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Ans 1:

$$p(y_i=1 / f(x_i)) = \frac{1}{1+e^{-f(x_i)}}$$

$$= \sigma(f(x_i)) \quad ; \sigma(x) \text{ is sigmoid function.}$$

$$p(y_i=-1 / f(x_i)) = 1 - \frac{1}{1+e^{-f(x_i)}}$$

$$= \frac{e^{-f(x_i)}}{1+e^{-f(x_i)}}$$

$$= \frac{1}{1+e^{f(x_i)}}$$

$$= \sigma(-f(x_i))$$

$$p(y_i / f(x_i)) = \sigma(y_i f(x_i))$$

Assuming data points to be independent and identically distributed (i.i.d):

$$\text{log likelihood, } L = \prod_{i=1}^{N} p(y_i / f(x_i))$$

$$\text{Negative log likelihood, } -\ln L = -\sum_{i=1}^{N} \ln [p(y_i / f(x_i))]$$

$$= -\sum_{i=1}^{N} \ln [\sigma(y_i f(x_i))]$$

$$= -\sum_{i=1}^{N} \ln \left[\frac{1}{1+e^{-y_i f(x_i)}} \right]$$

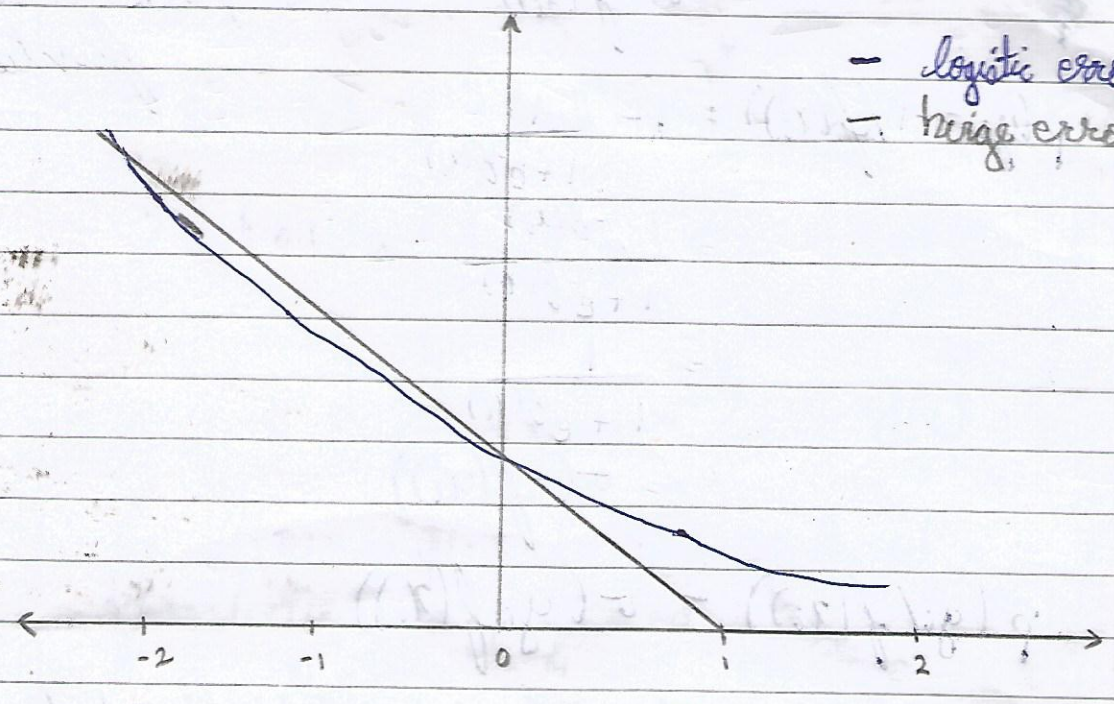
$$= \sum_{i=1}^{N} \ln [1+e^{-y_i f(x_i)}]$$

$$= \sum_{i=1}^{N} E_{LR}(y_i, f(x_i))$$

where $E_{LR}(y_i, f(x_i)) = \ln(1 + \exp(-y_i f(x_i)))$

With addition of a quadratic regularization term;

$$\rightarrow \sum_{i=1}^{N} E_{LR}(y_i, f(x_i)) + \lambda \|w\|^2$$



– logistic error function
–. hinge error function

~~log~~ Hinge error $= \sum_{n=1}^{N} E_{SV}(y_n t_n) + \lambda \|w\|^2$.

where $E_{SV}(y_n t_n) = [1 - y_n t_n]_+$.

