Register No.:								

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Odd Semester (2014- 2015) — **Examination: End Sem**
Course Code: MCA801 — **Course Name: Computer Algorithms**
Date:28/11/2014 — **Time:9:00 AM to 12.00 PM** — **Maximum Marks: 100**

INSTRUCTIONS:

1. Answer ALL questions.
2. Rough work/answers should NOT be written on the question paper.

Q.1. (A) What is the importance of Asymptotic notation? Define big-oh notation.

(B) Order the following list of functions by the big-oh notation in non decreasing order of their growth.
$6n \log n$, $\log \log n$, $\log^2 n$, $2^{\log n}$, $\lceil \sqrt{n} \rceil$, $5n$, $n^2 \log n$.

(C) For each function $f(n)$ and time t in the following table, determine the maximum problem size n of a problem that can be solved in time t assuming that the algorithm to solve the problem takes $f(n)$ microseconds.

Running Time $f(n)$	Maximum Problem size		
	1 Sec	1 minute	1 hour
$400n$	2500	?	?
$20n\lceil \log_2 n \rceil$	?	?	?
$2n^2$	?	?	?
n^4	?	?	?
2^n	?	?	?

Q.2. (A) Solve the following recurrence equation by recursion tree method and verify the result using substitution method.

$$T(n) = \begin{cases} 3T(n/2) + O(n) & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$$

(B) Characterize each of the following recurrence equation using Masters method.

(i) $T(n) = 2T(n/2) + \log n$
(ii) $T(n) = 9T(n/3) + n^3 \log n$
(ii) $T(n) = 8T(n/2) + n^2$

(C) You are given an array with integers (positive, negative, zero) and you are supposed to find the maximum contiguous sum in this array. Hence, you have to find a sub-array which results in the largest sum. For example, if the given array is $\{5, -6, 7, 12, -3, 0, -11, -6\}$, the answer would be, 19 (from the sub-array $\{7, 12\}$). Give an $O(n)$ time algorithm for solving this problem.

Q.3. (A) Differentiate between the Divide and conquer and Dynamic programming technique for solving problems.

(B) Define the LCS problem and explain the optimal substructure property for LCS problem.

(C) Give a Dynamic programming algorithm for the LCS problem and illustrate the LCS table for the two strings $X = skullandbones$, $Y = lullabybabies$.

Register No.:								

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

odd Semester (2015- 2016) — Examination: Mid Sem
Course Code: MCA801 — Course Name: Computer Algorithms
Date:10/09/2015 — Time:1.30 to 3.00 PM — Maximum Marks: 50

INSTRUCTIONS:

1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

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1. Consider a computer that can perform 10^8 instructions per second. In the table given below, function $f(n)$ represents the number of instructions in an algorithm. Complete the entries in the table by calculating the time taken by different algorithms for different n values in each row. [10]

	$f(n)$			
n	n	$n \log n$	n^2	n^4
10	$0.1\mu s$	?	?	?
50	?	?	?	?
100	?	?	?	?
1000	?	?	?	?
10^4	?	?	?	?
10^6	?	?	?	?

2. (a) Suppose you have algorithms with the following running times listed below. (Assume these are the exact running times). How much slower do each of these algorithms get when you double the input size? [5]

   i. n^2
   ii. n^3
   iii. $100n^2$
   iv. $n \log_2 n$
   v. 2^n

   (b) Find the total number of operation performed by the following code. Express the same in Θ notation. [5]

Algorithm Exponentiate(x, n)

```
{
m = n; power = 1; z = x;
while(m > 0) do
{
    while(m mod 2 == 0) do
    {
        m = ⌊(m/2)⌋; z = z * z;
    }
    m = m − 1; power = power * z;
} return power;
}
```

Q.3. Let $A[1 \ldots n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion of A.

