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Event-triggered Leader-following Consensus Control of Multiagent Systems Against DoS Attacks

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Abstract: This paper addresses the issue of secure leader-following consensus for multiagent systems (MASs) under denial-of-service (DoS) attacks. In such attacks, the communication channels of agents may be disrupted until the attack terminates. To tackle this challenge, first, an event-triggered communication (ETC) scheme is proposed to alleviate unnecessary information transmission in energy-limited and easily attackable networks. Then, the concept of effective DoS attack intervals is introduced, and the duration attack length rate is analyzed. By employing a Lyapunov function and the average dwell-time (ADT) method, the secure consensus can be achieved. Moreover, Zeno behavior is eliminated to ensure the feasibility of the event-triggered scheme. Finally, simulation results are provided to validate the effectiveness of the proposed update strategy and control protocol.

Keywords: Denial-of-service attack, event-triggered communication, leader-following consensus, multiagent systems, secure consensus.

1. INTRODUCTION

Multiagent systems (MASs) consist of multiple agents or robots that can communicate and collaborate with each other to accomplish various tasks. Over the past decade, MASs have received a lot of attention and demonstrated various uses in multiple fields, such as self-driving vehicles, wireless sensor networks, and intelligent mobile robots [1-5]. In MASs research, consensus is a core theme, which involves ensuring that all agents in the system eventually reach the same state or objective. In the process of consensus implementation, the traditional method is to use continuous control, which may result in unnecessary communications. Therefore, it is absolutely essential to develop effective control strategies and communication mechanisms in order to minimize communication expenses.

Event-triggered is a control strategy that can effectively address the aforementioned issues. Event-triggered strategy involve exchanging state information and updating controls only when predefined event conditions are met in the system state, aiming to reduce communication overhead and prolong system lifespan [6-9]. Compared to traditional continuous or periodic communication control, event-triggered control has garnered increasing attention in collaborative control of MASs [10]. Therefore, event-triggered control finds widespread application in addressing issues such as consensus and collaborative control in MASs [11-14]. For example, research in [15] addressed asynchronous periodic sampling consensus for leader-follower second-order MASs via an event-triggered mechanism, presenting a distributed control protocol to enhance consistency and minimize data transmission, showcasing exponential convergence with a directed spanning tree in the topology. In [16], an adaptive dynamic event-triggered scheme that does not rely on global information and can save energy is proposed for cooperative fault-tolerant output regulation of linear heterogeneous MASs with actuator faults. Reference [17] investigates a fully distributed event-triggering mechanism (ETM) that achieves fault-tolerant consensus control of linear MASs with disturbance suppression and asymptotic stability under intermittent communication.

The aforementioned approaches have addressed to some extent the issue of limited communication resources. However, they are all based on the assumption that communication occurs within a secure network environment, which is not realistic [18-20]. Due to bandwidth limitations, delays, data loss, and various types of network attacks, the implementation of consensus in MASs faces challenges [21]. In MASs, denial of service (DoS) attacks and false data injection (FDI) attacks are among the most common and severe threats to communication networks. DoS attacks disrupt the system by overwhelming the target network with a large volume of invalid requests or by consuming excessive bandwidth [22]. FDI attacks

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compromise the system's normal operation and decision-making by injecting false or erroneous data. There have been numerous significant research findings on the consensus of MASs affected by these attacks. For instance, [23] proposed an event-triggered control framework for modeling DoS attacks using deterministic sequences in general linear MASs, under which asymptotic consensus can be achieved. In [24], an improved dynamic linearization method was investigated for learning nonlinear MASs affected by DoS attacks. Reference [25] proposed an adaptive bipartite secure consensus control scheme that uses an event-triggered strategy to address the bipartite consensus control problem in unbalanced communication topology. In [26], by introducing a sort and filter method and utilizing a distributed secure state estimation strategy, sufficient conditions for tolerating a bounded number of malicious agents are provided.

Inspired by the above discussions, it is assumed that attackers target different intelligent agents asynchronously, and an attack on a specific agent will block the corresponding communication channel within an effective DoS interval. This implies that the communication topology of MASs is altered within effective DoS intervals. Thus, under the aforementioned assumptions, designing an event-triggered control protocol to address the variable topology while ensuring security consensus remains a challenge. To tackle this challenge, a ETC scheme is first proposed to alleviate unnecessary information transmission in energy-limited and vulnerable networks. Subsequently, the concept of effective DoS attack intervals is introduced, along with an analysis of attack duration rate. By employing Lyapunov functions and the ADT method, it is demonstrated that secure consensus can be achieved if the attack duration rate does not exceed a positive threshold.

The contributions of this article are as follows:

1) Unlike previous studies such as [27,28] and [29], this paper introduces the concept of attack length rate, which simplifies the description of attack sequences. This approach enables leader-following consensus in MASs even under irregular DoS attacks.
2) This paper introduces topological information parameters to distinguish between various attack scenarios. By dynamically adjusting the control method based on the current attack situation, this strategy effectively handles evolving threats, enhancing the system's adaptability to network changes.
3) The ETC method proposed in this paper accurately captures the cumulative sum of errors from neighboring agents, offering a precise depiction of the overall system state. Compared to [22], this method reduces unnecessary triggers and lowers communication overhead.

2. PRELIMINARIES

2.1. Notations

In this paper, $\mathbb{N}$ and $\mathbb{R}$ represent the sets of natural numbers and real numbers, respectively. $\mathbb{R}^q$ and $\mathbb{R}^{q\times q}$ stand for q-dimensional vectors and $q\times q$ real matrices, respectively. For a given matrix S, $S>0$, S^T, S^{-1}, $\lambda(S)$, and $\|S\|$ denote a positive definite matrix, its transpose, inverse matrix, eigenvalues, and induced matrix 2-norm, respectively. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ designate the smallest and largest eigenvalues, respectively. For matrices $A\in\mathbb{R}^{r\times m}$ and $B\in\mathbb{R}^{p\times q}$, $A\otimes B\in\mathbb{R}^{rp\times mq}$, where the symbol $\otimes$ denotes the Kronecker product operation. For given sets S_1 and S_2, $|S_1|$ indicates the cardinality of S_1, and S_1/S_2 represents the relative complement of S_1 in S_2.

