2019

BIG DATA ECONOMETRICS FINAL EXAM PROJECT (Instructor-Dr Zheng Li)

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Contents

Introduction:

This project has been completed as part of the coursework for my Master's in Financial Mathematics degree, ECG-590, BIG DATA ECONOMETRICS taught by Professor Zheng Li. I as a part of this exam project aim to apply all possible machine learning algorithms, statistical methods to reduce the Mean Squared Error of the test datasets.

Data:

Data was provided by the instructor himself so, we didn't have any freedom on the data part. Data has 50 independent variables and one dependent variables. Predictors are named as x01, x02,x03……..,x50. Target variable here is named as y. We have 400 datapoints .This is a snapshot of the data for the project.

```
[6] ExamData.head()
```

	y	x01	x02	x03	x04	x05	x06	x07	x08	x09	x10	x11	x12	x13	x14	x15	x16	x1
0	-3.987019	-1.150161	1.919942	-0.200096	-0.144521	-0.482146	-0.960322	-1.817256	-0.650505	-1.263704	-0.504251	-1.096154	1.802635	-0.230030	0.011708	-0.626882	-0.985122	-1.7794
1	4.385074	0.202677	1.184966	0.554749	1.440169	0.775009	0.645686	-0.504184	0.410975	-0.027140	-0.512595	0.165817	1.162804	0.704654	1.629204	0.714029	0.783973	-0.4688
2	8.190520	0.821439	1.066287	0.380227	0.115380	-0.057053	-1.254392	0.431652	1.063195	-2.030637	-0.220205	0.935704	1.027793	0.396658	0.153714	-0.110178	-1.375280	0.5669
3	0.718460	1.051590	-0.854382	0.460841	-1.645333	2.325046	0.840424	0.032657	0.082418	-0.401942	-0.247423	1.076781	-0.902341	0.460733	-1.702875	2.191417	0.788880	0.1404
4	-7.689298	-1.017523	-0.592204	-0.896561	1.946834	-0.740624	-0.814975	-0.011092	-0.593935	-2.546734	-1.984688	-1.051631	-0.578143	-0.890222	1.821483	-0.815850	-0.747625	-0.1224

```
[7] ExamData.shape
```

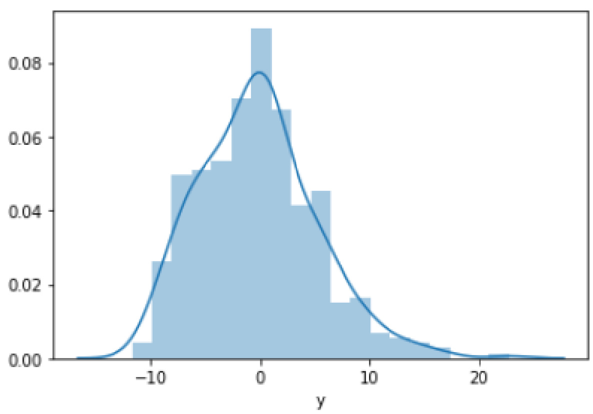
```
(400, 51)
```

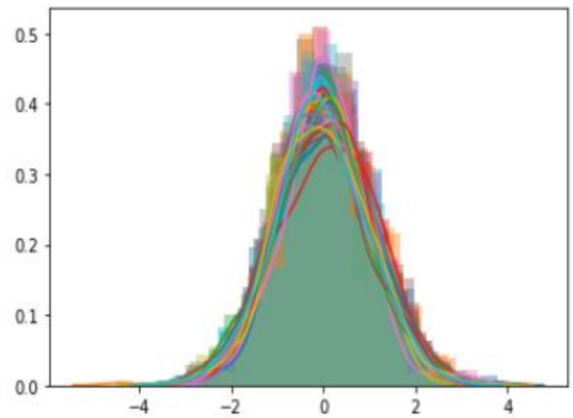
Train Test Split: To check the real performance of any model on unseen observations, data was divided into two halves, one to train the model and another to test the model. Thus, we would have 200 training datapoints and 200 test datapoints.

Data Exploration:

Distributions:

Let us see the distribution of both predictor and target variables to identify what kind of problem it is what can be available methodologies can be solve this problem.

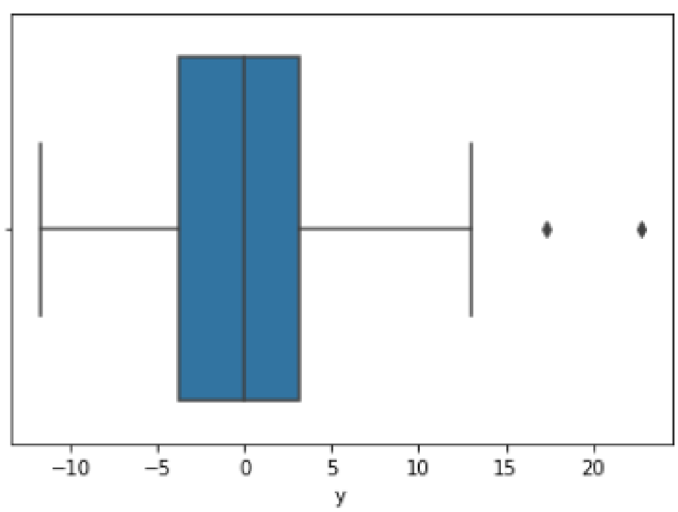




We can see y ranges from -10 to 20 and x ranges from -4 to 4 approximately. They are clearly continuous in nature and thus this is a regression problem.

Outliers:

We want to detect and identify the outliers in the training dataset so that model doesn't learn from outlier values. However, I would not attempt to remove outlier values from test dataset since, model performance should be measured in untampered unseen observations. We would use boxplot distribution to check if we have outliers in training data or not.



So, we have two outliers in positive end and beyond the 1.5 times of Inter Quantile Range. These two points are at index 243 and 104.

```
y_train[y_train > (Q3 + 1.5 * IQR)]

243    17.289235
104    22.790085
Name: y, dtype: float64
```

These two outlier points has been removed from training dataset leaving us with only 198 points to train the model.

Please note that no data transformations like scaling as Standardization or Normalization is needed in this dataset since all predictors are on same scale and ranges from -4 to 4.

Modeling to reduce Mean Squared Error:

We would start gradually from simpler model to more complex model and along the way we would do some variable selections, best subset selections , collinearity analysis and measure the MSE on test data. For Neural Network model ,we have included the Outliers in training data because Artificial Neutral Network Sequential Feed Forward model through enough number of iterations of learning from each batches and backpropagations is apt in learning from outliers too.

1. Null Model:

Null Model indicates only constant term and no variables used as predictors at all. We have MSE of 28.9.

```
#Null model, only a constant
Null_MSE=((y_test-y_train.mean())**2).mean()
print('Null model',Null_MSE)
```

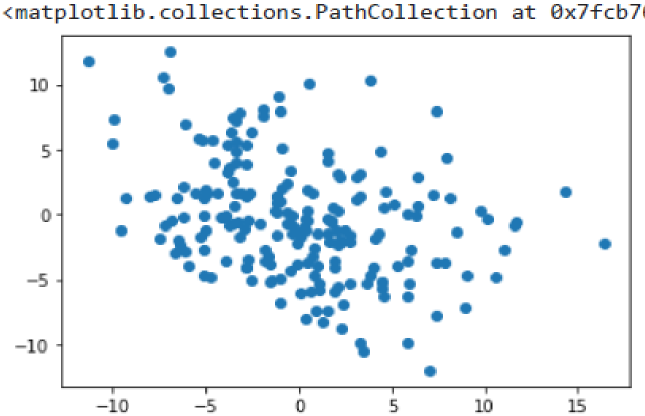
```
Null model 28.907721505285107
```

2. Linear Regression:

2.1. Linear Regression having all 50 predictors: MSE decreased to 23.78%.

```
plt.scatter(y_test-y_pred,y_pred)
```

```
<matplotlib.collections.PathCollection at 0x7fcb76808eb8>
```



Since, there is no pattern, linear regression model is good fit for the data

2.2. Multicollinearity Analysis using Variance Inflation factor:

As per rough standard VIF above 10 indicates multicollinearity. First 29 variables had very high VIF in the range of 200s and 100s. In our data taking 10 as threshold VIF we had first 29 variables in one Linear Regression Model and second set of remaining 21 variables in second Linear Regression model.

2.3. Linear Regression model on 2nd set of 21 variables:

```
lm.fit(X_train.iloc[:,29:],y_train)
```

```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

```
y_pred_1= lm.predict(X_test.iloc[:,29:])
print('MSE:', metrics.mean_squared_error(y_test, y_pred_1))
```

```
MSE: 28.270774956447195
```

2.4. Linear Regression on 1st set of 29 variables:

```
y_pred_2= lm.predict(X_test.iloc[:,:29])
print('MSE:', metrics.mean_squared_error(y_test, y_pred_2))
```

```
MSE: 20.50058173101595
```

3. Polynomial Quadratic Model:

MSE decreased to 18.73 in our Quadratic model on all 50 variables indicating, we have nonlinear relationship between all predictors and y.

```
from sklearn.preprocessing import PolynomialFeatures

poly = PolynomialFeatures(degree = 2)
X_poly = poly.fit_transform(X_train)

poly.fit(X_poly, y_train)
lm.fit(X_poly, y_train)
```

```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

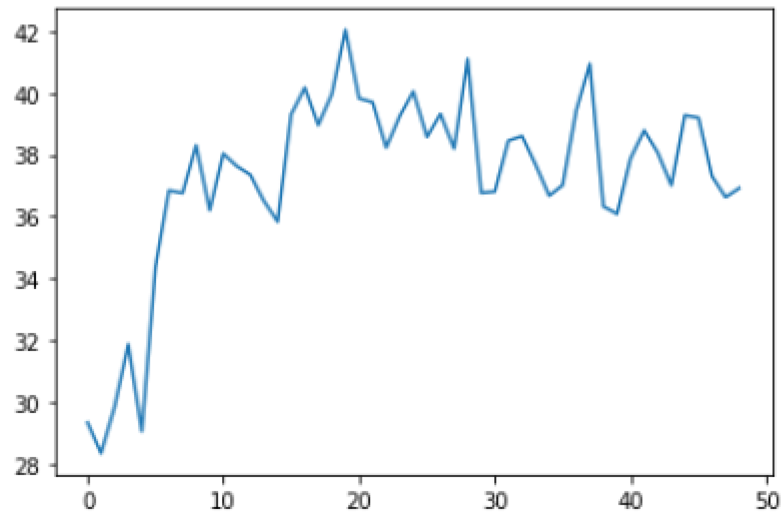
```
y_pred_3= lm.predict(poly.fit_transform(X_test))
print('MSE:', metrics.mean_squared_error(y_test, y_pred_3))
```

```
MSE: 18.733680182554075
```

When we tried to fit a quadratic model in selected features according to VIF as we did in Linear Regression but MSE increased to 95.93 indicating we are losing information by losing other features.

4. Decision Tree Regression:

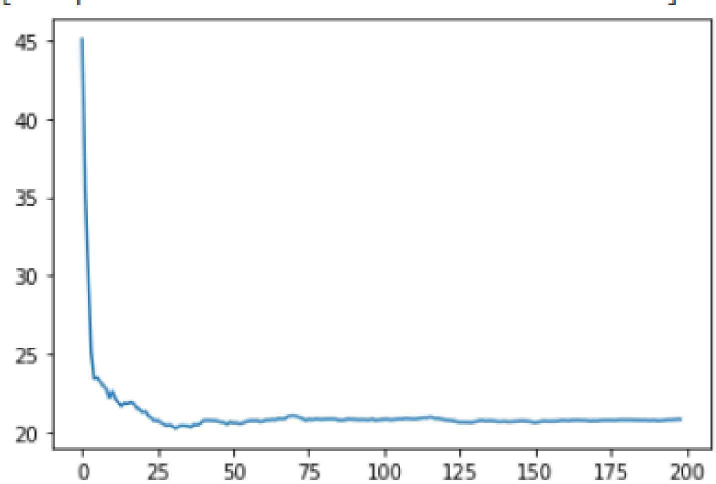
I tried to predict y for different depth level ranging from 1 to 50. This plot shows MSE for all the 50 depth levels. The minimum MSE we got here is 28.36% which is almost same as Null Model



5. Random Forest Regression:

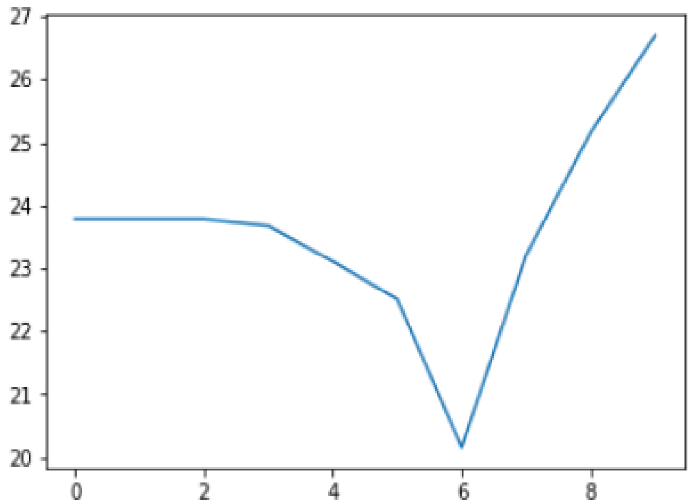
Random forest regression for number of trees ranging from 1 to 200 has following plot of MSEs with minimum of 20.27. It is certainly better than Decision Tree Regression model because of voting algorithm in Random Forest by several independent decision Trees in it.

```
[<matplotlib.lines.Line2D at 0x7fcb6b9f0828>]
```



6.Ridge Regression:

As we know Ridge Regression uses L2 norm to penalize parameter coefficients for different tuning parameter lambda. Here I have used 10 different penalty hyperparameter, [1e-15, 1e-10, 1e-8, 1e-4, 1e-3,1e-2, 1, 5, 10, 20]. Plot showing MSE for different parameter can be seen below. It has minimum MSE of 20.15 when penalty hyperparameter is 1.



7. Lasso Regression:

Lasso uses L1 norm to penalize predictors and thus be used to select variables as coefficients converges to zero at appropriate penalty hyperparameter. We have used Lasso here for both variable selection as well as model to reduce the MSE. Hyperparameter lambda to penalize the coefficient parameters are same 10 values used in Ridge Regression.

7.1. Lasso Regression to reduce MSE:

These were the MSEs for ten different values of Hyperparameters with minimum of 22.38 when penalty hyperparameter is 1.

```
23.782570634178445
23.782570598487528
23.782567066252813
23.747118911414262
23.454660531394467
22.381107314800232
27.480870793917646
28.907721505285117
28.907721505285117
28.907721505285117
```

7.2. Lasso Regression for variable selection:

It selected 20 variables out of 50 independent variables. These selected variables were feature_set=['x01','x02','x03','x04','x05','x06','x07','x09','x10','x11','x12','x15','x17','x18','x20','x21','x22','x24','x26','x29']

```
sel_.fit(scaler.transform(X_train),y_train)
```

```
SelectFromModel(estimator=LinearRegression(copy_X=True, fit_intercept=True,
                                           n_jobs=None, normalize=False),
                max_features=None, norm_order=1, prefit=False, threshold=None)
```

```
sel_.get_support()
```

```
array([ True, False,  True,  True,  True,  True,  True,  True, False,
        True,  True,  True,  True, False, False,  True, False,  True,
        True, False,  True,  True,  True, False,  True, False,  True,
       False, False,  True, False, False, False, False, False, False,
       False, False, False, False, False, False, False, False, False,
       False, False, False, False, False])
```

7.2.1. Linear Regression modeling using only selected variables.

Thus, MSE decreased to 20.76 which is quite improvement but not much. This indicates we have almost same information as Ridge or Linear Regression on first 29 variables. Compared to Linear Regression on all 50 variables, we had MSE of 23.77 and we are minimizing MSE by decreasing the number of variables from 50 to 20 which is great improvement

```
y_pred_sel= lm.predict(X_test_sel)
print('MSE:', metrics.mean_squared_error(y_test_sel, y_pred_sel))
```

```
MSE: 20.76247077216838
```

7.2.2. Random Forest on selected variables

```
#Random Forest on selected features

Acc_Random_sel=[]
for i in range(1,200):
  regressor = RandomForestRegressor(n_estimators = i, random_state = 0)
  regressor.fit(X_train_sel, y_train_sel)
  y_pred_sel_random=regressor.predict(X_test_sel)
  MSE=metrics.mean_squared_error(y_test_sel, y_pred_sel_random)
  Acc_Random_sel.append(MSE)
```

```
min(Acc_Random_sel)
```

```
24.443442944963035
```

Performance is worse than the Random forest regression on all 50 variables indicating we are losing information by losing other variables.