(a) List the five inversions of the array $\langle 2, 3, 8, 6, 1, 4 \rangle$ [2]

(b) What array with elements from the set $\{1, \ldots n\}$ has the most inversions? How many does it have? [2]

(c) Give an algorithm that determines the number of inversions in any permutation on n elements in $\Theta(n \log n)$ worst-case time. [6]

Q.4. Solve the following recurrence using recursion tree method [03]

$$T(n) = \begin{cases} 4T(n/2) + n^2 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

(b) Find the asymptotic tight bound for $T(n) = 8T(n/2) + n^2$ using Master method and verify the same using substitution method. [03]

(c) Find the asymptotic tight bound for the following: [04]

i. $T(n) = T(n/2) + T(n/4) + T(n/8) + n$

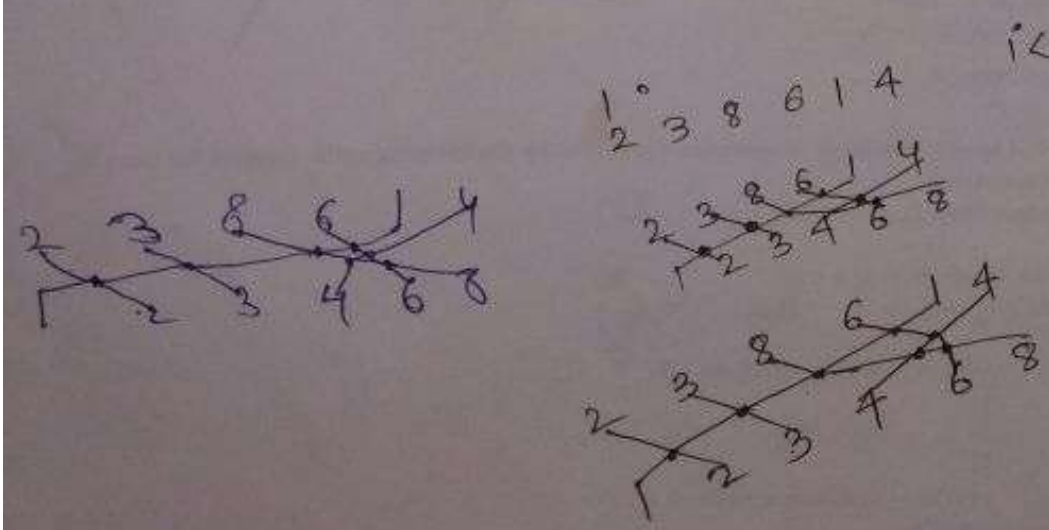
ii. $T(n) = T(n-1) + n$

Q.5. (a) Give an iterative algorithm for Random-Selection, that is to find the i^{th} smallest element in an unsorted array. Prove that it works in average case linear time. [05]

(b) Define the Rod cutting problem. How do you give a recursive solution for the problem? [02]

(c) Provide a bottom up solution for rod cutting problem along with the construction of optimal solution. [03]

(This question paper contains 2 page(s) and 5 Questions.)



Register No.:	1	2	C	0	3	7	

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Even Semester (2014- 2015) — Examination: Mid Sem
Course Code: MCA801 — Course Name: Computer Algorithms
Date:19/02/2015 — Time:10:30 to 12.00 AM — Maximum Marks: 50

INSTRUCTIONS:
1. Answer ALL questions
2. Rough work should NOT be done on the question paper.

1. Define Θ Notation. Prove that $f(n) = 5n^2 + 6n \geq 2 = \Theta(n^2)$ [05]

2. Consider the following list of functions and arrange them in ascending order of their growth rate using Big-oh notation $g_1(n) = 2\sqrt{(\log n)}$, $g_2(n) = 2^n$, $g_3(n) = n^{\frac{3}{2}}$, $g_4(n) = n(\log n)^3$, $g_5(n) = n^{\log n}$, $g_6(n) = 2^{2^n}$, $g_7(n) = 2^{n^2}$. [05]

3. Suppose you have four algorithms with the running time listed below. These running times are the exact number of instructions performed as a function of the input size n. Suppose you have a computer that can perform 10^{10} instructions per second. For each of the algorithms, what is the largest input size n for which you would be able to get the result within an hour? [10]

4. (A) Give a algorithm that returns the k^{th} smallest element in an unsorted array of n in average case linear time. [04]

(B) Trace the number of recursive calls in your algorithm for the following data (3, 5, 1, 6, 9, 2, 12, 8) [04]

5. Solve the following recurrence using recursion tree method [04]

$$T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

6. Find the asymptotic tight bounds for the following [06]

(i) $T(n) = 3T(n/2) + n \log n$

(ii) $T(n) = 2T(n/4) + n$

(iii) $T(n) = 2T(\sqrt{n}) + 1$

7. Let $x[1..n]$ and $y[1..n]$ contain 2 sets of integers, each sorted in nondecreasing order. Write an algorithm that finds the median of the $2n$ combined elements in $O(\log n)$ time. [08]

8. Prove that [04]

$$\sum_{0}^{n} 2^i = 2^{n+1} - 1$$

by mathematical induction.

