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Secure consensus of multiagent systems with DoS attacks via fully distributed dynamic event-triggered control

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Abstract—This article investigates the secure consensus problem of general linear multiagent systems with denial-of-service (DoS) attacks. Owing to the existence of DoS attacks, it is challenging to investigate the event-triggered control of multiagent systems in a fully distributed manner. This paper presents a novel dynamic event-triggered mechanism to alleviate the limited communication resources. Moreover, the designed mechanism is fully distributed and scalable owing to the introduction of some adaptive coupling weights. Since the DoS attacks have been considered in the event-triggered rule design, a novel adaptive parameter with an additional exponential term is adopted to deal with DoS attacks. The Lyapunov design also introduces such a term for the subsequent stability analysis. Sufficient conditions guaranteeing the asymptotic consensus of the studied system are developed in light of the controller's parameters, duration, and frequency of the DoS attacks. Strict proof is provided to demonstrate that a secure consensus can be reached effectively under the proposed control mechanism. Furthermore, it is shown that the Zeno behavior can be excluded from the designed triggering mechanism. Finally, simulations on a multiagent system consisting of interconnected unmanned intelligent vehicles are conducted to verify the results.

Index Terms—Multiagent systems, event-triggered mechanism, DoS attacks, consensus, fully distributed protocol

I. INTRODUCTION

The past two decades have witnessed rapidly growing in the field of cooperative control of multiagent systems due to its potential applications in many practical engineering systems [1]–[6]. As an essential topic of multiagent systems, plentiful pioneer results on the consensus problem have been obtained (see, for example, [7]–[16] and the references therein). To fulfill the consensus task, agents need to communicate with each other and eventually reach the same expected state. To overcome the drawback caused by continuous communication among agents, designing effective communication mechanism of the agents has received increasing attention.

Because of the bandwidth limitation of communication networks and popularization of embedded processors, the

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event-triggered mechanism has been developed as a discrete-sampling-based control method. Different from the time-triggered one, the controller updates in the event-triggered framework depend on some predefined events. This aperiodic triggering mechanism dramatically cuts down the number of controller updates on the premise of stability, thus saving communication resources. Therefore, the event-triggered mechanism has received great interest in the past decades. Such a mechanism has been introduced in [17] to solve the consensus problem for leaderless multiagent systems consisting of several single integrators. Then event-triggered results have been generalized to leaderless and leader-follower general linear multiagent systems in [18] and [19]. In [20], a dynamic triggering rule is presented to further cut down the triggering times and mitigates the communication burden for single systems by introducing a dynamic internal variable. The minimum inter-event time of such an event-based rule is larger than that of the static one. Such dynamic event-based rules are designed in [21]–[24] for leaderless and leader-follower multiagent systems.

In most works discussed above, global information for communication topology is used in the controller design process. For example, the knowledge of the Laplacian matrix is used in the triggering condition [17]. Its second smallest eigenvalue is used in [22], [23], and the number of agents is used in [21] to acquire the feedback gain matrix. Thus, those control laws can only be deemed as distributed rather than fully distributed. The entire network's global information is hard to acquire, especially for large-scale multiagent systems. Moreover, the controller that involves global information is not scalable in the case of time-varying topology. A fully distributed event-based control framework has recently received continuous attention to enhance the feasibility and scalability of the control strategy. In [25], the fully distributed event-triggered approach is achieved for the first time by introducing the adaptive coupling weights. Since then, fruitful results on fully distributed protocols have been designed [26]–[34]. In [26], multiagent systems with actuator failure are investigated under a fully distributed event-based approach. In [32], the authors develop a fully distributed protocol to resolve the bipartite formation tracking problem. A time-varying formation problem is investigated in [34], and the directed topology is considered in this paper. Due to their great adaptability, fully distributed protocols are applied to some engineering systems, like microgrids [28] and multi-robot systems [35].

The open setting of network communication medium in

most of the real-world multiagent systems brings a lot of security problems subject to cyber-attacks. DoS attack is one of the most common and effective cyber-attacks in multiagent systems, which can block the communication channels and worsen the system performance. It is thus crucial to consider the effects of DoS attacks in the process of controller design such that secure consensus can be achieved. However, global information is used in the majority of the existent works on secure consensus problems under the event-based framework. For example, leaderless and leader-follower multiagent systems with DoS attacks are investigated in [36], [37], and the event-based control laws are presented in these papers. However, to guarantee convergence, the second smallest eigenvalue of the Laplacian matrix is used to construct feedback gains and triggering conditions. Moreover, the number of agents is used in [38] to solve the output synchronization problem of heterogeneous multiagent systems with DoS attacks. The secure consensus of the multiple autonomous underwater vehicles is investigated in [39]. Though no global information is used in the control protocols, such information is required to limit the attack frequency and duration. Generally, such control protocols cannot be regarded as fully distributed. Thus, the absence of research that considers the secure consensus without using global information under DoS attacks motivates us to conduct this study.

In this article, a secure consensus problem for general linear multiagent systems with DoS attacks has been investigated under a fully distributed dynamic event-based control framework. Compared to the existent results, the difficulties and challenges of the above problem are as follows. Firstly, traditional adaptive coupling weights are usually designed in a secure and reliable communication environment. However, such adaptive parameters cannot solve the event-based consensus problem under insecure environments in a fully distributed way, and thus new adaptive parameters need to be developed. Secondly, owing to the introduction of the adaptive parameters, the traditional Lyapunov function is no longer applicable, and how to design an appropriate Lyapunov function for the studied consensus problem is another difficult issue. Thirdly, a positive exponential function like ae^{-bt} is usually introduced in most of the static event-triggered rules [25], [29] to avoid the Zeno behavior, which may bring difficulties in ensuring the exponential convergence of the Lyapunov function in the DoS attack case. Thus, a new event-triggered condition with the Zeno-free property is also required. Then, the main contributions can be generalized:

- 1) A novel adaptive law of coupling weights with the auxiliary exponential terms is designed to construct the fully distributed event-based control protocol. The communication environment is considered unreliable with DoS attacks, and the attack model is also proposed. A group of matrix inequalities without any usage of global information is given to obtain the feedback gain matrix.
- 2) A new event-triggered rule is developed over an unreliable communication network. It will be switched when the DoS attacks occur, such that the neighbors' state will not be used during attacked time intervals. Com-

pared with existing tracking results with DoS attacks for multiagent systems [36]–[38], no global information is used in the triggering function design. A dynamic internal variable is introduced to alleviate the limited communication bandwidth further. Proof of the Zeno-free property is also provided.

- 3) A well-designed Lyapunov function is provided to strictly demonstrate that secure asymptotic consensus can be reached under the developed control law. A sufficient quantitative relationship related to controller's parameters, the attack duration and the attack frequency is presented. At last, a simulation example based on unmanned intelligent vehicles is proposed to demonstrate the feasibility of the provided control law.

Some preliminaries, fundamental lemma, definitions, modeling of DoS attacks, and problem formulation are given in Section II. The fully distributed event-triggered mechanism designed for the multiagent systems under DoS attacks is given in Section III. A simulation example based on unmanned intelligent vehicles is conducted to demonstrate the credibility of attained results in Section IV. Finally, Section V concludes this paper.

Notations. \mathbb{R}^m denotes the set of m -dimensional real column vectors. Z_+^0 represents the set of positive integers. $\lambda_{\min}(H)$ and $\lambda_{\max}(H)$ are introduced to represent the minimum and maximum eigenvalues of a symmetric matrix H , respectively. Let H^T denote the transpose of matrix H . $\text{diag}\{d_1, \dots, d_N\}$ denotes a diagonal matrices with d_i being its diagonal entries. \otimes denotes the Kronecker production.

