Chapter 3

Continuous System Simulation

Continuous System Simulation

•A continuous system is one in which the predominant activities of the system cause smooth changes in the attributes of system entities

• When such a system is modeled mathematically, the variables of models representing the attributes are controlled by continuous functions.

• ***Continuous System Simulation*** describes how mathematical models of dynamic systems, usually described by sets of differential equations possibly coupled with algebraic equations can be simulated

Differential Equations

- A differential equation is a mathematical equation which relates some function with its derivatives, where functions usually represents physical quantities, the derivatives represent rate of change and equation represents relation between two.
- The typical example of linear differential equation with constant coefficients is used to describe the wheel suspension system of automobile.
- Dependent variable x, Independent variable t, Input F(t).

- If the dependent variable or any of its derivatives appear in any other form, such as being raised to a power, or are combined in any other way for example, by being multiplied together, the differential equation is said to be ***nonlinear*. [examples]**
- When more than one independent variable occurs in a differential equation, the equation is said to be ***partial differential equation*. [examples]**
- It can involve the derivatives of the same dependent variable with respect to each of the independent variables.

Significance of Differential Equations:

•The most physical and chemical process involves rate of change which requires differential equation for their mathematical description.

•Since a differential coefficient can also represent a growth rate, continuous models can also be applied to problems of social or economic nature where there is a need to understand general effects of growth trend.

•Differential equations can be used to represent different engineering problems as in describing behavior of automobile wheel suspension, with mechanical principles.

•Differential equations are extensively used to describe analytical solutions of different system.

•Differential equations are extensively used in simulating behavior of real operational system using electrical circuits.

Analog Computer

- An analog computer is a form of computer that uses continuously changeable aspects of the physical phenomena such as electrical, mechanical or hydraulic quantities to model the problem being solved.
- An **analog computer** is a **computer** which is used to process **analog** data.
- Analog computer does not use discrete values but rather continuous values.
- Processes cannot be reliably repeated with exact equivalence.
- **Examples of analog computers** are slide rule, nomogram, oscilloscope, television, **analog** sound processor, flight simulator etc.

Analog Computer

- The most widely used form of analog computer is the ***electronic analog computer,*** based on the use of high gain dc (direct current) amplifiers, called operational amplifiers.
- Voltages in the computer are equated to mathematical variables, and the operational amplifiers can add and integrate the voltages.
- With appropriate circuits, an amplifier can be made to add several input voltages, each representing a variable of the model, to produce a voltage representing the sum of the input variables.

The core mathematical operations used in an electric analog computer are:

- Addition
- Integration with respect to time
- Inversion
- Multiplication
- Exponentiation
- Logarithm
- Division
- Differentiation with respect to time is not frequently used, and in practice is avoided by redefining the problem when possible.

Analog Computer

- Different ***scale factors*** can be used on the inputs to represent coefficients of the model equations.
- Amplifiers that sums different terms are called ***summers.***
- Another circuit arrangement produces an ***integrator*** for which the output is the integral with respect to time of a single input voltage or the sum of several input voltages.
- All voltages can be positive or negative to correspond to the sign of the variable represented.
- To satisfy the equations of the model, it is sometimes necessary to use a sign ***inverter,*** which is an amplifier designed to cause the output to reverse the sign of the input.

Pros and Cons of Analog Computer

Advantages (Pros):

•The analog computer do not suffer quantization noise but are limited by analog noise.

•Analog computers are widely used in scientific and industrial applications where digital computers lack sufficient performance, like flight computer in aircrafts, for teaching control system in universities.

•The analog representation of a system is often more natural as it directly reflects the structure of system; thus simplifying both the setting up of a simulation and interpretation of results.

•The analog computer is faster than digital under certain circumstances as it could solve many equations in simultaneous manner while digital computer can be working on one equation at a time.

Disadvantages (Cons):

•An analog computer have limited accuracy and dedication to one particular problem, may not be significant.

•Electronic analog computer have limited accuracy due to following reasons-

•It is difficult to carry the accuracy of measuring a voltage beyond a certain point

•A number of assumptions are made in deriving relationship for operational amplifiers, none of which is strictly true.

•The assumption that there should be zero output voltage for zero input voltage is difficult to achieve.

•Operational amplifiers have limited dynamic range of outputs, so scale factor must be introduced to keep within range.

Advantages of Digital Computer

- Any degree of accuracy can be programmed virtually in digital computer with floating point representation of numbers, so an extremely wide range of variations can be tolerated.
- A digital computer has advantages of being easily used for many different problems.