2.2. Definitions and concepts

A communication network of MASs with N agents is described by a directed graph $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{W})$, where $\mathcal{V}=\{1, ..., N\}$ is the set of agents, $\mathcal{E}=\{(i,j): i,\ j\in\mathcal{V}\}$ is the set of communication edges, and $\mathcal{W}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is the adjacency matrix. Here, $a_{ii}=0$, $a_{ij}=1$ if $\{i,\ j\}\in\mathcal{E}$, and $a_{ij}=0$ otherwise. Let $\mathcal{L}$ be the Laplacian matrix with elements l_{ij} defined as follows: when $i\neq j$, $l_{ij}=-a_{ij}$, and when $i=j$, $l_{ij}=\sum_{j=1}^{N}a_{ij}$. A directed path from node i to node j is created by linking a sequence of edges (i,d_1), (d_1,d_2), $\ldots$, (d_{m-1},d_m), (d_m,j), where d_1, $\ldots$, d_m represent distinct nodes. If there exists a node (referred to as the root node) that has directed paths to other nodes, then the graph $\mathcal{G}$ is said to possess a directed spanning tree.

Lemma 1 [30]**:** Based on the Rayleigh quotient inequality, we can state the following: Let A be an n-order real symmetric matrix, where $\lambda_{max}(A)$ and $\lambda_{min}(A)$ denote the maximum and minimum eigenvalues of A, respectively. For any n-dimensional real vector X, the following inequalities hold

$$\lambda_{min}(A)\cdot X^TX\leq X^TAX\leq\lambda_{max}(A)\cdot X^TX.$$

Lemma 2 [31] **:** Based on Young's inequality, we obtain the resulting inequality: Let A be an n-order symmetric positive definite matrix, and a be a positive real number. For any n-dimensional real vectors X and Y, the following inequality holds

$$2X^TAY\leq aX^TAX+\frac{1}{a}Y^TAY.$$

Lemma 3 [32] **:** A non-singular M-matrix is a matrix with nonpositive off-diagonal elements, positive diagonal elements, and all eigenvalues having positive real parts. Let A be such a matrix, and consider the introduction of a vector $[\theta_1,\ \ldots,\ \theta_N]^T=A^{-T}\cdot\mathbf{1}_N$, where $\Theta=\mathrm{diag}\{\theta_1,\ \ldots,\ \theta_N\}>0$. Based on the properties of M-matrices, the following conclusion can be drawn

$$A+\Theta^{-1}A^T\Theta>0.$$

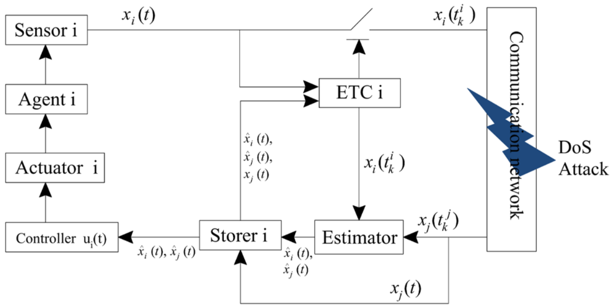

Fig. 1. Framework of MASs.

Consider MASs consisting of a leader and N followers, with the framework shown in Fig. 1, where the leader is denoted as node 0 and the followers are denoted as nodes $i = 1, ..., N$. The dynamics are represented by the following equations

$$\begin{aligned}\dot{x}_0(t) &= Ax_0(t),\\ \dot{x}_i(t) &= Ax_i(t) + Bu_i(t),\end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ represents the state, $u_i(t) \in \mathbb{R}^p$ is a controller designed against DoS attacks. The matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, and $C \in \mathbb{R}^{q\times n}$ are constants.

In the context of continuous communication, let s_0 denote the initial time point and s_k the moments when an attack begins. Due to limited attack resources, the attack stops at s_k^*, where $s_k^* \in (s_k,\ s_{k+1})$. Therefore, the periods when attacks occur can be represented as $t \in [s_1,\ s_1^*) \cup [s_2,\ s_2^*) \cup \ldots \cup [s_m,\ s_m^*)$ (abbreviated as $t \in \bigcup_{k=1}^m [s_k,\ s_k^*)$), and the non-attack periods are $t \in [s_0,\ s_1) \cup [s_1^*,\ s_2) \cup [s_2^*,\ s_3) \cup \ldots \cup [s_m^*, s_{m+1})$ (abbreviated as $t \in [s_0, s_1) \cup \big(\bigcup_{k=1}^m [s_k^*,\ s_{k+1})\big)$). During DoS attacks on the communication network, the communication links of the attacked agents are severed. After the attack ends, the network should return to normal. To describe the network's status during different periods, a piecewise constant function $\sigma(\cdot)$ is introduced, satisfying $[s_0, s_{m+1}] \to \mathbb{N}$. The values of $\sigma(t)$ during different periods are as follows: When $\sigma(t) = 1$, $t \in [s_0, s_1) \cup (\bigcup_{k=1}^m [s_k^*, s_{k+1}))$, which corresponds to non-attack periods. When $\sigma(t) \in \{2,\ \ldots,\ m+1\}$, $t \in \bigcup_{k=1}^m [s_k, s_k^*)$, which corresponds to attack periods. Different values of $\sigma(t)$ are used to distinguish between different attack scenarios. The changes in $\sigma(t)$ indicate the switching of network topological information.

The adjacency matrix of the communication network is denoted as $\mathcal{G}^{\sigma(t)}$, with its adjacency matrix represented as $\mathcal{A}^{\sigma(t)} = [a_{ij}^{\sigma(t)}] \in \mathbb{R}^{(N+1)\times(N+1)}$, where there exists a root node that does not receive information from any agent. The Laplacian matrix is represented as $\mathcal{L}^{\sigma(t)} = [l_{ij}^{\sigma(t)}] \in \mathbb{R}^{(N+1)\times(N+1)}$, in the form

$$\mathcal{L}^{\sigma(t)} = \begin{bmatrix} 0 & \mathbf{0}_N^T \\ -\mathbf{D} & \widehat{\mathcal{L}}^{\sigma(t)} \end{bmatrix},$$

where $\mathbf{D} = \left[a_{10}^{\sigma(t)},\ \ldots,\ a_{N0}^{\sigma(t)}\right]^T$.

Remark 1: In distributed systems, communication networks act as bridges connecting various nodes, enabling information exchange. When subjected to DoS attacks, attackers aim to disrupt communication by consuming resources such as bandwidth, computational power, or storage space. This can lead to: 1) Communication rerouting: To avoid the affected area, the system may need to recalculate routes, causing changes in communication paths. 2) Dynamic topology changes: The network might undergo dynamic adjustments to restore communication. Consequently, the network will switch between multiple configurations [27]. However, in practice, attack sequences may not follow a predictable stochastic pattern. This paper introduces the concept of attack length rate to characterize these sequences, providing a more practical and verifiable approach.

