Simulation Run Statistics

- Consider a single-server system in which the arrivals occur with a Poisson distribution and the service time has an exponential distribution.

- Suppose the study objective is to measure the mean waiting time, defined as the time entities spend waiting to receive service and excluding the service time itself.

- This system is commonly denoted by M/M/1 which indicates; first , that the inter-arrival time is distributed exponentially; second that the service time is distributed exponentially; and, third, that there is one server. The M stands for Markovian, which implies an exponential distribution.

Simulation Run Statistics

- In a simulation run, the simplest approach is to estimate the mean waiting time by accumulating the waiting time of n successive entities and dividing by n.
- This measure, the sample mean, is denoted by $\bar{x}(n)$ to emphasize the fact that its value depends upon the number of observations taken.
- If x_i (i=1,2,….,n) are the individual waiting times (including the value 0 for those entities that do not have to wait), then

$$\bar{x}(n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Simulation Run Statistics

- Whenever a waiting line forms, the waiting time of each entity on the line clearly depends upon the waiting time of its predecessors.

- Any series of data that has this property of having one value affect other values is said to be autocorrelated.

- The sample mean of autocorrelated data can be shown to approximate a normal distribution as the sample size increases.

Simulation Run Statistics

- The equation $\bar{x}(n)=\frac{1}{n}\sum_{i=1}^{n}x_i$ remains a satisfactory estimate for the mean of autocorrelated data.
- A simulation run is started with the system in some initial state, frequently the idle state, in which no service is being given and no entities are waiting.
- The early arrivals then have a more than normal probability of obtaining service quickly, so a sample mean that includes the early arrivals will be biased.

Simulation Run Statistics

- For a given sample size starting from a given initial condition, the sample mean distribution is stationary; but , if the distributions could be compared for different sample sizes, the distribution would be slightly different.

- The following figure is based on theoretical results, which shows how the expected value of sample mean depends upon the sample length, for the M/M/1 system, starting from an initial empty state, with a server utilization of 0.9.