II. PRELIMINARIES AND PROBLEM STATEMENT

In the paper, we use $\mathcal{G} = \{\mathcal{Y}, \mathcal{E}\}$ to denote an undirected graph with $\mathcal{Y} = \{1, 2, \dots, N\}$ and $\mathcal{E} \subseteq \mathcal{Y} \times \mathcal{Y}$ being the agent set and edge set, respectively. Let $\mathcal{N}_i = \{j \in \mathcal{Y} : (i, j) \in \mathcal{E}, i \neq j\}$ denote the neighboring set of agent i . Agents i and j can exchange information with each other if $\{i, j\} \in \mathcal{E}$, where $\{i, j\}$ denotes an edge of graph \mathcal{G} . Adjacency matrix of \mathcal{G} is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, where $a_{ii} = 0$, $a_{ij} = 1$, iff $\{i, j\} \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. We also define a diagonal matrix $\mathcal{D} = \text{diag}\{d_1, \dots, d_N\}$ with the diagonal element being $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. Then, the Laplacian matrix can be defined as $L = \mathcal{D} - \mathcal{A}$. The smallest nonzero eigenvalue of L is denoted as λ_2 . If there exists a path between every vertex i and j in graph \mathcal{G} , then \mathcal{G} is said to be connected.

The general linear multiagent system consisting of N agents is considered in this paper, whose dynamics are:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i \in \mathcal{Y}, \quad (1)$$

with $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m, i = 1, \dots, N$ being the states and control law for each agent, respectively. A and B are system and input matrices with compatible dimensions, respectively. Our study is based on the following two standard assumptions:

Assumption 1: Matrix pair (A, B) is stabilizable.

Assumption 2: The topology graph \mathcal{G} is undirected and connected.

Different from most existing relative works, unreliable networks with DoS attacks are considered in this article. Denote

$\{\mathbb{A}_p\}, p \in \mathbb{Z}_+^0$ as the time sequence when the attackers launch the DoS attacks. We define $\tau_p, p \in \mathbb{Z}_+^0$ as the duration time of DoS attacks, which jointly determines how long the attacks affect the network. Then, we can conclude that during span $\mathbb{I}_p = [\mathbb{A}_p, \mathbb{A}_p + \tau_p]$, the information exchange among agents will be blocked. We also define $\mathbb{U}(t)$ and $\mathbb{R}(t)$ as the time period during which an attack occurs or not over $[t_0, t]$ with $\mathbb{U}(t) = \cup_{p \in \mathbb{Z}_+^0} [\mathbb{A}_p, \mathbb{A}_p + \tau_p]$ and $\mathbb{R}(t) = [t_0, t] \setminus \mathbb{U}(t)$, respectively. Here, t_0 is the initial time.

Similar to [36], definitions of the constraints of frequency and duration time of DoS attacks are given as follows.

Definition 1: (Attack Frequency). For any given $t > t_0 \geq 0$, we define $n(t_0, t)$ as the total number of attacks that happened in communication networks during $[t_0, t]$. And the attack frequency $n_0 > 0$ is defined as

$$n_0 = \frac{n(t_0, t)}{(t - t_0)}.$$

Definition 2: (Attack Duration). Define $|\mathbb{U}(t)|$ as the total duration of DoS attacks during time interval $[t_0, t]$. And there exist an attack duration $\tau_0 > 0$ and a positive scalar ς such that

$$|\mathbb{U}(t)| \leq \varsigma + \tau_0(t - t_0),$$

where $t \geq t_0 \geq 0$.

Based on the above definition, the launched instant of the p -th attack is denoted as \mathbb{A}_p , and its duration is τ_p . Then we denote $\{t_k^i, t_{k+1}^i, \dots, t_{k+m}^i\}$ as the triggering instants of agent i suffering from attacks during the time intervals $[\mathbb{A}_p, \mathbb{A}_p + \tau_p]$. Over this period of time, the controller of agent i will be affected by DoS attacks, and such an effect lasts until the next triggering instant t_{k+m+1}^i . Thus, an agent's affected period is always longer than a successful attack period. To distinguish them, let Δ_p^i be the time interval from $\mathbb{A}_p + \tau_p$ to t_{k+m+1}^i and $\tilde{\mathbb{I}}_p = [\mathbb{A}_p, \mathbb{A}_p + \tau_p + \Delta]$ be the p -th affected period resulted by the p -th attack, where Δ is the maximum value of Δ_p^i . And we also define $\tilde{\mathbb{U}}(t) = \bigcup_{p \in \mathbb{Z}_+^0} \tilde{\mathbb{I}}_p \cap [t_0, t]$ and $\tilde{\mathbb{R}}(t) = [t_0, t] \setminus \tilde{\mathbb{U}}(t)$.

Now, we introduce the standard definition of secure asymptotic consensus for the studied multiagent system (1).

Definition 3: The secure asymptotic consensus problem of system (1) with DoS attacks is said to be resolved successfully as long as for any initial states, we have

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \quad i, j \in \mathcal{Y}. \quad (2)$$

The objectives of this article are twofold: a) develop a novel event-based protocols for multiagent systems (1) in a fully distributed framework; b) guarantee the Zeno-free property of the proposed event-triggered condition.

III. FULLY DISTRIBUTED EVENT-TRIGGERED MECHANISM DESIGN

To defend against DoS attacks, the control input $u_i(t)$ is developed with

$$u_i(t) = -K \sum_{j \in \mathcal{Y}} c_{ij}(t) a_{ij} y_{ij}(t). \quad (3)$$

$y_{ij}(t)$ is defined as

$$y_{ij}(t) = \begin{cases} \hat{x}_i(t) - \hat{x}_j(t) & t \in \mathbb{R}(t) \\ 0 & t \in \mathbb{U}(t), \end{cases} \quad (4)$$

where $\hat{x}_i(t) = e^{A(t-t_k^i)} x_i(t_k^i)$ is the estimated value of $x_i(t)$ and the triggering instant $t_k^i = \max \{t_s^i : t_s^i \leq t\}$. Here, K is the feedback gain matrix and is given as $K = B^T F$, with $F > 0$ being the solution of the following matrix inequalities

$$A^T F + FA - FBB^T F + \alpha I \leq 0 \quad (5)$$

and

$$A^T F + FA - \beta I \leq 0. \quad (6)$$

For simplicity, in what follows, we will denote $\mathbb{Q} = FBB^T F$. In addition, $c_{ij}(t)$ are adaptive coupling weights satisfying

$$\dot{c}_{ij}(t) = e^{\rho_1 t} \pi_{ij} a_{ij} y_{ij}(t)^T \mathbb{Q} y_{ij}(t), \quad (7)$$

where π_{ij} can be any positive constants, the initial value $c_{ij}(0) = c_{ji}(0) \geq 0$ and $\rho_1 > 0$ will be defined later. Then, the definitions of the consensus error and measurement error are given:

$$\delta_i(t) = x_i(t) - \frac{1}{N} \sum_{j \in \mathcal{Y}} x_j(t) \quad (8)$$

and

$$e_i(t) = \hat{x}_i(t) - x_i(t), \quad t \in [t_k^i, t_{k+1}^i], \quad (9)$$

respectively. In this article, the following triggering rule is developed

$$t_{k+1}^i = \inf \left\{ t > t_k^i \mid f_i(t) \geq 0 \right\}. \quad (10)$$

The triggering function for each agent is defined as follows

$$f_i(t) = \beta_i \left(\sum_{j \in \mathcal{Y}} a_{ij} c_{ij}(t) e_i^T(t) \mathbb{Q} e_i(t) - \theta_i \sum_{j \in \mathcal{Y}} a_{ij} y_{ij}^T(t) \mathbb{Q} y_{ij}(t) \right) - \Upsilon_i(t), \quad (11)$$

where β_i and θ_i are positive scalars, and $\Upsilon_i(t)$ satisfies

$$\dot{\Upsilon}_i(t) = -\eta_i \Upsilon_i(t) - \alpha_i \left\{ \sum_{j \in \mathcal{Y}} c_{ij}(t) a_{ij} e_i^T(t) \mathbb{Q} e_i(t) - \theta_i \sum_{j \in \mathcal{Y}} a_{ij} y_{ij}^T(t) \mathbb{Q} y_{ij}(t) \right\} \quad (12)$$

with $\Upsilon_i(t_0) > 0, 0 < \alpha_i < 1$ and $\eta_i > 0$. Let $\kappa_i = \eta_i - \frac{1-\alpha_i}{\beta_i}$.