Both logistic and hinge loss are continuous approximation of misclassification error. Hinge loss is more sparse and covers less area than logistic error function.

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Theory Extra Credit

Ans:

$$L(w, b, \xi_i \ldots \xi_N, \hat{\xi}_i \ldots \hat{\xi}_N) =$$

$$C\sum_{i=1}^{N}(\xi_i + \hat{\xi}_i) + \frac{1}{2}\|w\|^2 - \sum_{i=1}^{N}(\mu_i \xi_i + \hat{\mu}_i \hat{\xi}_i)$$

$$- \sum_{i=1}^{N} a_i(\varepsilon + \xi_i + f(x_i) - y_i) - \sum_{i=1}^{N} \hat{a}_i(\varepsilon + \hat{\xi}_i - f(x_i) + y_i) \quad (1)$$

$$\# \Rightarrow \frac{dL}{dw} = w - \sum_{i=1}^{N} a_i \phi(x_i) + \sum_{i=1}^{N} \hat{a}_i \phi(x_i) = 0$$

Here $f(x) = w^T\phi(x) + b$ was replaced and then differentiation was performed. ; $w = \sum_{i=1}^{N} \phi(x_i)[a_i - \hat{a}_i]$ (2)

$$\# \frac{dL}{db} = -\sum_{i=1}^{N} a_i + \sum_{i=1}^{N} \hat{a}_i = 0$$

$$\sum_{i=1}^{N}(\hat{a}_i - a_i) = 0 \quad (3)$$

$$\# \frac{dL}{d\xi_i} = C - \mu_i - a_i = 0$$

$$C = (\mu_i + a_i) \quad (4)$$

$$\# \frac{dL}{d\hat{\xi}_i} = C - \hat{\mu}_i - \hat{a}_i = 0$$

$$C = (\hat{\mu}_i + \hat{a}_i) \quad (5)$$

Now, combining like terms in 1;

$$\sum_{i=1}^{N} \xi_i (C - \mu_i - a_i) + \sum_{i=1}^{N} \hat{\xi}_i [C - \hat{\mu}_i - \hat{a}_i]$$

$$+ \varepsilon \left[-\sum_{i=1}^{N} a_i - \sum_{i=1}^{N} \hat{a}_i \right] + \left[\sum_{i=1}^{N} a_i y_i - \sum_{i=1}^{N} \hat{a}_i y_i \right]$$

$$+ \frac{1}{2} \|w\|^2 + \sum_{i=1}^{N} \left[\hat{a}_i w^T \phi(x_i) - a_i w^T \phi(x_i) \right]$$

Now, from 4 $\quad C = \mu_i + a_i$

from 5 $\quad C = \hat{\mu}_i + \hat{a}_i$

$\therefore$ (6) and (7) becomes 0, where (6) $\sum_{i=1}^{N} \xi_i (C - \mu_i - a_i) =$

(7) $\sum_{i=1}^{N} \hat{\xi}_i (C - \hat{\mu}_i - \hat{a}_i)$

Thus;

$$-\varepsilon \sum_{i=1}^{N} \left[a_i + \hat{a}_i \right] + \left[\sum_{i=1}^{N} (a_i - \hat{a}_i) y_i \right]$$

$$+ \frac{1}{2} \|w\|^2 + \sum_{i=1}^{N} \left[\hat{a}_i w^T \phi(x_i) - a_i w^T \phi(x_i) \right]$$