Analog methods: Example

- **1.Analog Computer Model of Automobile Suspension**

- The general method by which analog computers are applied can be demonstrated using the second-order differential equation that has already been discussed:
- Solving the equation for the highest order derivative gives:

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

$$M\ddot{x} = KF(t) - D\dot{x} - Kx$$

•Suppose a variable representing the input F(t) is supplied, and assume for the time being that there exist variables representing $-x$ and $-\dot{x}$.

•These three variables can be scaled and added with a summer to produce a voltage representing $M\ddot{x}$.

•Integrating this variable with a scale factor of 1/M produces $\dot{x}$.

•Changing the sign produces $-\dot{x}$, which supplies one of the variables initially assumed; and a further integration produces $-x$, which was the other assumed variable.

- For convenience, a further sign inverter is included to produce $+x$ as an output.
- A block diagram to solve the problem in this manner is shown in following Figure.

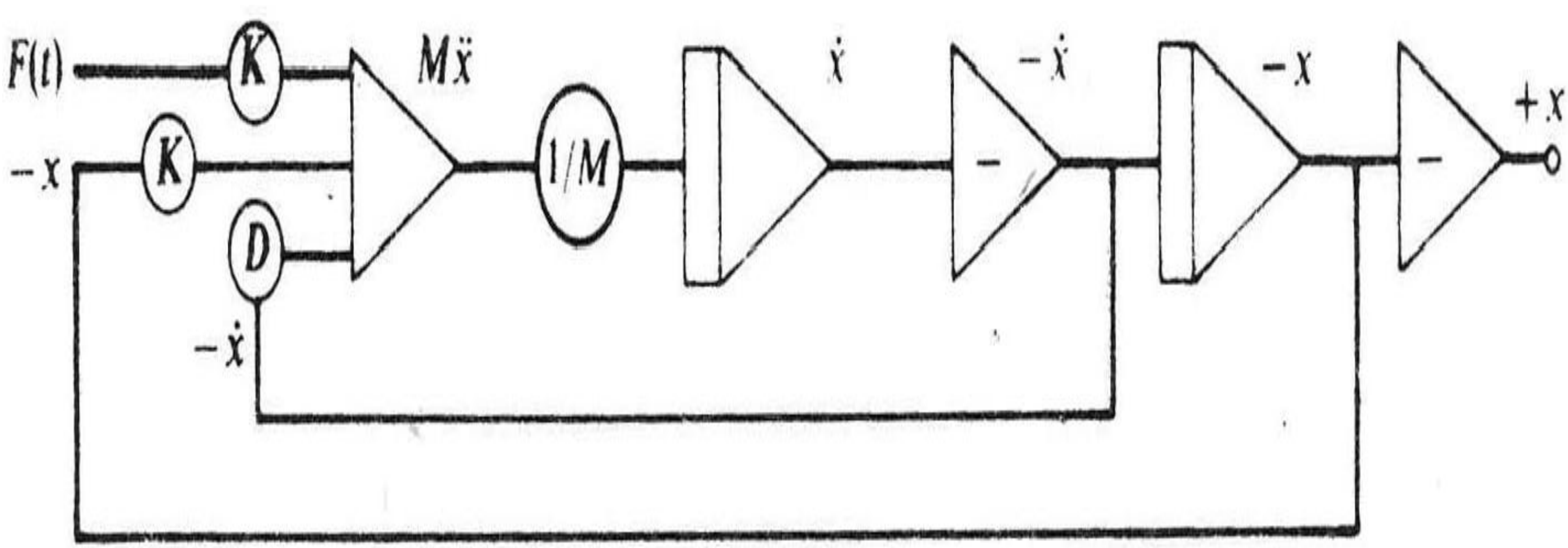


Figure: Analog Computer model for the automobile suspension problem

- The symbols used in the figure are ***standard symbols*** for drawing block diagrams representing analog computer arrangements.
- The circles indicate ***scale factors*** applied to the variables.
- The triangular symbol at the left of the figure represents the operation of ***adding*** variables.
- The triangular symbol with a vertical bar represents an ***integration***, and the one containing a minus sign is a ***sign changer.***