Now, we provide a lemma below, which will be used for stability analysis.

Lemma 1: For any given $\Upsilon_i(t_0) > 0, 0 < \alpha_i < 1$ and $\eta_i > 0$, the dynamic internal variable $\Upsilon_i(t)$ satisfies

$$\Upsilon_i(t) > 0, \quad i \in \mathcal{Y}. \quad (13)$$

Proof: According to (10)-(11), one has

$$\sum_{j \in \mathcal{Y}} a_{ij} c_{ij}(t) e_i^T(t) \mathbb{Q} e_i(t) - \theta_i \sum_{j \in \mathcal{Y}} a_{ij} y_{ij}^T(t) \mathbb{Q} y_{ij}(t) < \frac{\Upsilon_i(t)}{\beta_i}. \quad (14)$$

Based on the dynamics of $\Upsilon_i(t)$ in (12), one has that

$$-\frac{1}{\alpha_i} \dot{\Upsilon}_i(t) - \frac{\eta_i}{\alpha_i} \Upsilon_i(t) \leq \frac{\Upsilon_i(t)}{\beta_i}. \quad (15)$$

Then, we have

$$\dot{\Upsilon}_i(t) \geq -\frac{\alpha_i}{\beta_i} \Upsilon_i(t) - \eta_i \Upsilon_i(t). \quad (16)$$

Note that $\Upsilon_i(t_0) > 0$, $0 < \alpha_i < 1$, $\beta_i > 0$ and $\eta_i > 0$, according to the comparison lemma in [40], one gets $\Upsilon_i(t) \geq 0, \forall t > 0, i \in \mathcal{Y}$.

Remark 1: Plentiful results have been developed for the event-based consensus problem of multiagent systems with DoS attacks. However, global information like the smallest nonzero eigenvalue of L or the number of agents is used in [36]–[38], [41], respectively. Moreover, in [38], the next triggering instant when $t \in \mathbb{U}(t)$ is decided by the maximum attack duration π , which has to be detected or estimated. However, such accurate information about DoS attacks is difficult to be obtained. In [37], a positive constant θ is introduced to determine the next triggering instant, which can be seen as a time-triggered approach. However, how to design an appropriate θ is a difficult thing. If θ is large, a secure asymptotic consensus cannot be achieved, and if θ is too small, many invalid triggers will happen. In this paper, by setting $y_{ij}(t)$ to zero, a switched triggering condition for each agent when DoS attacks occur is designed without using π or any neighbors' information.

Now, we will introduce the main results of this article in the following theorem.

Theorem 1: If Assumptions 1-2 hold and $\eta_i > \frac{1-\alpha_i}{\beta_i}$ in (11)-(12), a secure asymptotic consensus of multiagent system (1) can be solved under fully distributed adaptive control law (3) and dynamic triggering condition (10)-(11) under the following condition

$$\tau_0 + \Delta n_0 < \frac{\rho_1}{\rho_2 + \rho_1}, \quad (17)$$

where $\rho_1 = \min \left\{ \frac{\alpha}{2\lambda_{\max}(F)}, \frac{\kappa_i}{2} \right\}$, $\rho_2 = \frac{\beta}{2\lambda_{\min}(F)}$. Moreover, Zeno behaviour of the close-looped system can be excluded.

Proof: Inspired by [35], for stability analysis, a Lyapunov function candidate is chosen as

$$W(t) = e^{\rho_1 t} V_1(t) + V_2(t), \quad (18)$$

with

$$V_1(t) = \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i(t)^T F \delta_i(t) + \sum_{i \in \mathcal{Y}} \Upsilon_i(t), \quad (19)$$

$$V_2(t) = \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \frac{(c_{ij}(t) - \varphi)^2}{8\pi_{ij}}, \quad (20)$$

where $\varphi > 0$ is a scalar that will be determined later. Therefore, the time derivative of $W(t)$ is

$$\dot{W}(t) = \dot{V}(t) + \rho_1 e^{\rho_1 t} V_1(t), \quad (21)$$

where $\dot{V}(t) = e^{\rho_1 t} \dot{V}_1(t) + \dot{V}_2(t)$. The following proof will be divided into two cases. For the simplicity purpose, the time t will be omitted hereafter unless there is a specification.

Case 1: when the studied multiagent system is free of attacks, we have

$$\dot{\delta}_i = A\delta_i + BK \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} \hat{\zeta}_{ji} \quad (22)$$

and

$$\begin{aligned} \dot{V} = & \left\{ \sum_{i \in \mathcal{Y}} \delta_i^T F \left[A\delta_i + BK \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} \hat{\zeta}_{ji} \right] \right. \\ & + \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \frac{c_{ij} - \varphi}{4} \hat{\zeta}_{ij}^T Q \hat{\zeta}_{ij} \\ & - \sum_{i \in \mathcal{Y}} \eta_i \Upsilon_i - \sum_{i \in \mathcal{Y}} \alpha_i \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T Q e_i \\ & \left. + \sum_{i \in \mathcal{Y}} \alpha_i \theta_i \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T Q \hat{\zeta}_{ij} \right\} e^{\rho_1 t}, \end{aligned} \quad (23)$$

where $\hat{\zeta}_{ij} = \hat{x}_i - \hat{x}_j$. According to the fact that $e^{\rho_1 t} > 0$, one gets

$$\begin{aligned} e^{-\rho_1 t} \dot{V} = & \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA) \delta_i - \sum_{i \in \mathcal{Y}} \eta_i \Upsilon_i \\ & + \sum_{i \in \mathcal{Y}} \delta_i^T FBK \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} \hat{\zeta}_{ji} \\ & + \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \frac{c_{ij} - \varphi}{4} \hat{\zeta}_{ij}^T Q \hat{\zeta}_{ij} \\ & - \sum_{i \in \mathcal{Y}} \alpha_i \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T Q e_i \\ & + \sum_{i \in \mathcal{Y}} \alpha_i \theta_i \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T Q \hat{\zeta}_{ij}. \end{aligned} \quad (24)$$

By the definition of c_{ij} and $c_{ij}(0) = c_{ji}(0)$, we can easily get that $c_{ij} = c_{ji}$. The studied communication graph is undirected, so we also have $a_{ij} = a_{ji}$. Thus,

$$\sum_{i \in \mathcal{Y}} \delta_i^T FBK \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} \hat{\zeta}_{ji} = -\frac{1}{2} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} [\delta_i - \delta_j]^T Q \hat{\zeta}_{ji}. \quad (25)$$

Substituting (25) into (24) yields that

$$\begin{aligned} e^{-\rho_1 t} \dot{V} = & \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA) \delta_i - \sum_{i \in \mathcal{Y}} \eta_i \Upsilon_i \\ & - \frac{1}{2} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} [\delta_i - \delta_j]^T Q \hat{\zeta}_{ij} \\ & + \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \frac{c_{ij} - \varphi}{4} \hat{\zeta}_{ij}^T Q \hat{\zeta}_{ij} \\ & - \sum_{i \in \mathcal{Y}} \alpha_i \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T Q e_i + \sum_{i \in \mathcal{Y}} \alpha_i \theta_i \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T Q \hat{\zeta}_{ij}. \end{aligned} \quad (26)$$

According to the definitions of the two error variables δ_i and e_i in (8)-(9), denoting $\zeta_{ij} = x_i - x_j$, we can easily get $\zeta_{ij} = \delta_i - \delta_j = \hat{\zeta}_{ij} - e_i + e_j$.