From 2;

$$\|w\|^2 = w^T w$$

$$\frac{1}{2} \sum_{i=1}^{N} \phi(x_i) \left[a_i - \hat{a}_i \right] \sum_{j=1}^{N} \phi(x_j)^T \left[a_j - \hat{a}_j \right]$$

$$+ \sum_{i=1}^{N}$$

$$\Rightarrow -\varepsilon\sum_{i=1}^{N}(a_i+\hat{a}_i)+\sum_{i=1}^{N}(a_i-\hat{a}_i)y_i$$

$$+\frac{1}{2}\left[\sum_{i=1}^{N}\phi(x_i)^T(a_i-\hat{a}_i)\sum_{j=1}^{N}\phi(x_j)(a_j-\hat{a}_j)\right]$$

$$+\sum_{i=1}^{N}\left[\hat{a}_i\omega^T\phi(x_i)-a_i\omega^T\phi(x_i)\right]$$

$$\Rightarrow -\varepsilon\sum_{i=1}^{N}(a_i+\hat{a}_i)+\sum_{i=1}^{N}(a_i-\hat{a}_i)y_i$$

$$+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}k(x_i,x_j)(a_i-\hat{a}_i)(a_j-\hat{a}_j)$$

$$+\sum_{i=1}^{N}\left[\phi(x_i)\sum_{j=1}^{N}\phi(x_j)(a_j-\hat{a}_j)-\phi(x_i)\sum_{j=1}^{N}\phi(x_j)\right]$$

$$+\sum_{i=1}^{N}\left[\hat{a}_i\phi(x_i)+\sum_{i=1}^{N}\phi(x_i)\left[\hat{a}_i\sum_{j=1}^{N}\phi(x_j)^T(a_j-\hat{a}_j)\right.\right.$$

$$\left.\left.-a_i\sum_{j=1}^{N}\phi(x_j)^T(a_j-\hat{a}_j)\right]\right.;\ k(x_i,x_j)=\phi(x_i)^T\phi(x_j)$$

$$\Rightarrow -\varepsilon\sum_{i=1}^{N}(a_i+\hat{a}_i)+\sum_{i=1}^{N}(a_i-\hat{a}_i)y_i$$

$$+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}k(x_i,x_j)(a_i-\hat{a}_i)(a_j-\hat{a}_j)$$

$$-\sum_{i=1}^{N}\sum_{j=1}^{N}\phi(x_j)^T\phi(x_i)(\hat{a}_j-a_j)(\hat{a}_i-a_i)$$

$$\Rightarrow -\varepsilon\sum_{i=1}^{N}(a_i+\hat{a}_i)+\sum_{i=1}^{N}(a_i-\hat{a}_i)y_i$$

$$+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}k(x_i,x_j)(a_i-\hat{a}_i)(a_j-\hat{a}_j)$$

$$-\sum_{i=1}^{N}\sum_{j=1}^{N}k(x_i,x_j)(a_i-\hat{a}_i)(a_j-\hat{a}_j)$$

$$\Rightarrow -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(a_i-\hat{a}_i)(a_j-\hat{a}_j)k(x_i,x_j)$$

$$-\varepsilon\sum_{i=1}^{N}(a_i+\hat{a}_i)+\sum_{i=1}^{N}(a_i-\hat{a}_i)y_i\ ;\ \text{Hence proved.}$$

Ans 2).

2.1).

First I have made two list of lists containing image of 28*28 pixels and label belonging to that image. Later I selected those image and labels where label is equal to 3 or 8. 2000 samples per class from the training set belonging to 3 or 8 and a test set of 500 samples per class belonging to 3 or 8 from the testing set were selected. Then soft-margin linear SVM formulation was used and five-fold cross-validation with grid search was performed for finding an appropriate regularization parameter, C. Soft-margin SVM was trained from that C.

