Simulation and Modelling

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Simulation and Modeling (3-1-3)

Evaluation:

	Theory	Practical	Total
Sessional	30	20	50
Final	50	-	50
Total	80	20	100

Course Objective:

- To introduce the details of modeling and simulation technologies to the students.
- To provide the students with the knowledge of discrete and continuous systems, generation of random variables, and analysis of simulation output and simulation languages.

Course Contents:

1. Introduction to Modeling and Simulation **(4 hrs)**

1.1 System concept
1.2 System Environment
1.3 Stochastic Activities
1.4 Continuous and Discrete System
1.5 System Modeling
1.6 Types of Models
1.7 Principles of Modeling
1.8 Area of application
1.9 Verification and Validation of model

2. System Simulation **(8 hrs)**

2.1 The Techniques of Simulation-Monte Carlo Method
2.2 Problems Depicting Monte Carlo Method
2.3 Comparison of simulation and analytical methods
2.4 Experimental nature of simulation
2.5 Types of system simulation
2.6 Distributed Lag Models
2.7 Cobweb Models
2.8 Steps of Simulation Study
2.9 Time advancement Mechanism
2.10 Queuing Models and its Characteristics
2.11 Queuing Discipline
2.12 Measures of queues, Single Server Queuing System

3. Continuous System **(8 hrs)**

3.1 Continuous system simulation and system dynamics
3.2 Continuous system models
3.3 Differential equations-Linear differential equation
3.4 Non linear differential equation

3.5 Partial differential equation
3.6 Analog computers
3.7 Components of analog computers
3.8 Analog methods
3.9 Hybrid computers
3.10 Digital analog simulators
3.11 Continuous system simulation language
3.12 CSMP III
3.12.1 Structure Statements
3.12.2 Data Statements
3.12.3 Control Statements
3.12.4 Hybrid Statements
3.13 Feedback System
3.14 Interactive system
3.15 Real time simulation
3.16 Predator pray model

4. Discrete System Simulation **(8 hrs)**

4.1 Discrete system simulation
4.2 Representation of time
4.3. Generation of arrival patterns
4.4 Simulation of telephone system
4.5 Gathering statistics
4.6 Counters and summary statistics
4.7 Measuring Utilization and Occupancy
4.8 Recording distribution and transit time
4.9 Discrete simulation languages

5. Probability Concepts and Random Number Generation **(5 hrs)**

5.1 Probability concepts in simulation- Stochastic variable
5.2 Discrete Probability function
5.3 Continuous Probability function
5.4 Random numbers
5.5 Properties of random numbers
5.6 Pseudo random number
5.7 Technique for generation of random number
5.8 Test for Random number generation
5.8.1 Uniformity test (K-S test and Chi-square test)
5.8.2 Independence test (Runs test and Auto Correlation test

6. Simulation languages **(6 hrs)**

6.1 Types of simulation languages
6.2 Discrete systems modeling and simulation with GPSS
6.3 GPSS programs applications
6.4 SIMSCRIPT –Organization of a SIMSCRIPT program
6.5 SIMSCRIPT programs.

7. **Analysis of Simulation Output** (6 hrs)

7.1 Nature of the Problem
7.2 Estimation methods
7.3 Simulation run statistics
7.4 Replication of run
7.5 Elimination of Initial Bias

Laboratory:

Develop a simulation model, the topic could be either initiated by the student or selected from a list provided by the instructor. An oral presentation with a demonstration should be part of the laboratory project report.

Text Books:

1. G. Gorden, *System Simulation,* Prentice Hall of India.
2. A.M. Law and W.D. Kelton, *Simulation Modeling and Analysis*, McGraw Hill, 1991

References:

1. J.A. Spriest and G.C. Vansteenkiste, *Computer-Aided Modeling and Simulation,* Academic Press.
2. A.M Law and R.F. Parry, *Simulation*: A *Problem-solving approach*, Addison Wesley Publishing Company.
3. Narsingh Deo, "*System Simulation with Digital Computer*"

Chapter 1: Introduction to Modelling and Simulation.

AKASH SHRESTHA

System Concept.

→ A system is defined as a group of components working together to form a single unit for acheiving a common goal.

→ A system is an aggregation of objects joined together to accomplish some task.

→ Example:-

A factory system, that makes and assembles parts into product can be taken as a system as shown in figure below:

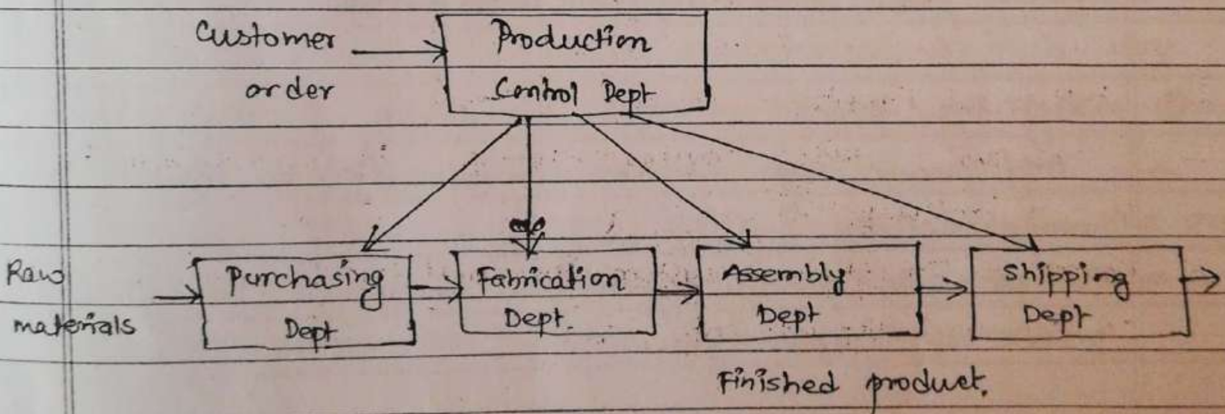


Fig:- Factory system

~~Too measure~~ Two major components of the above system are:

① Fabrication Dept: For making parts
② Assembly Dept: For producing the product.

However some other components also exists in the above system like.

1. purchasing Dept: For maintaing a supply of parts.
2. shipping Dept: For dispatching the finished product.
3. Production Control Dept: For receiving orders and assigning tasks to other department.

Components of System

1. Entity

→ It is an object of interest in a system.

→ Example:- In factory system, the entities are customer, order, parts, and products.

2. Attributes

→ It denotes the property of an entity.

→ Example:- In factory system, quantity for each order, ~~mae~~ no. of machine in production department etc can be example of attributes.

3. Activity / Event.

→ Any process that causes changes in a system is called activity.

→ Example:- In factory system, the activity is manufacturing process of department.

Some other examples of system are given below:

	System	Entity	Attributes	Activity
1	Supermarket	Customer	shopping list	checking out
2	Communication	Sender receiver message	msg. length / priority	Transmitting
3	Traffic	~~vehicle~~ Car / police man	speed distance	driving stopping

④	Bank	Banker, Account holder	Cash, cheque	Transaction

④ State

→ Collection of all entities, attributes, and activities at one point in time is called the state of the system.