Therefore, one can further achieve that

$$\begin{aligned} e^{-\rho_1 t} \dot{V} &= \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA) \delta_i - \sum_{i \in \mathcal{Y}} \eta_i \Upsilon_i \\ &\quad + \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \left(\alpha_i \theta_i + \frac{c_{ij} - \varphi}{4} \right) \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\quad - \sum_{i \in \mathcal{Y}} \alpha_i \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T \mathbb{Q} e_i \\ &\quad + \frac{1}{2} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} [e_i - e_j]^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\quad - \frac{1}{2} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij}. \end{aligned} \quad (27)$$

Moreover, it holds that

$$[e_i - e_j]^T \mathbb{Q} \hat{\zeta}_{ij} \leq \frac{1}{2} [e_i - e_j]^T \mathbb{Q} [e_i - e_j] + \frac{1}{2} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij}. \quad (28)$$

Clearly, one has that

$$\begin{aligned} &\sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} [e_i - e_j]^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\leq \frac{1}{2} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} + 2 \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T \mathbb{Q} e_i. \end{aligned} \quad (29)$$

Substituting (29) into (27), one gets

$$\begin{aligned} e^{-\rho_1 t} \dot{V} &\leq \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA) \delta_i - \sum_{i \in \mathcal{Y}} \eta_i \Upsilon_i \\ &\quad + \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \left(\alpha_i \theta_i + \frac{c_{ij} - \varphi}{4} \right) \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\quad - \sum_{i \in \mathcal{Y}} \alpha_i \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T \mathbb{Q} e_i + \frac{1}{4} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\quad + \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T \mathbb{Q} e_i - \frac{1}{2} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij}. \end{aligned} \quad (30)$$

The above equality (30) can then be further expressed as

$$\begin{aligned} e^{-\rho_1 t} \dot{V} &\leq \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA) \delta_i - \sum_{i \in \mathcal{Y}} \eta_i \Upsilon_i \\ &\quad - \sum_{i \in \mathcal{Y}} \left(\frac{\varphi}{4} - \theta_i \right) \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\quad - \sum_{i \in \mathcal{Y}} (1 - \alpha_i) \theta_i \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\quad + \sum_{i \in \mathcal{Y}} (1 - \alpha_i) \sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T \mathbb{Q} e_i. \end{aligned} \quad (31)$$

According to (10) and (11), we have

$$\beta_i \left[\sum_{j \in \mathcal{Y}} c_{ij} a_{ij} e_i^T \mathbb{Q} e_i - \theta_i \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \right] \leq \Upsilon_i. \quad (32)$$

Substituting (32) into (31) yields

$$\begin{aligned} e^{-\rho_1 t} \dot{V} &\leq \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA) \delta_i - \sum_{i \in \mathcal{Y}} \kappa_i \Upsilon_i \\ &\quad - \left(\frac{\varphi}{4} - \theta \right) \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij}, \end{aligned} \quad (33)$$

where $\kappa_i = \eta_i - \frac{1-\alpha_i}{\beta_i}$ and $\theta = \max \{\theta_i\}$. We can check that

$$\begin{aligned} &\sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\leq 2 \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} + 2 \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} [e_i - e_j]^T \mathbb{Q} [e_i - e_j] \\ &\leq 2 \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} + 8 \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} e_i^T \mathbb{Q} e_i. \end{aligned} \quad (34)$$

By (7), we can get that $\dot{c}_{ij} \geq 0$, which means $c_{ij} \geq c_{ij}(0)$ for any time $t \geq 0$. Let $\tilde{c} = \min \{c_{ij}(0)\}$. Then, we have

$$\tilde{c} \sum_{j \in \mathcal{Y}} a_{ij} e_i^T \mathbb{Q} e_i - \theta_i \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \leq \frac{\Upsilon_i}{\beta_i} \quad (35)$$

and

$$\begin{aligned} &\sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \\ &\leq 2 \sum_{i \in \mathcal{Y}} \left(1 + \frac{4\theta_i}{\tilde{c}} \right) \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} + \sum_{i \in \mathcal{Y}} \frac{8}{\beta_i \tilde{c}} \Upsilon_i. \end{aligned} \quad (36)$$

Define $q_0 = \max \left\{ 2 + \frac{8\theta}{\tilde{c}}, \frac{8(\frac{\varphi}{4} - \theta)}{\beta \tilde{c} q_1} \right\}$ with $\tilde{\beta} = \min \{\beta_i\}$ and $q_1 < \frac{1}{2} \left(\eta_i - \frac{1-\alpha_i}{\beta_i} \right)$, we can easily get

$$\sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \leq q_0 \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} + \frac{q_0 q_1}{\frac{\varphi}{4} - \theta} \sum_{i \in \mathcal{Y}} \Upsilon_i \quad (37)$$

which further implies

$$\sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} \geq \frac{1}{q_0} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} \hat{\zeta}_{ij}^T \mathbb{Q} \hat{\zeta}_{ij} - \frac{q_1}{\frac{\varphi}{4} - \theta} \sum_{i \in \mathcal{Y}} \Upsilon_i. \quad (38)$$

Substituting (38) into (33), one gets

$$\begin{aligned} e^{-\rho_1 t} \dot{V} &\leq \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA) \delta_i - \sum_{i \in \mathcal{Y}} \frac{\kappa_i}{2} \Upsilon_i \\ &\quad - \left(\frac{\varphi}{4} - \theta \right) \frac{1}{q_0} \sum_{i \in \mathcal{Y}} \sum_{j \in \mathcal{Y}} a_{ij} [\delta_i - \delta_j]^T \mathbb{Q} [\delta_i - \delta_j]. \end{aligned} \quad (39)$$

Denote $\delta = [\delta_1^T, \delta_2^T, \dots, \delta_N^T]^T$, so we can arrive at

$$\begin{aligned} e^{-\rho_1 t} \dot{V} &\leq \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA) \delta_i - \sum_{i \in \mathcal{Y}} \frac{\kappa_i}{2} \Upsilon_i \\ &\quad - \left(\frac{\varphi}{2} - 2\theta \right) \frac{1}{q_0} \delta^T (L \otimes FBB^T F) \delta \\ &\leq \frac{1}{2} \sum_{i \in \mathcal{Y}} \delta_i^T (A^T F + FA - \hat{\lambda} FBB^T F) \delta_i - \sum_{i \in \mathcal{Y}} \frac{\kappa_i}{2} \Upsilon_i \end{aligned} \quad (40)$$

where $\hat{\lambda} = \frac{(\varphi-4\theta)\lambda_2}{q_0}$. Choose a sufficient large φ such that $\varphi > \frac{q_0}{\lambda_2} + 4\theta$ and thus $\hat{\lambda} > 1$. By using matrix inequality (5) and definition of ρ_1 in Theorem 1, one can conclude that

$$\dot{V} \leq -e^{\rho_1 t} \rho_1 V_1, \quad (41)$$

which further shows that

$$\dot{W} = \dot{V} + \rho_1 e^{\rho_1 t} V_1 \leq 0. \quad (42)$$

For any time interval $[T_k, t) \subset \mathbb{R}(t_0, t)$, one can obtain that

$$W \leq W(T_k), \quad (43)$$

which implies that $e^{\rho_1 t} V_1 \leq W \leq W(T_k)$. Thus, we can finally conclude that

$$V_1 \leq e^{-\rho_1(t-T_k)} W(T_k). \quad (44)$$

Case 2: when the communication channel is suffering from attacks, then $y_{ij} = 0$ according to (4), we have

$$\begin{aligned} \dot{V}_1 &= \sum_{i \in \mathbb{Y}} \delta_i^T F A \delta_i + \sum_{i \in \mathbb{Y}} \dot{\Upsilon}_i \\ &\leq \sum_{i \in \mathbb{Y}} \delta_i^T F A \delta_i - \sum_{i \in \mathbb{Y}} \eta_i \Upsilon_i - \alpha_i \sum_{i \in \mathbb{Y}} \sum_{j \in \mathbb{Y}} c_{ij} a_{ij} e_i^T Q e_i \\ &\quad + \sum_{i \in \mathbb{Y}} \sum_{j \in \mathbb{Y}} c_{ij} a_{ij} e_i^T Q e_i. \end{aligned} \quad (45)$$