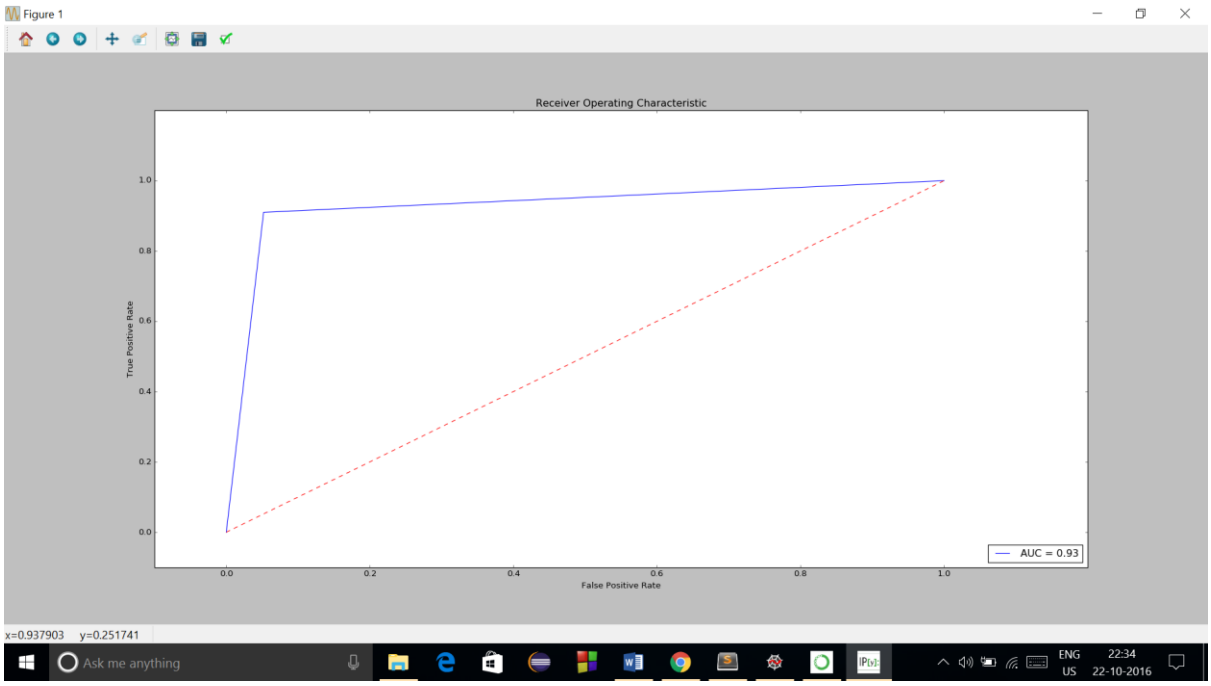
- grid.best_estimator_

SVC(C=0.1, cache_size=200, class_weight=None, coef0=0.0,

decision_function_shape=None, degree=3, gamma='auto', kernel='linear',

max_iter=-1, probability=False, random_state=None, shrinking=True,

tol=0.001, verbose=False)