8. Best Subset Selection:

Best Subset selection is statically very good method to find the model with set of features giving best accuracy but is computationally inefficient method. As the number of independent variables increases, it become more and more computationally harder.

```
models_best = pd.DataFrame(columns=["MSE", "model"])

tic = time.time()
for i in range(4):
    models_best.loc[i] = getBest(i)

toc = time.time()
print("Total elapsed time:", (toc-tic), "seconds.")
```

```
Processed 1 models on 0 predictors in 0.015178918838500977 seconds.
Processed 50 models on 1 predictors in 0.3023357391357422 seconds.
Processed 1225 models on 2 predictors in 7.206106901168823 seconds.
Processed 19600 models on 3 predictors in 117.38464117050171 seconds.
Total elapsed time: 125.45728993415833 seconds.
```

We can use Forward, Backward or Mixed Stepwise selection method instead and is computationally feasible too. I didn't do that in this project because we had already done variable selection through Lasso Regression.

9. Artificial Neural Network Model: Sequential Feed Forward Neural Network Model

Sequantial Feed Forward Neural Network model with activation fucntion as Linear since, this is Regression problem. Loss function used is Mean Squared Error and optmization method used is Stochastic Gradient Descent.

Since, we have 50 predictors so, outside layer would have 50 neurons. We would define one hidden layer with 20 neurons and since, there is one y target variable and is a regression problem, output layer containing one 1 neurons. Since, our data is small and has only 200 observations (Note that we are removing outliers in case of Neural Network model from training datset since, it is capable of learning from outliers by backpropagation), we have used epoch size of 1000.

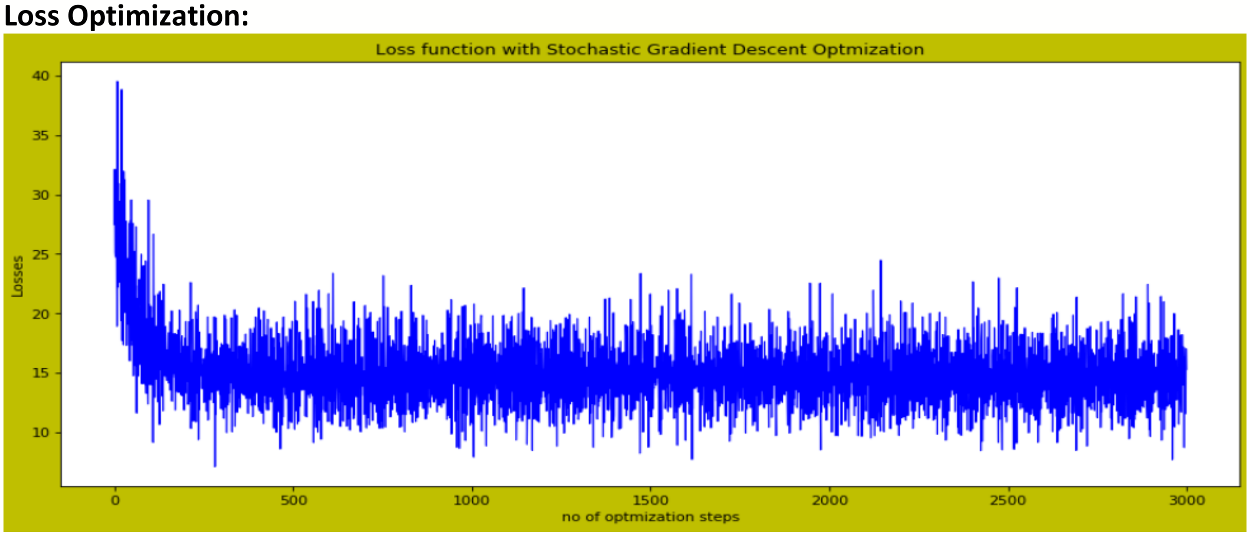
```
# design network
from keras import optimizers
sgd = optimizers.SGD(lr=0.01, clipnorm=1.)

model = Sequential()
model.add(Dense(number_of_neurons_layer1,input_shape=(dim, ), activation='linear'))
model.add(Dense(number_of_neurons_layer2, activation='linear'))
model.add(Dense(number_of_neurons_layer3, activation='linear'))
model.compile(loss='mean_squared_error', optimizer=sgd)

def train(data,label):
    model.fit(data, label, epochs=number_of_epochs, batch_size=72, validation_data=(data, label), verbose=0, shuffle=True,callbacks=[lr])

def score(data):
    return model.predict(data)
```

Loss Optimization:



Using Stochastic Gradient Descent to minimize the mean square error and backpropagation MSE on training data decreased to 6.615 but on test data, MSE was 22.83 which is not special and is same as compared to Linear Regression and Random Forest. However, this can be due to very less training datasets.

Conclusions:

1. Quadratic Model fitted best on this data with MSE of 18%

2. Random Forest performed better than than the Decision Tree or Linear Regression with all 5 0 features because of vote of several independent trees in the forest

3. Train Test split is a good method to measure the model accuracy on unseen data but since he re out data is small and has only 400 data points, 50% ratio is not good enough we would have as much data used to train the model. only 200 datapoints to train the model is not the best idea

4. There is multicollinearity in the variables but if we lose the variables expecially first 29 variabl es we are losing alot of information thus MSE increased from 23(Linear Regression when all 50 variables used) to 28(When last 21 variables used). VIF above 10 indicates multicollinearity

5. Ridge Regression with penalty parameter of 1 worked better than the Linear Regression as well as Lasso Regression

6. Lasso Regression is very important method to select the variables as it converges the parameter cofficient to zero at different penalty parameter lambda

7. Lasso Regression chose 20 variables out of 50 variables and MSE also improved from 23 when all 50 variables were used to 20 when only 20 variables were used in the model

8. Artificial Neural Network model redcuced the MSE to 6.615 on training dataset with only 200 points but performing not that good on test dataset. More training dataset might improve the model performance. By cross validation we can train the model on datapoints and also test the model on data, but we are not allowed to do that in this exam project.

9. Best subset can be a good algorithm to find best models but when we have too many idepen dent variables like here we have 50 variables , it becomes computationally unfeasible for 3 pred ictors only we have 19000 models. We can alternatively use forward, backward or mixed stepwise selection.

References:

Project Link:
https://github.com/deepak2025/BIG_DATA_ECONOMETRICS/blob/master/Big_Data_Econometrics_ECG_590_Final_Exam_Project%20(1).pdf

BIG DATA ECONOMETRIC FINAL EXAM PROJECT

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ECG-590

NORTH CAROLINA STATE UNIVERSITY

FINANCIAL MATHEMATICS, DEC'19

Goal of this project is to reduce the Mean squared Error using different Machine Learning and Deep Learning methods

Takeaway from this project:

1. Quadratic Model fitted best on this data with MSE of 18%
2. Random Forest performed better than than the Decision Tree or Linear Regression with all 50 features because of vote of several independent trees in the forest
3. Train Test split is a good method to measure the model accuracy on unseen data but since here out data is small and has only 400 data points, 50% ratio is not good enough we would have as much data used to train the model. only 200 datapoints to train the model is not the best idea
4. There is multicollinearity in the variables but if we lose the variables expecially first 29 variables we are losing alot of information thus MSE increased from 23(Linear Regression when all 50 variables used) to 28(When last 21 variables used). VIF above 10 indicates multicollinearity
5. Ridge Regression with penalty parameter of 1 worked better than the Linear Regression as well as Lasso Regression
6. Lasso Regression is very important method to select the variables as it converges the parameter cofficient to zero at different penalty parameter lambda
7. Lasso Regression chose 20 variables out of 50 variables and MSE also improved from 23% when all 50 variables were used to 20% when only 20 variables were used in the model
8. Artificial Neural Network model redcuced the MSE to 7.1% on training dataset with only 200 points but performing not that good on test dataset. More training dataset might improve the model performance. By cross validation we can train the model on datapoints and also test the model on data but we are not allowed to do that in this exam project.
9. Best subset can be a good algorithm to find best models but when we have too many idependent variables like here we have 50 variables , it becomes computationally unfeasible for 3 predictors only we have 19000 models. We can alternatively use forward, backward or mixed stepwise selection.

Let's start by importing libraries

In [0]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
%matplotlib inline
from sklearn.model_selection import train_test_split
```

Let's get the data file

In [4]:
```
from google.colab import files
uploaded = files.upload()
```

Choose Files No file chosen

Upload widget is only available when the cell has been executed in the current browser session. Please rerun this cell to enable.

```
Saving Data.csv to Data.csv
```

In [0]:
```
import io
ExamData = pd.read_csv(io.BytesIO(uploaded['Data.csv']))
```

Let's check the data once

In [6]:
```
ExamData.head()
```

Out[6]:

	y	x01	x02	x03	x04	x05	x06	x07	x08	x09	x10	
0	-3.987019	-1.150161	1.919942	-0.200096	-0.144521	-0.482146	-0.960322	-1.817256	-0.650505	-1.263704	-0.504251	-1.(
1	4.385074	0.202677	1.184966	0.554749	1.440169	0.775009	0.645686	-0.504184	0.410975	-0.027140	-0.512595	0.1
2	8.190520	0.821439	1.066287	0.380227	0.115380	-0.057053	-1.254392	0.431652	1.063195	-2.030637	-0.220205	0.9
3	0.718460	1.051590	-0.854382	0.460841	-1.645333	2.325046	0.840424	0.032657	0.082418	-0.401942	-0.247423	1.0
4	-7.689298	-1.017523	-0.592204	-0.896561	1.946834	-0.740624	-0.814975	-0.011092	-0.593935	-2.546734	-1.984688	-1.(

In [7]:
```
ExamData.shape
```

Out[7]: (400, 51)

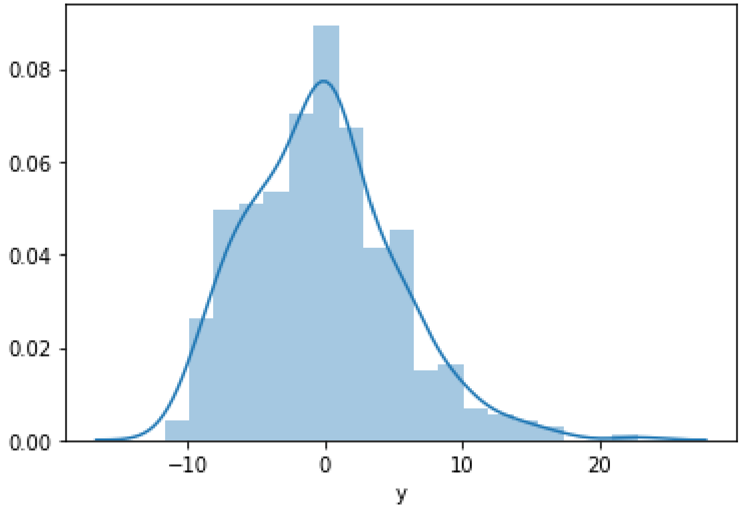
We can see we have 400 observations and 50 independent variables and 1 target variable y

Let's define x and y separately for modeling

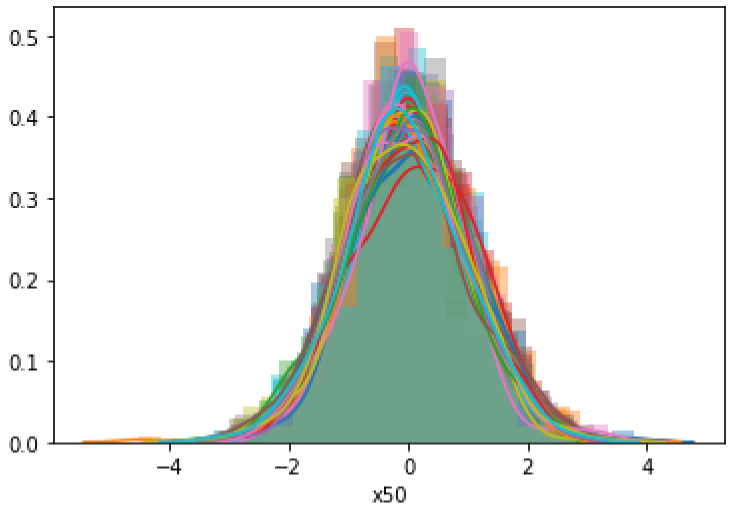
In [0]:
```
x=ExamData.iloc[:,1:]
y=ExamData.iloc[:,0]
```

In [14]:
```
sns.distplot(y)
```

Out[14]: <matplotlib.axes._subplots.AxesSubplot at 0x7ff805af4898>



In [15]:
```
for i in range(50):
  sns.distplot(x.iloc[:,i])
```



So, our target variable is continuous with values ranging from -10 to 21 approximately. So, clearly this is a regression problem. Also, almost all predictors ranges from -4 to 4 approx so, we don't really need to scaling like standardization or normalization

We are using Train Test Split with ratio of 50% for one half to be trained and another half to be used to check the model accuracy

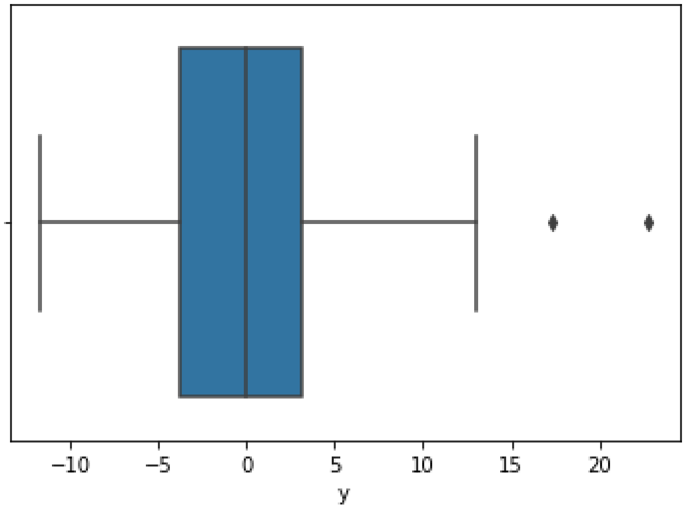
In [0]:
```
X_train, X_test, y_train, y_test = train_test_split(x, y, test_size=0.5, random_state=1)
```

Let's find the outliers by using boxplot method in training data and remove them so, that model doesn't learn from outlier values.

Let's check the outliers by having boxplot distribution of the target variable of trainig dataset only

In [17]:
```
sns.boxplot(y_train)
```

Out[17]: <matplotlib.axes._subplots.AxesSubplot at 0x7ff8063aa9b0>



We can see we have 2 outliers in the training y varaible which falls beyond the 1.5 times of interquartile range

Let's define the Interquartile range to detect the outliers

In [18]:
```
Q1 = y_train.quantile(0.25)
Q3 = y_train.quantile(0.75)
IQR = Q3 - Q1
print(IQR)
```

```
6.881305133
```

In [19]:
```
 y_train[y_train > (Q3 + 1.5 * IQR)]
```

Out[19]:
```
243    17.289235
104    22.790085
Name: y, dtype: float64
```

We have 2 outliers which is beyond positive 1.5 times of IQR in y_train and thus we would want to remove these two observations from X and Y training so, that model doesn't learn from them

In [20]:
```
y_train[y_train < (Q1 - 1.5 * IQR)]
```

Out[20]:
```
Series([], Name: y, dtype: float64)
```

In [21]:
```
print(type(y_train),type(X_train))
```

```
<class 'pandas.core.series.Series'> <class 'pandas.core.frame.DataFrame'>
```

In [0]:
```
y_train.drop([104,243],inplace=True)
```

In [23]:
```
y_train.shape
```

Out[23]:
```
(198,)
```

In [24]:
```
X_train.drop([104,243],inplace=True)
```

```
/usr/local/lib/python3.6/dist-packages/pandas/core/frame.py:4117: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#re
turning-a-view-versus-a-copy
  errors=errors,
```

In [25]:
```
X_train.shape
```

Out[25]:
```
(198, 50)
```

In [0]:
```
from sklearn import preprocessing
X_train_scaled = preprocessing.StandardScaler().fit(X_train).transform(X_train)
```

Goal of this project is to reduce the Mean squared Error using different algorithms so, we would start with Null model and then proceed to simpler algorithms to complex ones

Null Model

In [27]:
```
#Null model, only a constant
Null_MSE=((y_test-y_train.mean())**2).mean()
print('Null model',Null_MSE)
```

```
Null model 28.907721505285107
```

We have Mean Squared Error for Null model that is when we have only constant terms and no variables at all is 28.90

Linear Model

In [28]:
```
#Linear model
from sklearn.linear_model import LinearRegression
lm = LinearRegression()
lm.fit(X_train,y_train)
```

Out[28]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

In [0]:
```
y_pred= lm.predict(X_test)
```

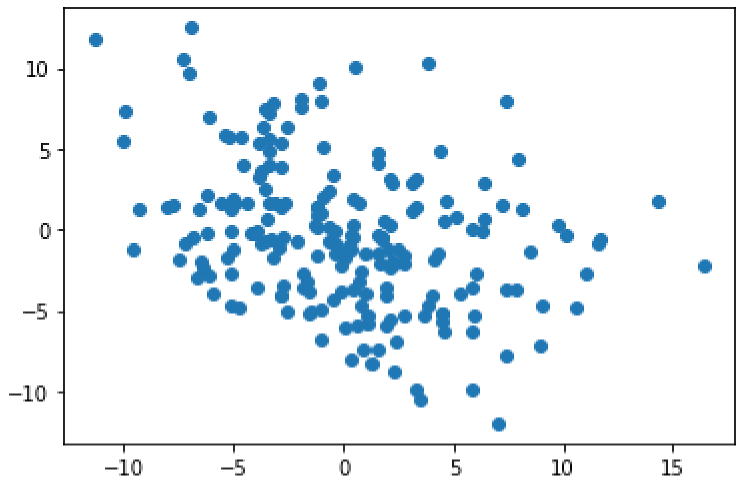
In [30]:
```
from sklearn import metrics
print('MSE:', metrics.mean_squared_error(y_test, y_pred))
```