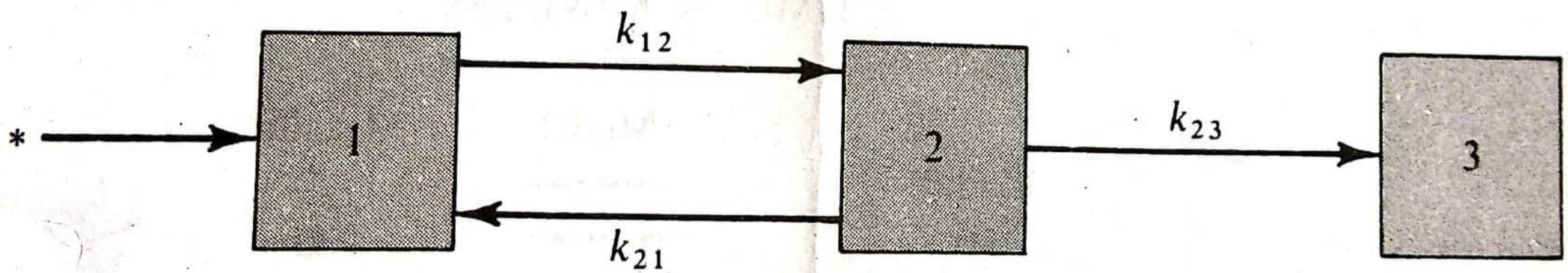
- The addition on the left, with its associated scaling factors, corresponds to the addition of the variables representing the three forces on the wheel, producing a variable representing $M\ddot{x}$.

- With an electronic analog computer, the variables that have been described would be voltages, and the symbols would represent operational amplifiers arranged as adders, integrators, and sign changers.

- The above Figure would then represent how the amplifiers are interconnected to solve the equation.
- It should be pointed out, however, that there can be several ways of drawing a diagram for a particular problem, depending upon which variables are of interest, and on the size of the scale factors.

2. Analog Computer Model of Liver

(First see the mathematical model of the liver)



$$\frac{dx_1}{dt} = -k_{12}x_1 + k_{21}x_2 \qquad x_1 = C_{11}e^{-b_1t} + C_{12}e^{-b_2t}$$

$$\frac{dx_2}{dt} = k_{12}x_1 - (k_{21} + k_{23})x_2 \qquad x_2 = C_{21}e^{-b_1t} + C_{22}e^{-b_2t}$$

$$\frac{dx_3}{dt} = k_{23}x_2 \qquad x_3 = C_{31} + C_{32}e^{-b_1t} + C_{33}e^{-b_2t}$$

Figure 2-10. Mathematical model of the liver.

- When a model has more than one independent variable, a separate block diagram is drawn for each independent variable and, where necessary, interconnections are made between the diagrams.
- As an example, the following Figure shows a block diagram for solving the model of the liver with 3 independent variables as x1,x2 and x3.