C = 0.1, Accuracy = 0.51

C = 1, Accuracy = 0.5

C = 10, Accuracy = 0.5

C = 100, Accuracy = 0.5

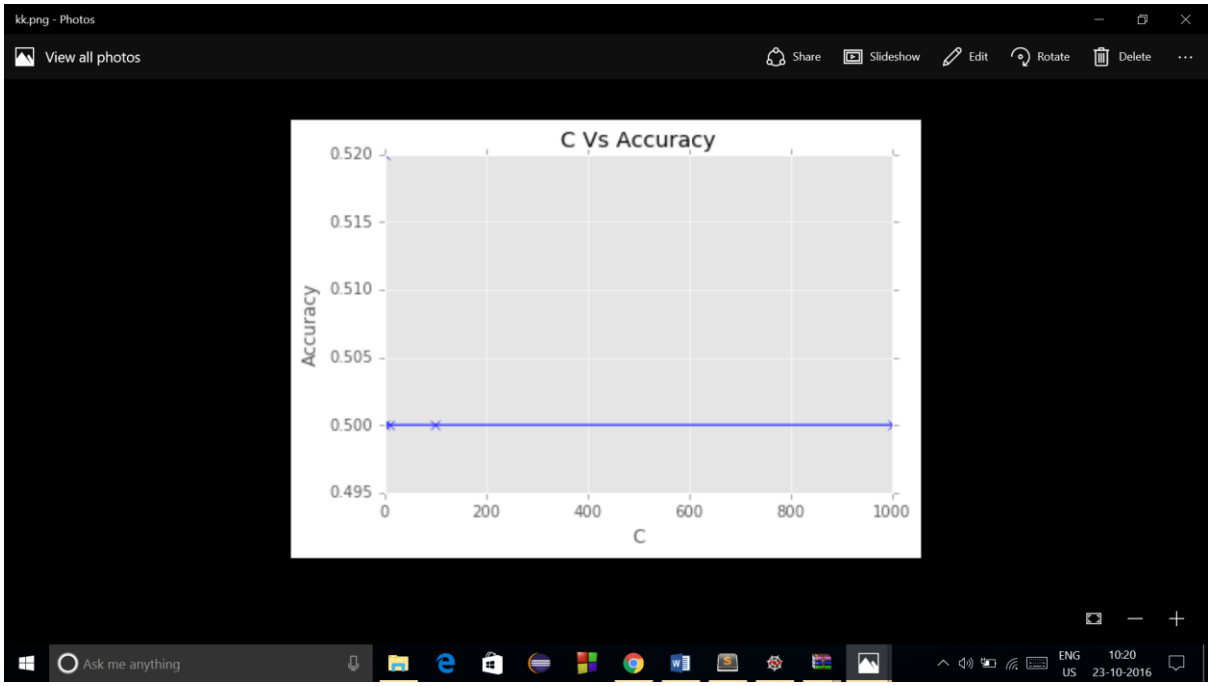
C = 1000, Accuracy = 0.5

param_grid = { 'kernel': ['linear'], 'C': [0.1, 1, 10, 100, 1000] }

svr = SVC()

grid = GridSearchCV(svr, param_grid, cv = 5)

grid.fit(trainimage, trainlabel)



2.2):

First I have made two list of lists containing image of 28*28 pixels and label belonging to that image. 2000 samples per class from the training set and a test set of 500 samples per class from the testing set were selected. Then soft-margin linear SVM formulation was used and five-fold cross-validation with grid search was performed for finding an appropriate regularization parameter, C. Soft-margin SVM was trained from that C. There was a gridsearch performed to find C.

param_grid = {

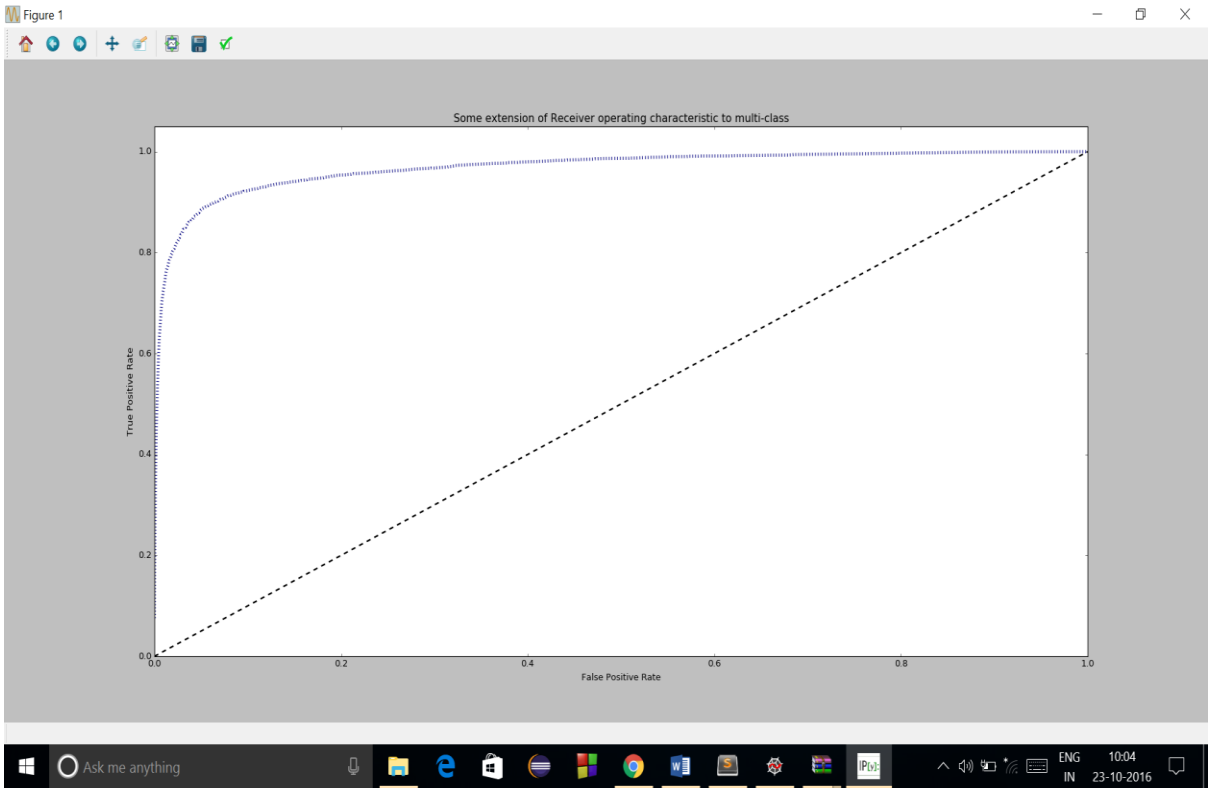
'estimator__C': [0.1, 1, 10, 100, 1000]