```
MSE: 23.782570634177702
```

MSE decreased from 28.9 to 23.78 when we are using all the variables in Linear Regression

In [31]:
```
plt.scatter(y_test-y_pred,y_pred)
```

Out[31]: <matplotlib.collections.PathCollection at 0x7ff805cd26a0>

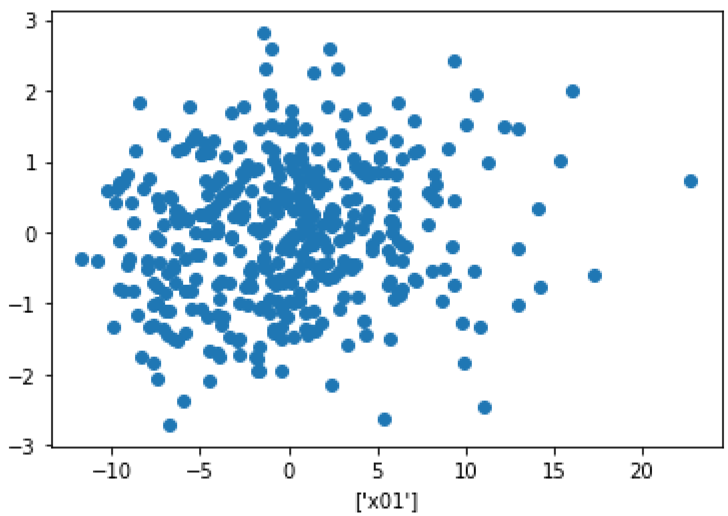


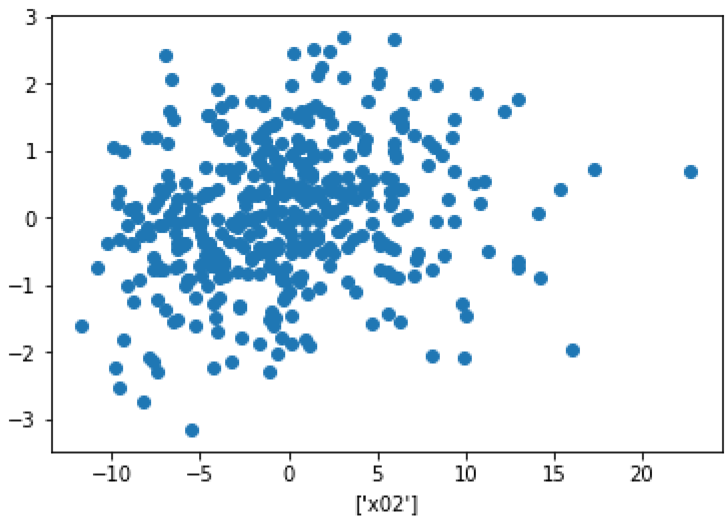
Since, there is no pattern, linear regression model is good fit for the data

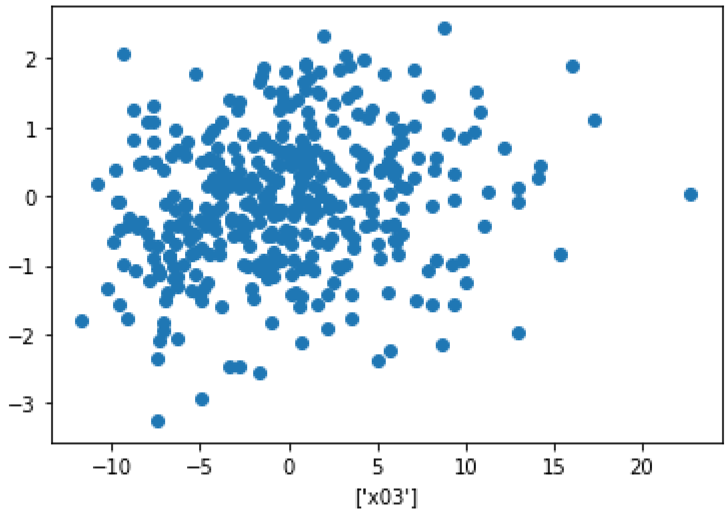
Let's see the plot of Y vs each of 50 predictors

In [32]:
```
for i in x.columns:
    plt.figure()
    plt.scatter(ExamData['y'],ExamData[i])
    plt.xlabel([i])
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:2: RuntimeWarning: More than 20 figures have bee
n opened. Figures created through the pyplot interface (`matplotlib.pyplot.figure`) are retained until explic
itly closed and may consume too much memory. (To control this warning, see the rcParam `figure.max_open_warni
ng`).
```



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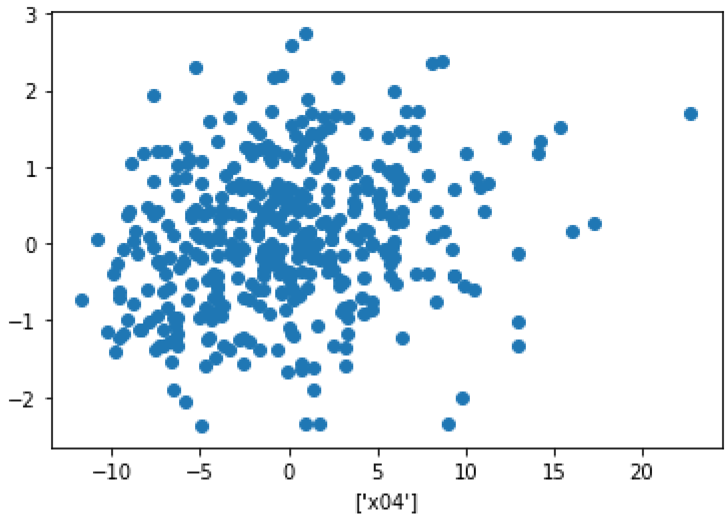
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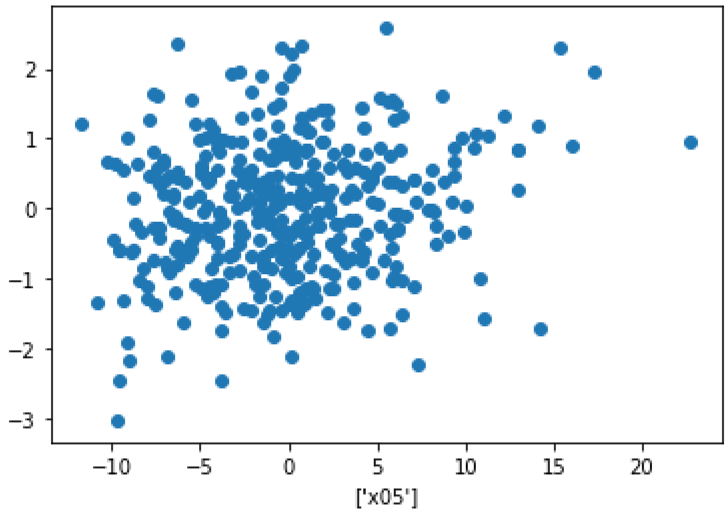
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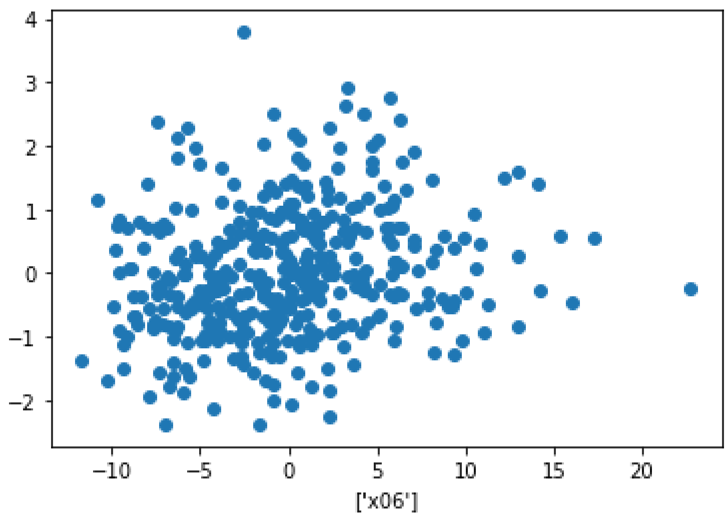
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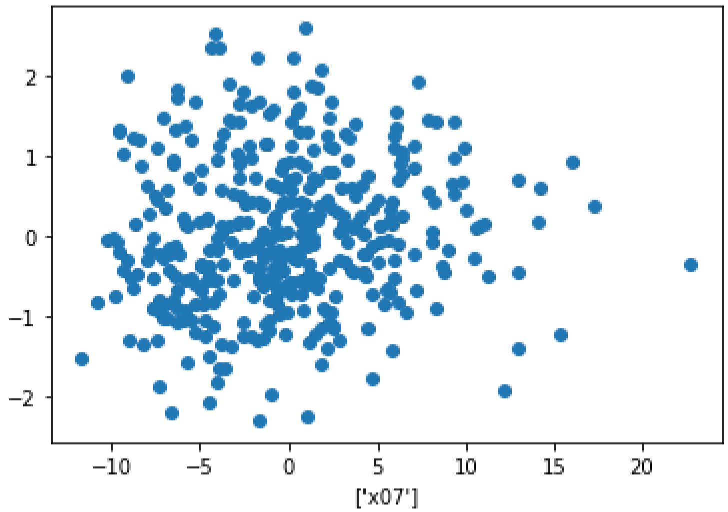
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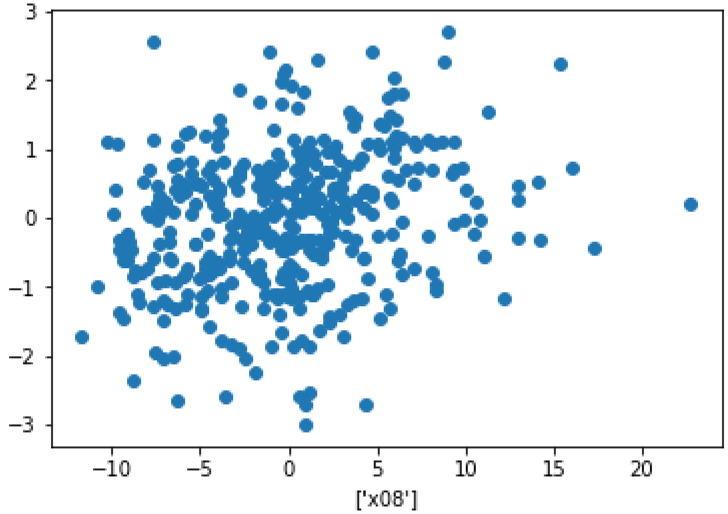
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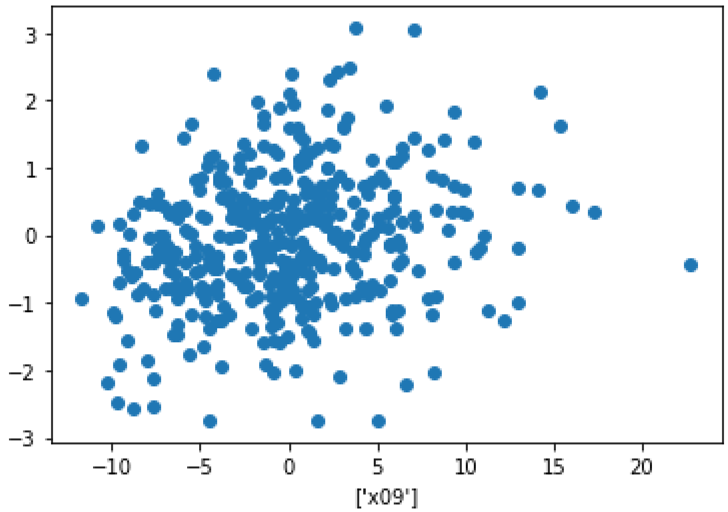
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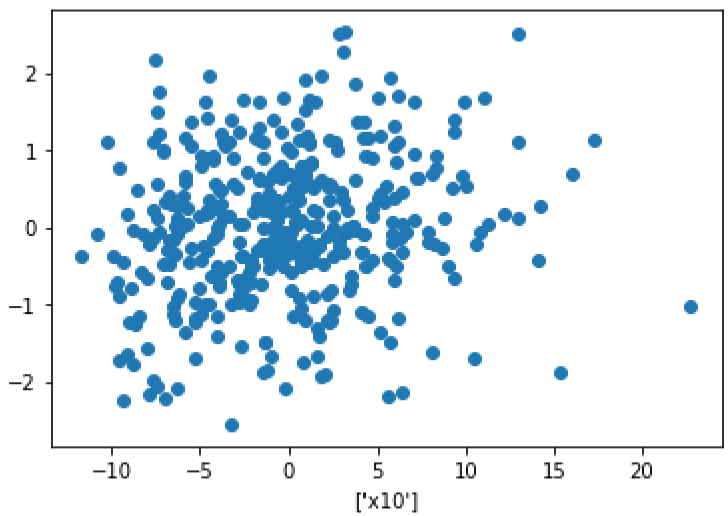
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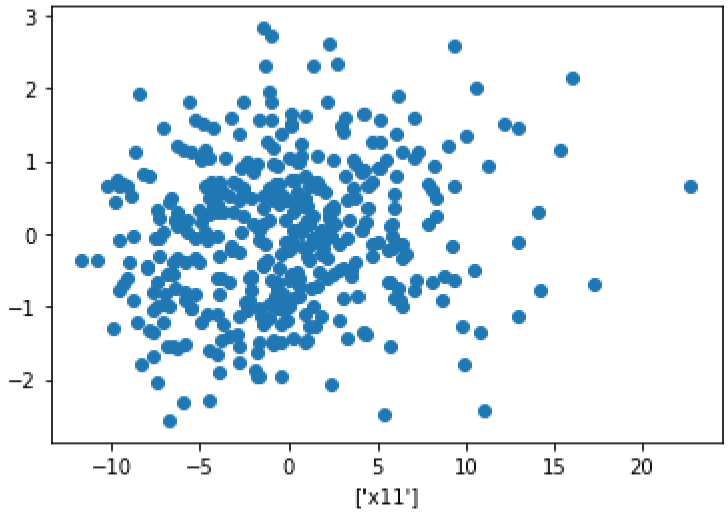
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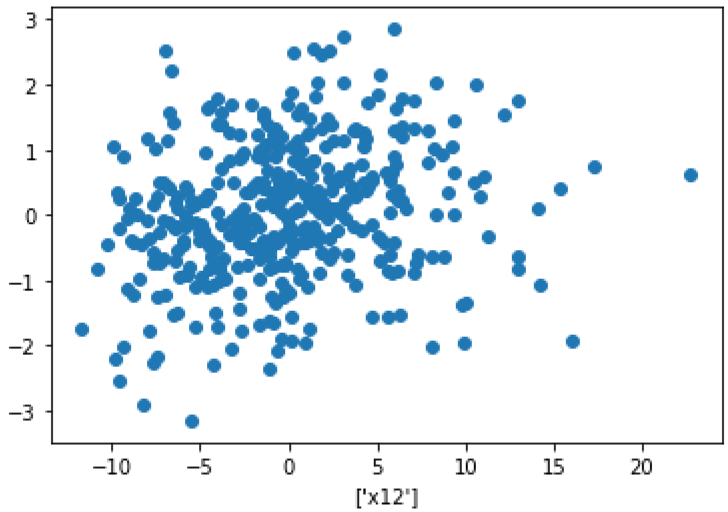
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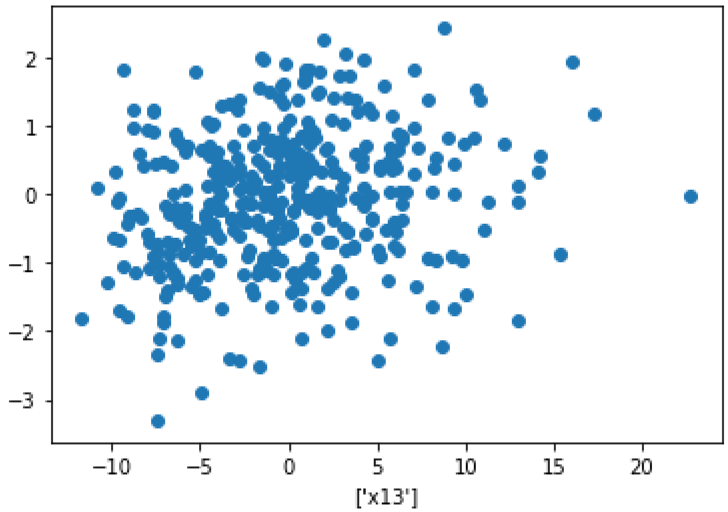
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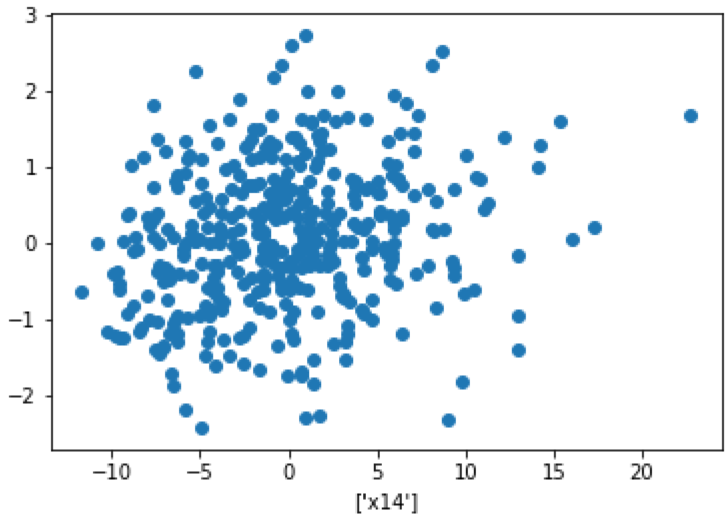
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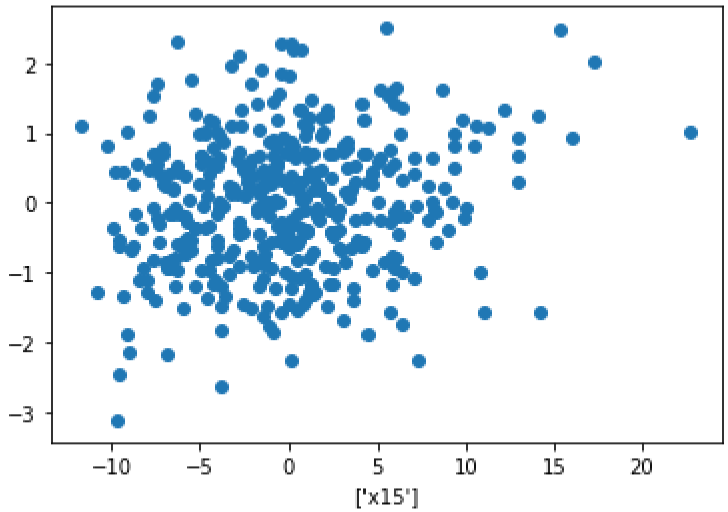
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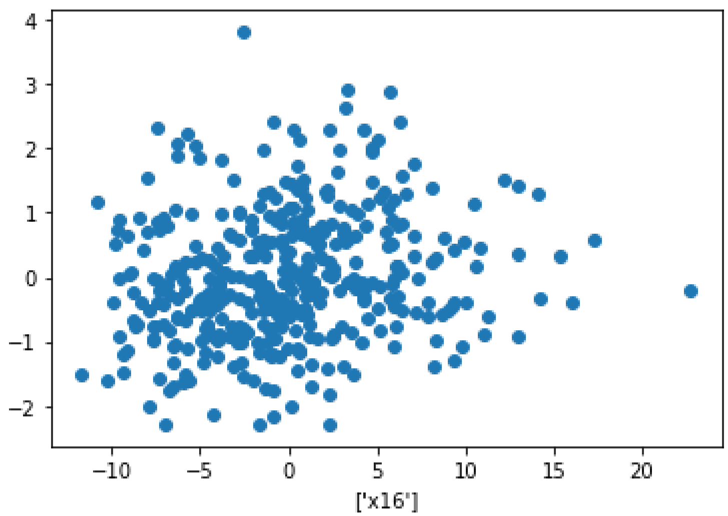
['x15']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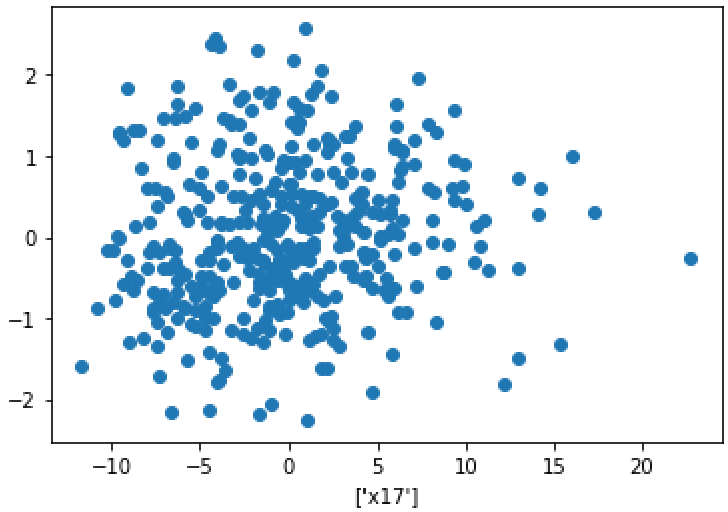
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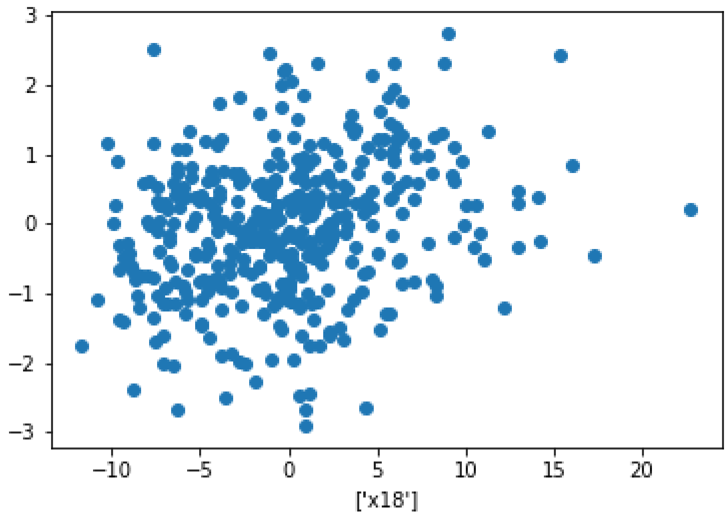
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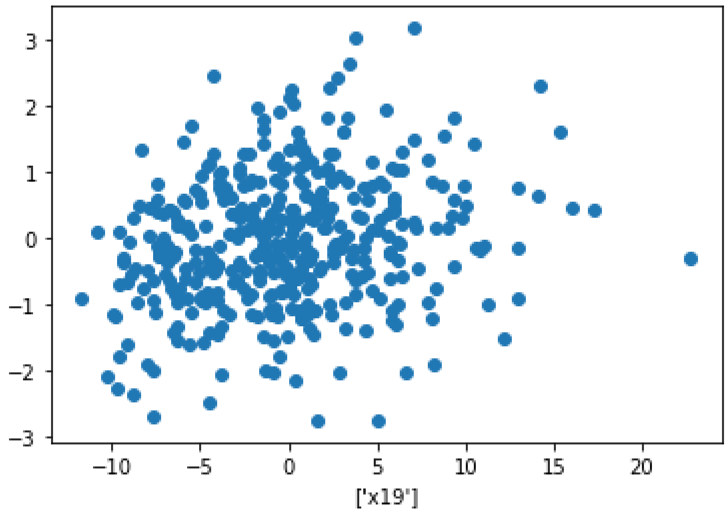
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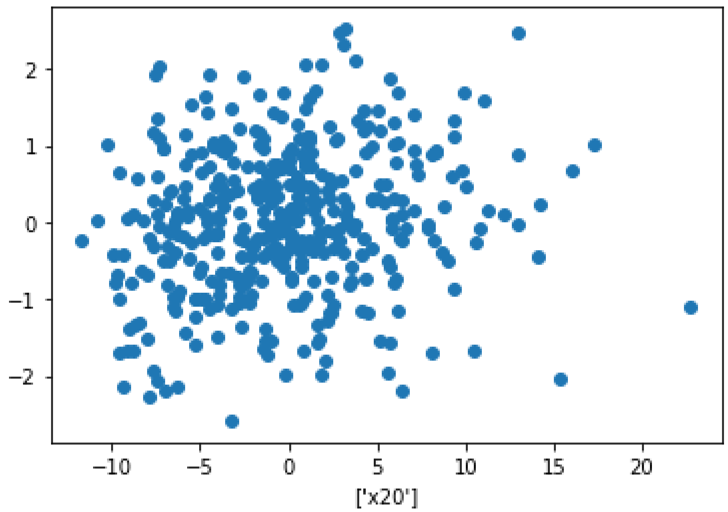
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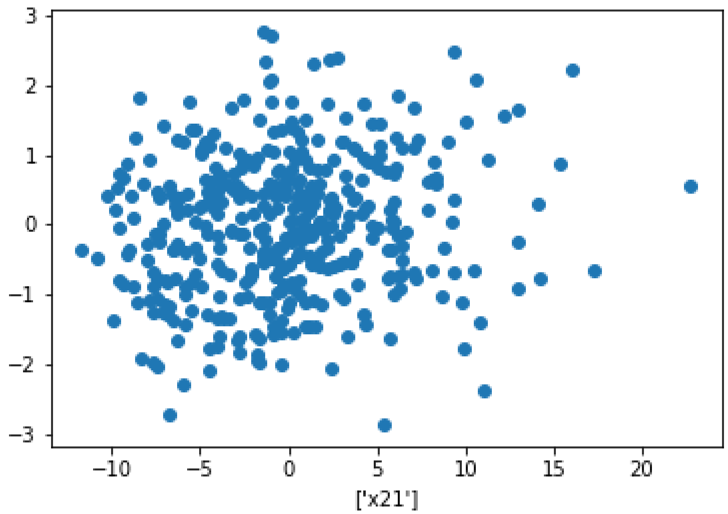
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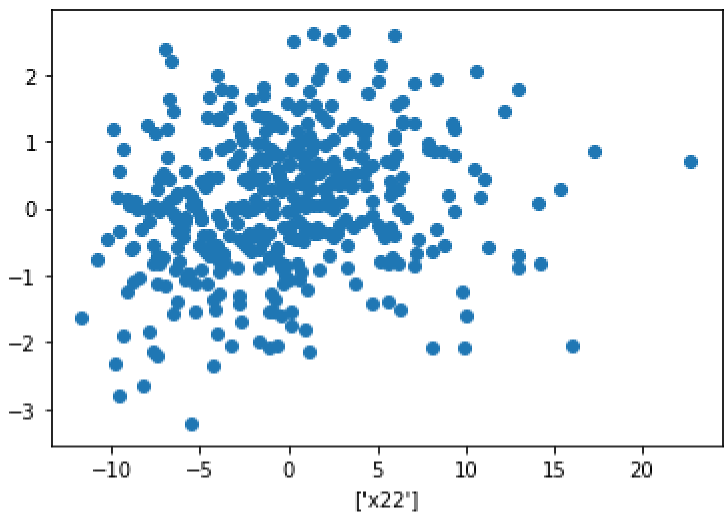
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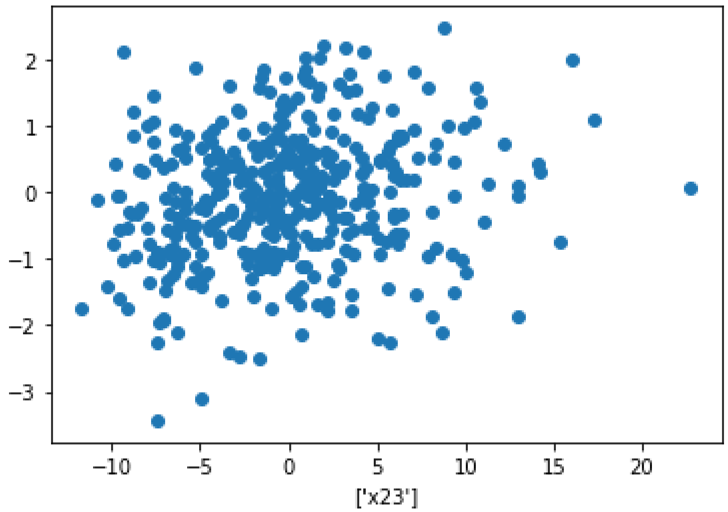
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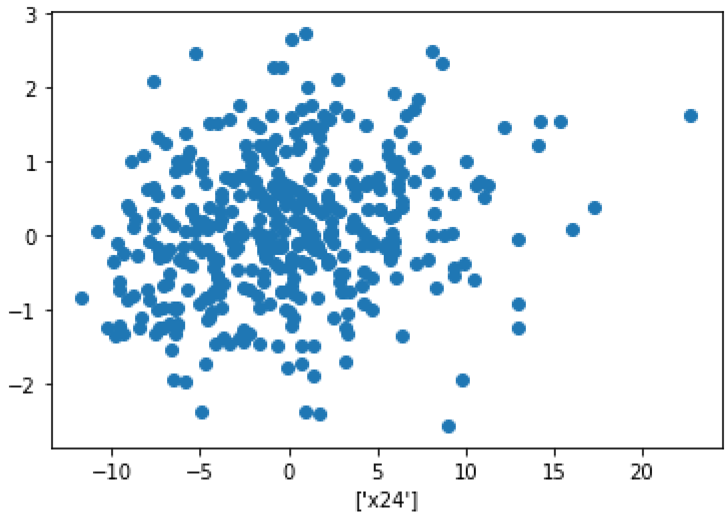