In the time interval $[s_0,\ s_{m+1}]$, the total attack duration S_a is defined as

$$S_a = \sum_{j=1}^{m} (s_j^* - s_j),$$

where s_j^* and s_j represent the end time and start time, respectively, of the j-th attack. At any given time $t > s_0$, the attack length rate is defined as: $S_a/(t - s_0)$.

Definition 1: For any given initial state $x(0) = [x_1^T(0),\ x_2^T(0),\ \ldots,\ x_N^T(0)]^T$, if $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0$, where $i = 1,\ 2,\ \ldots,\ N$, then the MASs (1) is said to globally asymptotically achieves a consensus.

3. MAIN RESULTS

For the leader-following system (1), the following control scheme is proposed

$$\begin{aligned} q_i(t) &= \sum_{j=1}^{N} a_{ij}^{\sigma(t)}(\hat{x}_i(t) - \hat{x}_j(t)) + d_i^{\sigma(t)}(\hat{x}_i(t) - x_0(t)),\\ u_i(t) &= Kq_i(t), \end{aligned} \tag{2}$$

where $K \in \mathbb{R}^{p\times n}$ is the feedback gain matrix to be determined. The introduction of $\sigma(t)$ is to ensure that the control scheme (2) remains effective during attacks. Additionally, $\hat{x}_i(t)$ is the estimated value of $x_i(t)$. Let $d_i^{\sigma(t)} = a_{i0}^{\sigma(t)}$, and $\hat{x}_i(t) = x_i(t_k^i)$ with the triggering instant $t_k^i = \max\{t_s^i : t_s^i \le t\}$. Here, t_s^i represents the moment of the s-th triggering event for agent i. Let $\delta_i(t) = x_i(t) - x_0(t)$ and $e_i(t) = \hat{x}_i(t) - x_i(t)$ represent the consensus error and measurement error, respectively. The following introduces some new functions: $\tilde{e}_i(t) = \sum_{j=1}^N \widehat{\mathcal{L}}_{ij}^{\sigma(t)}(\hat{x}_j(t) - x_j(t))$, $\hat{e}_i(t) = \sum_{j=1, j\neq i}^N \widehat{\mathcal{L}}_{ij}^{\sigma(t)}(\hat{x}_j(t) - x_j(t))$, and $\tilde{q}_i(t) = \tilde{e}_i(t) - q_i(t)$. Define $\mathcal{H}_1^i(t) = \tilde{e}_i^T(t)\Gamma\tilde{e}_i(t) - \eta\tilde{q}_i^T(t)\Gamma\tilde{q}_i(t)$, and $\mathcal{H}_2^i(t) = \hat{e}_i^T(t)\Gamma\hat{e}_i(t) - \eta\tilde{q}_i^T(t)\Gamma\tilde{q}_i(t)$, for the ETC scheme described below:

$$t_{k+1}^i = \inf_{l > t_k^i} \left\{ l : \mathcal{H}_1^i(t) > 0 \wedge \mathcal{H}_2^i(t) < 0,\ \forall t \in \left(t_k^i, l\right] \right\}, \tag{3}$$

in addition, $t_1^i = 0$, where $\Gamma = MBB^TM$(M given below), and η is a positive threshold to be determined.

Assumption 1: Matrix pair (A, B) is stabilizable.

Assumption 2: In $\mathcal{G}^1$, there exists a directed spanning tree rooted at the leader.

Remark 2: Due to the occurrence of DoS attacks during the intervals $t \in \bigcup_{k=1}^m [s_k, s_k^*)$, the network may gradually lose connectivity. Specifically, for $\sigma(t) \neq 1$, $\mathcal{G}^{\sigma(t)}$ may lack a directed spanning tree. This complication adds significant challenges to the problem, as having directed spanning trees in the communication network is essential for achieving consensus in MASs.

Theorem 1: When $t \in [s_0, s_1) \cup \left(\bigcup_{k=1}^m [s_k^*, s_{k+1})\right)$, by Assumption 2 and Lemma 3, $\widehat{\mathcal{L}}^1$ is a nonsingular M-matrix. Let $[\phi_1, \ldots, \phi_N]^T = (\widehat{\mathcal{L}}^1)^{-T} \cdot \mathbf{1}_N$ and $\Phi = \mathrm{diag}\{\phi_1, \ldots, \phi_N\}$. Additionally, as all the eigenvalues of $\widehat{\mathcal{L}}^1 + \Phi^{-1}\left(\widehat{\mathcal{L}}^1\right)^T\Phi$ are positive, define

$$\mu = \lambda_{\min}\left(\widehat{\mathcal{L}}^1 + \Phi^{-1}\left(\widehat{\mathcal{L}}^1\right)^T\Phi\right), \tag{4}$$

and

$$\nu = \min_{j=2,\ldots,m+1} \lambda_{\min}\left(\widehat{\mathcal{L}}^j + \Phi^{-1}\left(\widehat{\mathcal{L}}^j\right)^T\Phi\right). \tag{5}$$

It follows that $\mu > 0$, but ν is not necessarily greater than 0. Introducing new parameter r and s as follows:

$$r = \lambda_{\max}\left(\Phi^{-1}\left(\widehat{\mathcal{L}}^1\right)^T\Phi\widehat{\mathcal{L}}^1\right), \tag{6}$$

and

$$s = \max_{j=2,\ldots,m+1} \lambda_{\max}\left(\Phi^{-1}\left(\widehat{\mathcal{L}}^j\right)^T\Phi\widehat{\mathcal{L}}^j\right). \tag{7}$$

In the context of Assumption 1, consider the following two linear matrix inequalities:

$$MA + A^TM - \iota MBB^TM + \varrho_1 M < 0, \tag{8}$$

and

$$MA + A^TM - \tau MBB^TM - \varrho_2 M < 0, \tag{9}$$

where $M \in \mathbb{R}^{n\times n} > 0$, ϱ_1, and $\varrho_2 > 0$, ι is a positive constant, and τ may not necessarily be positive.

For a given MASs described by (1) under DoS attack, controlled by the controller (2) with $K = -B^TM$, and subject to the conditions $\iota < \mu - \frac{1}{\alpha}\eta r - \alpha$ and $\tau < \nu - \frac{1}{\alpha}\eta s - \alpha$, secure consensus can be achieved for any $t > s_0$ if the attack duration rate satisfies

$$\frac{S_a}{t - s_0} \leq \frac{\varrho_1 - \varrho^*}{\varrho_1 + \varrho_2}, \tag{10}$$

where $\varrho_1 > \varrho^* > 0$.