Simulation Run Statistics

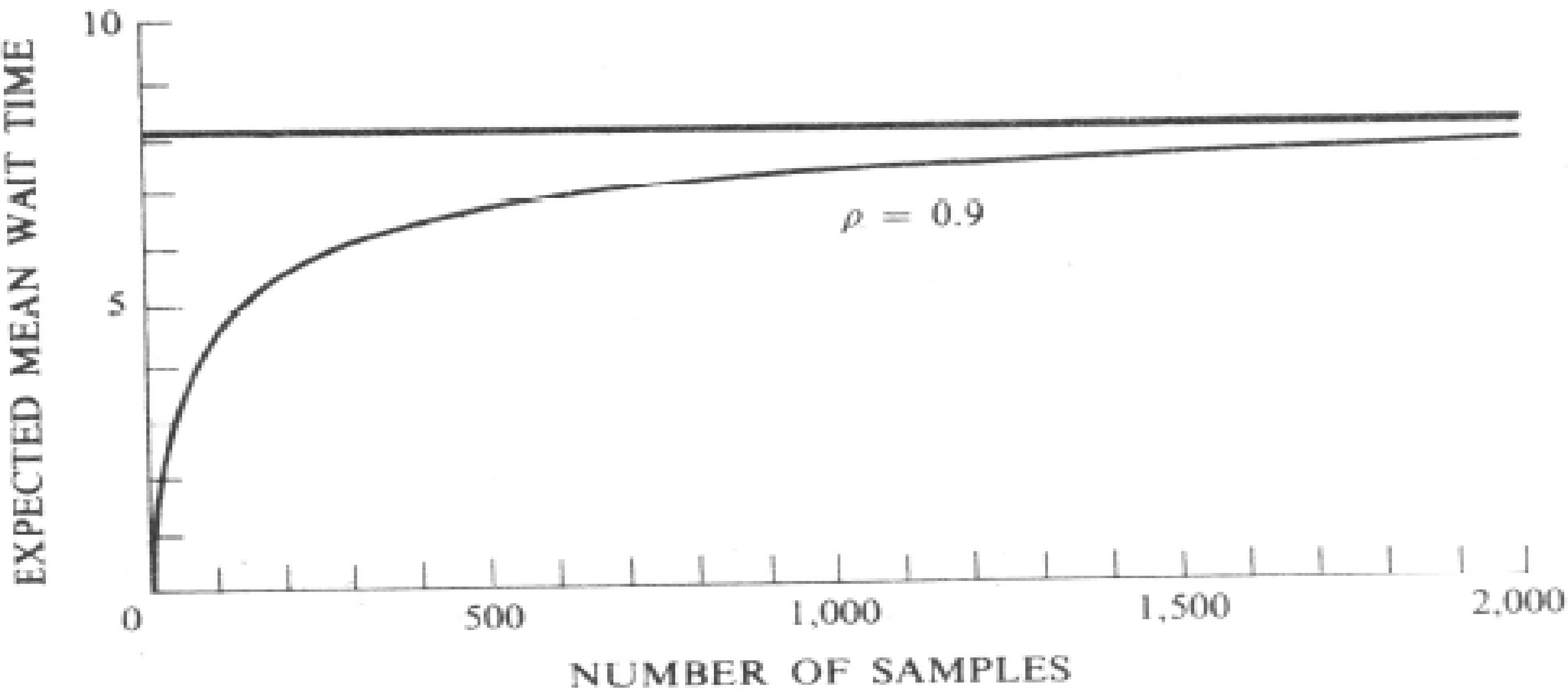

Figure 14-2. Mean wait time in M/M/1 system for different sample sizes.

Replications of Runs

- The precision of results of a dynamic stochastic can be increased by repeating the experiment with different random numbers strings.

- For each replication of a small sample size, the sample mean is determined.

- The sample means of the independent runs can be further used to estimate the variance of distribution. Let X_{ij} be the i^{th} observation in j^{th} run, then the sample mean and variance for the j^{th} run are:

Replications of Runs

$$\overline{x_j}(n) = \frac{1}{n}\sum_{i=1}^{n} x_{ij}$$

$$s_j^2 = \frac{1}{n-1}\sum_{i=1}^{n}[x_{ij} - \overline{x_j}(n)]^2$$

Replications of Runs

- **When we have similar means and variances for m independent measurements, then by combining them, the mean and variance for the population can be obtained as:**

Replications of Runs

$$\overline{x} = \frac{1}{p}\sum_{j=1}^{p}\overline{x_j}(n)$$

$$S^2 = \frac{1}{p}\sum_{j=1}^{p}s_j^2(n)$$

Replications of Runs

- **The following figure shows the result of applying the procedure to experimental results for the M/M/1 system.**