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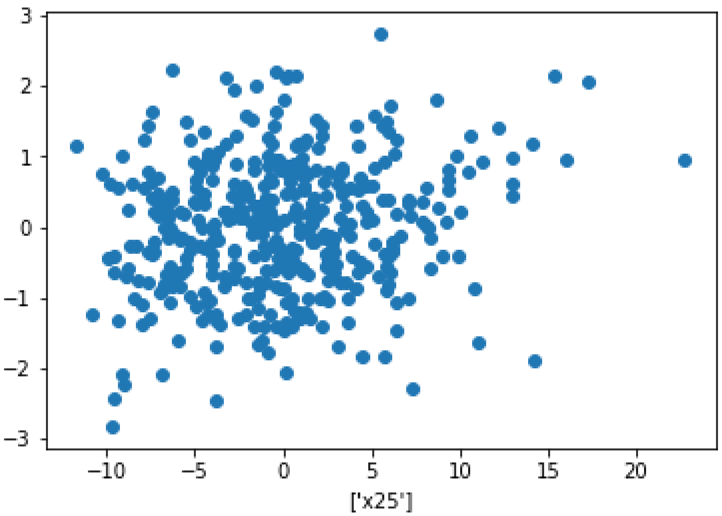

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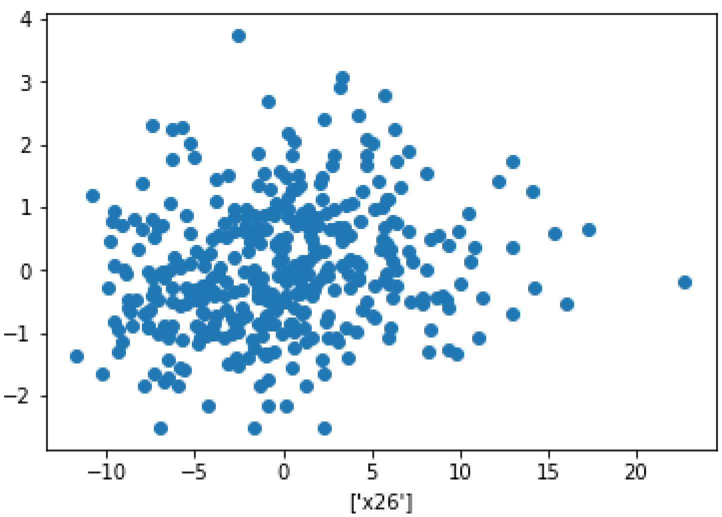

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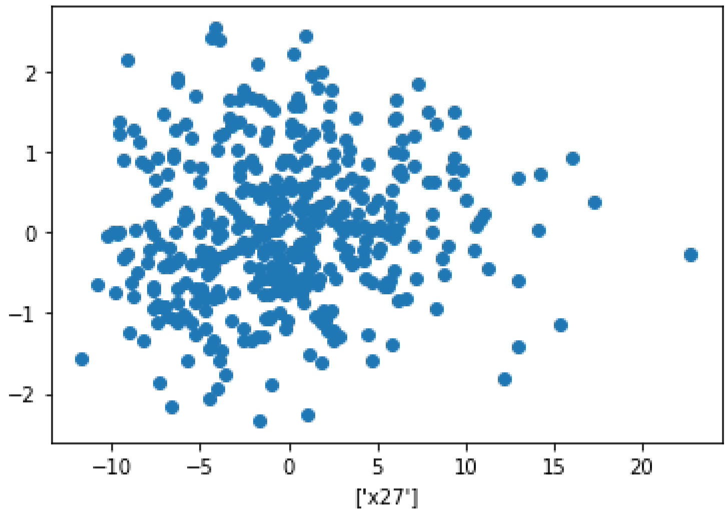
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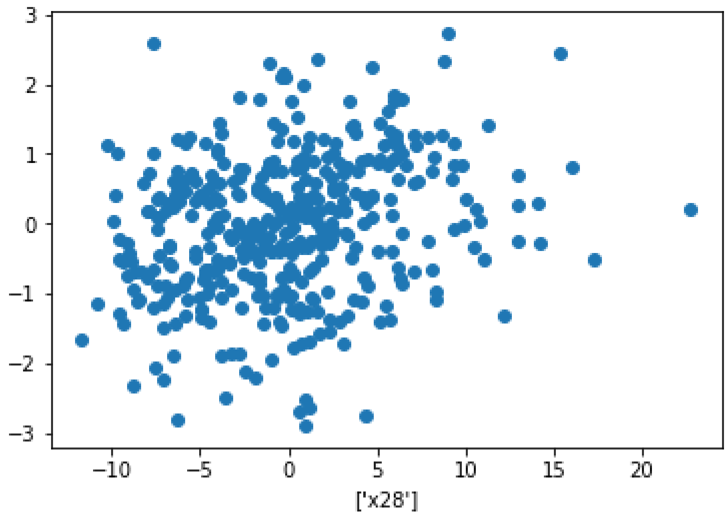
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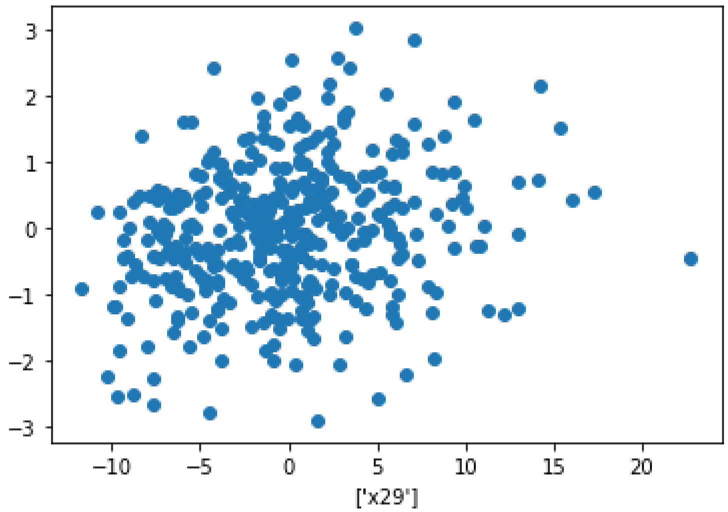
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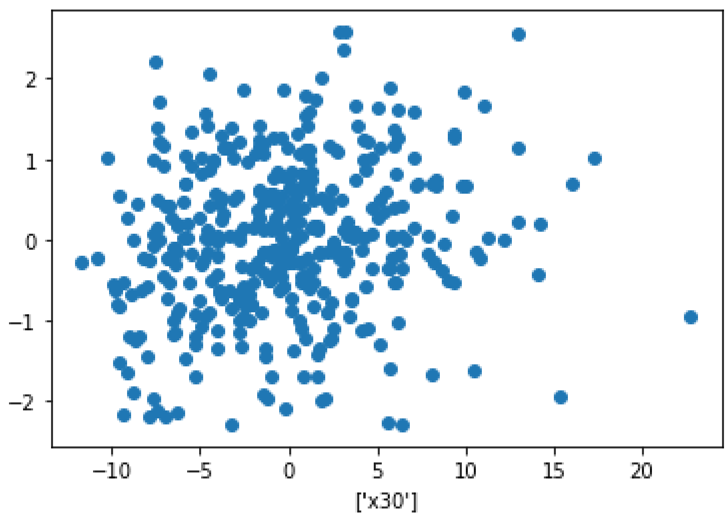
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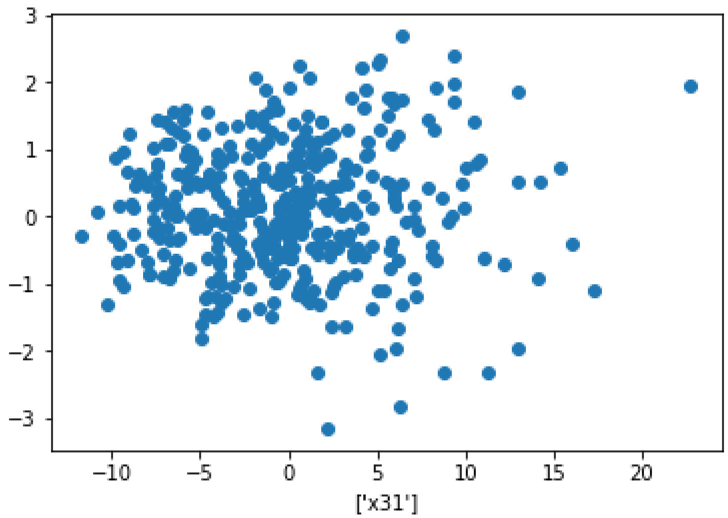

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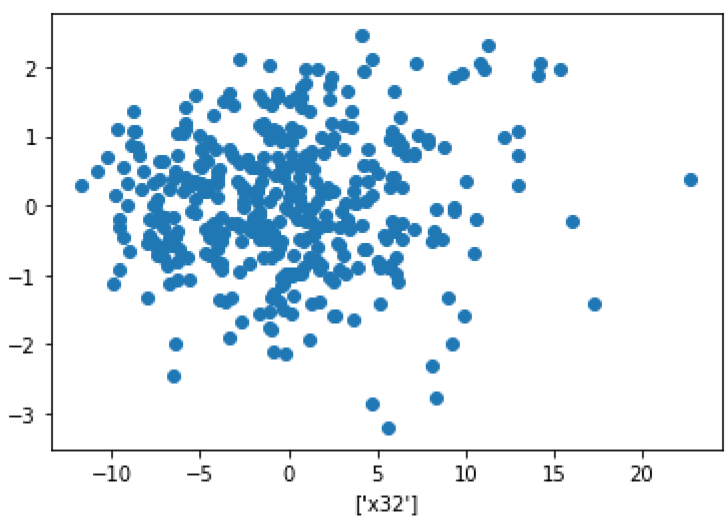

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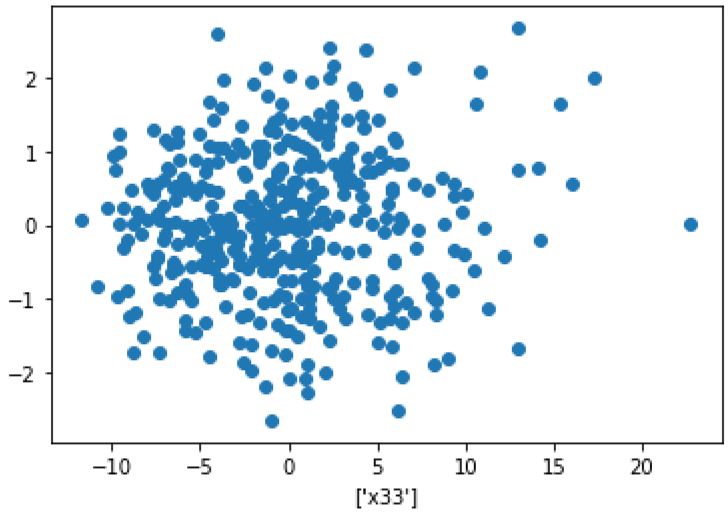
['x32']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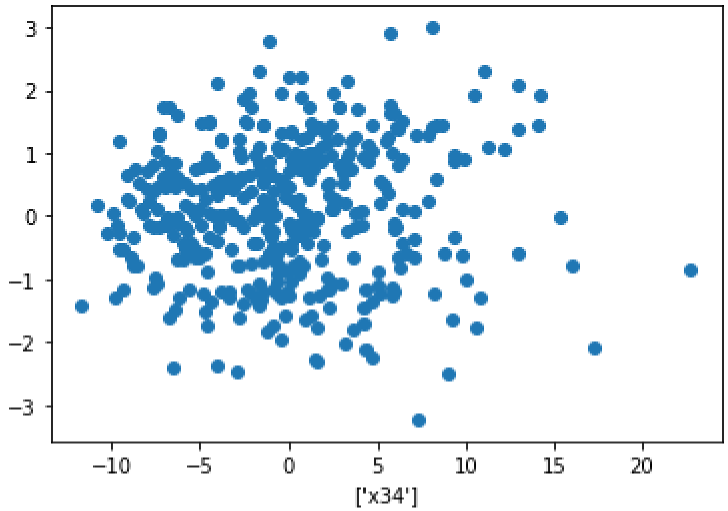
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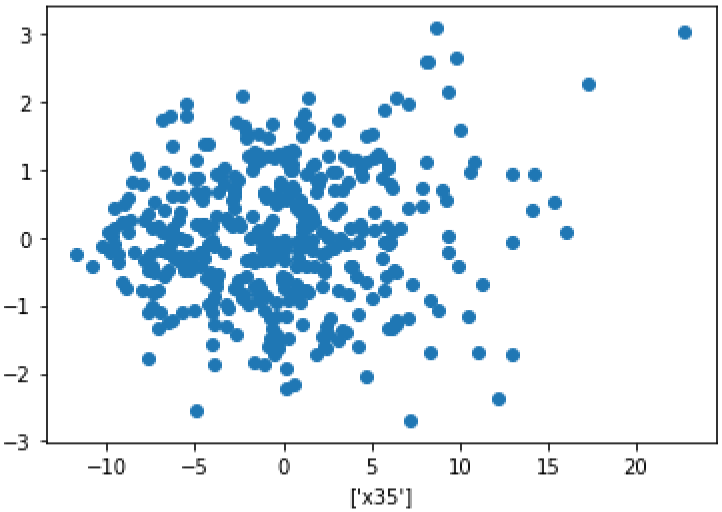
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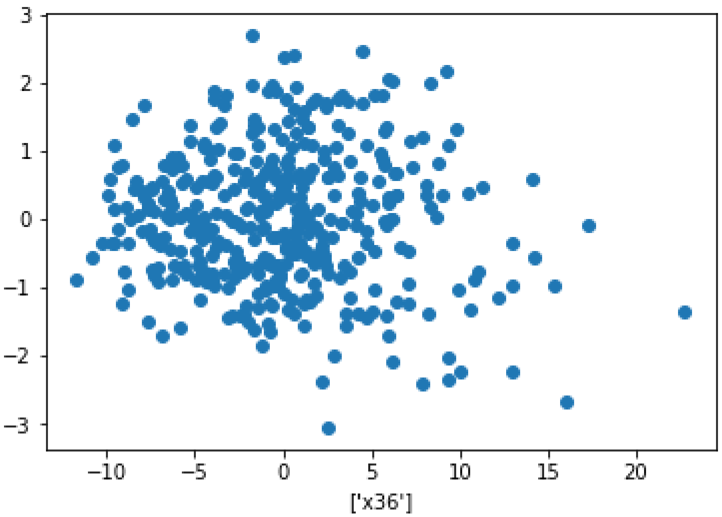
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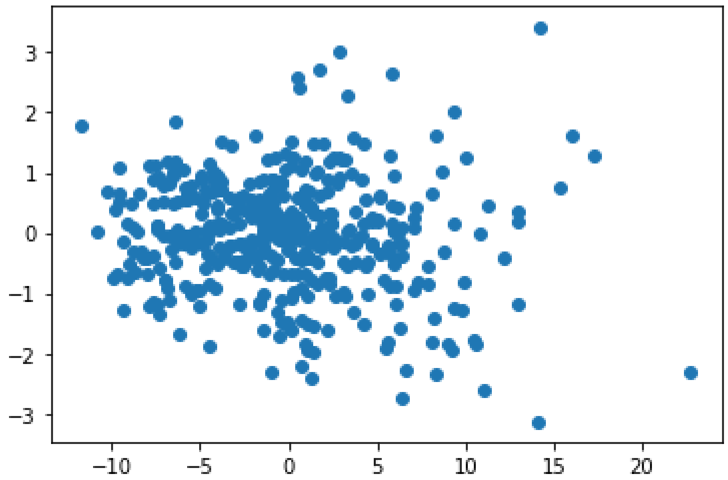

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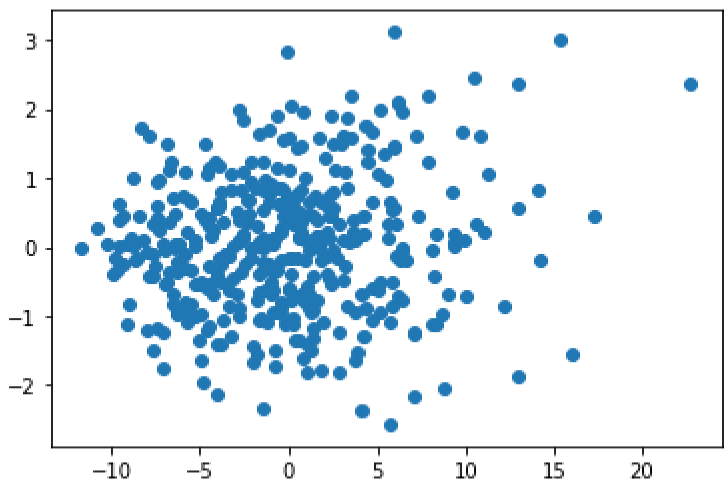

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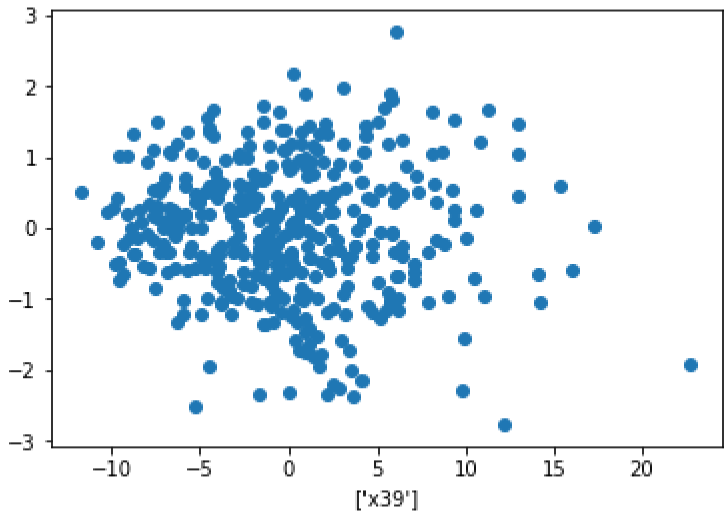
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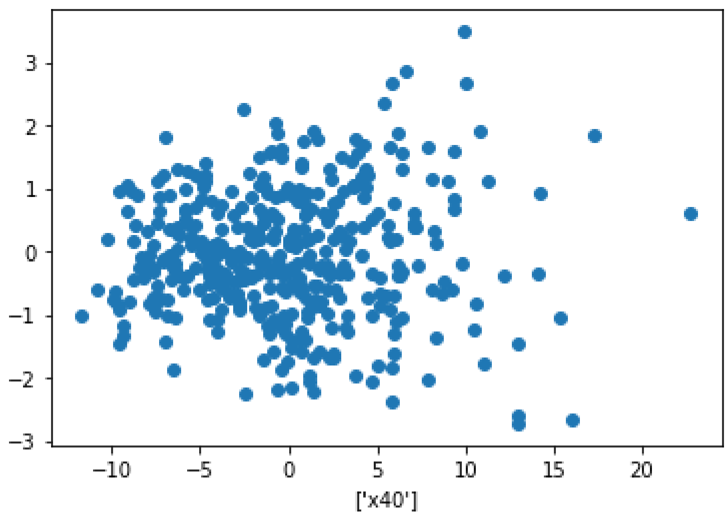
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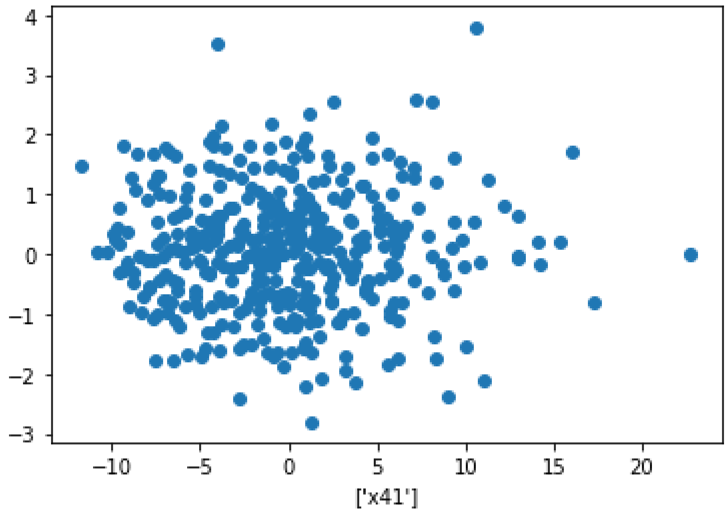
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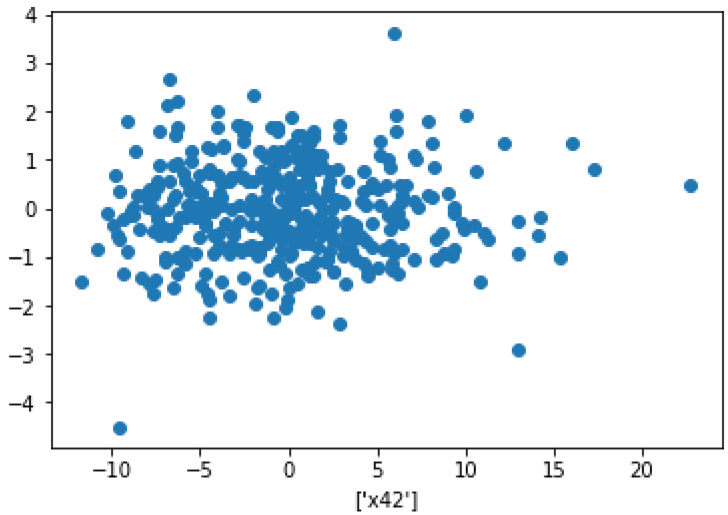
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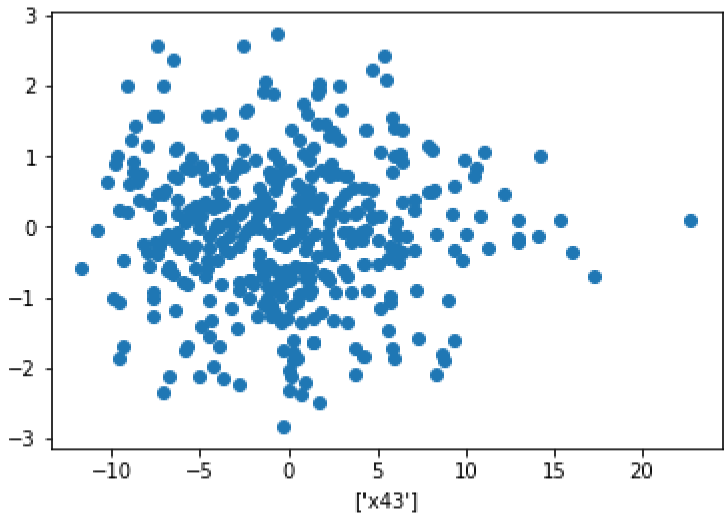
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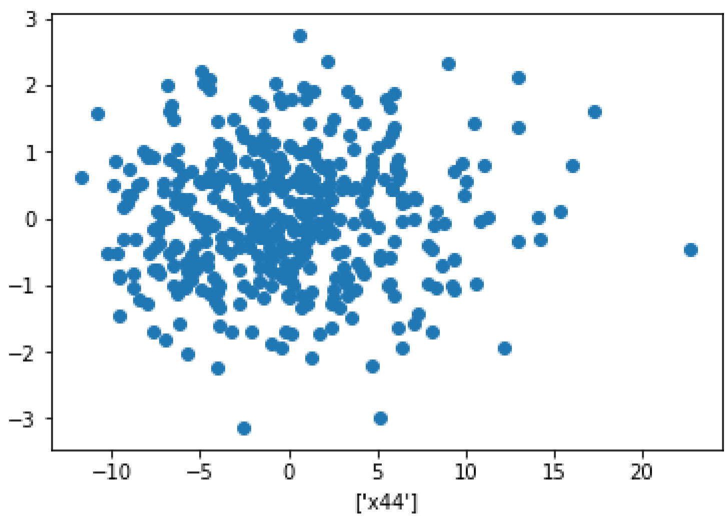
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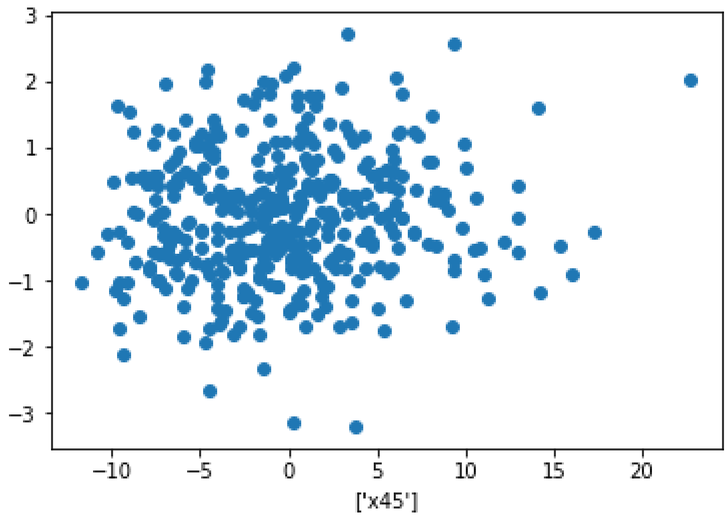
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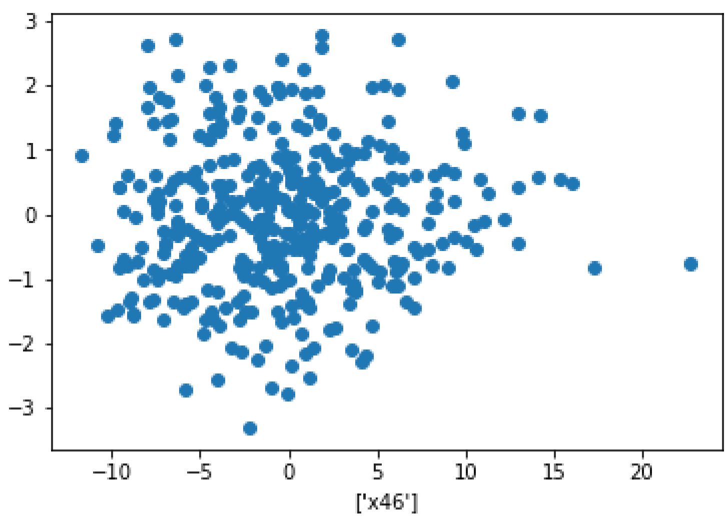
['x45']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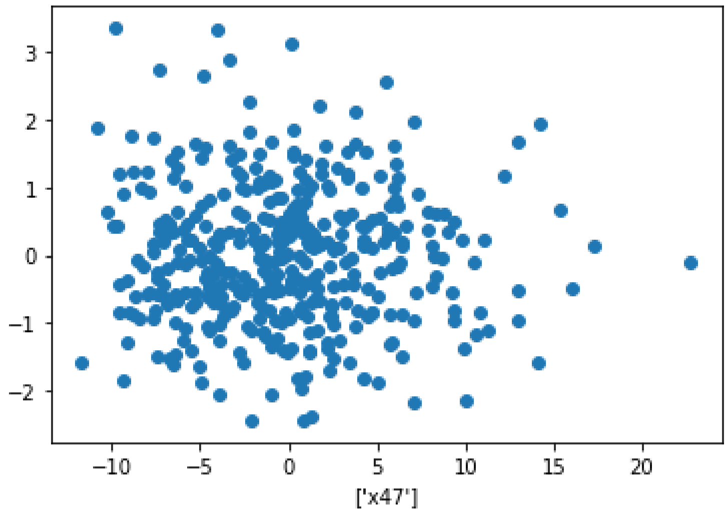
['x46']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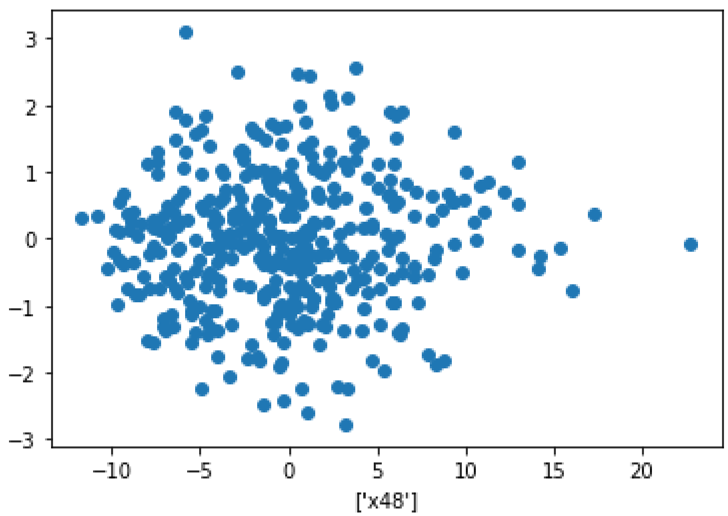
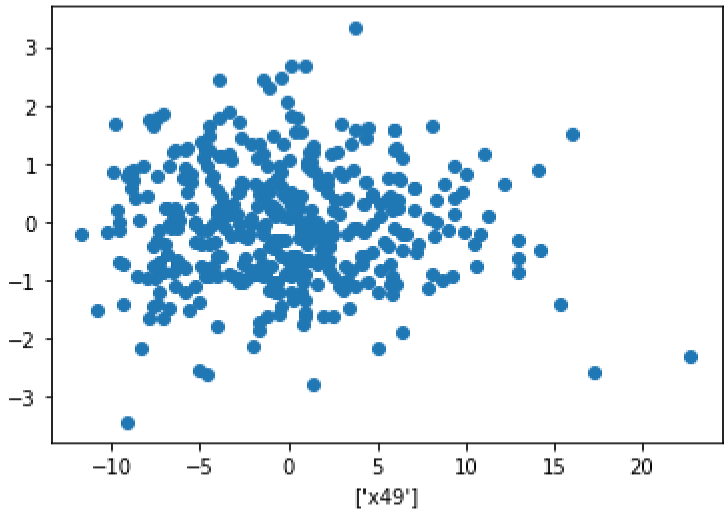
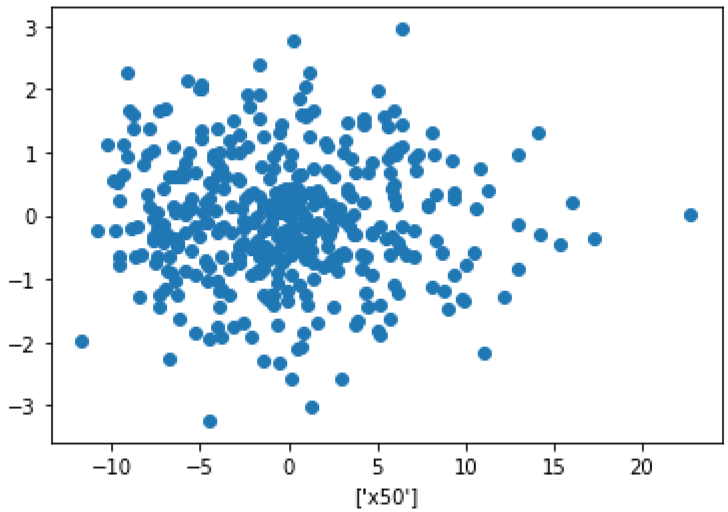

['x48']





By seeing the plot of each x predictor and target variable y, it looks like each of the predictor contributes some information in the prediction of y

Let's see if there is multicollinearity among the predictors

In [0]:
```
#correlation_matrix=x.corr()
#correlation_matrix[correlation_matrix>0.7]
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import variance_inflation_factor
```

In [34]:
```
vf=[]
for i in range(x.shape[1]):
    vif=variance_inflation_factor(x.values,i)
    vf.append(vif)
print(vf)
```

