-
a	[a]	[b]+[ab]	[ab]-[b]+[a]-[1]=[A]	4r	$[A]^2/4r$=SSA
b	[b]	[a]-[1]	[ab]+[b]-[a]-[1]=[B]	4r	$[B]^2/4r$=SSB
ab	[ab]	[ab]-[b]	[ab]-[b]-[a]+[1]=[AB]	4r	$[AB]^2/4r$=SSAB

SSE = TSS-(SSA+SSB+SSAB)

The analysis of variance table for 2^2 factorial design conducted in CRD

Source of variation	d.f	S.S	M.S.S	F
A	1	SSA	MSSA	$\frac{\text{MSSA}}{\text{MSSE}}$
B	1	SSB	MSSB	$\frac{\text{MSSB}}{\text{MSSE}}$
AB	1	SSAB	MSSAB	$\frac{\text{MSSAB}}{\text{MSSE}}$
Error	3(r-1)	SSE	MSSE	-
Total	4r-1	TSS	-	-

To obtain the sum of squares SSA, SSB, SSAB use yate's method:

Treatment/ combination	Total response	(3)	(4)	Divisor (5)	S.S (6)
(1)	56	56+76=132	132+102=234	4r=12	Grand total
a	76	36+66=102	20+30=50	12	$\frac{50^2}{12}=208.33$
b	36	76-56=20	102-132=-30	12	$\frac{(-30)^2}{12}=75$
ab	66	66-36=30	30-20=10	12	$\frac{(10)^2}{12}=8.33$
				Total	291.66

SSE=TSS-(SSA+SSB+SSAB)

=323-291.66

SSE=31.34

Analysis of variance table:

Source of variation	d.f	S.S	M.S.S	F	$F_{0.01}(1,6)$
A	1	208.33	208.33	53.15	13.75
B	1	75	75	19.13	
AB	1	8.33	8.33	2.09	
Error	3(r-1)=6	31.34	3.92		
Total	4r-1=11	323			

Critical value:

The table value of for (1,6) d.f at 1% Los is 13.75

Conclusion:

Since F> tabulated value of 'F' for the main effect A and B, we conclude that the main effects A and B both are significantly different at 1% Los