Replications of Runs

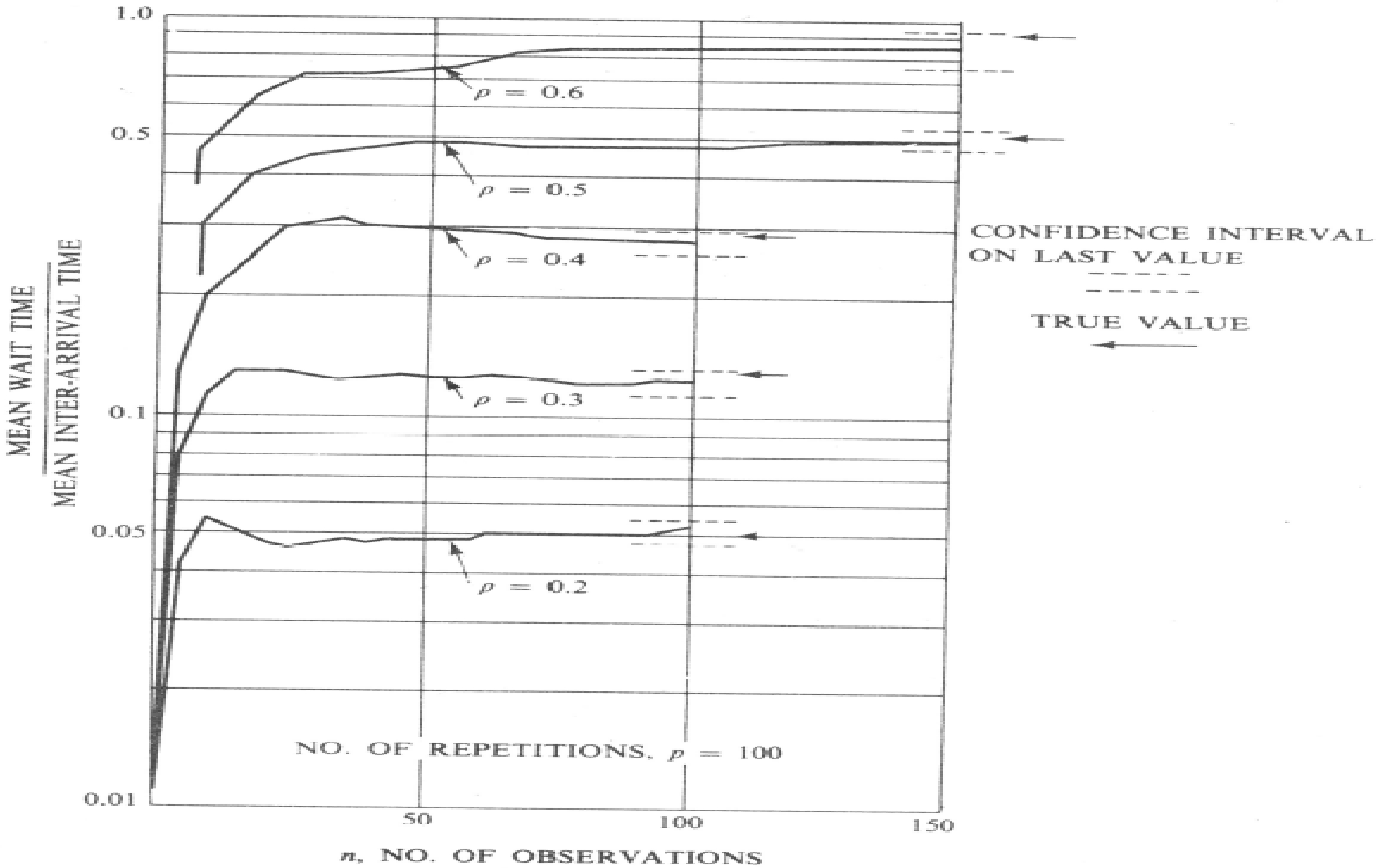

Figure 14-3. Experimentally measured wait time in M/M/1 system for different sample sizes.

Replications of Runs

- This variance can further be used to establish the confidence interval for p-1 degrees of freedom.
- The length of run of replications is so selected that all combined it comes to the sample size N. i.e. p.n=N.
- By increasing the number of replications and shortening their length of run, the confidence interval can be narrowed.
- But due to shortening of length of replication the effect of starting conditions will increase.
- The results obtained will not be accurate, especially when the initiazation of the runs is not proper.
- Thus, a compromise has to be made.
- There is no established procedure of dividing the sample size N into replications.
- However, it is suggested that the number of replications should not be very large, and that the sample means should approximate a normal distribution.

Elimination of Initial Bias

- Two general approaches can be taken to remove the bias: the system can be started in a more representative state that the empty state, or the first part of the simulation can be ignored.

- The ideal situation is to know the steady state distribution for the system, and select the initial condition from that distribution.

- In the study previously discussed, repeated the experiments on the M/M/1 system, supplying an initial waiting line for each run, selected at random from the known steady state distribution of waiting line.

Elimination of Initial Bias

- The case of 40 repetitions of 320 samples, which previously resulted in a coverage of only 9% was improved to coverage of 88%.

- The more common approach to removing the initial bias is to eliminate an initial section of the run.

- The run is started from an idle state and stopped after a certain period of time.

Elimination of Initial Bias

- The run is then restarted with statistics being gathered from the point of restart.
- It is usual to program the simulation so that statistics are gathered from the beginning, and simply wipe out the statistics gathered up to the point of restart.
- No simple rules can be given to decide how long an interval should be eliminated.

Elimination of Initial Bias

- The disadvantage of eliminating the first part of a simulation run is that the estimate of the variance, needed to establish a confidence limit, must be based on less information.

- The reduction in bias, therefore, is obtained at the price of increasing the confidence interval size.

Reference

- Geoffrey Gordon, System Simulation, Chapter 14, analysis of simulation output