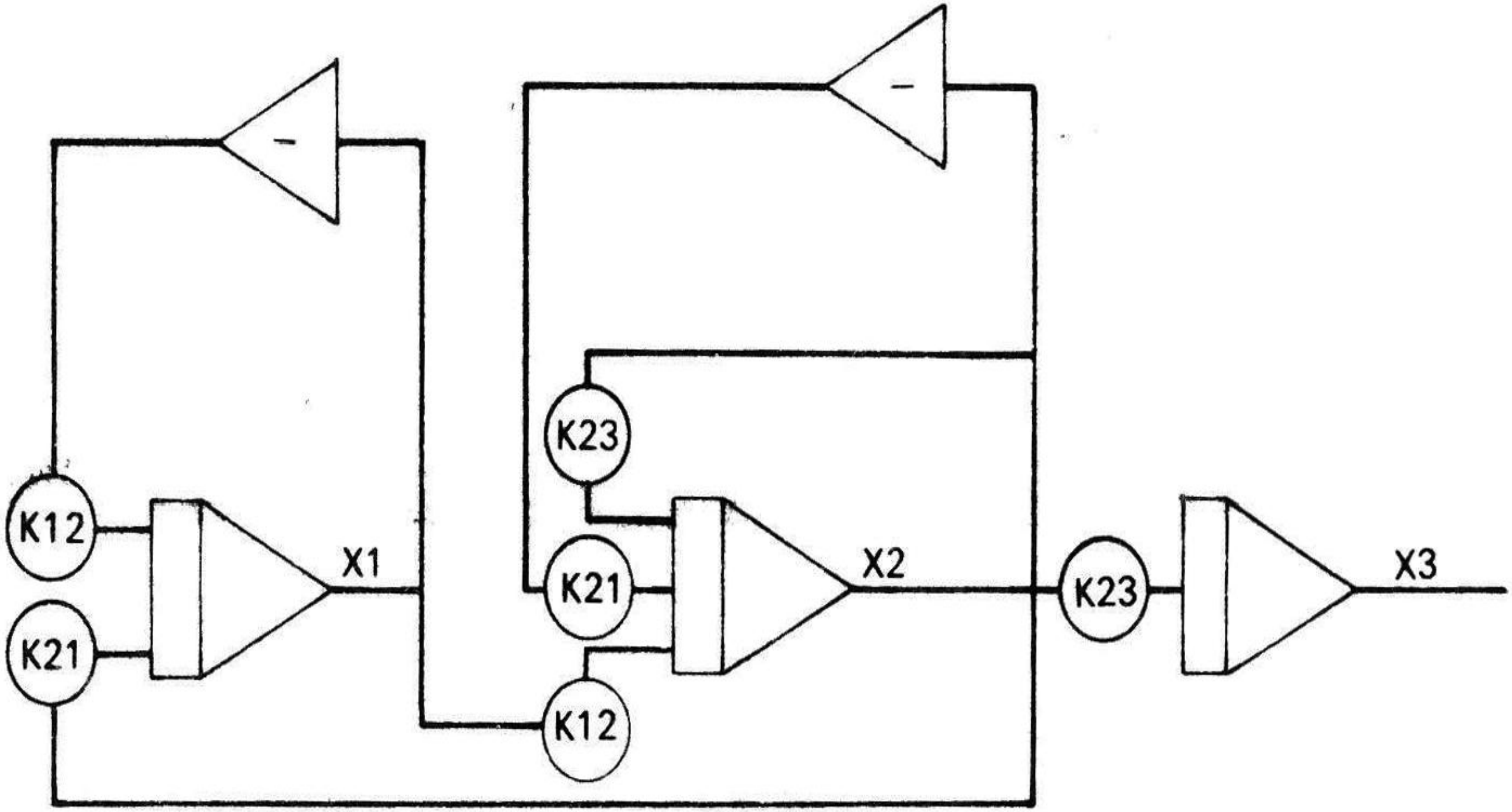


Figure: Analog computer model of the liver

- There are three integrators, shown at the bottom of the figure. Reading from left to right, they solve the equations for X_1, X_2, X_3.
- Interconnections between the three integrators, with sign changers where necessary, provide inputs that define the differential coefficients of the three variables.
- The first integrator, for example, is solving the equation

$$\dot{x}_1 = -K_{12}x_1 + K_{21}x_2$$

- The second integrator is solving the equation:

$$\dot{x}_2 = K_{12}x_1 - (K_{21} + K_{23})x_2$$

- The last integrator solves the equation:

$$\dot{x}_3 = K_{23}x_2$$

•In the second integrator, x2 is being used twice as an input to the integrator so that two coefficients k21 and k23 can be changed independently.

Hybrid Computers

•The hybrid computers are the combination of the traditional analog-computer elements, giving smooth continuous outputs and the elements carrying out such non linear, digital operations as storing values, switching and performing logical operations.

•Originally the term ‘hybrid computer’ had connotation of extending analog-computer capabilities, usually by the addition of special-purpose and often specially constructed devices.

•Hybrid computers may be used to simulate systems that are mainly continuous but in fact have some digital elements.

•For example, an artificial satellite for which both continuous equations of motions and the digital control signals must be simulated.

•It has the speed of analog computer and the memory and accuracy of digital computer.

Hybrid computers are used in several applications. Such as

•Petrol pump – Measurement convert fuel flow into currency rate
•Electrocardiogram machine
•ATM machine
•Control industrial process.
•Radar systems etc.

Continuous System Simulation Languages (CSSLs)

- CSSLs are the language which use familiar statement type of input for digital computers, allowing a problem to be programmed directly from equations of mathematical model, rather than requiring to be broken into functional elements.
- They could easily include macros, or subroutines that performs the functions of specific analog elements, so it is possible to incorporate convenience of a digital-analog simulator.
- The language contains macro instructions which carry out the action of adders, integrators and sign changers.

•CSSLs include a variety of algebraic and logical expressions to describe the relation between variables.

•Lets take Continuous System Modeling Program, Version III (CSMP III) to illustrate nature of these CSSLs.

•**CSMP III, or Continuous System Modeling Program IlI** is an early scientific computer software designed for modeling and solving differential equations numerically. This enables real-world systems to be simulated and tested with a computer.

•**Strucuture**

•Like all computer programs its analytical and simulation programs are constructed from three general types of statements:

•*Structural statements*, which define the model. They consist of FORTRAN-like programming language statements, and functional blocks of program code (procedures) designed for repeat operations that frequently occur in a model definition.

•*Data statements*, which assign numerical values to various changing parameters, constants, and initial conditions.

•*Control statements*, which specify options in assembly and execution of the program, and the choice of output of the results of the calculations performed.