→ Ex:- In banking system, the entities of the system are customers. The attributes are balance, account no, credit status etc and the activities are transaction i.e. deposit or withdraw of money.

System Environment.

→ Some time a system is affected by changes occuring outside the system. Such changes occuring outside the system are said to occur in system environment. So it is very important to study the boundary between the system and its environment while modelling a system.

→ Example:- Incase of factory system, factors controlling the arrival of orders may be considered to be outside the influence of factory system and therefore part of environment.

* Types of actives.

① Endogeneous Activity

→ The activity occuring within the system is called endogeneous activity.

:→ Example:- Manufacturing activity in factory system.

② Exogeneous Activity

→ The activity occuring outside the system i.e. in system environment is called exogeneous activity.

→ Example:- Arrival of order.

③ Stochastic Activity.

→ If output of an activity cannot be completely described in terms of its input or it vary randomly over a various possible outcome, the activity is said to be stochastic.

→ Example:- The randomness of stochastic activity would seem to apply the activity is depart of system environment.

→ The random output can often be measured and described in the form of probability distribution.

→ Example:- In case of factory system, the time taken for machining operation may need to be described by the probability distribution.

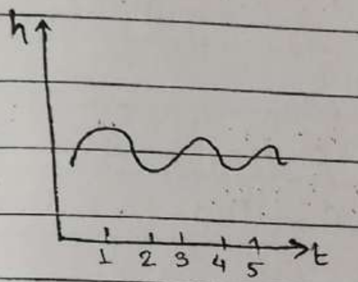
Basically, machining operation can be considered to be an endogeneous activity, however some time there might be power failure at a random interval of time hence, would be result of an exogeneous activity.

④ Deterministic Activity

→ If output of an activity can be completely described in terms of its input, then the activity is said to be deterministic.

* Types of System.

(1) Continuous System and Discrete System



-Fig:- Continuous System
(a)

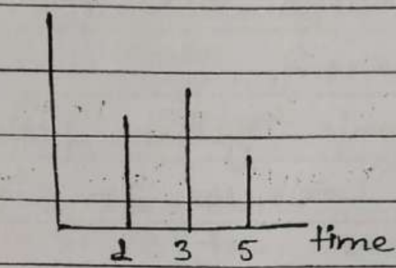


Fig:- Discrete ~~example~~ system
(b)

→ A system in which the state variable changes continuous over time is called continuous system.

→ Example:- Aircraft System

Fig (a)

The movement of aircraft occur smoothly over time.

→ So system such as aircraft, in which changes are smooth is called continuous system.

System in which state variable changes only at discrete set of points in time is called discrete set of point in time is called discrete system.

Example:- Banking system, because the no. of system changes only when a customer enters or leaves bank.

Fig (b)

② Static and Dynamic System.

→ Static system shows the value of system attributes when the system is in equillibrium or balanced.

→ Dynamic system Follows the changes over time that results from system activities.

③ Stochastic and Deterministic System.

→ The system in which the outcome of an activity can be completely described in terms of its input are called deterministic system.

→ The system in which the outcome of an activity cannot be completely described in terms of its input or vary over various possible outcome is called stochastic system.

④ Open and Close System.

→ A system for which there is no exogeneous activity is said to be closed system otherwise it is called closed system.

* System Modelling:

→ To study a system, sometime it is possible to experiment with the system itself. But sometime it is not possible to experiment with the system. While it is in hypothetical form.

→ So model is the substitute for the system and is also the simplification of system.

→ :(The relevant and necessary information gathered about a system for study purpose is called system model.

→ Studying of model instead of real system is much easier, faster, cheaper and safer.

→ Models ~~for~~ a system is always contradictory i.e. it is not unique as it depends on the study of system.

→ The tasks of deriving ~~of~~ a model of a system may be divided into following two subtasks.

(i) Establishing a model structure.

In this phase, it determines the system boundary and identifies the entities, attributes & activities.

(ii) Supplying the data.

In this phase, it provides the value of attributes and defines the relationship between the activities involved.

Consider the description of super market given below:

Shoppers needing several items of shopping arrive at super market. They get a basket if one available, carry out their shopping and then queue to checkout at one of the several counter. After checking out, they return the basket and leave.

Here in the assumption of super market. The entities are shoppers, basket, item & counter. The attributes are no. of items, availability of item, availability of basket, and no. of occupancy. The activities are ~~arriving &~~ arrival of customer, getting a basket, payment, returning a basket and leaving the supermarket.

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* Types of Model.

- Model
  - Physical
    - Static
    - Dynamic
  - Mathematical
    - Static
      - Numerical
      - Analytical
    - Dynamic
      - Analytical
      - Numerical

① Physical Model

→ In physical model of a system, the system attributes are represented by measurment such as voltage or position of shaft.

→ Here the system activities are deflected in physical logic that derived a model.

→ Ex: The rate at which the shaft of DC motor rotates depends on the voltage applied to the motor. If the applied voltage is used to represent the velocity of vehicle, then the no. of revolution of shaft is measure of distonce the vehicle has travelled.

② Mathematical model

→ Under mathematic model, this model uses symbolic notations and mathematical equattons to represent a system.

→ In this model, attributes are represented by variable and activities are represented by mathematical fun^n that interrelates variable.

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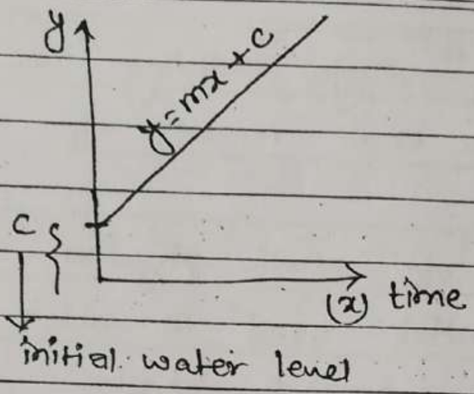
→ Eg:-



→ Suppose we wish to construct a model to show how water level changes with time so as to predict how long it will take to fill the whole water tank.

→ We can model this system by using simple linear functions as below,

$$y = mx + c$$

where, c is the initial water level,
x is time
m is the slope that determines the rate of flowing of water and
y is the water level at any time x.

② Static Physical Model.

→ A model that can only show the value that system attributes can take when the system is in balance is called static model.

→ These model looks like a real system which are to be modeled.

→ The best example of static physical model is 'scale model'.

A 'scale models' are the representation or copy of the objects that is larger or smaller than the actual size of object.

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→ In construction of college building, we use scale model, for determining, the exact model of class room, library, ground etc.

→ Another example of static physical model is the model that scientist used in which sphere represent atoms and rods represent bonds.

