Register No.:								
		1	9	C	A	3	8	

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Odd Semester (2015- 2016) — Examination: End Sem
Course Code: MCA801 — Course Name: Computer Algorithms
Date:30/11/2015 — Time:2.00 PM to 5.00 PM — Maximum Marks: 100

INSTRUCTIONS:
1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

Q.1. (a) Prove by Mathematical induction that [05]

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

(b) Solve the following recurrence using recursion tree method [05]

$$T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

(c) Find the asymptotic tight bound for [05]

i. $T(n) = 3T(n/2) + n \log n$

ii. $T(n) = 2T(\sqrt{n}) + \log n$

(d) Explain an algorithm to find the second largest in an unsorted array that takes minimum possible number of comparisons. How many comparisons does it make to find find second smallest in an array of size 64. [05]

Q.2. (a) Differentiate between the divide and conquer and dynamic problem techniques for solving problems. [05]

(b) Assume that you are enough Indian currency notes of denominations $\langle 10, 20, 50, 100, 500, 1000 \rangle$. Design an algorithm that returns the minimum number of denominations for a given amount, which is a multiple of 10. Eg: $5880 = \langle 1000\times5, 500\times1, 100\times3, 50\times1, 20\times1, 10\times1 \rangle$ [05]

(c) State and prove the principle of optimality in computing a minimum spanning tree. [05]

(d) With respect to KMP algorithm for string matching, compute the prefix function Π for the pattern *ababaababbabab* [05]

Q.3. (a) Explain the principle of optimality in Matrix chain multiplication problem. [05]

(b) Define optimal binary search tree problem. Determine the cost and structure of an optimal BST for a set $n = 7$ with the following probabilities: [10]

i	0	1	2	3	4	5	6	7
p_i		0.04	0.06	0.08	0.02	0.10	0.12	0.14
q_i	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05

(c) Find the LCS of "CGATAATTGAGA" and "GTAGAAG". Show the value of each sub-problem. [05]

Q.4. (a) Explain the Depth first search traversal Algorithm. Illustrate the construction of DFS tree on the graph given in Figure 1. [5]

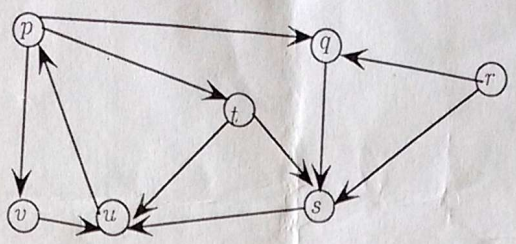

Figure 1: A directed graph

(b) Illustrate the parenthesis theorem in DFS for the Figure 1. [5]

(c) Obtain all possible topological ordering for the graph in Figure 2. [5]

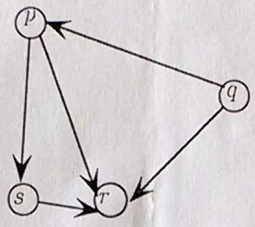

Figure 2: A directed acyclic graph

(d) Write the algorithm to compute the strongly connected component in a graph. Illustrate the same on the graph given in Figure 1. [5]

Q.5. (a) Explain the steps in proving a problem to be NP-Complete. [5]

(b) Explain (i) the vertex cover problem and (iii) Clique problem in a graph. [5]

(c) Explain how is Clique problem is reduced to the Vertex cover problem. [5]

(d) Write an approximation algorithm for Vertex-cover problem. Derive the approximation guarantee of your algorithm. [5]

(*This question paper contains* 2 *page(s) and* 5 *Questions.*)

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Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Even Semester (2015- 2016) — Examination: Mid Sem
Course Code: MCA801 — Course Name: Computer Algorithms
Date:18/02/2016 — Time:10.30 AM to 12.00 AM — Maximum Marks: 50

INSTRUCTIONS:

1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

Q.1. Consider a computer that can perform 10^7 instructions per second. In the table given below, function $f(n)$ represents the number of instructions in an algorithm. Complete the entries in the table by calculating the time taken by different algorithms for different n values in each row. [10]

	$f(n)$			
n	n	$n \log n$	n^2	n^4
10	$1\mu s$	?	?	?
50	?	?	?	?
100	?	?	?	?
1000	?	?	?	?
10^4	?	?	?	?
10^5	?	?	?	?