And we can easily check that $\dot{V}_2 = 0$ due to $\dot{c}_{ij} = 0$. Therefore,

$$\dot{V} \leq e^{\rho_1 t} \sum_{i \in \mathbb{Y}} \delta_i^T F A \delta_i - e^{\rho_1 t} \kappa_i \sum_{i \in \mathbb{Y}} \Upsilon_i. \quad (46)$$

Since $\kappa_i > 0$ and $\rho_2 = \frac{\beta}{2\lambda_{\min}(F)}$, according to (6), we have

$$\dot{V} \leq e^{\rho_1 t} \rho_2 V_1. \quad (47)$$

Therefore,

$$\dot{W} = \dot{V} + \rho_1 e^{\rho_1 t} V_1 \leq e^{\rho_1 t} (\rho_1 + \rho_2) V_1 \leq (\rho_1 + \rho_2) W. \quad (48)$$

Thus, we can obtain that

$$W \leq e^{(\rho_1 + \rho_2)(t-T_{k'})} W(T_{k'}). \quad (49)$$

for any time interval $[T_{k'}, t) \subset \mathbb{U}(t)$. Since $e^{\rho_1 t} V_1 \leq W$, we have

$$V_1 \leq e^{\rho_2(t-T_{k'})} W(T_{k'}). \quad (50)$$

Remark 2: The stability analysis is divided into two cases when $t \in \mathbb{R}(t)$ and $t \in \mathbb{U}(t)$. Note that the conclusion in (44) and (50) is still accessible when the time interval change to $\tilde{\mathbb{R}}(t)$ and $\tilde{\mathbb{U}}(t)$. We can find that $\tilde{\mathbb{R}}(t)$ is a subset of $\mathbb{R}(t)$, so that (44) is still true. In such a case, $\mathbb{U}(t)$ is expanded to $\tilde{\mathbb{U}}(t)$, but (44) is a sufficient condition of (50). Therefore, (44) and (50) are still accessible when $t \in \tilde{\mathbb{R}}(t)$ and $t \in \tilde{\mathbb{U}}(t)$, respectively.

Then, we will consider V_1 over $[t_0, t]$. Note that (44) is satisfied when $t \in \tilde{\mathbb{R}}(t)$, while (50) is satisfied when $t \in \tilde{\mathbb{U}}(t)$ due to the above analysis. Without loss of generality, consider $t \in [\mathbb{A}_p + \tau_p + \Delta, \mathbb{A}_{p+1})$. We have

$$\begin{aligned} V_1 &\leq e^{-\rho_1[t-(\mathbb{A}_p+\tau_p+\Delta)]} W(\mathbb{A}_p + \tau_p + \Delta) \\ &\leq e^{-\rho_1[t-(\mathbb{A}_p+\tau_p+\Delta)]} e^{\rho_2(\mathbb{A}_p+\tau_p+\Delta-\mathbb{A}_p)} W(\mathbb{A}_p) \\ &\quad \dots \\ &\leq e^{-\rho_1(t-t_0-|\tilde{\mathbb{U}}(t)|)} e^{\rho_2|\tilde{\mathbb{U}}(t)|} W(t_0) \end{aligned} \quad (51)$$

and for $t \in [\mathbb{A}_p, \mathbb{A}_p + \tau_p + \Delta]$, one has

$$\begin{aligned} V_1 &\leq e^{\rho_2(t-\mathbb{A}_p)} W(\mathbb{A}_p) \\ &\leq e^{\rho_2(t-\mathbb{A}_p)} e^{-\rho_1[\mathbb{A}_p-(\mathbb{A}_{p-1}+\tau_{p-1}+\Delta)]} W(\mathbb{A}_{p-1} + \tau_{p-1} + \Delta) \\ &\quad \dots \\ &\leq e^{\rho_2|\tilde{\mathbb{U}}(t)|} e^{\rho_1(t-t_0-|\tilde{\mathbb{U}}(t)|)} W(t_0) \end{aligned} \quad (52)$$

Thus, we have

$$V_1 \leq e^{-\rho_1(t-t_0-|\tilde{\mathbb{U}}(t)|)} e^{\rho_2|\tilde{\mathbb{U}}(t)|} W(t_0) \quad \forall t \geq t_0 \quad (53)$$

From the definition of Δ and $\tilde{\mathbb{I}}_p = \mathbb{I}_p + \Delta$, we have

$$|\tilde{\mathbb{U}}(t)| = |\mathbb{U}(t)| + \Delta n(t_0, t) \quad (54)$$

Substituting (54) into (53) and using Definitions 1-2, we can obtain that

$$\begin{aligned} V_1 &\leq W(t_0) e^{-\rho_1(t-t_0-|\tilde{\mathbb{U}}(t)|)+\rho_2|\tilde{\mathbb{U}}(t)|} \\ &\leq W(t_0) e^{-\rho_1(t-t_0)} e^{(\rho_1+\rho_2)|\tilde{\mathbb{U}}(t)|} \\ &= W(t_0) e^{-\rho_1(t-t_0)} e^{(\rho_1+\rho_2)[|\mathbb{U}(t)|+\Delta n(t_0, t)]} \\ &\leq W(t_0) e^{-\rho_1(t-t_0)} e^{(\rho_1+\rho_2)[\varsigma+\tau_0(t-t_0)+\Delta n_0(t-t_0)]} \\ &= W(t_0) e^{(\rho_1+\rho_2)e^{[(\rho_1+\rho_2)(\tau_0+\Delta n_0)-\rho_1](t-t_0)}} \end{aligned} \quad (55)$$

According to (17), we know that $(\tau_0+\Delta n_0)(\rho_1+\rho_2)-\rho_1 < 0$. Thus, we can conclude that $V_1 \rightarrow 0$ when $t \rightarrow \infty$, implying that the asymptotic consensus of the multiagent system is achieved.

Remark 3: Note that in some existing works, such as [42], asymptotic exponential convergence of W is always considered. However, such a traditional consideration is difficult to be realized in a fully distributed way. Although in [42], proof has been fulfilled by discarding variable φ , a strict sufficient condition is required for the system and input matrix in (1). In this article, we only consider the convergence of V_1 , which also implies convergence of consensus error δ_i .

After that, we will prove the Zeno-free property of the closed-looped system under the designed triggering rule (10) and (11). Firstly, according to (11) and (12), one has

$$\Upsilon_i \geq \Upsilon_i(t_0) \exp \left\{ - \left(\eta_i + \frac{\alpha_i}{\beta_i} \right) (t - t_0) \right\}. \quad (56)$$

Since the asymptotic consensus of the multiagent system (1) is achieved and c_{ij} , Υ_i and x_i is continuous corresponding to t , we define positive scalars $\hat{c} > 0$, s.t. $|c_{ij}| \leq \hat{c}$. Moreover, $\|\Upsilon_i\|$ and $\|\zeta_{ij}\|$ are also bounded. The evolution of e_i can be obtained as

$$\dot{e}_i = A e_i - \sum_{j \in \mathbb{Y}} a_{ij} c_{ij} B K y_{ij}. \quad (57)$$

The time derivative of $\|e_i\|$ satisfies

$$\begin{aligned} \frac{d \|e_i\|}{dt} &= \frac{e_i^T}{\|e_i\|} \dot{e}_i \leq \|\dot{e}_i\| \\ &\leq \|A\| \|e_i\| + \sum_{j \in \mathbb{Y}} a_{ij} c_{ij} \|B K\| \|y_{ij}\| \\ &\leq \|A\| \|e_i\| + \varrho_i, \end{aligned} \quad (58)$$

where ϱ_i is the upper bound of $\sum_{j \in \mathcal{Y}} a_{ij} c_{ij} \|BK\| \|y_{ij}\|$. Consider a function $\phi(t)$ which is non-negative and satisfies the following equation

$$\dot{\phi} = \|A\|\phi + \varrho_i, \quad \phi(0) = \|e_i(t_k^i)\| = 0. \quad (59)$$