(This question paper contains 1 page(s) and 5 Questions)

4. (A) Explain the principle used in KMP algorithm for pattern matching in tetail along with pseudo code. [10]

(B) Illustrate the KMP algorithm on the text $T = abacaabaccabacabaabb$ and the pattern $P = abacab$ [06]

(C) Find the number of comparisons used by the Naive algorithm for the text and pattern given in part B. [04]

5. (A) What is the best way to multiply a chain of matrices with dimensions that are 10×5, 5×2, 2×20, 20×12, 12×4 and 4×6. Show the steps and the cost of each sub problem. [10]

(B) Explain Optimal Binary search tree (BST) and an algorithm to compute Optimal BST. Illustrate your algorithm for an input containing 6 keys.(Assume that the search is always successful). [10]

(This question paper contains 2 page(s) and 5 Questions.)

Register No.:	1	4	C	A	7	9	

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Odd Semester (2015- 2016)
Course Code: MCA801
Date:30/11/2015
Time:2.00 PM to 5.00 PM
Examination: End Sem
Course Name: Computer Algorithms
Maximum Marks: 100

INSTRUCTIONS:

1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

Q.1 (a) Prove by Mathematical induction that [05]

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

(b) Solve the following recurrence using recursion tree method [05]

$$T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n \geq 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

(c) Find the asymptotic tight bound for [05]

i. $T(n) = 3T(n/2) + n \log n$

ii. $T(n) = 2T(\sqrt{n}) + \log n$

(d) Explain an algorithm to find the second largest in an unsorted array that takes minimum possible number of comparisons. How many comparisons does it make to find find second smallest in an array of size 64. [05]

Q.2 (a) Differentiate between the divide and conquer and dynamic problem techniques for solving problems. [05]

(b) Assume that you are enough Indian currency notes of denominations (10, 20, 50, 100, 500, 1000). Design an algorithm that returns the minimum number of denominations for a given amount, which is a multiple of 10. Eg: $5880 = \langle 1000 \times 5, 500 \times 1, 100 \times 3, 50 \times 1, 20 \times 1, 10 \times 1 \rangle$ [05]

(c) State and prove the principle of optimality in computing a minimum spanning tree. [05]

(d) With respect to KMP algorithm for string matching, compute the prefix function Π for the pattern *ababaabbbabab* [05]

Q.3 (a) Explain the principle of optimality in Matrix chain multiplication problem. [05]

(b) Define optimal binary search tree problem. Determine the cost and structure of an optimal BST for a set $n = 7$ with the following probabilities: [10]

i	0	1	2	3	4	5	6	7
p_i		0.04	0.06	0.08	0.02	0.10	0.12	0.14
q_i	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05

(c) Find the LCS of "CGATAATTGAGA" and "GTAGAAG". Show the value of each sub-problem. [05]

Q.4 (a) Explain the Depth first search traversal Algorithm. Illustrate the construction of DFS tree on the graph given in Figure 1. [5]

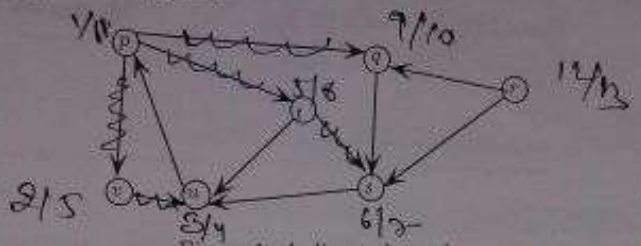


Figure 1: A directed graph

(b) Illustrate the parenthesis theorem in DFS for the Figure 1. [5]

(c) Obtain all possible topological ordering for the graph in Figure 2. [5]

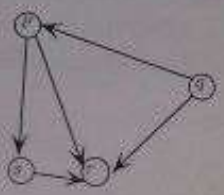


Figure 2: A directed acyclic graph

(d) Write the algorithm to compute the strongly connected component in a graph. Illustrate the same on the graph given in Figure 1. [5]

Q.5. (a) Explain the steps in proving a problem to be NP-Complete. [5]

(b) Explain (i) the vertex cover problem and (ii) Clique problem in a graph. [5]

(c) Explain how is Clique problem is reduced to the Vertex cover problem. [5]

(d) Write an approximation algorithm for Vertex-cover problem. Derive the approximation guarantee of your algorithm. [5]

(This question paper contains 2 page(s) and 5 Questions.)