}

svr = OneVsRestClassifier(LinearSVC())

grid = GridSearchCV(svr, param_grid, cv = 5)

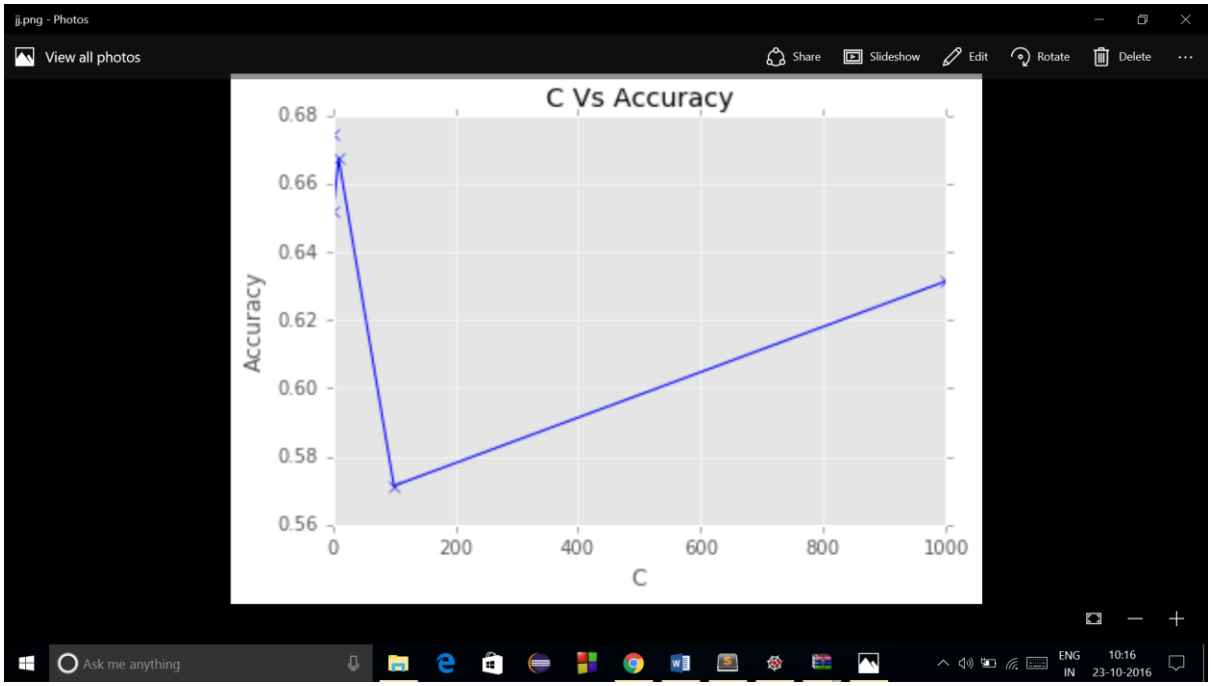
grid.fit(trainimage, trainlabel)



