Derivatives

Basic Properties/Formulas/Rules

$\frac{d}{dx}\left(cf\left(x\right)\right)=cf'\left(x\right)$, c is any constant. $\left(f\left(x\right)\pm g\left(x\right)\right)'=f'\left(x\right)\pm g'\left(x\right)$

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$, n is any number. $\frac{d}{dx}\left(c\right)=0$, c is any constant.

$\left(f\,g\right)'=f'\,g+f\,g'$ – **(Product Rule)** $\left(\frac{f}{g}\right)'=\frac{f'\,g-f\,g'}{g^2}$ – **(Quotient Rule)**

$\frac{d}{dx}\left(f\left(g\left(x\right)\right)\right)=f'\left(g\left(x\right)\right)g'\left(x\right)$ **(Chain Rule)**

$\frac{d}{dx}\left(\mathbf{e}^{g(x)}\right)=g'\left(x\right)\mathbf{e}^{g(x)}$ $\frac{d}{dx}\left(\ln g\left(x\right)\right)=\frac{g'\left(x\right)}{g\left(x\right)}$

Common Derivatives

Polynomials

$\frac{d}{dx}\left(c\right)=0$ $\frac{d}{dx}\left(x\right)=1$ $\frac{d}{dx}\left(cx\right)=c$ $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$ $\frac{d}{dx}\left(cx^n\right)=ncx^{n-1}$

Trig Functions

$\frac{d}{dx}\left(\sin x\right)=\cos x$ $\frac{d}{dx}\left(\cos x\right)=-\sin x$ $\frac{d}{dx}\left(\tan x\right)=\sec^2 x$

$\frac{d}{dx}\left(\sec x\right)=\sec x\tan x$ $\frac{d}{dx}\left(\csc x\right)=-\csc x\cot x$ $\frac{d}{dx}\left(\cot x\right)=-\csc^2 x$

Inverse Trig Functions

$\frac{d}{dx}\left(\sin^{-1} x\right)=\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\cos^{-1} x\right)=-\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\tan^{-1} x\right)=\frac{1}{1+x^2}$

$\frac{d}{dx}\left(\sec^{-1} x\right)=\frac{1}{\left|x\right|\sqrt{x^2-1}}$ $\frac{d}{dx}\left(\csc^{-1} x\right)=-\frac{1}{\left|x\right|\sqrt{x^2-1}}$ $\frac{d}{dx}\left(\cot^{-1} x\right)=-\frac{1}{1+x^2}$

Exponential/Logarithm Functions

$\frac{d}{dx}\left(a^x\right)=a^x\ln\left(a\right)$ $\frac{d}{dx}\left(\mathbf{e}^x\right)=\mathbf{e}^x$

$\frac{d}{dx}\left(\ln\left(x\right)\right)=\frac{1}{x},\ x>0$ $\frac{d}{dx}\left(\ln\left|x\right|\right)=\frac{1}{x},\ x\neq 0$ $\frac{d}{dx}\left(\log_a\left(x\right)\right)=\frac{1}{x\ln a},\ x>0$

Hyperbolic Trig Functions

$\frac{d}{dx}\left(\sinh x\right)=\cosh x$ $\frac{d}{dx}\left(\cosh x\right)=\sinh x$ $\frac{d}{dx}\left(\tanh x\right)=\operatorname{sech}^2 x$

$\frac{d}{dx}\left(\operatorname{sech} x\right)=-\operatorname{sech} x\tanh x$ $\frac{d}{dx}\left(\operatorname{csch} x\right)=-\operatorname{csch} x\coth x$ $\frac{d}{dx}\left(\coth x\right)=-\operatorname{csch}^2 x$

Exercise : 1

Calculus for machine learning

Questions:

1. write the formula for finding a derivative of a function $f(x)$.

2. write the formula for finding a derivative of a function $g(x)$.