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Even Semester (2014- 2015)
Course Code: MCA801 **Examination: End Sem**
Date:30/04/2015 **Time:09.00 AM to 12.00 AM** **Course Name: Computer Algorithms** **Maximum Marks: 100**

INSTRUCTIONS:
1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

Q.1. (a) Suppose you have algorithms with the following running times listed below. (Assume these are the exact running times). (1)How much slower do each of these algorithms run when you double the input size? (2)How much faster do each of these algorithms run when the input size is reduced to one third? [10]

i. n^2
ii. n^3
iii. $100n^2$
iv. $n \log_2 n$
v. 2^n

(b) Solve the following recurrence using recursion tree method [06]

$$T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

(c) Find the asymptotic tight bound for [04]

i. $T(n) = 3T(n/2) + n \log n$
ii. $T(n) = 2T(n/4) + n$

Q.2. (a) Consider an array $a_1 \ldots a_n$. A pair (a_i, a_j) is called an *inversion* if $a_i > a_j$ for two indices i and j such that $i < j$. For example in the sequence $\langle 2, 4, 1, 3, 5 \rangle$ there are 3 inversions $(2, 1)$, $(4, 1)$ and $(4, 3)$. Design a divide and conquer based algorithm to count the number of inversions in a given array. [10]

(b) Suppose you are given an array A with n entries, with each entry holding a distinct number. A sequence of values is *unimodal*, if for some index p between 1 and n, the values in the array entries increase upto position p in A and then decrease the remainder of the way until position n. For example in the array $\langle 1, 4, 8, 5, 2 \rangle$, the peak entry is 8 which is in position 3. Write an algorithm to find the peak entry in such an array in $O(\log n)$ time. [06]

(c) Differentiate between the divide and conquer and dynamic problem techniques for solving problems. [04]

3. (a) Explain the principle of optimality in Longest common subsequence (LCS) problem. [04]

(b) Find the LCS of 'CGATAATTGAGA' and 'GTAGAAG'. Show the value of each sub-problem. [08]

(c) What are prefix codes? Draw the frequency table and Huffman tree for the following string: [08]
'dogs do not spot hot pots or cats'

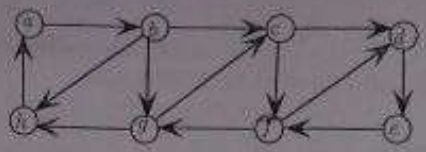


Figure 1: A directed graph

Q.4 (a) Explain the algorithm to find the strongly connected component in a graph. [06]

(b) Find the strongly connected components in the graph in Fig. 1. [08]

(c) Explain the parenthesis theorem with respect to the DFS algorithm. [06]

Q.5. ~~greedy algorithms~~

(a) Define a minimum cost spanning tree (MST). State and prove the principle of optimality in computing an MST. [08]

(b) Obtain the topological ordering for the graph in Fig. 2. [08]

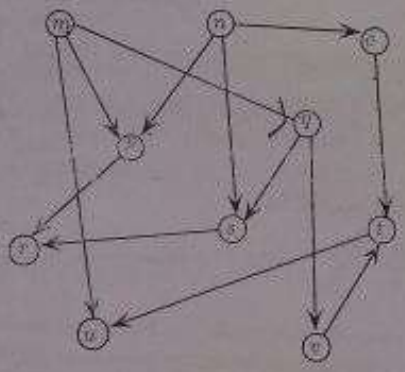


Figure 2: A directed acyclic graph

(c) Give the adjacency list representation for a complete binary tree having 7 vertices. Assume that the vertices are numbered from 1 to 7. Give an equivalent adjacency matrix representation for the same. [04]

(This question paper contains 2 page(s) and 5 Questions.)

Register No.