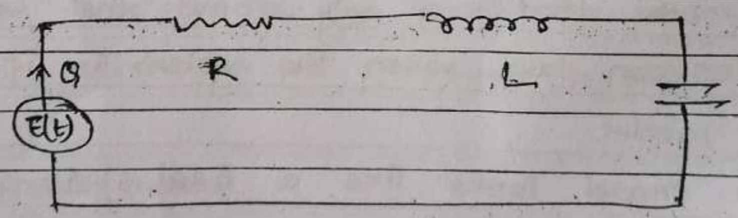
(b) Dynamic Physical Model.

→ A model that can show the value of sytem attributes that changes over time as a result of system activities is called dynamic model.

→ It is the physical model that represents the behaviour of the system when it is not in equilibrium.

→ Example:-

Below represents the electronic circuit with an inductor L and register R and capacitor C connec is series with voltage source that varies with time according to time E(t). If Q is the charge on capaci then the behaviour of the circuit can be defined by the following differential equation.



$$E(t) = V_R + V_L + V_C$$

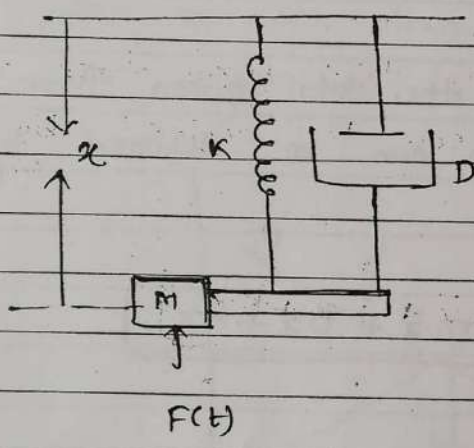
$$= iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$= \frac{dq}{dt} R + L \cdot \frac{d}{dt}\left(\frac{dq}{dt}\right) + \frac{1}{c}\int \frac{dq}{dt} \cdot dt$$

$$E(t) = \dot{q}R + L\,\ddot{q} + \frac{1}{c} \cdot q$$

$$\therefore \; E(t) = L\ddot{q} + R\dot{q} + \frac{q}{c}$$

→ In some case, when the behaviour of mechanical system is equivalent to the electrical system behaviour, it is easier to use the equivalent electrical system as electrical system are easier to modify, than the mechanical system. In this case, electrical system is model for mechanical system.



Here, in the above figure, m is the mass where certain force which gives upward motion when force f is applied to it varying with time t.

K is the stiffness of spring and D is the damping force of shock absorber.

Here, force applied on mass m is

$$F_1 = m \times a$$

$$\text{or } F_1 = m \cdot \frac{d^2x}{dt^2}$$

$= m\ddot{x}$

Damping force exerts by shock absorber

i.e. DF $\propto$ velocity.

$$F_2 = D \cdot \frac{dx}{dt}$$

$$= D\dot{x}$$

Spring Force is proportional to its extension or contraction.

i.e. $F_3 \propto x$

$$F_3 = Kx$$

If F(t) is the total system force varying with time t, the whole system can be defined by the following differential eqn.

$$\boxed{K\,F(t) = m\ddot{x} + D\dot{x} + Kx}$$

So, we see that above two system are analogus in manner i.e. displacement equivalent to charge, mass equivalent inductance L, velocity $\dot{x}$ equivalent to current i i.e. $\dot{q}$ and force f equivalent to voltage E.

:

So, we can use the above electrical model to study the mechanical system.

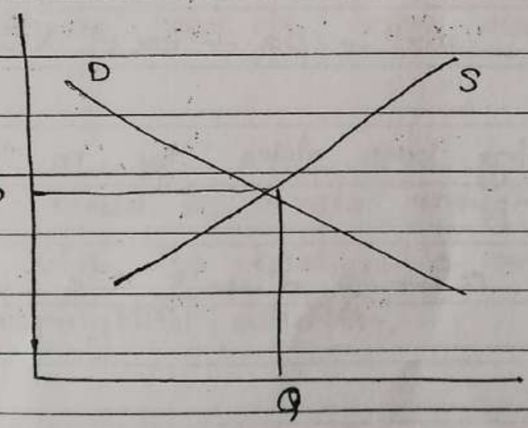
→ It is simple to modify the electrical system than changing the mechanical system.

→ Example : If car wheel is considered to bounce too much for particular suspension system, the electrical model will demonstrate by showing charge oscillating ~~access~~ excessively.

⑤ static ~~mathematical~~ models.

→ This model gives the relationship between system attributes when the system is in equillibrium.

→ In marketing commodity there is an equillibrium point between demand D and supply S. Both demand and supply depend on price. Demand will be low when price is high and supply increases as ~~supp~~ price increases. This relationship can be represented by two straight line that cross at a point to each other as shown in figure below.



The relationship is linear, thus a complete market model can be written as

$$D = a - bp$$
$$S = c + dp$$
$$S = D$$

Example: -

$a = 600$

$b = 3,000$

$c = 150$

$d = 2,000$

Find p and S at equillibrium

At Eq,

$S = D$

$c + dp = a - bp$

Changes of system attributes to be derived

Derivations may be made with analytical solutions and numerical wheel suspension system is a dynamic mathematical model i.e.

$$mx'' + Dx' + kx = k\,f(t)$$

Dividing both sides by m

$$\ddot{x} + \frac{D}{m}\dot{x} + \frac{k}{m}x = \frac{k}{m}F(t)$$

$$x'' + \sigma\omega x' + \omega^2 x = \omega^2 F(t)$$

where $\frac{k}{m} = \omega^2$ and $\frac{D}{m} = \sigma\omega$ and ς = damping ratio.

From the above model. We see that the condition for the motion to occur without oscillation required that,

$$\zeta \geq 1$$

Otherwise the motion is oscillated.

Solution to mathematical model

Once a mathematical model is build, it must be their examined to see how it can be used to answer the question of intrest about the system.

The two methods used to solve mathematical models are:-

→ Analytical Model

If the model is simple enough, it may be possible to work with its relationship and quanties to get an exact analytical solution.

→ This method ~~uso~~ uses deductive reasoning of mathematical theory to solve a model.

: → Let us consider

$$d = vt$$

If we know the distance 'd' to be travelled and velocity 'v', then we can work with the model to get $t = \frac{d}{v}$ to calculate the time taken.

→ Numerical method/ Simulation method.

- If an analytical solution to mathematical model is available and computationaly efficient, it is usually desirable to study the model using analytical method.

- However, many systems are highly complex so valid mathematical model of them are also complex. In such case, the model must be studied by means of simulation, i.e. numerically exercising the model for input in questions to see how they affect the output measure of performance.

- It produces solution in each step, each step gives solution. For once, at a condition and calculation is repeated until a final solution is obtained.