10^7 — 1 sec
$10 \log 10$ — ? $\frac{1}{10^7} \times 10 \log 10$

Q.2. (a) Suppose you have algorithms with the following running times listed below. (Assume these are the exact running times). How much slower do each of these algorithms get when you double the input size? [5]

i. n^2
ii. n^3
iii. $100n^2$
iv. $n \log_2 n$
v. 2^n

$4n^2$

(b) Find the total number of operation performed by the following code. Express the same in Θ notation. [5]

Algorithm Exponentiate(x, n)
{
$m = n$; $power = 1$; $z = x$;
while($m > 0$) do
{
  while($m \mod 2 == 0$) do
  {
    $m = \lfloor (m/2) \rfloor$; $z = z * z$;
  }
  $m = m - 1$; $power = power * z$;
} return $power$;
}

n^n
m
$\log_2 m$
$6 \log n^2$

Q.3. (a) Solve the following recurrence using recursion tree method [06]

$$T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

(b) Find the asymptotic tight bound for [04]

i. $T(n) = 3T(n/2) + n \log n$

ii. $T(n) = 2T(n/4) + n$

Q.4. (a) Suppose you are given an array A with n entries, with each entry holding a distinct number. A sequence of values is *unimodal*, if for some index p between 1 and n, the values in the array entries increase upto position p in A and then decrease the remainder of the way until position n. For example in the array $\langle 1, 4, 8, 5, 2 \rangle$, the peak entry is 8 which is in position 3. Write an algorithm to find the peak entry in such an array in $O(\log n)$ time. [04]

(b) Write an algorithm PARTITION(A, p, r), which partitions a given array of n elements around the element $A[r]$. Parameters p and r are the start and end index respectively in array A. Trace your algorithm for the data $\langle 5, 10, 12, 16, 2, 8, 15, 6 \rangle$ [06]

Q.5. (a) Explain the principle of optimality in Matrix chain multiplication problem. [04]

(b) Find the optimal parenthesization for the chain $\langle 10, 20, 5, 12 \rangle$. Show the cost each sub-problem. [06]

(*This question paper contains 2 page(s) and 5 Questions.*)

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Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Even Semester (2015- 2016) — Examination: End Sem
Course Code: MCA801 — Course Name: Computer Algorithms
Date:27/04/2016 — Time:2.00 PM to 5.00 PM — Maximum Marks: 90

INSTRUCTIONS:

1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

Q.1. (a) Estimate the time complexity of the following function in terms of Θ notation. Find the return value k when $n = 16$. [5]

```
int unknown(int n){
int i, j, k = 0;
        for (i = n/2; i <= n; i++)
                for (j = 2; j <= n; j = j * 2)
                        k = k + n/2;
        return (k);
}
```

(b) Find the asymptotic tight bound for [10]

i) $T(n) = 3T(n/2) + n \log n$
ii) $T(n) = 2T(n/4) + n$
iii) $T(n) = T(n-1) + n$
iv) $T(n) = T(\sqrt{n}) + 1$
v) $T(n) = 27T(n/3) + n^3$

Q.2. (a) Differentiate between the divide and conquer and dynamic problem techniques for solving problems. [5]

(b) Define the problem of cutting rods profitably. Explain a bottom up solution for the problem. [5]

(c) Assume that you are enough Indian currency notes of denominations $\langle 10, 20, 50, 100, 500, 1000 \rangle$. Design an algorithm that returns the minimum number of denominations for a given amount, which is a multiple of 10. Eg: $5880 = \langle 1000\times5, 500\times1, 100\times3, 50\times1, 20\times1, 10\times1 \rangle$ [10]

Q.3. (a) Explain the principle of optimality in Longest Common Subsequence (LCS) problem. [5]

(b) Find the LCS of "CGATAATTGAGA" and "GTAGAAG". Show the value of each sub-problem. [5]

(c) Explain the main principle used in KMP pattern matching algorithm. Illustrate the computation of Π function in KMP algorithm. [5]

Q.4. (a) Explain the Depth first search(DFS) traversal Algorithm. [5]

(b) Illustrate the construction of DFS tree on the graph given in Figure 1 indicate $d[v]$ and $f[v]$ for each node. [5]

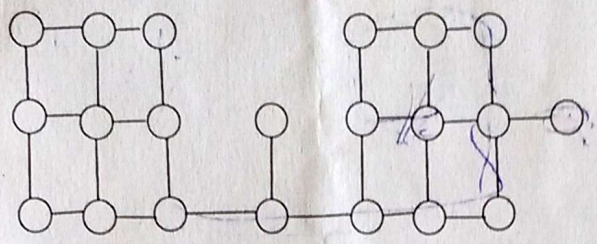

Figure 1: An undirected graph

(c) What is the maximum possible recursion depth including the initial call for the graph in Figure 1. Trace this path clearly for this figure. [5]

(d) Construct a BFS tree for the graph in Figure 1. Indicate clearly the source vertex and the distance of each vertex from the source. [5]

Q.5. (a) Define the class P, NP and NP-Complete problems. [5]

(b) Explain the steps in proving a problem to be NP-Complete. [5]

(c) Explain the 3CNF-SAT and the Vertex Cover problem. How is the 3CNF-SAT problem reduced to vertex cover problem. [10]

(This question paper contains 2 page(s) and 5 Questions.)