Proof: Construct a Lyapunov function

$$V(t) = \delta^T(t)[\Phi \otimes M]\delta(t).$$

From (2), it follows that

$$q_i(t) = \sum_{j=1}^N a_{ij}^{\sigma(t)}(\delta_i(t) - \delta_j(t)) + d_i^{\sigma(t)}\delta_i(t) + \sum_{j=1}^N a_{ij}^{\sigma(t)}(e_i(t) - e_j(t)) + d_i^{\sigma(t)}e_i(t).$$

Let $q(t) = [q_1^T(t), \ldots, q_N^T(t)]^T$, and similarly for $\delta(t)$ and $e(t)$. Rewriting the above yields

$$q(t) = [\widehat{\mathcal{L}}^{\sigma(t)} \otimes I_n]\delta(t) + [\widehat{\mathcal{L}}^{\sigma(t)} \otimes I_n]e(t).$$

From $\dot{\delta}_i(t) = A\delta_i(t) - BB^TMq_i(t)$, we have

$$\dot{\delta}(t) = [I_N \otimes A - \widehat{\mathcal{L}}^{\sigma(t)} \otimes BB^TM]\delta(t) - [\widehat{\mathcal{L}}^{\sigma(t)} \otimes BB^TM]e(t).$$

Given that $\tilde{e}_i(t) = \sum_{j=1}^N \widehat{\mathcal{L}}_{ij}^{\sigma(t)}(\hat{x}_j(t) - x_j(t))$, we define $\tilde{e}(t) = [\tilde{e}_1^T(t), \ldots, \tilde{e}_N^T(t)]^T$. Consequently, $\tilde{e}(t)$ can be expressed as $\tilde{e}(t) = [\widehat{\mathcal{L}}^{\sigma(t)} \otimes I_n]e(t)$. The time derivative of $V(t)$ is given by

$$\begin{aligned}
\dot{V}(t) &= 2\delta^T(t)[\Phi \otimes M]\dot{\delta}(t) \\
&= 2\delta^T(t)[\Phi \otimes MA - \Phi\widehat{\mathcal{L}}^{\sigma(t)} \otimes MBB^TM]\delta(t) \\
&\quad - 2\delta^T(t)[\Phi\widehat{\mathcal{L}}^{\sigma(t)} \otimes MBB^TM]e(t) \\
&= \delta^T(t)[\Phi \otimes (MA + A^TM)]\delta(t) \\
&\quad - \delta^T(t)[(\Phi\widehat{\mathcal{L}}^{\sigma(t)} + (\widehat{\mathcal{L}}^{\sigma(t)})^T\Phi) \otimes MBB^TM]\delta(t) \\
&\quad - 2\delta^T(t)[\Phi \otimes MBB^TM]\tilde{e}(t).
\end{aligned} \tag{11}$$

According to Lemmas 1 and 2, the following inequalities hold

$$\begin{aligned}
&- \delta^T(t)[(\Phi\widehat{\mathcal{L}}^{\sigma(t)} + (\widehat{\mathcal{L}}^{\sigma(t)})^T\Phi) \otimes MBB^TM]\delta(t) \\
&\leq -\lambda_{\min}(\widehat{\mathcal{L}}^{\sigma(t)} + \Phi^{-1}(\widehat{\mathcal{L}}^{\sigma(t)})^T\Phi) \\
&\quad \times \delta^T(t)[\Phi \otimes MBB^TM]\delta(t),
\end{aligned}$$

and

$$\begin{aligned}
&- 2\delta^T(t)[\Phi \otimes MBB^TM]\tilde{e}(t) \\
&\leq \alpha\delta^T(t)[\Phi \otimes MBB^TM]\delta(t) \\
&\quad + \frac{1}{\alpha}\tilde{e}^T(t)[\Phi \otimes MBB^TM]\tilde{e}(t).
\end{aligned}$$

Here, α is a positive scalar. Substituting these into (11) results in

$$\begin{aligned}
\dot{V}(t) &\leq \delta^T(t)[\Phi \otimes (MA + A^TM)]\delta(t) \\
&\quad + (\alpha - \lambda_{\min}(\widehat{\mathcal{L}}^{\sigma(t)} + \Phi^{-1}(\widehat{\mathcal{L}}^{\sigma(t)})^T\Phi)) \\
&\quad \times \delta^T(t)[\Phi \otimes MBB^TM]\delta(t)
\end{aligned}$$

$$+\frac{1}{\alpha}\tilde{e}^T(t)[\Phi\otimes MBB^TM]\tilde{e}(t). \tag{12}$$

Let $\tilde{q}(t)=[\tilde{q}_1^T(t),\ \ldots,\ \tilde{q}_N^T(t)]^T$. Given $\tilde{q}_i(t)=\tilde{e}_i(t)-q_i(t)$, it follows that $\tilde{q}(t)=-[\widehat{\mathcal{L}}^{\sigma(t)}\otimes I_n]\delta(t)$. Thus, from (3), we derive

$$\begin{aligned}&\tilde{e}^T(t)[\Phi\otimes MBB^TM]\tilde{e}(t)\\&\le\eta\delta^T(t)\big[(\widehat{\mathcal{L}}^{\sigma(t)})^T\Phi\widehat{\mathcal{L}}^{\sigma(t)}\otimes MBB^TM\big]\delta(t).\end{aligned}$$

According to Lemma 1, one can further obtain

$$\begin{aligned}&\tilde{e}^T(t)[\Phi\otimes MBB^TM]\tilde{e}(t)\\&\le\eta\delta^T(t)[\Phi\otimes MBB^TM]\delta(t)\\&\quad\times\lambda_{\max}(\Phi^{-1}(\widehat{\mathcal{L}}^{\sigma(t)})^T\Phi\widehat{\mathcal{L}}^{\sigma(t)}).\end{aligned} \tag{13}$$

When $t\in[s_0,\ s_1)\cup\big(\bigcup_{k=1}^m[s_k^*,\ s_{k+1})\big)$, substituting (4), (6), and (13) into (12), we obtain

$$\begin{aligned}\dot{V}(t)\le\ &\delta^T(t)[\Phi\otimes(MA+A^TM\\&-(\mu-\alpha-\eta r/\alpha)MBB^TM)]\delta(t).\end{aligned}$$