Principle used in modelling

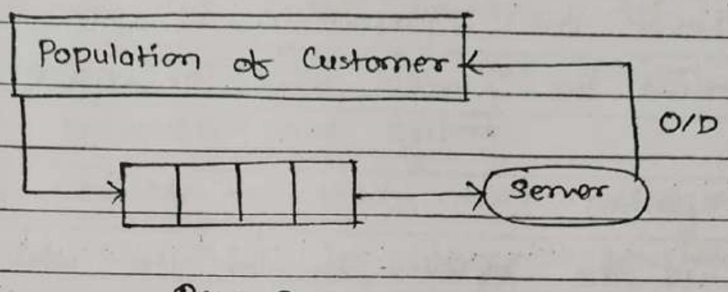


Fig: Queuing model

This provides the guidelines that provide different viewpoints to be judged while defining a model.

ⓐ Building block

→ Description of system should be organised in series of block.
→ Each block describes part of a system.
→ Each block results in few input parameters and result is few output parameters.
→ System is described as interconnection of these blocks.
→ The system can be represented graphically as a simple block diagram. Eg: above figure.

ⓑ Relevance

→ Models should include relevant aspect of system.
→ If irrelevant informations are considered in our models then it may not harm our model but the complexity of the model increases.
→ Example:- Considering the employee in our factory system may be considered irrelevant.

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(c) Accuracy

→ Whatever the information is gathered for the model, it must be correct and accurate.

(d) Aggregation

→ It is the extend ~~of the~~ to which the number of individual entities can be grouped together into larger entities.

→ Example:- In case of Factory system, the general manager of the Factory may be satisfied with the description that can be given. However, the production control manager, may want to consider the shops of the department as an individual entity.

Introduction to Simulation

→ It is one of the most powerful tool available to decision maker responsible for the design and operation of complex processes and system.

→ It makes possible to study, analysis and evaluation of situation that wouldn't be otherwise possible.

→ Simulation can be defined as process of designing a model of real system and conducting experiments with these models and evaluating various strategies for the operation of system.

* Advantages of Simulation

→ We can test new design without assigning resources to their implementations.

→ It can be used to explore new staffing policy, operating procedures, design rules, organizational structure etc without disturbing any ongoing operations.

→ Time can be compressed or expanded according to need under investigation.

→ It's great strength is its ability to let us experiment with new ~~child~~ and unfamiliar situations.

→ It allows us to gain the idea about how a system actually works and provides understanding of which parameters are important upto performance.

* Disadvantages of Simulation

→ Building a proper model requires specialized training. It is an art that is learnt over time and through experience.

→ Gathering highly reliable input data can be time consuming and expensive.

→ Simulations are used in cases where analytical solutions are possible.

→ Lack of knowledge about simulation method in people.

→ Writing computer program to define and execute a model is time consuming task.

Verifications and Validations

→ A model is the substitute for the system to be studied and also is the simplification of system.

→ The relevent and necessary information gathered about the system for study purposes is called model.

→ One of the most important and difficult task for a model developer is verification and validation of simulation on model.

→ verification deals with building model right and validation deals with building right model.

Process of Building a model.

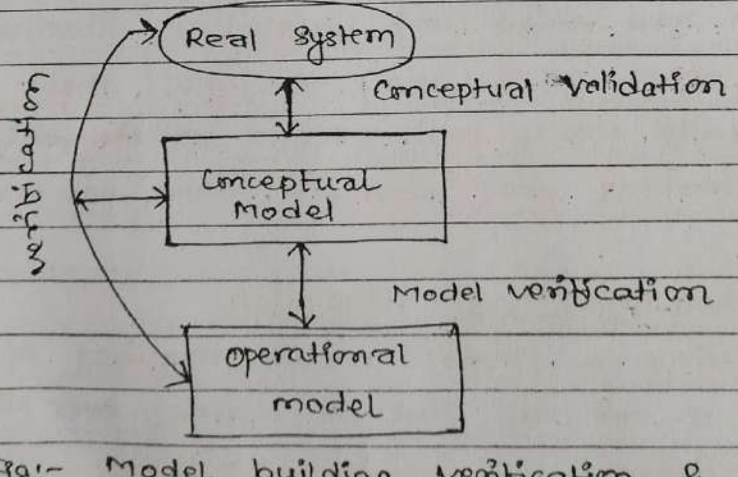

Fig:- Model building verification & validation.

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→ The process ~~of~~ or facility of intrest is called system.

→ In order to study it, we have to make sets of assumption on how it works.

→ These assumptions which usually take a form of mathematical or logical relationship, consitute a model and this model can be studied to understand how the system behaves.

→ The figure above lists the steps to build a model of a system.

Following are the steps involved in designing a model.

1) The first step of model building consist of observing the real system and interaction among its various components and collecting data on its behaviour.

→ Person like operator, technicians etc familiar with the system should be questioned to take advantage of their special knowledge.

→ As model development proceeds, new questions may arise and model developer may need to return this step.

2) The second step is the construction of the conceptual model. The conceptual model defines

(a) assumption of system components.

(b) structural assumptions which defines the interaction between system components.

(c) Input parameters and data assumptions.

3) The third step is the translation of conceptual model into an operational model. i.e. computer recognizable form. Hence, A model building is a non linear process with three steps as a model builder could return to each of these

steps many times while building verifying and validating the model.

Verification and Validation are the most important tasks in model development.

* Verification

→ It is concerned with building model right.

→ It is utilized in the comparision of the conceptual model to the computer representation that implements the conception.

→ The main purpose of model verification is to assure that the conceptual model is reflected accurately in computerised representation.

→ It asks the following questions

① Is the model implemented correctly in the computer?

② Are the input parameters and structure of the model correctly represented?

→ Some of the task that can be used for verification process are:

(i) Have a computerized representation checked by some other than its developer.

(ii) Closely examine the model output for reasonableness under variety of setting of input parameters.

(iii) Give small descriptions / comments on every variable used and each major section of code block.

(iv) Make a flow diagram which includes logically possible action system can take when an event occurs, and then follow the model logic for each action for each event type.

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(v) If the computerized representation is animated, verify that what is seen in the animation imitates the real system.

* Validation

→ Validation deals with building right model.

→ It is utilized to determine that a model is an accurate representation of real system.

→ Validation is usually achieved through the calibration of the model, which is an iterative process of comparing the model to the actual system behaviour and using the difference two to improve the model.

→ This process is repeated untill the model accuracy is judged to be acceptable.

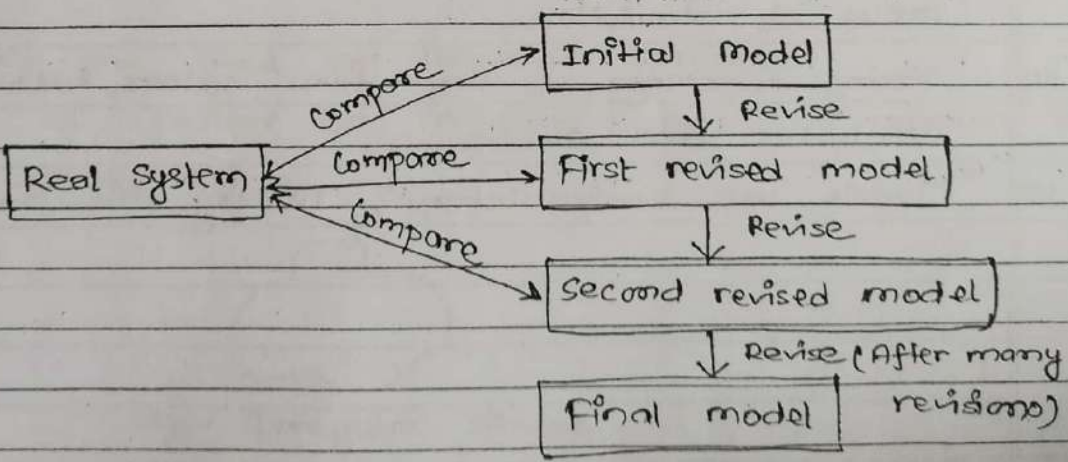
→ Three step approach for model validation is:-

(i) Build a model that has high face validity.