OneVsRestClassifier(estimator=LinearSVC(C=0.1, class_weight=None, dual=True, fit_intercept=True,
intercept_scaling=1, loss='squared_hinge', max_iter=1000,
multi_class='ovr', penalty='l2', random_state=None, tol=0.0001,
verbose=0),
n_jobs=1

roc_auc:

0.96792355555555565



C = 0.1, Accuracy = 0.6744

C = 1, Accuracy = 0.6516

C = 10, Accuracy = 0.6674

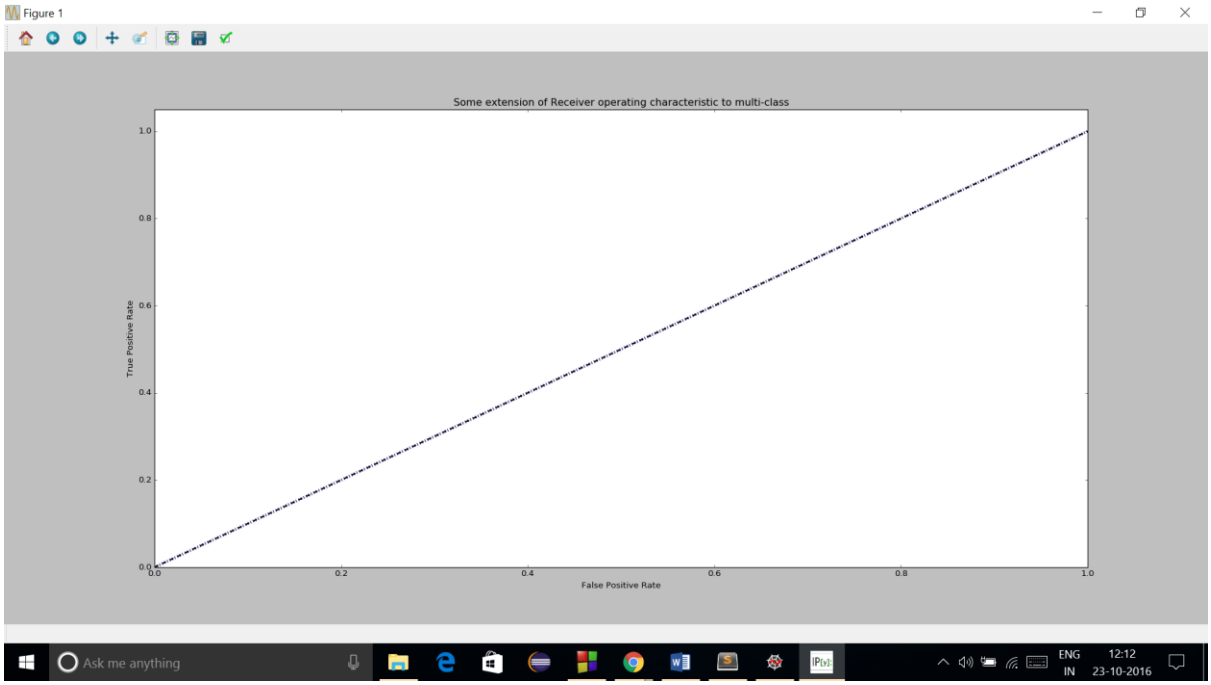
C = 100, Accuracy = 0.5714

C = 1000, Accuracy = 0.6314

Auc:

0: 0.99516133333333323, 1: 0.99492133333333321, 2: 0.96737022222222213, 3: 0.96847066666666659, 4: 0.98848044444444438, 5: 0.96452977777777782, 6: 0.98314533333333343, 7: 0.97833422222222222, 8: 0.88355511111111107, 9: 0.95334311111111114, 'macro': 0.96792355555555565}

2.3).