•Structural statements can make use of the operations of addition, subtraction, multiplication, division, and exponentiation, using the same notation and syntax rules as are used in FORTRAN.

•If, for example, the model includes the equation

X=6Y/W+(Z-2)^2 the following statement will be used to define the variables and calculus performed:

X=6.0*Y/W+(Z-2.0)2.0**

[real constant also in decimals]

•1.2E-4 represents 0.00012

- Examples:

- $$Y = \int_0^1 X dt + IC$$

Y=INTGRL(IC,X)
Y(0)=IC
INTEGRATOR

$$Y = e^x$$

Y=EXP(x)
EXPONENTIAL

- Y=ln(x)

Y=ALOG(x)
EXPONENTIAL

- Y=sin(X)

Y=SIN(X)
TRIGONOMETRIC SIGN

Feedback Systems

- In a ***feedback system***, a portion of the system output is fed back into the system, thus introducing a level of dependencies among input and output signals in the system.
- With the use of feedback in communication systems, satisfactory response and robust performance can generally be achieved.
- In a system with feedback, also known as a *closed-loop control system*, the past output may influence the present or future outputs.
- Most physical systems embody some form of feedback.
- In a system with feedback, a closed sequence of cause-and-effect relations exists between system variables.

•There are two types of feedback systems. In a ***positive feedback system***, the feedback is used to increase the input signal level, thus generally making the system unstable.

•Positive feedback is widely used in oscillatory circuits such as oscillators and timing circuits.

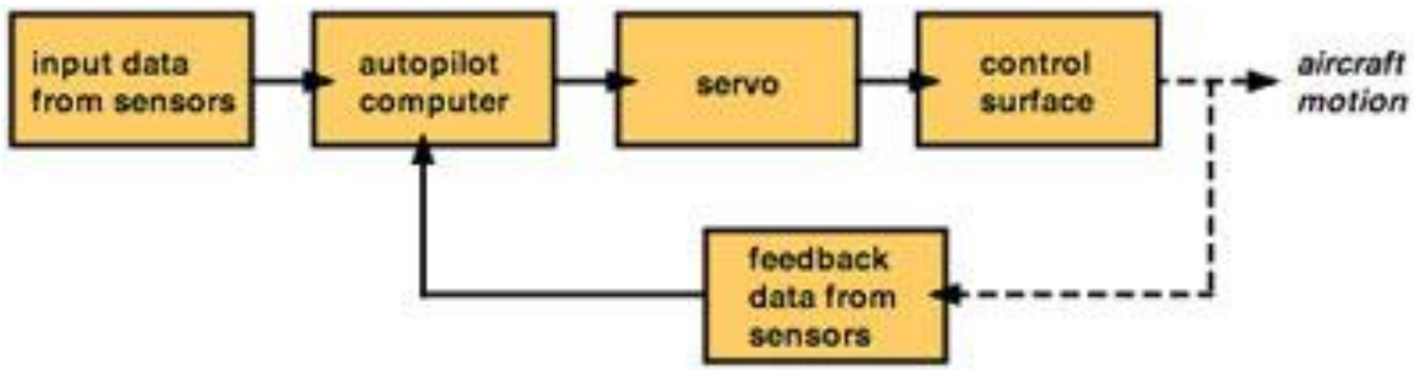
•In a ***negative feedback system***, the feedback is used to decrease the input signal level to ensure system stability.

• Tendency toward oscillation or instability is an important characteristic of feedback, and the issue of instability in all feedback systems thus needs to be fully addressed.

- A home heating system controlled by a thermostat is a simple example of a feedback system.
- The system has a furnace whose purpose is to heat a room, and the output of the system can be measured as room temperature.
- Depending upon whether the temperature is below or above the thermostat setting, the furnace will be turned on or off, so that information is being fed back from the output to the input.
- In this case, there are only two state, either the furnace is on or off.

Feedback System and Autopilot

- An example of a feedback system in which there is continuous control is the aircraft system.
- Here, the input is a desired aircraft heading and the output is the actual heading.
- The gyroscope of the auto pilot is able to detect the difference between the two headings.
- A feedback is established by using the difference to operate the control surface, since change of heading will then affect the signal being used to the heading.
- The difference between the desired heading and actual heading is called the error signal, since it is a measure of the extent to which the system deviates from the desired condition.
- The feedback in the autopilot is said to be negative feedback.
- The more the system output deviates from the desired value stronger is the force to drive it back.



•Many simulation studies of continuous system are concerned primarily with the study of servomechanism, which is the general name given to the devices that rely upon feedback for their operation.