(ii) Validation of model assumption. (Structural / Data assumptions)

(iii) Validating the I/O transformation.//

Calibration & Validation of Model



→ Callibration is the iterative process of comparing the model to real system, making adjustments to model, comparing revised model to real system, comparing again and so on until a final model is obtained.

→ This process is repeated until the model accuracy reaches to acceptable model.

→ Above figure shows the iterative process of callibrating the model

→ The comparision of the model to the real system is carried out by following two test:-

(i) Subjective Test

This test involves people who are knowledgeable about one or more aspects of system making judgement about the model and its inputs.

(ii) Objective Test

This test always require data on system behaviour plus the corresponding data produced by the model.

Assignment 1//

(1) When and when not to use simulation? Explain different areas of simulation.

(2) Taking reference of any five systems, identify the entities, attributes and events.

(3) "Models are contradictory". Justify this statement.

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Monte Carlo Simulation

→ It is a technique of experimental sampling with random number or method of trial which can be used to solve many problems which are otherwise difficult or sometimes impossible.

→ This method cannot be used to attempt high accuracy and hence is suitable for those problems that doesn't require high degree of accuracy.

→ Monte Carlo method are stocastic technique and make use of random number and probability to solve the problem.

→ It can be used to solve both stocastic and deterministic problems. When we solve the deterministic problem using monte carlo method, we first convert the deterministic model into stocastic model. Eg:- Calculating the value of pi (π).

→ Some example of problems solved using Monte Carlo simulation are:-

i) To find the area of irregular surfaces.
ii) Numerical Integration
iii) Random walk problem

i) Finding the area of irregular surfaces

→ Let us consider an irregular shape as shown in the figure below.

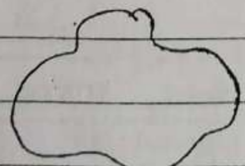
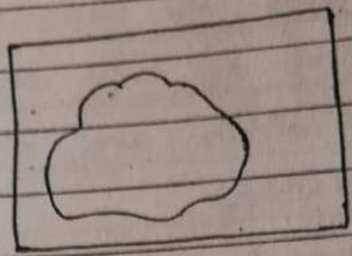
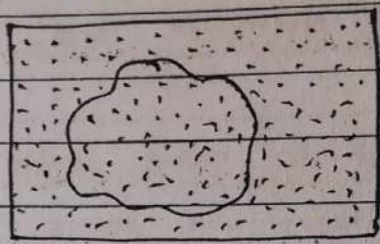


Fig:- Irregular shape

→ Enclose the shape with rectangle as below



→ Inside rectangle mark random 'N' numbers of dots as shown in figure below.



Let there be 'm' dots inside the irregular surface and 'N' be the total no. of dots inside rectangle.

→ Count the number of dots inside irregular figure, let 'm'. If 'F' be the area of irregular figure then, we can write $\frac{F}{A} = \frac{M}{N}$

where, F = area of irregular figure
A = area of rectangle.
M = Dot's inside irregular figure.
N = Dot's inside rectangle.

→ Larger the number of dots, more accurate will be the result.

Eg:- Suppose N = 50, M = 25, A = 20. Find F we have,

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$$\frac{F}{A} = \frac{M}{N}$$

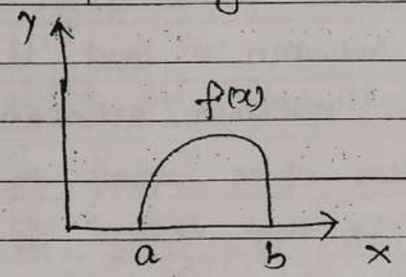
or, $$\frac{F}{20} = \frac{25}{50}$$

or. $F = 10$

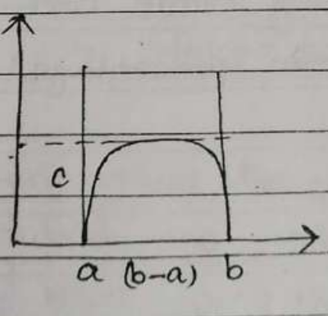
* Numerical Integration.

→ The integration of single variable over a given range gives the area under graph representing the function procedure.

→ let us consider a function f(x) having lower and upper bound a and b respectively.



→ Now draw a rectangle as shown in figure below. The side of the rectangle are (b−a) and c.



→ Here area of rectangle = $(b-a) \times c$

→ Then mark random number of dots inside the rectangle and count dots. Let 'm' be the number of dots inside the curve and 'N' be the total no. of dots inside the rectangle. The approximately,

$$\frac{\text{Area of irregular surface}}{\text{Area of rectangle}} = \frac{M}{N}$$

i.e. $$\frac{\text{Area of Curve (F)}}{\text{Area of rectangle}} = \frac{M}{N}$$

i.e. $$\boxed{\int_a^b f(x)\,dx = \frac{M}{N} \times (b-a)\,c}$$

⇒ The accuracy improves as the number of 'N' increas

⇒ For any random point x, y. The value of x is selecte at random between 'a' and 'b' and value of y is selected at random between 0 and c.

⇒ If y_0 is less than equals to $f(x_0)$, the point x_0, y_0 is accepted in the count of M. Otherwise it is rejected.