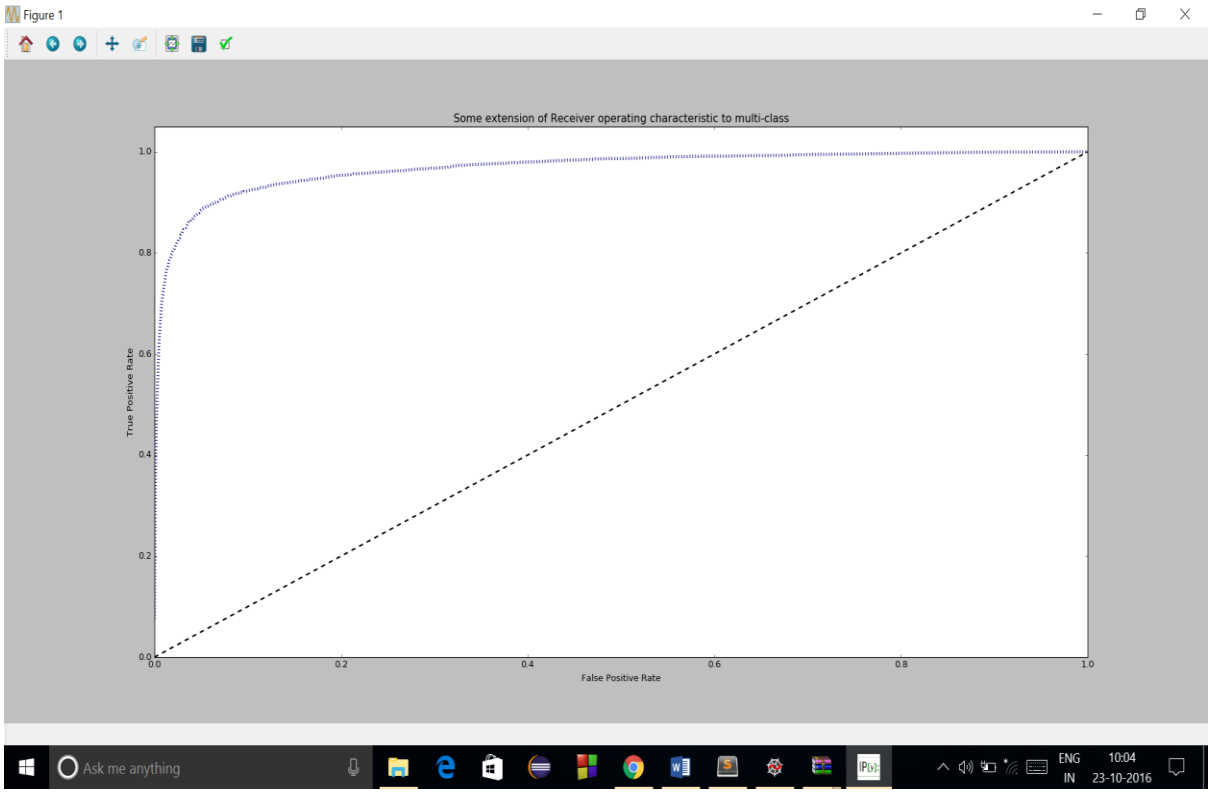
0: 0.5, 1: 0.5, 2: 0.5, 3: 0.5,

4: 0.5, 5: 0.5, 6: 0.5, 7: 0.5, 8: 0.5, 9: 0.5, 'macro': 0.51

C = 1, Gamma = 0.000001

I have chosen this C and gamma as others were giving me lower accuracy through function estimator.score().

Accuracy of 2 and 3 together:



Ans 3).

Chose 150 randomly selected samples per class from the training set. Then a test set of 100 samples per class was selected. Then KPCA and KNN was performed with k = 3. Five-fold cross-validation was performed with grid search to estimate the gamma. Code is provided in the source section.

gamma: [0.1, 1, 10, 100, 1000], then gamma = 0.1 as the best one.

0.290666666667

Predict_proba returns this:

```
array([[ 0.33333333,  0.33333333,  0.        , ...,  0.        ,
         0.        ,  0.        ],
       [ 0.33333333,  0.33333333,  0.        , ...,  0.        ,
         0.        ,  0.        ],
       [ 0.66666667,  0.33333333,  0.        , ...,  0.        ,
         0.        ,  0.        ],
       ...,
       [ 0.33333333,  0.33333333,  0.        , ...,  0.        ,
         0.        ,  0.33333333],
```

```
[ 0.        , 0.33333333, 0.        , ..., 0.        ,
  0.        , 0.33333333],
[ 0.        , 0.33333333, 0.        , ..., 0.        ,
  0.        , 0.33333333]])
```