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

ven Semester (2014- 2015) — Examination: End Sem
ourse Code: MCA801 — Course Name: Computer Algorithms
ate:30/04/2015 — Time:09.00 AM to 12.00 AM — Maximum Marks: 100

NSTRUCTIONS:
1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

(a) Suppose you have algorithms with the following running times listed below. (Assume these are the exact running times) (1)How much slower do each of these algorithms run when you double the input size? (2)How much faster do each of these algorithms run when the input size is reduced to one third? [10]

i. n^2
ii. n^3
iii. $100n^2$
iv. $n \log_2 n$
v. 2^n

(b) Solve the following recurrence using recursion tree method [06]

$$T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

Find the asymptotic tight bound for [04]

i. $T(n) = 3T(n/2) + n \log n$
ii. $T(n) = 2T(n/4) + n$

(a) Consider an array $a_1 \ldots a_n$. A pair (a_i, a_j) is called an *inversion* if $a_i > a_j$ for two indices i and j such that $i < j$. For example in the sequence $(2, 4, 1, 3, 5)$ there are 3 inversions $(2, 1)$, $(4, 1)$ and $(4, 3)$. Design a divide and conquer based algorithm to count the number of inversions in a given array. [10]

(b) Suppose you are given an array A with n entries, with each entry holding a distinct number. A sequence of values is *unimodal*, if for some index p between 1 and n, the values in the array entries increase upto position p in A and then decrease the remainder of the way until position n. For example in the array $(1, 4, 8, 5, 2)$, the peak entry is 8 which is in position 3. Write an algorithm to find the peak entry in such an array in $O(\log n)$ time. [06]

Differentiate between the divide and conquer and dynamic problem techniques for solving problems. [04]

Explain the principle of optimality in Longest common subsequence (LCS) problem. [04]

Find the LCS of 'CGATAATTGAGA' and 'GTAGAAG'. Show the value of each sub-problem. [08]

What are prefix codes? Draw the frequency table and Huffman tree for the following string: [08]
'dogs do not spot hot pots or cats'

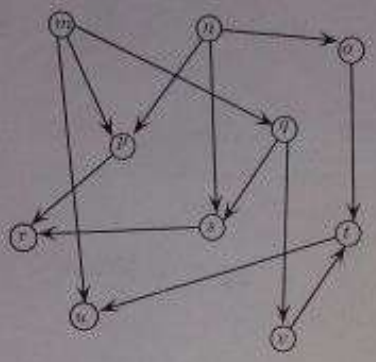


Figure 1: Directed acyclic graph

Repeat

4. Obtain the topological ordering for the graph in Fig. 1. (4)

5. Obtain the Strongly connected component for the graph in Fig 2. (4)

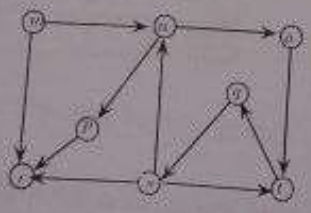


Figure 2: Directed graph

Department of MACS, NITK, Surathkal
Quiz-1

Reg. No. 14CA35 Name:
Subject: MCA801 Computer Algorithms
Date& Time: 16 Nov 2015, 9:00AM

Duration: 45 mins
Max. Marks: 20

1. Find the LCS of $X = \langle ABABBACCAB \rangle$ and $Y = \langle ABCBA \rangle$. Show value of each subproblem. (4)

2. Find solution to following instance of 0-1 Knapsack problem with capacity $W = 8$. Derive the value of each subproblem and arrive at final solution.

Item	Weight	Value
1	1	8
2	3	6
3	5	5

(4)

3. With respect to KMP algorithm for string matching Compute the prefix function Π for the pattern *ababaababbaba* (4)

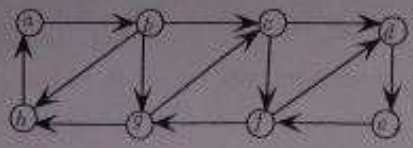


Figure 1: A directed graph

Q.4. (a) Explain the algorithm to find the strongly connected component in a graph. [0
(b) Find the strongly connected components in the graph in Fig. 1. [0
(c) Explain the parenthesis theorem with respect to the DFS algorithm. [06]

Q.5. ~~greedy algorithms~~

(a) Define a minimum cost spanning tree (MST). State and prove the principle of optimality in computing an MST. [08]

(b) Obtain the topological ordering for the graph in Fig. 2. [08]

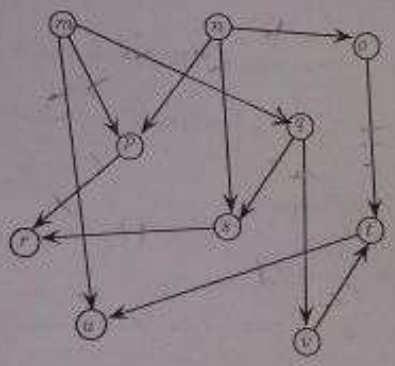


Figure 2: A directed acyclic graph

(c) Give the adjacency list representation for a complete binary tree having 7 vertices. Assume that the vertices are numbered from 1 to 7. Give an equivalent adjacency matrix representation for the same. [04]

(This question paper contains 2 page(s) and 5 Questions.)

