

Gelfand Correspondence Program in Mathematics

Introductory Assignment

Thank you for your interest in our Program. Below are some problems we would like you to try. These problems have different degrees of difficulty, and you are not expected to be able to solve all of them. If you decide to participate in GCPM, please solve these problems and mail us your solutions. This will only serve us to get acquainted with your abilities; you will be accepted to GCPM independently of your results. When writing down your solutions, please include all the explanations to your solutions and not only the answers to problems. If you cannot solve a problem but have some ideas or a partial solution, write them down. Try to explain your ideas and answers as clearly as possible. Please keep this page for future reference and do all your work on separate sheets of paper, numbering each problem. Send your work to: **Gelfand Correspondence Program in Mathematics, Department of Mathematics, Rutgers the State University, 110 Frelinghuysen Road, Piscataway, N.J. 08855-8019.**

Please see [directions](#) before you start

1. Which is bigger, $\frac{100000001}{100000002}$ or $\frac{200000001}{200000002}$?

Problems 2 and 3 deal with the inhabitants of the city Boole. Some of the inhabitants of the city Boole are liars and always lie, all others always tell the truth.

2. Once ten inhabitants of the city Boole met in a room and each one said: "all the rest of you are liars". How many people in the room were liars?

3. Once several inhabitants of the city Boole met in a room. Three of them made the following statements:

The first one said:

- a) There are no more than three people here.
b) All of us are liars.

The second one said:

- a) There are no more than four people here.
b) Not everybody here is a liar.

The third one said:

- a) There are five people here.
b) Three of us are liars.

How many people were in the room and how many of them were liars?

4. Each of the equations below is missing a pair of numerators:

$$\text{a) } \frac{?}{7} - \frac{?}{5} = \frac{1}{35}, \quad \text{b) } \frac{?}{5} - \frac{?}{7} = \frac{1}{35}.$$

Assume that the numerators are positive integers. Find as many pairs of numerators as you can. Do not forget that mixed fractions are allowed.

Example: Let us check whether 2 and 4 form a solution for a).

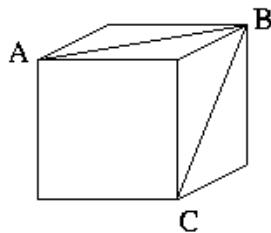
$$\frac{2}{7} - \frac{4}{5} = \frac{10}{35} - \frac{28}{35} = -\frac{18}{35} \neq \frac{1}{35}.$$

So the pair (2,4) is not a solution for a).

5. Is the sum $1+2+3+4+\dots+98+99+100$ even or odd?

Note: The " \dots " stands for the missing terms, and there are 100 terms in all.

6. What is the measurement, in degrees, of the angle between the diagonals, AB and BC, of the adjacent faces of the cube?



7. Remove the parentheses (multiply out):

$$(1-x)(1+x+x^2+x^3+x^4+x^5+\dots+x^{99}+x^{100}) .$$

For example: $(1+x)(1+x+x^2) = 1+x+x^2+x+x^2+x^3 = 1+2x+2x^2+x^3.$

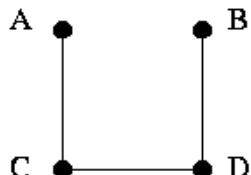
8. The difference of two numbers is 0.01. Is it possible that the difference of their squares is more than 1000?

9. In a box, there are fresh cucumbers which weigh 100 pounds. Each cucumber is composed of 99% water. After some time, the cucumbers dried out. Now each cucumber is composed of 98% water. How much do the cucumbers weigh now?

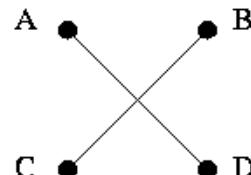
Below are three more challenging problems:

1. Four points, A,B,C and D, are the corners of a square. Each side of the square is 10 feet long. Draw a system of straight lines connecting the four points so that the total length of the lines is 28 feet or less.

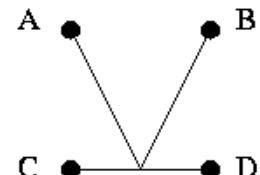
Below are three examples that do NOT work:



Total length is 30 feet



Total length is more than 28.2 feet

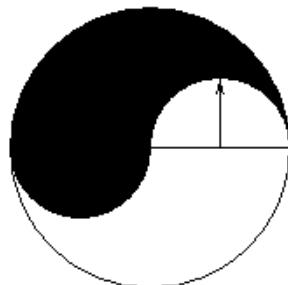


Total length is more than 28 feet

2. Remove the parentheses (multiply out):

$$(1-x)^2(1+2x+3x^2+4x^3+5x^4+\dots+99x^{98}+100x^{99}+101x^{100}) .$$

3. The shaded region in the figure is bounded by three semi-circles. Cut this region into four congruent parts, i.e. parts of equal size and shape.



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