Q. Solve $\int_2^4 x^2\,dx$ using Monte Carlo method. Also estimate the error percentage.

⇒ Here, $y = f(x) = x^2$

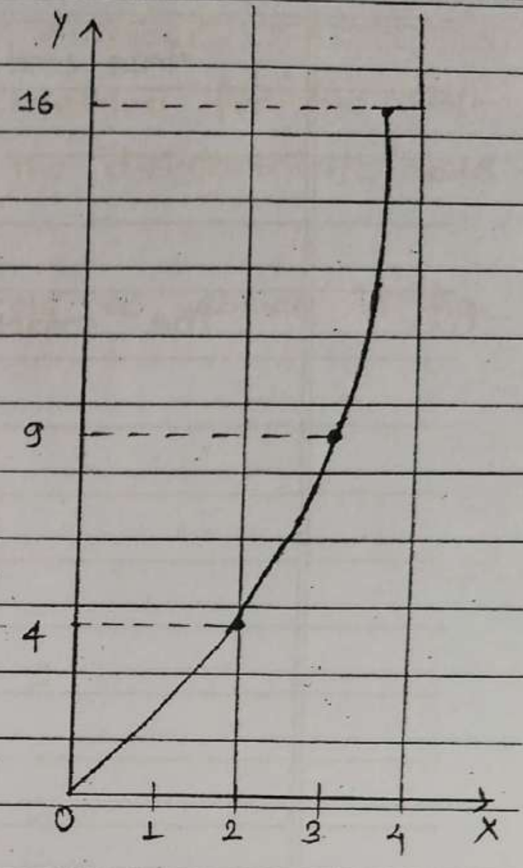
lower limit = 2

upper limit = 4

x	2	3	4
y	4	9	16

Area of rectangle = $(4-2) \times 16$

$= 32$



Suppose, we take $N = 5$ random dots (x, y) inside the rectangle such that

$2 \leq x \leq 4$

$0 \leq y \leq 16$

$(x_1, y_1) = (2, 0)$

$(x_2, y_2) = (2, 2)$

$(x_3, y_3) = (4, 5)$

$(x_4, y_4) = (3, 10)$

$(x_5, y_5) = (2, 15)$

Now, we can write the equation of curve as

$$y \leq x^2$$

putting value of x and y in above equation.

For $(2,0)$; $0 \leq 4$, so $(2,0)$ is accepted.

For $(2,2)$; $2 \leq 4$, so $(2,0)$ is accepted.

For $(4,5)$; $5 \leq 16$, so $(4,5)$ is accepted.

For $(3,10)$; $10 \leq 9$, is false so rejected.

For $(2,15)$; $15 \leq 4$, is false so rejected.

Since, the accepted values lies inside the curve and rejected values lies ~~out~~ outside the curve. We got

$N = 5$

$M = 3$

From, Monte Carlo method we have

$$\frac{\text{Area under curve}}{\text{Area under rectangle}} = \frac{M}{N}$$

$\therefore$ Area under curve $= \frac{3}{5} \times 32$

$= 19.2$

The exact solution is,

$$\int_2^4 x^2 \, dx$$

$$= \left[\frac{x^3}{3}\right]_2^4$$

$$= \left[\frac{64}{3} - \frac{8}{3}\right]$$

$$= 18.67$$

% Error $= \frac{19.2 - 18.67}{19.2} \times 100\%$

$= 2.76\%$ //

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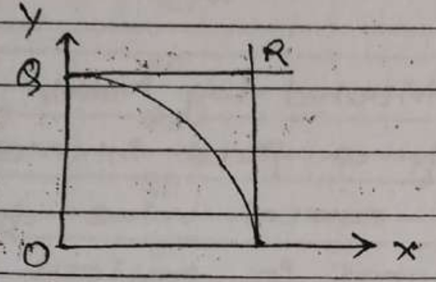
Deterministic problem through random number.

→ The application of Monte Carlo method for the evaluation of pi (π) requires converting the deterministic model into stocastic model.

→ Consider a quadrant of unit circle as shown in fig. below.



→ All point satisfy the eqn

$x^2+y^2 \leq 1,\ x, y \geq 0$ lie inside the circle quadrant.

The eqn can be written as

$$y^2 \leq 1-x^2$$

$$\therefore y \leq \sqrt{1-x^2}$$

Now if (r_1, r_2) is a pair of random number in range $r(0,1)$, we call this pair acceptable if

$$r_2 \leq \sqrt{1-r_1^2}$$

Distributed Lag Model.

When a model is large or complex, we perform numeric computation technique using computer. However there is a simple technique in simulation that can be applied without difficulty even for large model.

Properties of Distributed Lag Model.

→ Model changes only at fixed interval of time.

→ Model is based on current value of variable and values that occured in previous intervals. Eg:- Census data.

→ This model consist the rule of linear algebric expression.

→ They represent continuous systems, but one in which data is available at fixed point in time.

→ If current value of variable can be represented in terms of values then such variable is called lagged variable.

$$X = A + BX_{-1}$$

where, X = current value of X

and X_{-1} = previous value of X

So, if an intial value of X is given a new value of X can be computed.

Q. Consider the following mathematical model of National Economy.

:

$$I = 2 + 0.1\,Y_{-1}$$

$$Y = 45.45 + 2.27(I + G)$$

$$T = 0.2Y$$

$$C = 20 + 0.7(Y - T)$$

where, C the consumption, I be investment, T be taxes and Y be national income. For the above equations, the values of G are supplied to 5 years as below.

Year	G
1991	15
1992	20
1993	25
1994	30
1995	35

If initial value of Y_{-1} is 75, calculate consumption for 1st and 2nd year.

→ Solution,

For 1st year

$I = 2 + 0.1 \times Y_{-1}$

$= 2 + 0.1 \times 75$

$= 9.5$

$Y = 45.45 + 2.27 (I + G)$

$= 45.45 + 2.2 + (9.5 + 15)$

$= 101.065$

$T = 0.2 \times Y$

$= 0.2 \times 101.065$

$= 20.213$

$C = 20 + 0.7 (101.063 - 20.213)$

$= 76.396$

For 2^{nd} year

$$I = 2 + 0.1 \times Y_{-1}$$
$$= 2 + 0.1 \times 101.065$$
$$= 12.1065$$

$$Y = 45.45 + 2.27(X + G)$$
$$=$$

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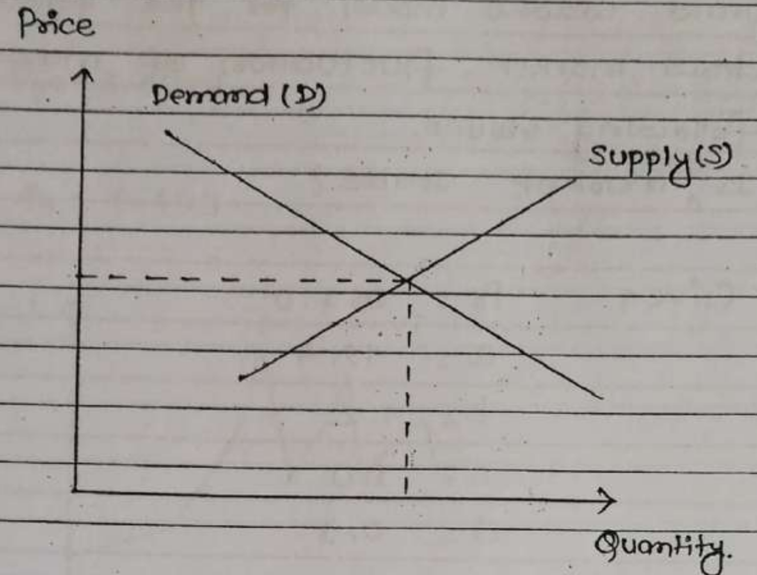
CobWeb Model



→ Consider the static mathematical model for market, demand and supply model. We have,

$$D = a - bp$$
$$S = c + dp$$
$$D = S$$

→ Generally, supply should be dependent on demand from previous marketing period.