Based on $\iota<\mu-\frac{1}{\alpha}\eta r-\alpha$, we can derive

$$\dot{V}(t)<\delta^T(t)[\Phi\otimes(MA+A^TM-\iota MBB^TM)]\delta(t).$$

Based on (8), we have

$$\dot{V}(t)<-\varrho_1\delta^T(t)[\Phi\otimes M]\delta(t)=-\varrho_1V(t). \tag{14}$$

When $t\in\bigcup_{k=1}^m[s_k,\ s_k^*)$, substituting (5), (7), and (13) into (12), we obtain

$$\begin{aligned}\dot{V}(t)\le\ &\delta^T(t)[\Phi\otimes(MA+A^TM\\&-(\nu-\alpha-\eta s/\alpha)MBB^TM)]\delta(t).\end{aligned}$$

Based on $\tau<\nu-\frac{1}{\alpha}\eta s-\alpha$, we can derive

$$\dot{V}(t)<\delta^T(t)[\Phi\otimes(MA+A^TM-\tau MBB^TM)]\delta(t).$$

Based on (9), we have

$$\dot{V}(t)<\varrho_2\delta^T(t)[\Phi\otimes M]\delta(t)=\varrho_2V(t). \tag{15}$$

Through (14) and (15), we can conclude

$$V(s_{m+1})<\exp(-\varrho_1(s_{m+1}-s_m^*))V(s_m^*), \tag{16}$$

and

$$V(s_m^*)<\exp(\varrho_2(s_m^*-s_m))V(s_m). \tag{17}$$

Combining (16) and (17), results in

$$\begin{aligned}V(s_{m+1})&<\exp(-\varrho_1(s_{m+1}-s_m^*))V(s_m^*)\\&\quad\cdots\\&<\exp\Bigg(-\varrho_1(s_1-s_0)-\varrho_1\sum_{j=1}^m(s_{j+1}-s_j^*)\\&\quad+\varrho_2\sum_{j=1}^m(s_j^*-s_j)\Bigg)V(s_0).\end{aligned} \tag{18}$$

For any $t\ge s_0$, there is an m such that $t\in[s_m,s_{m+1})$. Simplifying (18) yields

$$V(t)<\exp(-\varrho_1(t-s_0-S_a)+\varrho_2S_a)V(s_0).$$

Based on (10) in Theorem 1, we can further obtain

$$V(t)<\exp(-\varrho^*(t-s_0))V(s_0). \tag{19}$$

Due to $\varrho^*>0$, this implies $\lim_{t\to\infty}\|\delta(t)\|=0$. Therefore, secure consensus is achieved. □

The main challenge in implementing the ETC strategy is eliminating Zeno behavior. Zeno behavior occurs when events are triggered infinitely within a finite time, causing constant communication. Therefore, it is crucial to demonstrate that the proposed ETC (3) can eliminate this behavior.

Theorem 2: With the consensus condition stated in Theorem 1, Zeno behavior of MASs (1) can be excluded, indicating that there is no infinite triggering within finite time.

Proof: Since the MASs (1) achieve consensus over time, $\|q_i(t)\|$ is bounded. The result of computing the right-hand derivative of $\|e_i(t)\|$ is

$$\begin{aligned}D^+\|e_i(t)\|&\le\|\dot{x}_i(t)\|\\&=\|Ax_i(t)+BKq_i(t)\|\\&\le\|A\|\|e_i(t)\|+\|A\hat{x}_i(t)\|+\|BK\|\|q_i(t)\|\\&\le\|A\|\|e_i(t)\|+\Omega_i(t),\end{aligned} \tag{20}$$

where $\Omega_i(t)$ represents the upper bound of $\|A\hat{x}_i(t)\|+\|BK\|\|q_i(t)\|$.

Case 1: If $\|A\|\neq0$, Since $\|e_i(t_k^i)\|=0$, it follows from (20) that

$$\|e_i(t)\|\le\frac{\Omega_i(t)}{\|A\|}\left(e^{\|A\|(t-t_k^i)}-1\right). \tag{21}$$

According to the ETC (3), the next triggering moment t_{k+1}^i satisfies

$$\|\tilde{e}_i^T(t_{k+1}^i)MB\|^2>\eta\|\tilde{q}_i^T(t_{k+1}^i)MB\|^2, \tag{22}$$

and

$$\|\hat{e}_i^T(t_{k+1}^i)MB\|^2<\eta\|\tilde{q}_i^T(t_{k+1}^i)MB\|^2. \tag{23}$$

According to (23), let

$$\sqrt{\eta}\|\tilde{q}_i^T(t_{k+1}^i)MB\|-\|\hat{e}_i^T(t_{k+1}^i)MB\|=\Lambda(t_{k+1}^i),$$

where $\Lambda(t_{k+1}^i)>0$. Then, from (21), we obtain

$$\big\|\tilde{e}_i^T(t)MB\big\|$$

$$
\begin{aligned}
&= \left\| \sum_{\substack{j=1\\ j\neq i}}^{N} \widehat{\mathcal{L}}_{ij}^{\sigma(t)} e_j^T(t)MB + \widehat{\mathcal{L}}_{ii}^{\sigma(t)} e_i^T(t)MB \right\| \\
&\leq \left\| \sum_{\substack{j=1\\ j\neq i}}^{N} \widehat{\mathcal{L}}_{ij}^{\sigma(t)} e_j^T(t)MB \right\| + \left\| N e_i^T(t)MB \right\| \\
&\leq \left\| \sum_{\substack{j=1\\ j\neq i}}^{N} \widehat{\mathcal{L}}_{ij}^{\sigma(t)} e_j^T(t)MB \right\| \\
&\quad + \frac{N\Omega_i(t)\|MB\|}{\|A\|}\left(e^{\|A\|(t-t_k^i)}-1\right).
\end{aligned}
$$

Combining with (22), it follows that

$$
\begin{aligned}
&\sqrt{\eta}\|\tilde{q}_i^T(t_{k+1}^i)MB\| \\
&< \left\| \sum_{\substack{j=1\\ j\neq i}}^{N} \widehat{\mathcal{L}}_{ij}^{\sigma(t)} e_j^T(t_{k+1}^i)MB \right\| \\
&\quad + \frac{N\Omega_i(t_{k+1}^i)\|MB\|}{\|A\|}\left(e^{\|A\|(t_{k+1}^i-t_k^i)}-1\right).
\end{aligned} \tag{24}
$$

This implies that $t_{k+1}^i - t_k^i > \Delta$, and

$$
\Delta = \frac{1}{\|A\|}\ln\left(1+\frac{\Lambda(t_{k+1}^i)\|A\|}{N\Omega_i(t_{k+1}^i)\|MB\|}\right).
$$

Therefore, the proof is concluded.