```
[228.23739680781165, 208.6134404078362, 204.1758564663967, 216.8728092202751, 209.1648111033765, 218.93881421
883754, 203.213959240962, 205.11312020568704, 234.43372296688415, 190.52145081030676, 114.1277753920412, 107.
11963734135139, 123.7630227284136, 111.26405791459301, 111.50367521790403, 118.3957142378508, 108.74463396709
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3091655994863, 106.82970713526485, 99.77598867444435, 121.73801668577076, 107.10447553095682, 99.219606517405
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27286054638, 1.1800239437090227, 1.1180221663554775, 1.1367699872530346, 1.1325418569843526, 1.18249156457189
23]
```

Since, if VIF is more than 10 then it represent multicollinearity. Let's see how many variables has VIF more than 10

In [35]:
```
vf=pd.DataFrame(vf)
vf[vf<10]
```

Out[35]:

	0
0	NaN
1	NaN
2	NaN
3	NaN
4	NaN
5	NaN
6	NaN
7	NaN
8	NaN
9	NaN
10	NaN
11	NaN
12	NaN
13	NaN
14	NaN
15	NaN
16	NaN
17	NaN
18	NaN
19	NaN
20	NaN
21	NaN
22	NaN

	0
23	NaN
24	NaN
25	NaN
26	NaN
27	NaN
28	NaN
29	NaN
30	1.139046
31	1.134770
32	1.239837
33	1.122934
34	1.170706
35	1.107772
36	1.137746
37	1.095952
38	1.106509
39	1.141327
40	1.129977
41	1.140734
42	1.150238
43	1.187005
44	1.155543
45	1.180024
46	1.118022

	0
47	1.136770
48	1.132542
49	1.182492

We can see in our variables, only x30 to x50 variables have low VIF around 1 thus no multicollinearity. We are going to use variables only from 30 to 50 in that case

In [36]:
```
lm.fit(X_train.iloc[:,29:],y_train)
```

Out[36]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

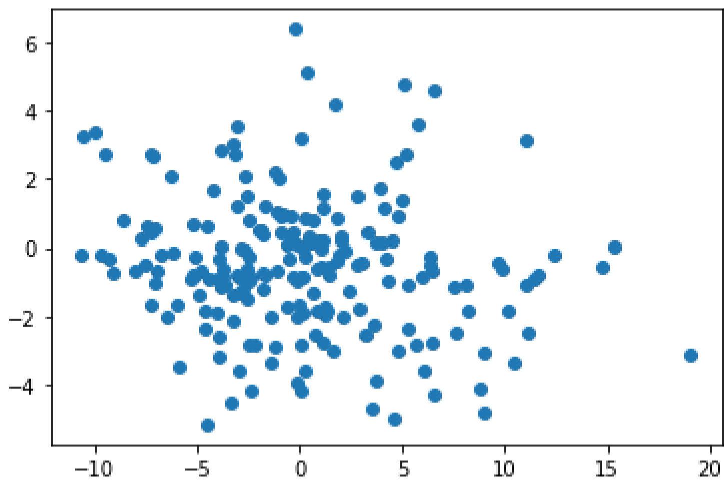
In [37]:
```
y_pred_1= lm.predict(X_test.iloc[:,29:])
print('MSE:', metrics.mean_squared_error(y_test, y_pred_1))
```