Then, we can conclude that $\|e_i\| \leq \phi(t - t_k^i)$, where

$$\phi = \frac{\varrho_i}{\|A\|} \left(e^{\|A\|t} - 1 \right) \quad (60)$$

is the solution to (59). Assume that the initial time is $t_0 = 0$. Then, a sufficient condition for (10)-(11) is

$$\|e_i\|^2 \leq \frac{\Upsilon_i(0) \exp \left\{ - \left(\eta_i + \frac{\alpha_i}{\beta_i} \right) t \right\}}{\beta_i \hat{c} \|\mathbb{Q}\| d_i}. \quad (61)$$

Let us define the lower bound of $t_{k+1}^i - t_k^i$ as ζ , and clearly ζ is the time span for ϕ evolving from 0 to right-side-hand of (61). Therefore, ζ satisfies the following inequality:

$$\frac{\varrho_i^2}{\|A\|^2} \left(e^{\|A\|\zeta} - 1 \right)^2 \geq \frac{\Upsilon_i(0) \exp \left\{ - \left(\eta_i + \frac{\alpha_i}{\beta_i} \right) (t_k^i + \zeta) \right\}}{\beta_i \hat{c} \|\mathbb{Q}\| d_i} \quad (62)$$

So we can conclude that

$$\begin{aligned} t_{k+1}^i - t_k^i &\geq \zeta \\ &\geq \frac{1}{\|A\|} \ln \left(1 + \frac{\|A\|}{\varrho_i} \sqrt{\frac{\Upsilon_i(0) \exp \left\{ - \left(\eta_i + \frac{\alpha_i}{\beta_i} \right) (t_k^i + \zeta) \right\}}{\beta_i \hat{c} \|\mathbb{Q}\| d_i}} \right) \end{aligned} \quad (63)$$

Since the lower bound of ζ in (63) approaches zero only when $t \rightarrow \infty$. Then we can conclude that ζ always exists and is strictly positive in any limited time. Thus, the Zeno behaviour is avoided.

Remark 4: The main challenge of our work is that the fully distributed event-triggered analysis under the DoS attacks framework is considered. Note that most of the existent results of fully distributed event-triggered controller design in [25]–[31], [33] cannot be easily generalized in unreliable communication networks. To overcome the limitation of traditional design of fully distributed protocols, a novel adaptive law of parameter c_{ij} and a delicately designed Lyapunov function with additional exponential term $e^{\rho_1 t}$ is provided in this article. Strict proof is provided to demonstrate that the secure consensus can be achieved.

Remark 5: Compared with [35], estimators are used to estimating the states of agents, which brings difficulties for the stability analysis. Two advantages of introducing this scheme have been concluded in [36]: a) continuous communication in both event-triggered rule (10)-(11) and control protocol (3) can be avoided; b) by using model-based estimators, the system matrix A in (1) is permitted to be unstable. New decreasing rate ρ_1 and increasing rate ρ_2 of V_1 , which is different from [35], are presented in this paper because of the new control law. Moreover, the usage of estimators leads to a new form of measurement error e_i , which further needs to construct a new trigger function. Hence, a different approach to exclude Zeno behaviour and a novel minimum inter-event time is provided

in this article. It is also worth to mention that if the dynamics of multiagent systems (1) degenerate to several integrators, the event-based mechanism in [35] can be covered by our results.

IV. SIMULATION EXAMPLE

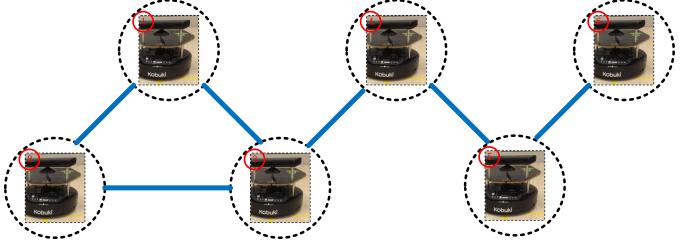


Fig. 1. Topology structure of the studied multiagent system.

In this section, a multiagent system consisting of a group of unmanned intelligent vehicles is presented to prove the credibility of the designed control protocol.

Here, six agents are considered, and the topology of the communication network among intelligent vehicles is portrayed in Fig. 1. According to [43], each intelligent vehicle can be modeled by

$$\begin{cases} \dot{C}_i^x(t) = v_i(t) \cos(\Theta_i(t)) \\ \dot{C}_i^y(t) = v_i(t) \sin(\Theta_i(t)) \\ \dot{\Theta}_i(t) = \omega_i(t) \end{cases} \quad (64)$$

where $C_i^x(t)$ and $C_i^y(t)$ are the horizontal and vertical coordinates of vehicles' centroid, respectively. $\Theta_i(t)$ is the heading angle; $v_i(t)$ is the linear velocity and $\omega_i(t)$ is angular velocity. By applying the dynamic feedback linearization method described in [44], unmanned intelligent vehicle model (64) can be transformed into (1) with

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

We consider an insecure communication environment under DoS attack with $\tau_0 = 0.0375$ and $n_0 = 0.375$. By solving the matrix inequalities (5)-(6) and choosing $\alpha = 1, \beta = 3$ we have

$$F = \begin{pmatrix} 1.7321 & 1 & 0 & 0 \\ 1 & 1.7321 & 0 & 0 \\ 0 & 0 & 1.7321 & 1 \\ 0 & 0 & 1 & 1.7321 \end{pmatrix}.$$

By using $K = B^T F$ we can further calculate

$$K = \begin{pmatrix} 1 & 1.7321 & 0 & 0 \\ 0 & 0 & 1 & 1.7321 \end{pmatrix}.$$

The initial state $x_i(0)$ of each agent is randomly chosen from $[-2, 2]$. As discussed in Section III, the $\eta_i, \beta_i, \theta_i, \alpha_i, \pi_{ij}$ and $\Upsilon_i(0)$ should be positive scalars. α_i and η_i should satisfy $0 < \alpha_i < 1$ and $\eta_i > \frac{1-\alpha_i}{\beta_i}$, respectively. Therefore, we choose $\eta_i = 2, \beta_i = 1, \theta_i = 0.1, \alpha_i = 0.1, \pi_{ij} = 1, \Upsilon_i(0) = 0.5$ in this simulation example. By using the designed method, the consensus errors of 6 agents are shown in Fig. 2. Note that

we take the state of agent 1 minus the states of other agents, and this value goes to zero. It means that the states of each agent tend to the identical value. That is, the secure consensus is achieved. The red line depicts the DoS attack sequence in Fig. 3. The triggering instants of 6 agents are also shown in Fig. 3. We can obtain from the simulation that $\Delta = 0.1073$. And we can also calculate that $\rho_1 = 0.1830$ and $\rho_2 = 2.0490$. So, $(\tau_0 + \Delta n_0) = 0.0777 < \rho_1 / (\rho_1 + \rho_2) = 0.0820$, which conforms to the condition in Theorem 1. The maximum attack frequency and duration that can be handled by our method are limited by the convergence rate ρ_1 and ρ_2 . Therefore, the upper bounds of τ_0 and n_0 can be relaxed by adjusting the values of ρ_1 and ρ_2 . However, the upper bounds are also limited by the Δ , whose value cannot be changed arbitrarily. And it may cause a conservative proportion of the DoS attacks in some cases. The triggering numbers of 6 agents are 125, 93, 80, 76, 65 and 105, respectively. Zeno behaviour is not exhibited in the close-looped system. Time evolutions of adaptive parameters $c_{ij}(t)$ and dynamic internal variables $\Upsilon_i(t)$ are shown in Fig. 4 and Fig. 5.