Case 2: If $\|A\| = 0$, one has

$$
\|e_i^T(t)MB\| \leq \Omega_i(t)\|MB\|(t-t_k^i).
$$

Similar to Case 1, it can be derived that

$$
\begin{aligned}
\left\| \sum_{j=1}^{N} \widehat{\mathcal{L}}_{ij}^{\sigma(t)} e_j^T(t)MB \right\| &\leq \left\| \sum_{\substack{j=1\\ j\neq i}}^{N} \widehat{\mathcal{L}}_{ij}^{\sigma(t)} e_j^T(t)MB \right\| \\
&\quad + N\Omega_i(t)\|MB\|(t-t_k^i).
\end{aligned} \tag{25}
$$

From (22), it follows that $t_{k+1}^i - t_k^i > \Delta$, where

$$
\Delta = \frac{\Lambda(t_{k+1}^i)}{N\Omega_i(t_{k+1}^i)\|MB\|}.
$$

The proof is concluded.

Based on Cases 1 and 2, Zeno behavior can be ruled out. This completes the proof. □

Remark 3: It can also be proven by contradiction that Zeno behavior is excluded. Suppose an agent i exhibits Zeno behavior, implying that agent i triggers an infinite number of events within a finite time span. Consequently, $\lim_{m\to\infty}\sum_{k=0}^{m}\Delta_k^i$ converges, where $\Delta_k^i = t_{k+1}^i - t_k^i$. This implies $\lim_{t\to\infty}(t_{k+1}^i - t_k^i) = 0$. Substituting this conclusion into (24) yields $\sqrt{\eta}\|\tilde{q}_i^T(t_{k+1}^i)MB\| < \|\hat{e}_i^T(t_{k+1}^i)MB\|$. This contradicts (23). Therefore, $\lim_{m\to\infty}\sum_{k=0}^{m}\Delta_k^i = \infty$, which diverges. Hence, Zeno behavior is excluded.

4. NUMERICAL EXAMPLE

In this section, two numerical examples are provided to validate the theoretical results.

Example 1: Consider a multi-submarine robot system consisting of one leader submarine robot and six follower submarine robots. Each submarine robot is equipped with a three-chamber system for buoyancy and attitude control. The gas pressures in the three chambers are interconnected through the top gas pipes to maintain consistency. There are bottom-end bidirectional pipeline valves between the first and second chambers, as well as between the first and third chambers, controlling the fluid flow between these chambers. It is assumed that the fluid flow in the pipelines is described by Darcy's law, where the flow rate is proportional to the hydraulic pressure difference, and the pipelines have the same flow resistance.

Darcy's law relates the fluid flow rate in a pipeline to the hydraulic pressure difference through the equation $Q = \frac{\Delta P}{R}$. Here, Q represents the fluid flow rate, ΔP is the liquid pressure difference between two chambers, and R denotes the flow resistance of the pipeline, assumed to be 1 Pa·s/m^3. If the volume of each chamber remains constant and the bottom area of each chamber is S, the fluid flow rate can be expressed as $Q = S\cdot\left(\frac{\Delta h}{\Delta t}\right)$. This flow rate is directly proportional to the rate of change of liquid pressure over time, where $\frac{dP}{dt} = \rho\cdot g\cdot\left(\frac{\Delta h}{\Delta t}\right)$, with a proportionality coefficient $K = 1$. Assuming that at $t = 0$ two bidirectional pipeline valves are opened, for each chamber of the leader submarine robot, the following fluid dynamics equations can be written

$$
\begin{cases}
\dfrac{dP_{01}}{dt} = -\dfrac{K}{R}(P_{01}-P_{02}) - \dfrac{K}{R}(P_{01}-P_{03}),\\
\dfrac{dP_{02}}{dt} = \dfrac{K}{R}(P_{01}-P_{02}),\\
\dfrac{dP_{03}}{dt} = \dfrac{K}{R}(P_{01}-P_{03}).
\end{cases}
$$

Let $x_0 = [P_{01}, P_{02}, P_{03}]^T$. According to the above, we can write: $\dot{x}_0(t) = Ax_0$, where $A = \begin{bmatrix} -2 & 1 & 1\\ 1 & -1 & 0\\ 1 & 0 & -1\end{bmatrix}$. For the follower submarine robots, the control input is designed as $u_i(t) = [u_{i1}, u_{i2}]^T$. The fluid dynamics equations for each follower submarine robot are as follows:

$$
\begin{cases}
\dfrac{dP_{i1}}{dt} = -\dfrac{K}{R}(P_{i1}-P_{i2}) - \dfrac{K}{R}(P_{i1}-P_{i3}) - u_{i1},\\
\dfrac{dP_{i2}}{dt} = \dfrac{K}{R}(P_{i1}-P_{i2}) + u_{i1} - u_{i2},\\
\dfrac{dP_{i3}}{dt} = \dfrac{K}{R}(P_{i1}-P_{i3}) + u_{i1} - u_{i2}.
\end{cases}
$$

Let $x_i = [P_{i1}, P_{i2}, P_{i3}]^T$, where $i = 1, 2, \ldots, 6$. Based on the above, we can write: $\dot{x}_i(t) = Ax_i + Bu_i$, where

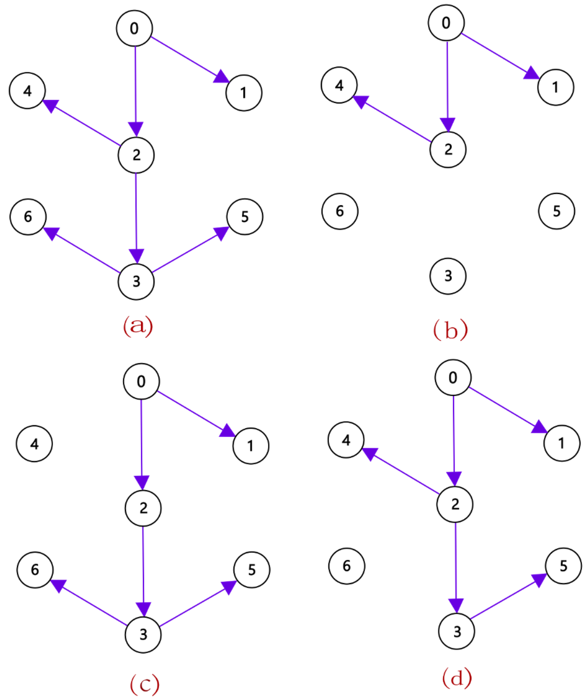

Fig. 2. Communicalion topology.