```
MSE: 28.270774956447195
```

This MSE is worst than even the Linear Regression indicating that by losing the variables from x01 to x29, we are losing much information of y

In [38]:
```
plt.scatter(y_test-y_pred_1,y_pred_1)
```

Out[38]: <matplotlib.collections.PathCollection at 0x7ff805ed84e0>



Let's fit the same linear model but with first 29 variables instead of last 31 variables and see how model performs

In [39]:
```
x_2=X_train.iloc[:,:29]
lm.fit(x_2,y_train)
```

Out[39]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)

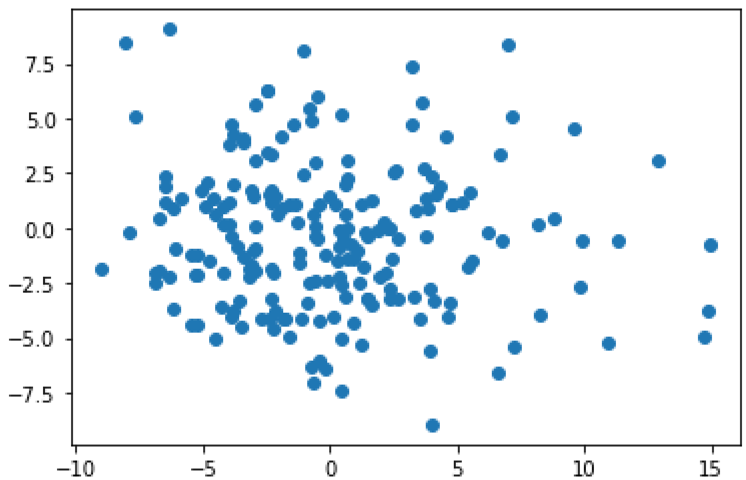
In [40]:
```
y_pred_2= lm.predict(X_test.iloc[:,:29])
print('MSE:', metrics.mean_squared_error(y_test, y_pred_2))
```

MSE: 20.50058173101595

We managed to decrease the MSE to 20.5 incidating that first 29 variables has more information of y than 2nd set of 21 variables

In [41]:
```
plt.scatter(y_test-y_pred_2,y_pred_2)
```

Out[41]: <matplotlib.collections.PathCollection at 0x7ff8059f4be0>



Again since there is no clear pattern, linear regression seems to be a good fit for the data

Quadratic Model

In [42]:
```
from sklearn.preprocessing import PolynomialFeatures