Remark 6: It should be pointed out that the dynamic variable $\Upsilon_i(t)$ is always positive, which has been proved in Lemma 1. Note that $\Upsilon_i(t)$ is a term of $f_i(t)$ with a negative sign and the new events will be generated when $f_i(t) \geq 0$. Therefore, the introduce of $\Upsilon_i(t)$ makes it more difficult to generate the next events by raising the trigger threshold, and the number of trigger instants is reduced compared to the static event-triggered rule. Moreover, the introduction of $\Upsilon_i(t)$ is also used to ensure the secure consensus, which means the static trigger rule may not achieve the secure consensus (at least not in this article). Thus, we cannot compare the simulation's static and dynamic event-triggered rules.

V. CONCLUSION

In this paper, the secure consensus problem for general linear multiagent systems with DoS attacks is solved under a fully distributed event-based control law. A group of adaptive coupling weights with neoteric adaptive law is developed in this law so that global information is no longer needed. A new dynamic event-triggered rule has been presented to alleviate the limited bandwidth. Strict proof of the convergence has been provided by using a well-designed Lyapunov function. Moreover, the Zeno behavior of the close-looped system is proved to be avoided. Finally, a group of unmanned intelligent vehicles is provided as an example to illustrate the correctness of the proposed method. In this paper, the time delays among communication networks are not considered. Sometimes, time delays will deteriorate the system performance. Our future direction is to investigate the fully distributed event-triggered control under DoS attacks and time delays.

REFERENCES

- [1] S. Gherairi, "Design and implementation of an intelligent energy management system for smart home utilizing a multi-agent system," *Ain Shams Engineering Journal*, p. 101897, 2022.
- [2] J. Y. Siu, N. Kumar, and S. K. Panda, "Command authentication using multiagent system for attacks on the economic dispatch problem," *IEEE Transactions on Industry Applications*, vol. 58, no. 4, pp. 4381–4393, 2022.
- [3] B. Ács, L. Dóra, O. Jakab, A. Jüttner, P. Madarasi, and L. Z. Varga, "Optimizations of a multi-agent system for a real-world warehouse problem," *SN Computer Science*, vol. 3, no. 6, pp. 1–12, 2022.
- [4] Q. Chen, Y. Sun, M. Zhao, and M. Liu, "Consensus-based cooperative formation guidance strategy for multiparafoil airdrop systems," *IEEE Transactions on Automation Science and Engineering*, vol. 18, no. 4, pp. 2175–2184, 2021.

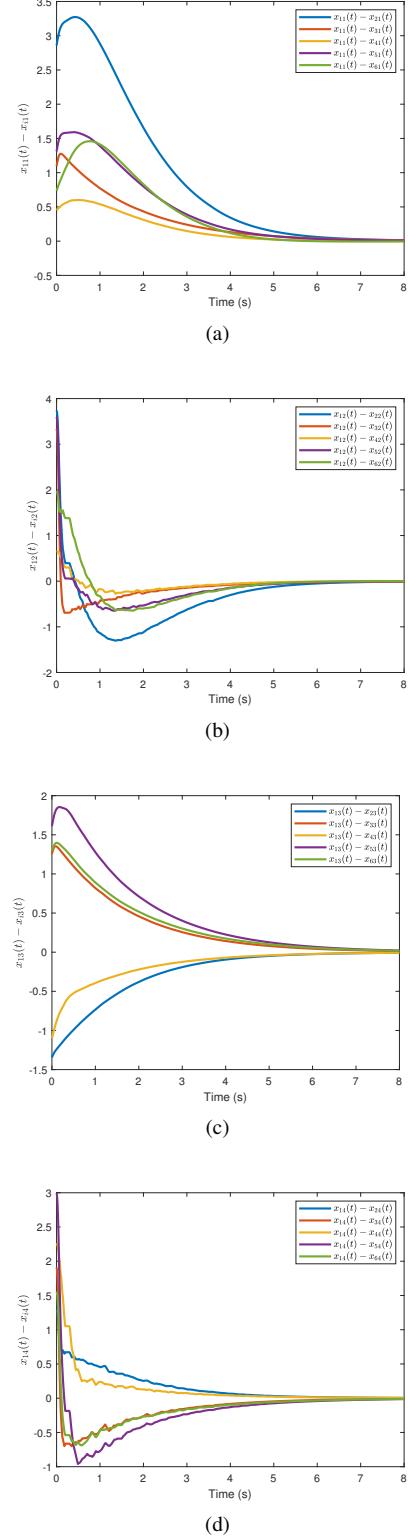


Fig. 2. Consensus error of 6 agents.

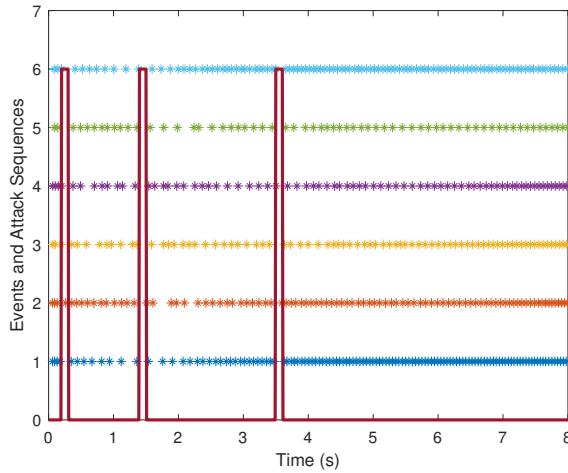


Fig. 3. Attack sequences and triggering instants.

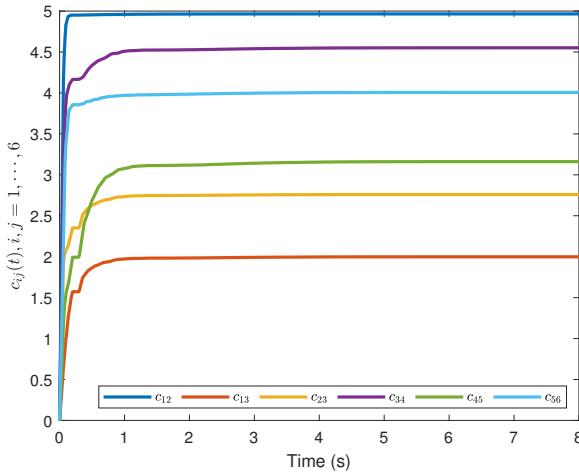


Fig. 4. Adaptive coupling weights $c_{ij}(t)$.

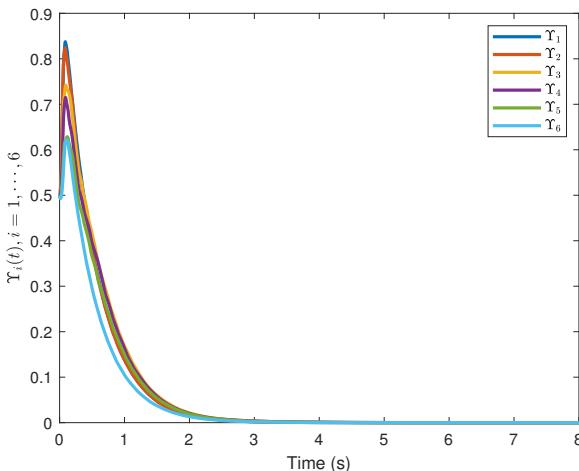


Fig. 5. Dynamic internal variables $\Upsilon_i(t)$.