Q.3. Let $A[1 \ldots n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion of A.

(a) List the five inversions of the array $\langle 2, 3, 8, 6, 1, 4 \rangle$ [2]

(b) What array with elements from the set $\{1, \ldots, n\}$ has the most inversions? How many does it have? [2]

(c) Give an algorithm that determines the number of inversions in any permutation on n elements in $\Theta(n \log n)$ worst-case time. [6]

Q.4. (a) Solve the following recurrence using recursion tree method [03]

$$T(n) = \begin{cases} 4T(n/2) + n^2 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

(b) Find the asymptotic tight bound for $T(n) = 8T(n/2) + n^2$ using Master method and verify the same using substitution method. [03]

(c) Find the asymptotic tight bound for the following: [04]

i. $T(n) = T(n/2) + T(n/4) + T(n/8) + n$

ii. $T(n) = T(n-1) + n$

Q.5. (a) Give an iterative algorithm for Random-Selection, that is to find the i^{th} smallest element in an unsorted array. Prove that it works in average case linear time. [05]

(b) Define the Rod cutting problem. How do you give a recursive solution for the problem? [02]

(c) Provide a bottom up solution for rod cutting problem along with the construction of optimal solution [03]

(This question paper contains 2 page(s) and 5 Questions.)

Q.4. (a) Explain the Depth first search traversal Algorithm. Illustrate the construction of DFS tree on the graph given in Figure 1. [5]

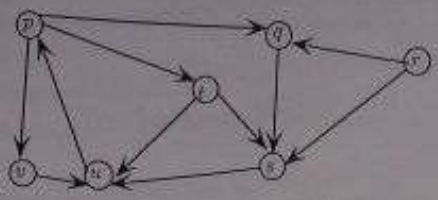


Figure 1: A directed graph

(b) Illustrate the parenthesis theorem in DFS for the Figure 1. [5]

(c) Obtain all possible topological ordering for the graph in Figure 2. [5]

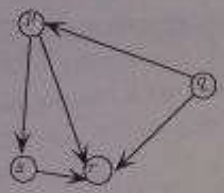


Figure 2: A directed acyclic graph

(d) Write the algorithm to compute the strongly connected component in a graph. Illustrate the same on the graph given in Figure 1. [5]

Q.5. (a) Explain the steps in proving a problem to be NP-Complete. [5]

(b) Explain (i) the vertex cover problem and (ii) Clique problem in a graph. [5]

(c) Explain how is Clique problem is reduced to the Vertex cover problem. [5]

(d) Write an approximation algorithm for Vertex-cover problem. Derive the approximation guarantee of your algorithm. [5]

(This question paper contains 2 page(s) and 5 Questions.)

Register No.:	1	4	C	A	3	5	

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Odd Semester (2015- 2016) | Examination: End Sem
Course Code: MCA801 | Course Name: Computer Algorithms
Date:30/11/2015 | Time:2.00 PM to 5.00 PM | Maximum Marks: 100

INSTRUCTIONS:
1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

Q.1 (a) Prove by Mathematical induction that [05]

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

(b) Solve the following recurrence using recursion tree method [05]

$$T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

(c) Find the asymptotic tight bound for [05]
   i. $T(n) = 3T(n/2) + n \log n$
   ii. $T(n) = 2T(\sqrt{n}) + \log n$

(d) Explain an algorithm to find the second largest in an unsorted array that takes minimum possible number of comparisons. How many comparisons does it make to find find second smallest in an array of size 64. [05]

Q.2 (a) Differentiate between the divide and conquer and dynamic problem techniques for solving problems. [05]

(b) Assume that you are enough Indian currency notes of denominations $\{10, 20, 50, 100, 500, 1000\}$. Design an algorithm that returns the minimum number of denominations for a given amount, which is a multiple of 10. Eg: $5880 = \{1000 \times 5, 500 \times 1, 100 \times 3, 50 \times 1, 20 \times 1, 10 \times 1\}$ [05]