poly = PolynomialFeatures(degree = 2)
X_poly = poly.fit_transform(X_train)

poly.fit(X_poly, y_train)
lm.fit(X_poly, y_train)
```

Out[42]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)

In [43]:
```
y_pred_3= lm.predict(poly.fit_transform(X_test))
print('MSE:', metrics.mean_squared_error(y_test, y_pred_3))
```

```
MSE: 18.733680182554075
```

Let's try to fit the quadratic model on selected 1st 29 features and see if perfoance improves or not.

In [44]:
```
X_poly = poly.fit_transform(x_2)

poly.fit(X_poly, y_train)
lm.fit(X_poly, y_train)
```

Out[44]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

In [46]:
```
y_pred_quad_set= lm.predict(poly.fit_transform(X_test.iloc[:,:29]))
print('MSE:', metrics.mean_squared_error(y_test, y_pred_quad_set))
```

```
MSE: 95.93603207246629
```

MSE decreases to even lower number of 95.93 indicating we are losing information

We can see MSE decreased to 18.733% indicating quadratic model is better fit

Decision Tree Regression

In [0]:
```
#Let's do Decision Tree regression
from sklearn.tree import DecisionTreeRegressor
```

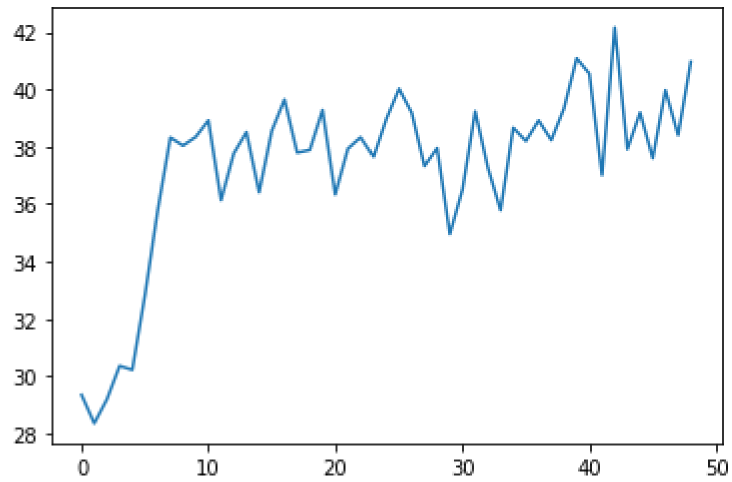
In [0]:
```
Acc_Decision=[]
for i in range(1,50):
  regr_1 = DecisionTreeRegressor(max_depth=i)
  regr_1.fit(X_train, y_train)
  y_pred_4 = regr_1.predict(X_test)
  MSE=metrics.mean_squared_error(y_test, y_pred_4)
  Acc_Decision.append(MSE)
```

In [49]:
```
min(Acc_Decision)
```

Out[49]: 28.36175947896122

In [50]:
```
plt.plot(Acc_Decision)
```

Out[50]: [<matplotlib.lines.Line2D at 0x7ff805f1c7f0>]



We tried to use the Decision Tree regression perormance with depth ranging from 1 to 50, is not that great and is infact worst than the Linear regression also. Our aim is to reduce MSE by applying different modeling methods

Random Forest Regression

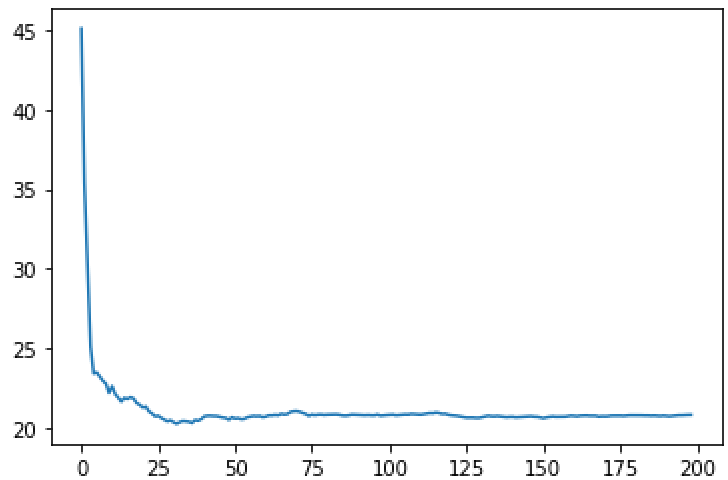
In [0]:
```
from sklearn.ensemble import RandomForestRegressor
Acc_Random=[]
for i in range(1,200):
  regressor = RandomForestRegressor(n_estimators = i, random_state = 0)
  regressor.fit(X_train, y_train)
  y_pred_5=regressor.predict(X_test)
  MSE=metrics.mean_squared_error(y_test, y_pred_5)
  Acc_Random.append(MSE)
```

In [52]:
```
min(Acc_Random)
```

Out[52]:
```
20.279687867097145
```

In [53]:
```
plt.plot(Acc_Random)
```

Out[53]:
```
[<matplotlib.lines.Line2D at 0x7ff7fe157c50>]
```



We can see the MSE has reached the minimum MSE of 20.279 and remains constant even if number of decreased

Ridge Regression

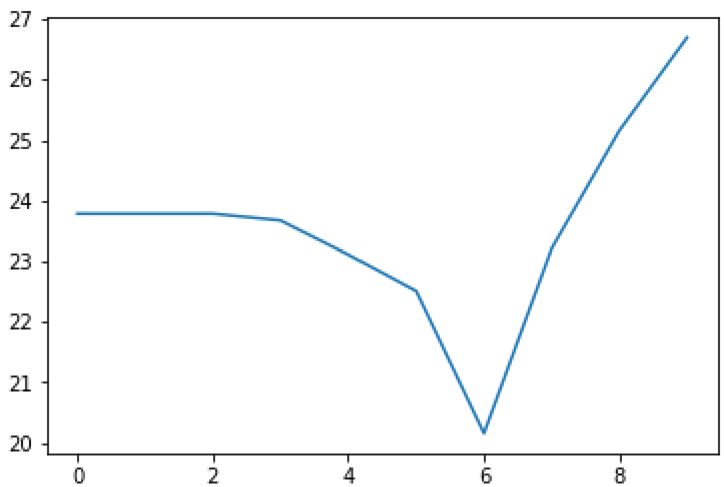
In [0]:
```
from sklearn.linear_model import Ridge
Acc_Ridge=[]
for i in [1e-15, 1e-10, 1e-8, 1e-4, 1e-3,1e-2, 1, 5, 10, 20]:
  ridge=Ridge(alpha=i,normalize=True)
  ridge.fit(X_train,y_train)
  y_pred_6=ridge.predict(X_test)
  MSE=metrics.mean_squared_error(y_test, y_pred_6)
  Acc_Ridge.append(MSE)
```

In [55]:
```
min(Acc_Ridge)
```

Out[55]: 20.158611518654055

In [56]:
```
plt.plot(Acc_Ridge)
```

Out[56]: [<matplotlib.lines.Line2D at 0x7ff7fe7c67b8>]



Thus Ridge regression has minimum MSE of 20.158 when penalty tuning parameter is 1. Please note that Ridge Regression uses L2 norm to panalize the predictor's cofficient thus can't be used to do variable selection

In [57]:
```
from sklearn.model_selection import GridSearchCV
parameters={'alpha':[1e-15, 1e-10, 1e-8, 1e-4, 1e-3,1e-2, 1, 5, 10, 20]}
ridge=Ridge()
ridge_regressor=GridSearchCV(ridge,parameters,scoring='neg_mean_squared_error',cv=2)
ridge_regressor.fit(x,y)
```

Out[57]:
```
GridSearchCV(cv=2, error_score='raise-deprecating',
             estimator=Ridge(alpha=1.0, copy_X=True, fit_intercept=True,
                             max_iter=None, normalize=False, random_state=None,
                             solver='auto', tol=0.001),
             iid='warn', n_jobs=None,
             param_grid={'alpha': [1e-15, 1e-10, 1e-08, 0.0001, 0.001, 0.01, 1,
                                   5, 10, 20]},
             pre_dispatch='2*n_jobs', refit=True, return_train_score=False,
             scoring='neg_mean_squared_error', verbose=0)
```

In [58]:
```
print(ridge_regressor.best_params_, ridge_regressor.best_score_)
```

```
{'alpha': 20} -20.571701795778154
```

This Ridge Regression using Grid Search using 2-Fold cross validation is same as Ridge regression on 50% train test split and thus has almost same MSE

Lasso Regression

In [0]:
```
from sklearn.linear_model import Lasso
from sklearn.feature_selection import SelectFromModel
from sklearn.preprocessing import StandardScaler
```

In [60]:
```
scaler = StandardScaler()
scaler.fit(X_train)
```

Out[60]:
```
StandardScaler(copy=True, with_mean=True, with_std=True)
```

Let's do the variable selection using LASSO Regression. Lasso uses L1 norm to penalize predictors and thus it can be used to select variables.

In [61]:
```
sel_ = SelectFromModel(LinearRegression())
sel_.fit(scaler.transform(X_train),y_train)
```

Out[61]:
```
SelectFromModel(estimator=LinearRegression(copy_X=True, fit_intercept=True,
                                           n_jobs=None, normalize=False),
                max_features=None, norm_order=1, prefit=False, threshold=None)
```

In [62]:
```
sel_.get_support()
```

Out[62]:
```
array([ True, False,  True,  True,  True,  True,  True,  True, False,
        True,  True,  True,  True, False, False,  True, False,  True,
        True, False,  True,  True,  True, False,  True, False,  True,
       False, False,  True, False, False, False, False, False, False,
       False, False, False, False, False, False, False, False, False,
       False, False, False, False, False])
```

Those with True values are selected variables by Lasso and one with False has been reduced to 0

Let's try to build the model using selected Features set

In [0]:
```
feauture_selected=pd.DataFrame(sel_.get_support(),columns=['feature'])
[feauture_selected['feature']==True]
feature_set=['x01','x02','x03','x04','x05','x06','x07','x09','x10','x11','x12','x15','x17','x18','x20','x21',
'x22','x24','x26','x29']
x_selected=x[feature_set]
```

In [65]:
```
X_train_sel, X_test_sel, y_train_sel, y_test_sel = train_test_split(x_selected, y, test_size=0.5, random_stat
e=1)

lm.fit(X_train_sel, y_train_sel)
```

Out[65]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

In [66]:
```
y_pred_sel= lm.predict(X_test_sel)
print('MSE:', metrics.mean_squared_error(y_test_sel, y_pred_sel))
```

```
MSE: 20.76247077216838
```

Thus MSE has decreased to 20.76 which is quite improvement but not much. which indicates we have almost same information as Ridge or Linear Regression on first 29 variables. Compared to Linear Regression on all the 50 variables, we had MSE of 23.77 and we are minimizing MSE by decreasing the number of variables from 50 to 20 which is great improvement

In [0]:
```
#Random Forest on selected features

Acc_Random_sel=[]
for i in range(1,200):
  regressor = RandomForestRegressor(n_estimators = i, random_state = 0)
  regressor.fit(X_train_sel, y_train_sel)
  y_pred_sel_random=regressor.predict(X_test_sel)
  MSE=metrics.mean_squared_error(y_test_sel, y_pred_sel_random)
  Acc_Random_sel.append(MSE)
```

In [68]:
```
min(Acc_Random_sel)
```

Out[68]:
```
24.443442944963035
```

Random Forest on selected Features of 20 predictors is even worst than the random Forest on all 50 features indicatig we are losing information

Let's use the Lasso Regression to predict Y and see it's MSE instead of variable selection. Here number of iterations to converge the parameters is 100000

In [69]:
```
for i in [1e-15, 1e-10, 1e-8, 1e-4, 1e-3,1e-2, 1, 5, 10, 20]:
  model_lasso = Lasso(alpha=i,max_iter = 100000)
  model_lasso.fit(X_train, y_train)
  pred_test_lasso= model_lasso.predict(X_test)
  print(metrics.mean_squared_error(y_test,pred_test_lasso))
```

```
/usr/local/lib/python3.6/dist-packages/sklearn/linear_model/coordinate_descent.py:475: ConvergenceWarning: Ob
jective did not converge. You might want to increase the number of iterations. Duality gap: 482.462601424491
5, tolerance: 0.5062479822606639
  positive)
```

```
23.782570634178445
23.782570598487528
23.782567066252813
23.747118911414262
23.454660531394467
22.381107314800232
27.480870793917646
28.907721505285117
28.907721505285117
28.907721505285117
```

MSE isn't decreaing much on even 100000 iterations. This performance is worst than even Ridge Regression

Let's build a Neural Network model using Keras. This would be Sequantial Feed Forward Neural Network model with activation fucntion as Linear since, this is Regression problem. Loss function used is Mean Squared Error and optmization method used is Stochastic Gradient Descent.

Neural Network Model

Let's import all the libraries to build the model

In [70]:
```
#Neural Network model
from keras.models import Sequential
from keras.layers import Dense, Dropout
from keras.layers import LSTM
from keras.callbacks import Callback
from keras.models import Sequential
from keras.layers import LSTM, Dense, Activation
import pickle
from mpl_toolkits.mplot3d import Axes3D
import sys
from queue import Queue
import json
%matplotlib inline
```

```
Using TensorFlow backend.
```

The default version of TensorFlow in Colab will soon switch to TensorFlow 2.x.
We recommend you upgrade (https://www.tensorflow.org/guide/migrate) now or ensure your notebook will continue to use TensorFlow 1.x via the `%tensorflow_version 1.x` magic: more info (https://colab.research.google.com/notebooks/tensorflow_version.ipynb).

Let's use a class to see how Loss is being reduced with each learning from each batch of 72

In [0]:
```
class LossHistory(Callback):
    def on_train_begin(self, logs={}):
        self.losses = []

    def on_batch_end(self, batch, logs={}):
        sys.stdout.write(str(logs.get('loss'))+str(', '))
        sys.stdout.flush()
        self.losses.append(logs.get('loss'))

lr = LossHistory()
```

Let's define the structure of Neural Network model. Since, we have 50 predictors so, outside layer would have 50 neurons. We would define one hidden layer with 20 neurons and since, there is one y target variable and is a regression problem, output layer containing one 1 neurons. Since, our data is small and has only 200 observations (Note that we are removing outliers in case of Neural Network model from training datset since, it is capable of learning from outliers by backpropagation), we have used epoch size of 1000

In [0]:
```
number_of_neurons_layer1 = 50
number_of_neurons_layer2 = 20
number_of_neurons_layer3 = 1
number_of_epochs = 1000
```

In [0]:
```
dim =  50
samples = 200
```

In [0]:
```
X_train, X_test, y_train, y_test = train_test_split(x, y, test_size=0.5, random_state=1)
```

In [0]:
```
# design network
from keras import optimizers
sgd = optimizers.SGD(lr=0.01, clipnorm=1.)

model = Sequential()
model.add(Dense(number_of_neurons_layer1,input_shape=(dim, ), activation='linear'))
model.add(Dense(number_of_neurons_layer2, activation='linear'))
model.add(Dense(number_of_neurons_layer3, activation='linear'))
model.compile(loss='mean_squared_error', optimizer=sgd)

def train(data,label):
    model.fit(data, label, epochs=number_of_epochs, batch_size=72, validation_data=(data, label), verbose=0,
shuffle=True,callbacks=[lr])

def score(data):
    return model.predict(data)
```

In [78]:
```
train(X_train,y_train)
```