- [5] S. Gude and K. C. Jana, "A multiagent system based cuckoo search optimization for parameter identification of photovoltaic cell using lambert w-function," *Applied Soft Computing*, vol. 120, p. 108678, 2022.
- [6] C. B. Saner, A. Trivedi, and D. Srinivasan, "A cooperative hierarchical multi-agent system for ev charging scheduling in presence of multiple charging stations," *IEEE Transactions on Smart Grid*, vol. 13, no. 3, pp. 2218–2233, 2022.
- [7] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.
- [8] W. Yu, G. Chen, and M. Cao, "Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems," *Automatica*, vol. 46, no. 6, pp. 1089–1095, 2010.
- [9] W. Wang, X. Chen, H. Fu, and M. Wu, "Model-free distributed consensus control based on actor-critic framework for discrete-time nonlinear multiagent systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 11, pp. 4123–4134, 2020.
- [10] Q. Van Tran, M. H. Trinh, and H.-S. Ahn, "Discrete-time matrix-weighted consensus," *IEEE Transactions on Control of Network Systems*, vol. 8, no. 4, pp. 1568–1578, 2021.
- [11] D. Zhang and G. Feng, "A new switched system approach to leader-follower consensus of heterogeneous linear multiagent systems with dos attack," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 2, pp. 1258–1266, 2021.
- [12] H. Zhang, Z. Gao, Y. Wang, and Y. Cai, "Leader-following exponential consensus of fractional-order descriptor multiagent systems with distributed event-triggered strategy," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 6, pp. 3967–3979, 2022.
- [13] G. Wen, P. Wang, Y. Lv, G. Chen, and J. Zhou, "Secure consensus of multi-agent systems under denial-of-service attacks," *Asian Journal of Control*, 2022.
- [14] L. Wang and Z. Wu, "Distributed secure consensus for second-order multi-agent systems under replay attacks," in *2021 13th International Conference on Machine Learning and Computing*, 2021, pp. 265–270.
- [15] H. Chu, D. Yue, L. Gao, and X. Lai, "Consensus of multiagent systems with relative state saturations," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 4, pp. 2391–2402, 2021.
- [16] S. Du, L. Zhu, H. Han, and J. Qiao, "Cooperative relay pursuit with input saturation constraint and external disturbances," *International Journal of Robust and Nonlinear Control*, vol. 32, no. 18, pp. 9939–9956, 2022.
- [17] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2012.
- [18] Z. Zhang, F. Hao, L. Zhang, and L. Wang, "Consensus of linear multiagent systems via event-triggered control," *International Journal of Control*, vol. 87, no. 6, pp. 1243–1251, 2014.
- [19] B. Zhou, X. Liao, T. Huang, and G. Chen, "Leader-following exponential consensus of general linear multi-agent systems via event-triggered control with combinational measurements," *Applied Mathematics Letters*, vol. 40, pp. 35–39, 2015.
- [20] A. Girard, "Dynamic triggering mechanisms for event-triggered control," *IEEE Transactions on Automatic Control*, vol. 60, no. 7, pp. 1992–1997, 2015.
- [21] S.-L. Du, T. Liu, and D. W. C. Ho, "Dynamic event-triggered control for leader-following consensus of multiagent systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 9, pp. 3243–3251, 2020.
- [22] W. He, B. Xu, Q.-L. Han, and F. Qian, "Adaptive consensus control of linear multiagent systems with dynamic event-triggered strategies," *IEEE Transactions on Cybernetics*, vol. 50, no. 7, pp. 2996–3008, 2020.
- [23] S. Hu, J. Qiu, X. Chen, F. Zhao, and X. Jiang, "Dynamic event-triggered control for leader-following consensus of multiagent systems with the estimator," *IET Control Theory & Applications*, vol. 16, no. 4, pp. 475–484, 2022.
- [24] C. Xu, H. Xu, Z.-H. Guan, and Y. Ge, "Observer-based dynamic event-triggered semiglobal bipartite consensus of linear multi-agent systems with input saturation," *IEEE Transactions on Cybernetics*, pp. 1–14, 2022.
- [25] B. Cheng and Z. Li, "Fully distributed event-triggered protocols for linear multiagent networks," *IEEE Transactions on Automatic Control*, vol. 64, no. 4, pp. 1655–1662, 2019.
- [26] B. Chang, J. Fang, and X. Mu, "Fully distributed dynamic event-triggered control for multi-agent systems with input saturation and actuator failure," *European Journal of Control*, vol. 60, pp. 1–10, 2021.
- [27] W. Xu, W. He, D. W. Ho, and J. Kurths, "Fully distributed observer-based consensus protocol: Adaptive dynamic event-triggered schemes," *Automatica*, vol. 139, p. 110188, 2022.

- [28] D. Ghaderyan, F. L. Pereira, and A. P. Aguiar, "A fully distributed method for distributed multiagent system in a microgrid," *Energy Reports*, vol. 7, pp. 2294–2301, 2021.
- [29] X. Li, Y. Tang, and H. R. Karimi, "Consensus of multi-agent systems via fully distributed event-triggered control," *Automatica*, vol. 116, p. 108898, 2020.
- [30] X. Feng, Y. Yang, and D. Wei, "Adaptive fully distributed consensus for a class of heterogeneous nonlinear multi-agent systems," *Neurocomputing*, vol. 428, pp. 12–18, 2021.
- [31] H. Wang, W. Ren, W. Yu, and D. Zhang, "Fully distributed consensus control for a class of disturbed second-order multi-agent systems with directed networks," *Automatica*, vol. 132, p. 109816, 2021.
- [32] W. Li, H. Zhang, Y. Zhou, and Y. Wang, "Bipartite formation tracking for multi-agent systems using fully distributed dynamic edge-event-triggered protocol," *IEEE/CAA Journal of Automatica Sinica*, vol. 9, no. 5, pp. 847–853, 2022.
- [33] J. Zhang, H. Zhang, S. Sun, and Z. Gao, "Leader-follower consensus control for linear multi-agent systems by fully distributed edge-event-triggered adaptive strategies," *Information Sciences*, vol. 555, pp. 314–338, 2021.
- [34] Y. Zhao, Q. Duan, G. Wen, D. Zhang, and B. Wang, "Time-varying formation for general linear multiagent systems over directed topologies: A fully distributed adaptive technique," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 1, pp. 532–541, 2021.
- [35] S. Li, K. Liang, and W. He, "Fully distributed event-triggered secure consensus of general linear multi-agent systems under sequential scaling attacks," *ISA Transactions*, vol. 127, pp. 146–155, 2022.
- [36] Z. Feng and G. Hu, "Secure cooperative event-triggered control of linear multiagent systems under DoS attacks," *IEEE Transactions on Control Systems Technology*, vol. 28, no. 3, pp. 741–752, 2020.
- [37] W. Xu, D. W. C. Ho, J. Zhong, and B. Chen, "Event/self-triggered control for leader-following consensus over unreliable network with DoS attacks," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 10, pp. 3137–3149, 2019.
- [38] S. Du, W. Xu, J. Qiao, and D. W. C. Ho, "Resilient output synchronization of heterogeneous multiagent systems with DoS attacks under distributed event-/self-triggered control," *IEEE Transactions on Neural Networks and Learning Systems*, pp. 1–10, 2021.
- [39] L. Wen, S. Yu, Y. Zhao, and Y. Yan, "Event-based secure consensus of multiple AUVs under DoS attacks," *Nonlinear Dynamics*, vol. 107, no. 3, pp. 2407–2419, 2022.
- [40] H. K. Khalil, "Nonlinear systems third edition," *Prentice Hall*, vol. 115, 2002.
- [41] S. Du, Y. Wang, L. Dong, and X. Li, "Secure consensus of multiagent systems with DoS attacks via a graph-based approach," *Information Sciences*, vol. 570, pp. 94–104, 2021.
- [42] Y. Gong, B. Zhang, and M. Xiong, "Fully distributed dynamic event-triggered consensus control for multi-agent systems under DoS attacks," *2021 China Automation Congress (CAC)*, pp. 2698–2703, 2021.
- [43] X. Liu, S. S. Ge, and C.-H. Goh, "Vision-based leader–follower formation control of multiagents with visibility constraints," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 3, pp. 1326–1333, 2019.
- [44] T. Liu and Z.-P. Jiang, "Distributed formation control of nonholonomic mobile robots without global position measurements," *Automatica*, vol. 49, no. 2, pp. 592–600, 2013.