→ But the demand however will respond to current price.

So, writing the market model in distributed lag model, we get

$$D = a - bp$$
$$S = c + dp_{-1}$$
$$D = S$$

→ So, given an initial value of price, P_0. The value of 'S' can be calculated. This determines the value of D.

→ Model of this type are called cobweb model because when model is solved graphically, the combinations of spiral, supply and demand curve take from of Cobweb shape.

Q. Draw Cobweb model for the following market. Also, draw market fluctuation of market price for the following values.
Is the market stable?

⇒ Given $P_0 = 1.0$

$a = 12.4$

$b = 1.2$

$c = 1.0$

$d = 0.9$

The market model is given as

$$D = a - bp$$
$$S = c + dp_{-1}$$
$$D = S$$

At equillibrium $S = D$

$$c + dp_{-1} = a - bp$$

$$c + dp_0 = a - bp_1$$
$$\boxed{dp_0 + bp_1 = 11.4}$$
$$1.0 + 0.9 \times 1.0 = 12.4 - 1.2 \times p_1$$

$$\therefore \quad p_1 = 8.75$$

Now, for P_2

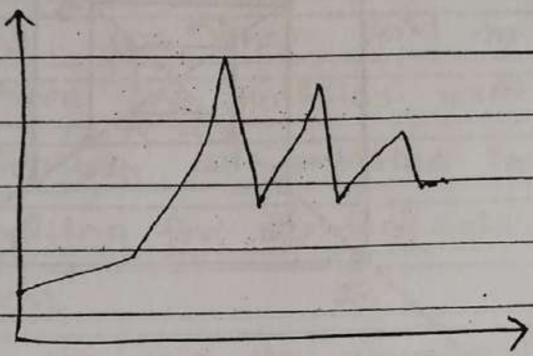
$$dp_1 + bp_2 = 11.4$$

$$P_2 = 2.9375$$

For P_3, $P_8 = 7.296$

For P_4, $P_4 = 4.0273$

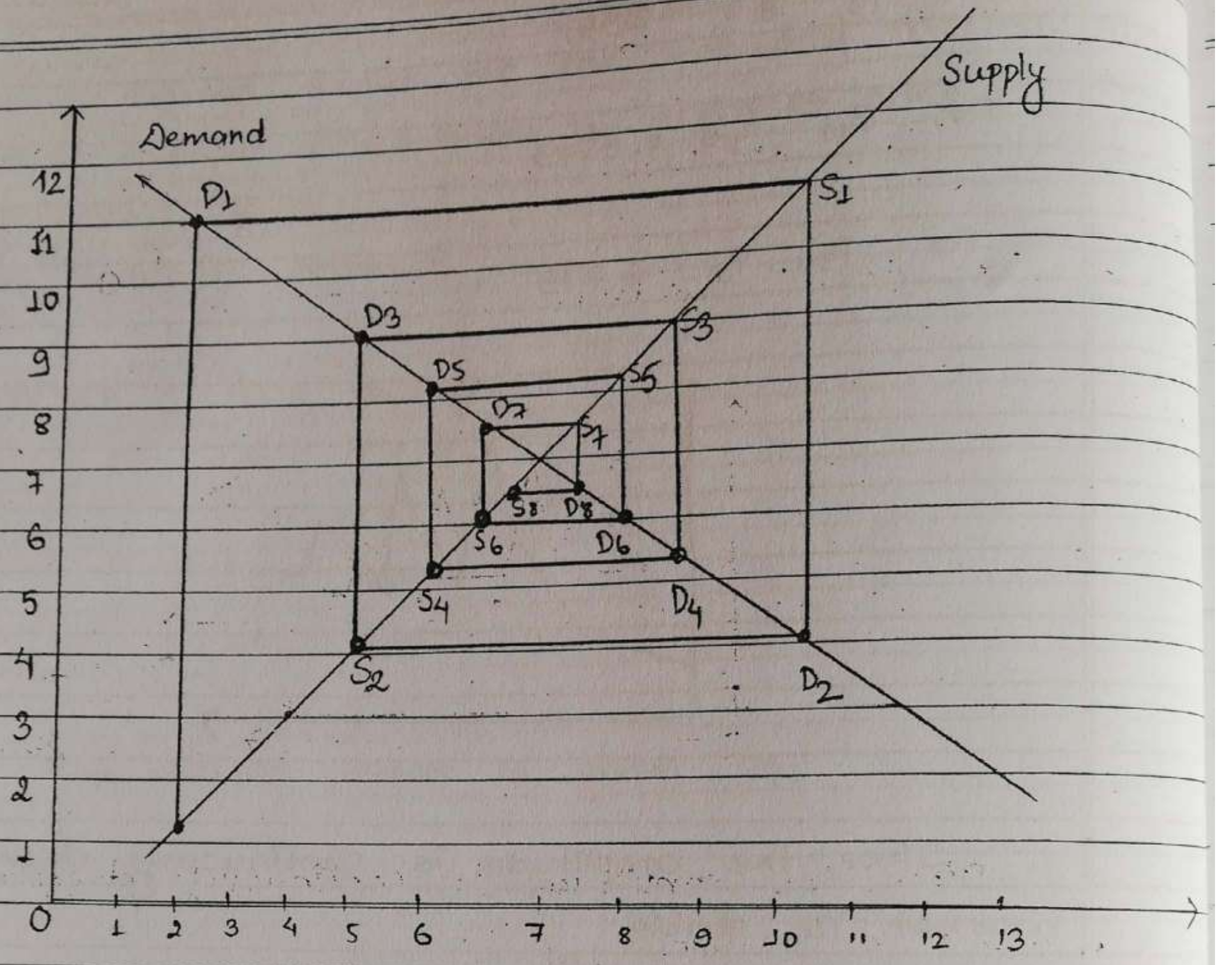
For P_5, $P_5 = 6.479$



Since, the amplitude is continuously decreasing, the market is stable.

Now,

Price	Supply	Demand
$P_0 = 1.0$	$1+0.9\times 1 = 1.9$	$D_0 = 11.2$
$P_1 = 8.75$	$1+0.9\times 8.75 = 8.875$	$D_1 = 1.9$
$P_2 = 2.9$	$1+0.9\times 2.9 = 3.61$	$D_2 = 8.83$
$P_3 = 7.3$	$1+0.9\times 7.3 = 7.56$	$D_3 = 3.64$
$P_4 = 4.62$	$1+0.9\times 4.62 = 5.158$	$D_4 = 7.6$
$P_5 = 6.5$	$1+0.9\times 6.5 = 6.85$	$D_5 = 4.6$
$P_6 = 4.64$	$1+0.9\times 4.64 = 5.176$	$D_6 = 6.83$
$P_7 = 6.01$	$1+0.9\times 6.01 = 6.409$	$D_7 = 5.19$
$P_8 = 4.95$	$1+0.9\times 4.95 = 5.455$	$D_8 =$



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Queuing System

→ Most systems of intrest in a simulation study contain a process in which there is a demand for services.