3. Find $\frac{df}{dx}$ for $f(x) = x^2 + 6$

1. Formula for finding derivative of $f(x)$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2. $$\frac{dg}{dx} = \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

3. $f(x) = x^2+6$

$\therefore f(x+\Delta x) = (x+\Delta x)^2 + 6$

w.k.t., $$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^2 + 6\right] - \left[x^2+6\right]}{\Delta x}$$

Using the formula:

$$(a+b)^2 = a^2+b^2+2ab$$

$$= \lim_{\Delta x \to 0} \left[\cancel{x^2} + (\Delta x)^2 + 2x\,\Delta x \cancel{+6}\right] - \cancel{x^2} - \cancel{6}$$

$$= \lim_{\Delta x \to 0} \frac{[\cancel{x^2} + (\Delta x)^2 + 2x\Delta x + \cancel{6}] - \cancel{x^2} - \cancel{6}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(\Delta x)^2 + 2x\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(\Delta x)\,[\Delta x + 2x]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (\Delta x + 2x)$$

Applying limit as $\Delta x \longrightarrow 0$
we get,

$$\boxed{\frac{df}{dx} = 2x}$$

Find the Derivaties of all the below functions :

(Refer calculus Quick Reference attachment for help)

1. x^5
2. $x^{2/3}$
3. $\frac{1}{\sqrt{x}}$
4. x^{-2}
5. π (pie)
6. 50
7. 45
8. πx^2
9. $995x^5$
10. $10x^2$
11. $99x^3 + 75x^2$
12. $12x + 25x^2 + 30x^5 - 20x^2$
13. $225x^2 + \frac{15}{\sqrt{x}} - \frac{1}{3}x^{-4} + 25x^3$

Solution for Exercise : 2

1. x^5

wkt,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\therefore \frac{d}{dx}(x^5) = 5x^{5-1} = \underline{\underline{5x^4}}$$

2. $x^{2/3}$

$$\frac{d}{dx} x^{2/3}$$

$$= \frac{2}{3} x^{2/3 - 1}$$

$$= \frac{2}{3} x^{\frac{2-3}{3}} \Rightarrow \underline{\underline{\frac{2}{3} x^{-1/3}}}$$

3. $\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$

$$\frac{d}{dx}(x^{-1/2}) = \frac{-1}{2} x^{-1/2 - 1}$$

$$= \underline{\underline{\frac{-1}{2} x^{-3/2}}}$$

4. x^{-2}

$$\frac{d}{dx}(x^{-2}) = -2x^{-2-1}$$

$$= \underline{\underline{-2x^{-3}}}$$

5. π

wkt, $\frac{d}{dx}(c) = 0$ where, c = Constant

$$\therefore \underline{\underline{\frac{d}{dx}(\pi) = 0}}$$

6. 50

$$\frac{d}{dx}(50) = \underline{\underline{0}}$$

7. 45

$$\frac{d}{dx}(45) = \underline{\underline{0}}$$

8. πx^2

wkt, $\frac{d}{dx}(c.f(x)) = c.\frac{d}{dx} f(x)$

$$\therefore \frac{d}{dx}(\pi x^2) = \pi . \frac{d}{dx}(x^2)$$

dx $\qquad$ dx

$$= \pi . 2x^{2-1}$$

$$= \underline{\underline{2\pi x}}$$

9. $995x^5$

$$\frac{d}{dx}(995x^5) = 995 \times [5x^{5-1}]$$

$$= 995 \times 5 \times x^4$$

$$= \underline{\underline{4975x^4}}$$

10. $10x^2$

$$\frac{d}{dx}(10x^2) = 10 \times 2 \times x^{2-1}$$

$$= \underline{\underline{20x}}$$

11. $99x^3 + 75x^2$

wkt $\frac{d}{dx}[f(x) \pm g(x)]$

$$= \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\therefore \frac{d}{dx}[99x^3 + 75x^2] = \frac{d}{dx}[99x^3] + \frac{d}{dx}[75x^2]$$

$$= 99 \times 3x^{3-1} + 75 \times 2 \times x^{2-1}$$

$$= \underline{297x^2 + 150x}$$

12. $12x + 25x^2 + 30x^5 - 20x^2$

$$\frac{df}{dx} = 12 \times 1 \times \underbrace{x^{1-0}}_{1} + 25 \times 2 \times x^{2-1} + 30 \times 5 \times x^{5-1} - 20 \times 2 \times x^{2-1}$$

$$= 12 + 50x + 150x^4 - 40x$$

13. $255x^2 + \frac{15}{\sqrt{x}} - \frac{1}{3}x^{-4} + 25x^3$

$$\frac{df}{dx} = 255 \times 2 \times x^{2-1} + 15 \times \left(\frac{-1}{2}\right) x^{-1/2-1} - \frac{1}{3} \times (-4)\, x^{(-4-1)}$$

$$+ 25 \times 3 \times x^{3-1}$$

$$= 510x - \frac{15}{2}x^{-3/2} + \frac{4}{3}x^{-5} + 75x^2$$

Exercise 3:

Find the derivatives of the following:

1. $y = e^x \sin x$

2. $y = x^2 e^x \cos x$

[Hint: Consider $f(x) = x^2$, $g(x) = e^x$ and $h(x) = \cos x$]

& apply the formula

$$\frac{d}{dx}\left[f(x) \cdot g(x) \cdot h(x)\right] = \left\{\frac{d}{dx}\left(f(x)\right)\right\} \times g(x) \cdot h(x) + f(x) \cdot \left\{\frac{d}{dx}\left(g(x)\right)\right\} h(x) + f(x) \cdot g(x) \left\{\frac{d}{dx}\left(h(x)\right)\right\}$$

Solution for Exercise 3:

① $y = e^x \sin x$

Let $f(x) = e^x \quad g(x) = \sin x$

Product Rule:

$$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f(x) \cdot \frac{d}{dx} g(x) + g(x) \cdot \frac{d}{dx} f(x)$$

$$\Rightarrow \frac{d}{dx}\left[e^x \cdot \sin x\right] = e^x \cdot \cos x + \sin x \cdot e^x$$

$$= e^x \cos x + e^x \sin x$$

② $y = x^2 e^x \cos x$

$$\frac{d}{dx}\left[f(x) \cdot g(x) \cdot h(x)\right] = \left\{\frac{d}{dx}(f(x))\right\} g(x) \cdot h(x) + f(x) \cdot \left\{\frac{d}{dx} \cdot (g(x))\right\} h(x) + f(x) \cdot g(x) \cdot \left\{\frac{d}{dx}(h(x))\right\}$$

On substituting we get,

$$\frac{d}{dx}\left[x^2 e^x \cos x\right] = 2x \times e^x \times \cos x + x^2 \times e^x \times \cos x + x^2 \times e^x \times (-\sin x)$$

$$= 2x e^x \cos x + x^2 e^x \cos x - x^2 e^x \sin x$$

Exercise 4 : Chain Rule

Find the derivatives of the following functions:

1. e^{3x^2+12}

2. $\sin^3 2x$

3. $(5x-2)^3$

4. $(x^2-3x+5)^3$

Solution for Exercise 4 :

① $y = e^{3x^2 + 12}$

Let $u = 3x^2 + 12$

$\therefore y = e^u \qquad u = 3x^2 + 12$

$y = f(g(x))$

wkt., $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

$y = e^u$

$\dfrac{dy}{du} = e^u$

$(u = 3x^2 + 12)$

$\dfrac{du}{dx} = 3 \times 2 \times x + 0$

$\dfrac{du}{dx} = 6x$

On substituting we get,

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = e^u \cdot 6x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^{u} \cdot 6x$$

Substituting for 'u'

$$\frac{dy}{dx} = e^{(3x^2+12)} \cdot 6x$$

② $y = \sin^3 2x$

$y = (\sin 2x)^3$

Let, $\sin 2x = u$

$\therefore y = u^3$, $u = \sin x$

Let, $v = 2x$

$y = u^3 \quad u = \sin v \quad v = 2x$

$y = f(g(h(x)))$

$$\frac{dy}{du} = 3u^2 \qquad \frac{du}{dv} = \cos v \qquad \frac{dv}{dx} = 2$$

wkt, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = 3u^2 \cdot \cos v \cdot \times 2$

Substituting for 'u' & 'v' we get

$$\frac{dy}{dx} = 6(\sin^2 \underline{v}) \cdot \cos 2x$$

$$= 6(\sin^2 2x)(\cos 2x)$$

$$\frac{dy}{dx} = 6(\sin 2x)^2 \cos 2x$$

③ $y = (5x - 2)^3$

let, $(5x - 2) = u$

$\therefore y = u^3 \qquad u = (5x - 2)$

$$y = f(g(x))$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{du} = 3u^2$$

$$\frac{du}{dx} = 5 - 0 = 5$$

$$\therefore \frac{dy}{dx} = 3u^2 \times 5$$

$$= 15u^2$$

$$= \underline{\underline{15(5x-2)^2}}$$

(4) $y = (x^2 - 3x + 5)^3$

let, $(x^2 - 3x + 5) = u$

$y = u^3 \qquad u = x^2 - 3x + 5$

$\frac{dy}{du} = 3u^2 \qquad \frac{du}{dx} = 2x - 3 + 0$

$= 2x - 3$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 3u^2 \cdot (2x-3)$

$\frac{dy}{dx} = 3 \cdot \underline{\underline{(x^2 - 3x + 5)^2 (2x-3)}}$