```
25.28822, 32.1681, 37.339314, 24.857685, 30.013388, 38.246815, 29.575705, 32.01464, 27.468153, 25.548523, 40.
93689, 18.960724, 35.34854, 21.070946, 29.872519, 23.434183, 27.07907, 35.426914, 27.727484, 27.457832, 27.55
5311, 21.745632, 24.24398, 37.393536, 28.835226, 24.141514, 26.603512, 31.350393, 17.59992, 30.031834, 35.537
86, 19.260353, 20.761034, 31.00826, 23.51952, 19.458273, 20.9521, 32.92854, 18.75256, 25.415955, 26.538483, 1
9.706142, 24.99003, 17.135939, 30.784842, 31.833366, 19.868925, 17.057491, 21.992432, 18.70217, 29.90968, 17.
260132, 27.653086, 23.200886, 28.22744, 18.20826, 19.861288, 20.303488, 22.958263, 22.573927, 19.684753, 17.9
52837, 28.663464, 22.853241, 25.360779, 13.974365, 25.718283, 21.264387, 14.489476, 29.059095, 11.932207, 21.
189932, 18.39994, 16.201057, 28.429544, 18.492886, 22.680122, 19.020939, 19.134045, 19.273468, 21.577885, 19.
891323, 18.80097, 20.24629, 22.39971, 16.02381, 19.663235, 21.13102, 18.410482, 17.371801, 19.691862, 18.9743
75, 17.650358, 20.407982, 15.874342, 19.948532, 22.39183, 17.297995, 14.784863, 14.791033, 17.68857, 23.35785
3, 16.15442, 19.482445, 18.576204, 14.210124, 24.66663, 13.824416, 16.446344, 13.409368, 24.850973, 24.51871
9, 13.535472, 13.6883, 17.562191, 18.162384, 16.15308, 22.638712, 13.148387, 15.495734, 15.44133, 14.94167, 2
1.975584, 19.77463, 16.817987, 13.531019, 22.158634, 13.102633, 14.835672, 16.638792, 15.138546, 18.922169, 1
8.634054, 16.580116, 14.097758, 19.320665, 14.380768, 15.68226, 20.284458, 14.704885, 13.708962, 14.241448, 1
7.880638, 17.158838, 13.634084, 19.70151, 15.236677, 14.808332, 18.029213, 15.644937, 15.960829, 13.173888, 2
0.127008, 12.959908, 18.734016, 16.601772, 15.745268, 16.502607, 15.60832, 14.26689, 19.359253, 13.624877, 1
5.284496, 18.94095, 12.715173, 12.697978, 18.190912, 16.796577, 17.814562, 14.648424, 14.602107, 15.935057, 1
1.929655, 20.383932, 15.494361, 14.658617, 17.264967, 12.374532, 21.47721, 12.369688, 13.695973, 16.10254, 1
7.461006, 18.938826, 15.006707, 11.984602, 17.456757, 14.959032, 13.877481, 14.413622, 19.098854, 12.417058,
18.78262, 11.88676, 15.828282, 14.009479, 17.073212, 15.233498, 13.335844, 15.097946, 18.519785, 13.1562195,
14.459797, 19.461882, 16.052177, 19.000751, 9.825661, 16.29626, 17.205925, 11.736868, 13.395454, 13.841223, 1
9.752995, 14.060717, 17.11837, 14.533429, 14.386257, 14.179025, 17.853548, 18.51861, 15.594855, 10.636785, 1
7.18201, 15.80161, 12.021575, 18.208006, 14.268683, 12.592975, 13.598133, 14.539208, 18.178251, 15.4728565, 1
6.115683, 13.667821, 20.258705, 15.081872, 8.752866, 13.962954, 17.455177, 13.717741, 12.69615, 16.431238, 1
6.708334, 17.600193, 14.024885, 13.358778, 15.991451, 14.274955, 15.106745, 15.309292, 15.03889, 14.976426, 1
7.028145, 11.58839, 17.15154, 13.302226, 14.488475, 18.17802, 21.984669, 11.980446, 10.159707, 22.115456, 13.
318937, 8.2628765, 17.532877, 13.691584, 13.664025, 15.133626, 17.237192, 12.176755, 14.657728, 13.727439, 1
7.22878, 17.993723, 13.136601, 13.675345, 10.172514, 16.762394, 19.008375, 19.417763, 10.61526, 14.995265, 1
5.018923, 15.929445, 13.817588, 18.610064, 12.348105, 13.759414, 14.684467, 10.613819, 21.026968, 14.537818,
11.511883, 20.063242, 10.62088, 16.204227, 19.052652, 17.995594, 9.611455, 17.966152, 12.983304, 16.888102, 1
5.134622, 17.62019, 14.281899, 12.408835, 17.506752, 14.410634, 12.380137, 17.975138, 12.195547, 14.600738, 1
7.64347, 11.322446, 16.239613, 16.113054, 14.21729, 14.428794, 16.078081, 15.26083, 13.053129, 15.322625, 10.
403339, 20.29473, 17.200457, 15.243029, 11.604073, 9.306104, 15.702747, 21.209452, 12.865431, 19.944254, 11.1
37532, 16.196512, 12.678639, 16.24779, 17.852211, 10.3672285, 17.005352, 15.81584, 17.327547, 10.662447, 12.3
51477, 16.290592, 16.489883, 9.439708, 17.542095, 18.646524, 16.039326, 12.652621, 16.36947, 13.543517, 13.63
1627, 18.337942, 15.460545, 10.2680435, 20.211239, 12.175045, 18.61018, 13.6083355, 17.699532, 11.234109, 16.
019445, 15.174066, 14.851383, 14.65651, 14.667033, 11.540543, 19.512173, 14.129803, 16.703796, 13.564225, 10.
441517, 18.999105, 15.310347, 19.099813, 11.9661665, 13.274362, 17.803524, 13.438834, 13.061301, 12.586422, 1
9.180176, 12.361376, 13.62843, 13.834001, 17.91368, 12.666924, 16.736435, 15.321418, 17.297567, 13.072798, 1
4.189596, 13.911819, 14.145203, 17.156057, 16.458326, 12.266294, 16.22826, 13.941934, 18.646492, 11.259042, 1
7.060144, 13.399685, 13.980641, 14.279994, 12.370867, 18.864939, 20.155823, 9.452743, 14.990102, 13.900097, 1
5.500285, 15.343698, 17.776464, 11.699627, 15.230102, 12.575498, 19.431648, 11.960092, 16.777292, 12.2069645,
15.975023, 13.073662, 14.560835, 17.579859, 14.937737, 13.347309, 16.837744, 13.363096, 17.156515, 13.868135,
```

```
15.073072, 16.3917, 12.646945, 11.724125, 16.643673, 16.614332, 17.16364, 14.320042, 12.644684, 10.880729, 1
8.980312, 14.716359, 18.258339, 13.805546, 11.826669, 16.752192, 15.925724, 11.088282, 9.38462, 19.695421, 1
5.682587, 14.28243, 13.888289, 16.816998, 14.735312, 14.9795885, 14.875532, 15.185008, 17.519844, 10.94819, 1
1.742957, 13.523357, 20.6878, 17.758888, 11.38347, 15.584435, 15.188646, 8.949619, 22.064596, 13.743838, 15.8
37605, 14.982583, 18.622227, 13.1730175, 12.140763, 10.002997, 19.69942, 14.799428, 10.349631, 13.551051, 22.
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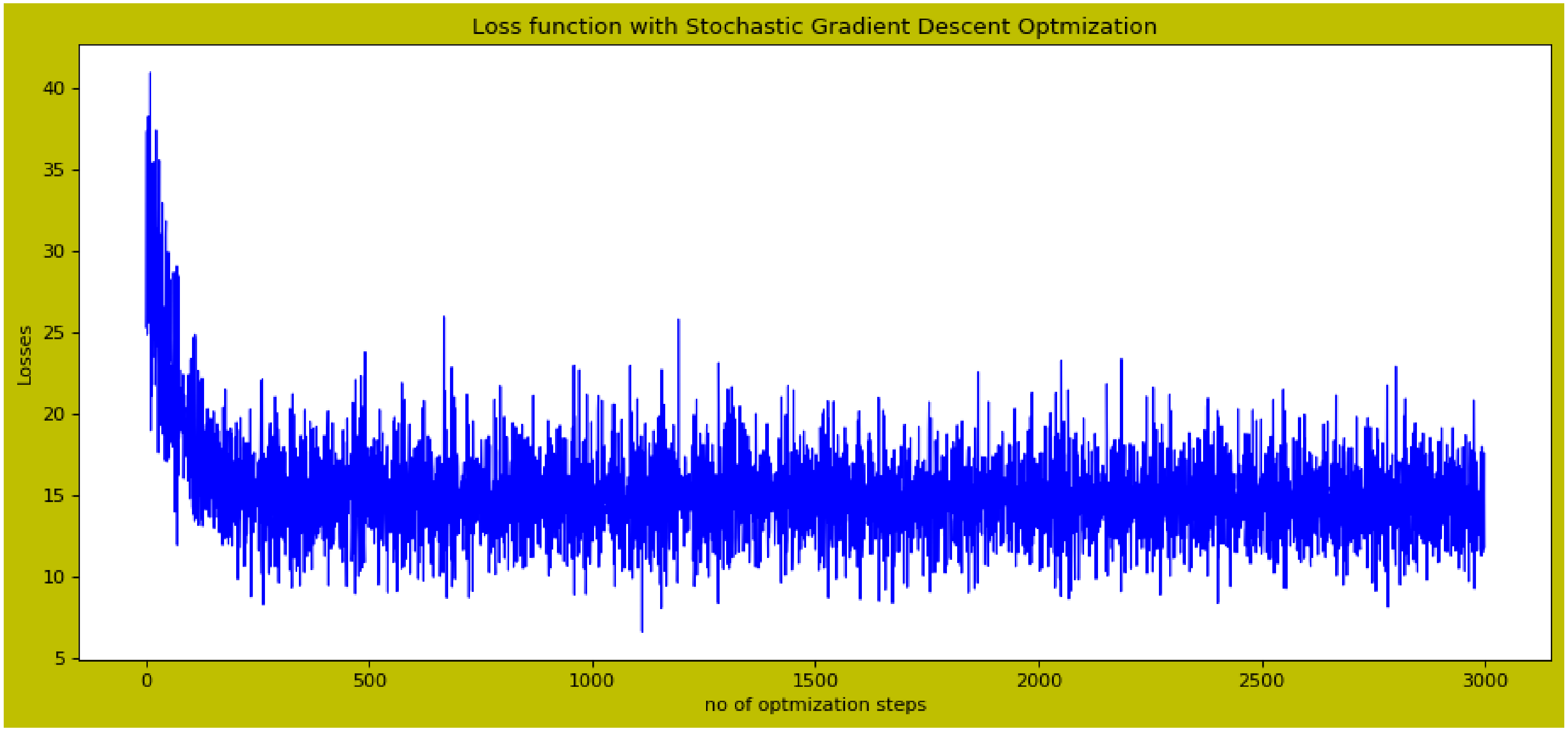
11.772151,

In [80]:
```
fig, ax = plt.subplots(num=None, figsize=(14, 6), dpi=80, facecolor='y', edgecolor='k')
size = len(lr.losses)
ax.plot(range(0,size), lr.losses, '-', color='blue', animated = True, linewidth=1)
plt.xlabel('no of optmization steps')
plt.ylabel('Losses')
plt.title('Loss function with Stochastic Gradient Descent Optmization')
```

Out[80]: Text(0.5, 1.0, 'Loss function with Stochastic Gradient Descent Optmization')



We can see how model's loss is almost constant after 500th batch.

In [81]:
```
min(lr.losses)
```

Out[81]: 6.6165

Minimum Loss fucntion we achieved is 7.1 approximately and thus accuracy of 92.9%. But unfortunately this result is on training dataset not on test dataset.

In [82]:
```
accuracy=100-min(lr.losses)
accuracy
```

Out[82]: 93.38350009918213

In [0]:
```
y_pred_nn=score(X_test)
```

In [85]:
```
print('MSE:', metrics.mean_squared_error(y_test, y_pred_nn))
```

```
MSE: 22.83741110622265
```

Let's try to do the Best Subset selection

In [0]:
```
import itertools
import time
import statsmodels.api as sm
```

In [0]:
```
def processSubset(feature_set):
  # Fit model on feature_set and calculate MSE
  model = sm.OLS(y_train,X_train)
  regr = model.fit()
  MSE = ((y_test-regr.predict(X_test)) ** 2).mean()
  return {"model":regr, "MSE":MSE}
```

In [0]:
```
def getBest(k):
  tic = time.time()
  results = []
  for combo in itertools.combinations(X_train.columns, k):
    results.append(processSubset(combo))
  # Wrap everything up in a nice dataframe
  models = pd.DataFrame(results)

  # Choose the model with the minimum MSE
  best_model = models.loc[np.argmin(np.array(models['MSE']))]
  toc = time.time()
  print("Processed", models.shape[0], "models on", k, "predictors in", (toc-tic), "seconds.")

  # Return the best model, along with some other useful information about the model
  return best_model
```

In [92]:
```
models_best = pd.DataFrame(columns=["MSE", "model"])

tic = time.time()
for i in range(4):
    models_best.loc[i] = getBest(i)

toc = time.time()
print("Total elapsed time:", (toc-tic), "seconds.")
```

```
Processed 1 models on 0 predictors in 0.015178918838500977 seconds.
Processed 50 models on 1 predictors in 0.3023357391357422 seconds.
Processed 1225 models on 2 predictors in 7.206106901168823 seconds.
Processed 19600 models on 3 predictors in 117.38464117050171 seconds.
Total elapsed time: 125.45728993415833 seconds.
```

In [93]:
```
print(models_best.loc[1, "model"].summary())
```

OLS Regression Results

Dep. Variable:	y	R-squared (uncentered):	0.527
Model:	OLS	Adj. R-squared (uncentered):	0.370
Method:	Least Squares	F-statistic:	3.347
Date:	Sun, 15 Dec 2019	Prob (F-statistic):	7.00e-09
Time:	00:59:16	Log-Likelihood:	-547.20
No. Observations:	200	AIC:	1194.
Df Residuals:	150	BIC:	1359.
Df Model:	50		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
x01	-3.1188	5.203	-0.599	0.550	-13.400	7.163
x02	-0.1291	4.746	-0.027	0.978	-9.507	9.248
x03	7.1450	4.637	1.541	0.125	-2.017	16.307
x04	2.6646	5.096	0.523	0.602	-7.404	12.733
x05	-2.8229	5.043	-0.560	0.576	-12.787	7.141
x06	3.2452	4.827	0.672	0.502	-6.292	12.782
x07	-3.2488	5.110	-0.636	0.526	-13.346	6.848
x08	4.7795	4.944	0.967	0.335	-4.989	14.548
x09	0.0828	4.776	0.017	0.986	-9.353	9.519
x10	-4.0881	5.181	-0.789	0.431	-14.325	6.149
x11	0.7750	3.406	0.228	0.820	-5.955	7.505
x12	1.7651	3.356	0.526	0.600	-4.867	8.397
x13	-2.2081	3.323	-0.664	0.507	-8.774	4.358
x14	-1.8011	3.587	-0.502	0.616	-8.889	5.287
x15	-1.2930	3.804	-0.340	0.734	-8.809	6.222
x16	-2.5538	3.544	-0.721	0.472	-9.556	4.449
x17	-1.0052	3.736	-0.269	0.788	-8.386	6.376
x18	-2.2132	3.821	-0.579	0.563	-9.764	5.337
x19	2.5080	3.502	0.716	0.475	-4.412	9.428
x20	-1.3930	3.301	-0.422	0.674	-7.916	5.130
x21	3.5559	3.595	0.989	0.324	-3.547	10.658
x22	-0.2649	3.510	-0.075	0.940	-7.200	6.670
x23	-3.6777	3.639	-1.011	0.314	-10.868	3.512
x24	0.4457	3.471	0.128	0.898	-6.413	7.304
x25	5.3167	3.282	1.620	0.107	-1.168	11.802
x26	0.7936	3.565	0.223	0.824	-6.251	7.838
x27	4.8582	3.840	1.265	0.208	-2.729	12.445
x28	-1.2869	3.189	-0.404	0.687	-7.588	5.015
x29	-1.9092	3.131	-0.610	0.543	-8.095	4.277

```
x30            6.5661      3.781      1.737      0.084      -0.904      14.036
x31            0.0796      0.358      0.223      0.824      -0.627       0.786
x32           -0.2485      0.359     -0.692      0.490      -0.958       0.461
x33            0.2262      0.367      0.616      0.539      -0.500       0.952
x34            0.7586      0.310      2.449      0.015       0.147       1.371
x35            0.9340      0.354      2.639      0.009       0.235       1.633
x36            0.2610      0.353      0.740      0.461      -0.436       0.958
x37           -1.0088      0.366     -2.755      0.007      -1.732      -0.285
x38            0.9213      0.338      2.730      0.007       0.254       1.588
x39           -0.3646      0.352     -1.036      0.302      -1.060       0.331
x40           -0.1663      0.354     -0.470      0.639      -0.865       0.532
x41            0.2125      0.340      0.625      0.533      -0.459       0.884
x42           -0.2185      0.368     -0.593      0.554      -0.946       0.509
x43            0.0813      0.337      0.241      0.810      -0.585       0.748
x44           -0.0183      0.358     -0.051      0.959      -0.726       0.689
x45            0.2680      0.355      0.756      0.451      -0.433       0.968
x46            0.2159      0.339      0.637      0.525      -0.453       0.885
x47            0.0775      0.339      0.229      0.819      -0.592       0.747
x48            0.3978      0.356      1.117      0.266      -0.306       1.102
x49           -0.2881      0.358     -0.804      0.422      -0.996       0.420
x50           -0.2767      0.364     -0.759      0.449      -0.997       0.443
==============================================================================
Omnibus:                       23.387   Durbin-Watson:                   2.126
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               28.905
Skew:                           0.793   Prob(JB):                     5.29e-07
Kurtosis:                       3.975   Cond. No.                         51.0
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

In [94]:
```
models_best
```

Out[94]:

	MSE	model
0	24.526909	<statsmodels.regression.linear_model.Regressio...
1	24.526909	<statsmodels.regression.linear_model.Regressio...
2	24.526909	<statsmodels.regression.linear_model.Regressio...
3	24.526909	<statsmodels.regression.linear_model.Regressio...