$B = \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$. Assuming the above equations satisfy (2) and (3), and the communication network of the multi-submarine robot system is subjected to a DoS attack. In Fig. 2, subfigures (a), (b), (c), and (d) correspond to the topology under $\sigma(t) = 1, 2, 3, 4$, respectively. Fig. 3 shows the graph of $\sigma(t)$ changing over time, where $\sigma(t) = 1$ represents the non-attack scenario, and $\sigma(t) = 2$, 3, 4 represent three different attack scenarios. Set the initial states of $x_0(t)$ and $x_i(t)$ as shown in Fig. 4. Based on the above, we obtain

$$\Phi = \begin{bmatrix} 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}.$$

Through calculation, we get $\mu = 0.8298$, $\nu = 0$, $r = 2.7573$, and $s = 2.6456$. Design $M = \begin{bmatrix} 1.2121 & 0 & 0 \\ 0 & 1.2121 & 0 \\ 0 & 0 & 1.2121 \end{bmatrix}$. Given $\eta = 0.03$, $\alpha = 0.284$, $\iota = 0.2545$, and $\tau = -0.5635$, we verify $\iota < \mu - \frac{1}{\alpha}\eta r - \alpha$ and $\tau < \nu - \frac{1}{\alpha}\eta s - \alpha$, resulting in $\iota < 0.254536$ and $\tau < -0.5635$. These inequalities are successfully met. We define $\varrho_1 = 0.462$ and $\varrho_2 = 1.459$, and confirm the satisfaction of (8) and (9). Setting $\varrho^* = 0.001$ ensures

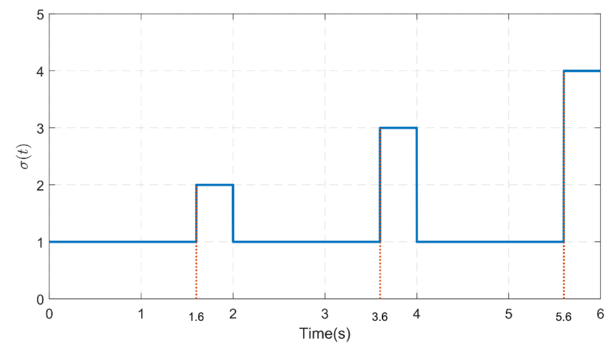

Fig. 3. The graph of $\sigma(t)$ changing over time.

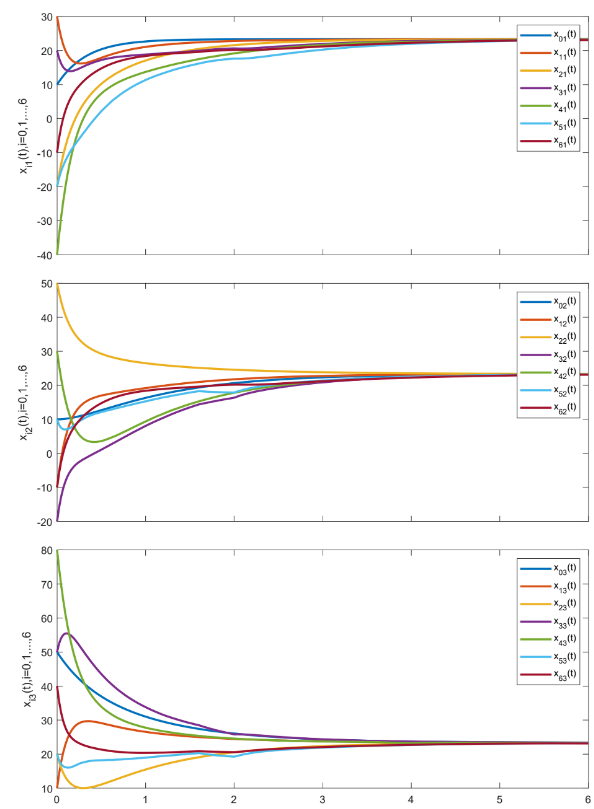

Fig. 4. State changes of $x_i(t)$ in Example 1.

$\varrho_1 > \varrho^* > 0$. We set the attack duration as $S_a = 1.20$ seconds and $t - s_0 = 6.00$ seconds, ensuring compliance with the attack length rate inequality $\frac{S_a}{t-s_0} \leq \frac{\varrho_1-\varrho^*}{\varrho_1+\varrho_2}$.

According to the simulation, Fig. 5 illustrates the triggering instants computed by each robot based on (3). In Fig. 6, the evolution of $\|\tilde{e}_i^T(t)MB\|$, $\|\hat{e}_i^T(t)MB\|$, and $\sqrt{\eta}\|\tilde{q}_i^T(t)MB\|$ over time can be observed. Fig. 4 shows the changes in pressure states within each chamber of the submarine robot. It is evident that $x_{i1}(t) - x_{i3}(t)$ achieves secure consensus over time, indicating that the multi-submarine robots eventually achieve consensus in depth

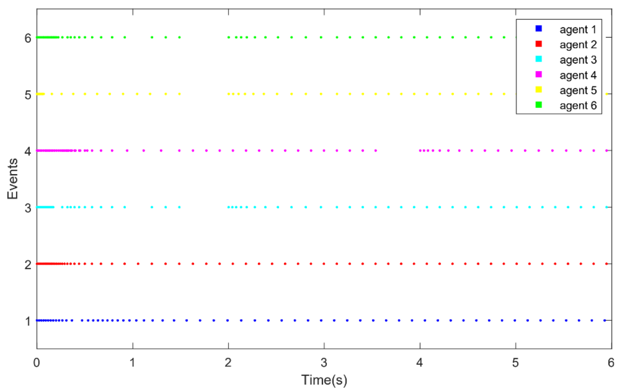

Fig. 5. Triggering instants in Example 1.

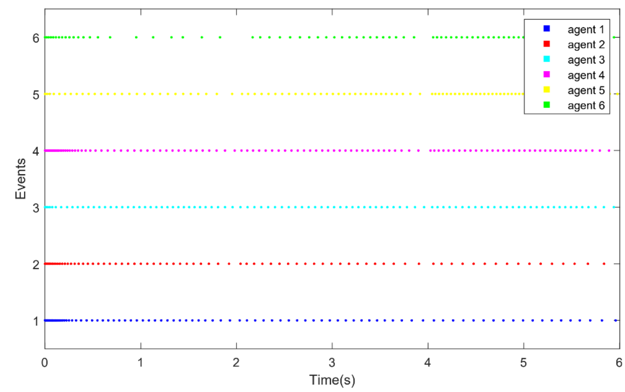

Fig. 7. Triggering instants in Example 2.