(c) State and prove the principle of optimality in computing a minimum spanning tree. [05]

(d) With respect to KMP algorithm for string matching, compute the prefix function Π for the pattern *ababaababbabab* [05]

Q.3 (a) Explain the principle of optimality in Matrix chain multiplication problem. [05]

(b) Define optimal binary search tree problem. Determine the cost and structure of an optimal BST for a set $n = 7$ with the following probabilities: [10]

i	0	1	2	3	4	5	6	7
p_i		0.04	0.06	0.08	0.02	0.10	0.12	0.14
q_i	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05

(c) Find the LCS of 'CGATAATTGAGA' and 'GTAGAAG'. Show the value of each sub-problem. [05]

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Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Even Semester (2013- 2014) — Examination: Mid Sem
Course Code: MCA801 — Course Name: Computer Algorithms
Date:12/09/2014 — Time:10:30 AM to 12.00 PM — Maximum Marks: 50

INSTRUCTIONS:

1. Answer ALL questions.
2. Rough work/answers should NOT be written on the question paper.

Q.1. (A) Consider 2 programs A and B whose running times are $T_A = 100n$ and $T_B = 2n^2$ respectively. Suppose that both these running times are the number of milliseconds taken on a particular computer on an input of size n. Find the range of values for n for which program B runs faster than A. Also plot the running times against n values. [06]

(B) Consider the programs A and B above. Complete the entries in the following table by finding the maximum problem size allowed for progrms A and B for the time (sec) given in column 1. [04]

Time sec.	Maximum Problem size Solvable with program A	Maximum Problem size Solvable with program B
1	10	?
10	?	?
100	?	?
1000	?	?

(C) Use Mathematical induction to prove that [04]

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

Q.2. (A) Write a non recursive procedure for Randomized Select. [05]

(B) Explain your code to find the 4th largest in the array $\langle 12, 3, 6, 1, 5, 10, 9, 7 \rangle$. [04]

Q.3. (A) Find the asymptotic upper bound for the recurrence [04]
$T(n) = 2T(n-1) + O(n)$

(B) Solve the following recurrence using recursion tree method [06]

$$T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

Q.4. An algorithm SELECT finds the i^{th} smallest by dividing the input elements into groups of 5.

(A) Will the algorithm work in linear time if they are divided into groups of 7? [04]

(B) Prove that SELECT does not run in linear time if groups of 3 are used. [04]

(C) Use Masters method to solve the recurrence $T(n) = 2T(n/2) + O(n \log n)$. [02]

Q.5. Let $X[1..n]$ and $Y[1..n]$ be two arrays, each containing n numbers already in sorted order. Give an $O(\log n)$ time algorithm to find the median of all $2n$ elements in arrays X and Y. [06]

(This question paper contains 1 page(s) and 5 Questions.)

Register No.: 14CA35

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

odd Semester (2015- 2016)
Course Code: MCA801
Date:10/09/2015

Examination: Mid Sem
Course Name: Computer Algorithms
Time:1.30 to 3.00 PM
Maximum Marks: 50

INSTRUCTIONS:

1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the Question Paper.

Q.1 Consider a computer that can perform 10^8 instructions per second. In the table given below, function $f(n)$ represents the number of instructions in an algorithm. Complete the entries in the table by calculating the time taken by different algorithms for different n values in each row. [10]

	$f(n)$			
n	n	$n \log n$	n^2	n^4
10	$0.1\mu s$	?	?	?
50	?	?	?	?
100	?	?	?	?
1000	?	?	?	?
10^4	?	?	?	?
10^6	?	?	?	?

Q.2 (a) Suppose you have algorithms with the following running times listed below. (Assume these are the exact running times.) How much slower do each of these algorithms get when you double the input size? [5]

i. n^2
ii. n^3
iii. $100n^2$
iv. $n \log_2 n$
v. 2^n

(b) Find the total number of operation performed by the following code. Express the same in Θ notation. [5]

Algorithm Exponentiate(x, n)

```
{
m = n; power = 1; z = x;
while(m > 0) do
{
    while(m mod 2 == 0) do
    {
        m = ⌊(m/2)⌋; z = z * z;
    }
    m = m - 1; power = power * z;
} return power;
}
```

$1 + \log n + 4$
$= 7 + \log n$
$\Theta(\log n)$