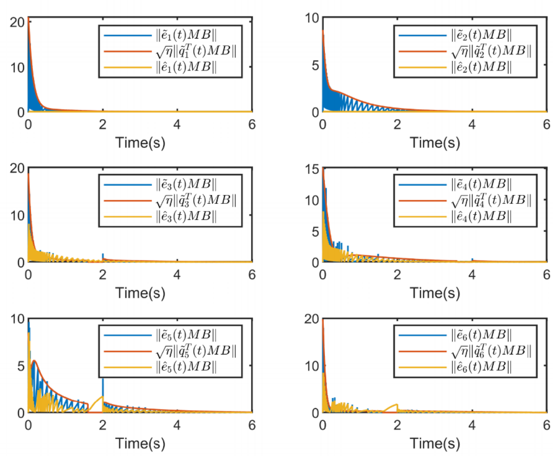

Fig. 6. The changes of $\|\tilde{e}_i^T(t)MB\|$, $\|\hat{e}_i^T(t)MB\|$, and $\sqrt{\eta}\|\tilde{q}_i^T(t)MB\|$ over time.

Table 1. Comparison of numbers of triggering times for different control methods.

Control method	**Agent 1**	**Agent 2**	**Agent 3**	**Agent 4**
From (2) and (3)	61	65	59	69
From [22]	96	77	96	112
Control method	**Agent 5**	**Agent 6**	**Total number**	**Mean time interval**
From (2) and (3)	54	63	371	0.0970 seconds
From [22]	107	80	568	0.0634 seconds

and attitude.

Remark 4: Theorem 1 demonstrates that when the attack duration rate does not exceed a specified positive threshold, using controller (2) enables achieving exponential stability with a convergence rate of ϱ^*. In (8), the stability of the matrix pair (A,B) ensures the feasibility of M, which in turn determines the value of K. To achieve a larger ϱ^* and increase the exponential convergence rate in (19), maximizing ϱ_1 in (8) is advisable. Due to $\frac{1}{\alpha}\eta r+\alpha \geq \sqrt{\eta r}$, we have $\imath < \mu - 2\sqrt{\eta r}$. In (3), choosing a smaller η reduces the constraint on $\imath$, thereby obtaining a larger ϱ_1 in (8) and consequently increasing the exponential convergence rate in (19). However, in (3), a smaller η will increase the number of triggering times, thereby increasing the communication burden. Therefore, it is necessary to choose an appropriate value for η to balance the trade-off between convergence rate and communication burden.

Remark 5: By adjusting the initial or unaffected communication network topology of MASs, the positive definiteness of Φ can be modified to enhance μ. Similarly, with DoS attacks, if the directed spanning tree is maintained, achieving larger ν values becomes feasible. According to Remark 4, setting α values near $\sqrt{\eta r}$ and $\sqrt{\eta s}$ alleviates constraints on $\imath$ and τ, thereby enabling higher values of $\imath$ and τ. Subsequently, according to the linear matrix inequalities (8) and (9), higher ϱ_1 and lower ϱ_2 can be obtained. Consequently, the condition on the attack duration rate $\frac{S_a}{t-s_0} \leq \frac{\varrho_1-\varrho^*}{\varrho_1+\varrho_2}$ can be more easily satisfied, thereby enhancing the resilience of MASs to attacks.

Example 2: This example employs the control strategy introduced in [22]: $u_i(t) = K\sum_{j\in N_i} a_{ij}\big(\hat{x}_i(t)-\hat{x}_j(t)\big)+d_i\big(\hat{x}_i(t)-\hat{x}_0(t)\big)$, with other relevant values drawn from Example 1. In this strategy, attack scenarios illustrated in Figs. 2 and 3 are considered, with $u_i(t)=0$ during $\sigma(t)=2,3,4$. Fig. 8 demonstrates that the control strategy outlined in [22] achieves secure consensus. Fig. 7 shows the triggering instants of this control strategy. By comparing Table 1, it can be concluded that the control method used in this paper triggers fewer times than the control strategy in [22], indicating that this method reduces unnecessary triggers and lowers communication overhead.

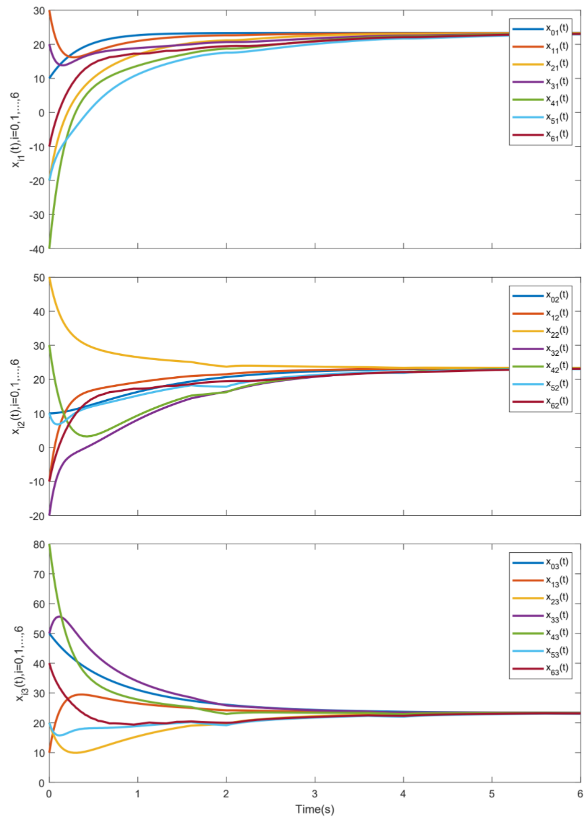

Fig. 8. State changes of $x_i(t)$ in Example 2.

5. CONCLUSION

This paper addresses the challenge of securing leader-following consensus in MASs amidst DoS attacks. We introduce an event-triggered scheme tailored for energy-limited networks under attack, reducing unnecessary information transmission. Additionally, we introduce the concept of effective DoS attack intervals and analyze attack duration and its impact. Secure consensus is demonstrated to be achievable through the utilization of the Lyapunov function and ADT method, with the elimination of Zeno behavior ensuring the feasibility of the event-triggered scheme. Simulation results confirm the effectiveness of our proposed update strategy and control protocol. Future work could further optimize the event-triggered update strategy to reduce computational burden, while exploring leaderless consensus control protocols to enhance the applicability of the proposed method.

CONFLICT OF INTEREST

The authors declare that there are no competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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