→ The system may service entities (customer, packets) at a rate which is lesser than the rate at which the entities arrive.

→ The entities are then said to join in waiting line. The line where the entities wait is called queue.

→ The combination of all entities in system being served and being waiting for service will be called as queuing system.

Queuing results due to congenstion in the system.

Problem in Queuing System

→ A queuing problem is essentially a problem of balancing the cost of waiting time against the cost of idle time for services facilities in the system.

→ This balance requires an analysis of the queuing system like idle time, average waiting time, queue length etc.

→ The problem arises due to stocastic nature of time between the arrival of customers as well as time it takes to serve the each customer.

→ This problem can be solved with the help of Simulation.

Elements / characteristics of Queuing System

:

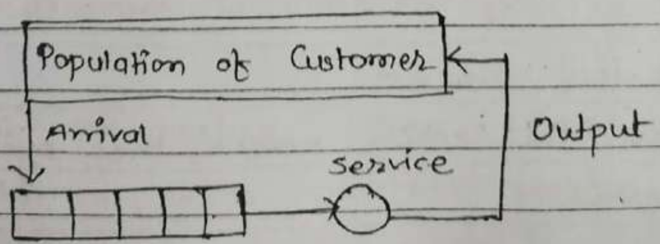


Fig: Single Server Queuing Model.

→ In the above figure, customer can be student waiting for registration number, packets waiting for route in router etc.

→ If the customer after arriving can enter the system center, there is no problem. Otherwise they have to wait for service and form a queue.

→ They remain in queue until they are provided the service.

→ But sometimes, the queue become too long, at that time they will leave the queue without being served.

→ In order to model a queuing system, we need to have indepth about what constitutes a queuing system.

→ There are three basic elements common to all queuing system.

i) Arrival Pattern

→ In a queuing system, before entities can be processed or subjected to waiting, they must enter the system.

→ Depending upon the environment, entities can arrive smoothly or in unpredictable form.

→ They also can arrive one at a time or in batch.

→ They arrive independently.

→ So, arrival pattern simply defines the way how entities enters into a system.

Eg:- A special arrival process which is useful for modelling process is poisson's arrival process.

ii) Service Process

→ Once the entity enters the system, they must be served.

→ An entity who arrives and finds the server idle enters the service immediately and service time S_1, S_2, ... S_n of the successive entities are independent.

→ An entity who arrives and finds the server busy joins the end of the queue.

→ Then the entities are processed in FCFS order or according to some kind of priority rule.

iii) Queuing Discipline

→ It represents the way, queue is organized or is the order in which service is provided such as

* ~~FIFO~~ LIFO

According to this rule, the service is offered to the customer who has arrived recently.

* ~~LIFO~~ FIFO

According to this rule, the service is offered ~~to the customer~~ on the basis of arrival time of customer. A customer who comes first gets the service first.

* SIRO (Serve in random order)

Here a customer is randomly picked up from the waiting queue for service.

* Priority

A special number called priority is assigned to each customer waiting in queue. Then according to this number, customer is choosen for service.

* SPTF (Shortest processing Time First)

Here customer with shortest service time ~~is~~ will be choosen for the service.

Assignment 2.

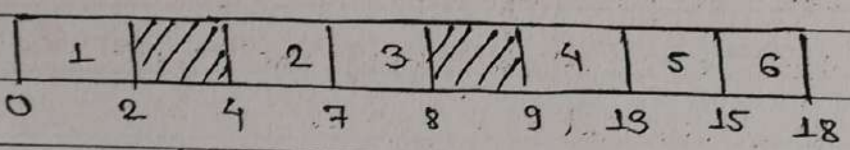
Q. Explain the poisson's arrival process in details.

i.v. imp

Q. Calculate the following parameter for SSQM when inter-arrival time and service time are given.

Customer	Inter-Arrival time	Arrival Time	Service Time
1	–	0	2
2	4	4	3
3	1	5	1
4	4	9	4
5	2	11	2
6	4	15	3

⇨ Gantt chart of above table:-



Time Service begin	Waiting in Queue	Time service end	Time Customer spends in System	Idle time in queue
0	0	2	0	0
4	4-4=0	7	3	0
7	7-5=2	8	3	2
9	9-9=0	13	4	0
13	13-11=2	15	4	1
15	15-15=0	18	3	0
	Total = 4		Total Time spent = 19	

① Average waiting time

$$= \frac{\text{waiting in queue for each customer}}{\text{total no. of customer}}$$

$$= \frac{4}{6} \text{ time units}$$

② Probability that the customer has to wait in queue.

$$= \frac{\text{No. of customer who has to wait}}{\text{total no. of customer}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

③ Probability of idle server

$$= \frac{\text{total runtime of idle server}}{\text{total runtime of simulation}}$$

$$= \frac{3}{18} = \frac{1}{6}$$

④ Probability of busy server

$$= 1 - \text{probability of idle server}$$

$$= \frac{5}{6}$$

⑤ Average service time

$$= \frac{\text{total service time}}{\text{total no. of customer}}$$

$$= \frac{15}{6} = \frac{5}{2}$$

⑥ Average waiting time

$$= \frac{\text{total waiting time}}{\text{no. of customer who waits}}$$

$$= \frac{4}{2} = 2$$

⑦ Average time customer spends in system.

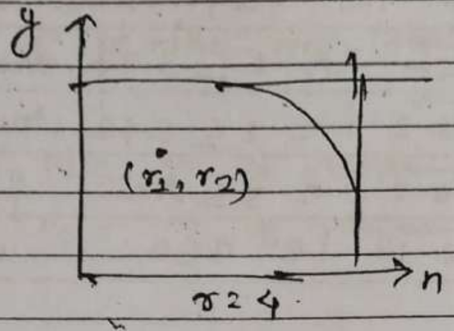
$$= \frac{\text{total time customer spends in system}}{\text{total no. of customers}}$$

$$= \frac{19}{6}$$

$= 3.1667$

Find the value of pi(π) using monte carlo method.

Let us consider a quadrant of unit circle of radius '4' as



According to Monte Carlo Method

$$\frac{\text{Area under curve}}{\text{Area under rectangle}} = \frac{\text{No. of dots inside curve}}{\text{No. of dots inside rectangle}}$$

$$\frac{\pi}{4} = \frac{m}{N}$$

Suppose, we have $N=5$ random dots (x,y) such that

$0 \le x \le 4$

$0 \le y \le 4$

$(x,y) = (0,0)$ $\quad$ $(x_3, y_3) = (2,2)$

$(x_4, y_4) = (3,3)$

$(x_5, y_5) = (4,4)$

Now the equation of circle can be written as

$$x^2 + y^2 \le 4^2$$

$y^2 = 16 - x^2$ putting random value

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(x, y) in above eqn

we get

$0 \le \sqrt{16}$ i.e. $0 \le 4$, true so accepted.

$1 \le \sqrt{16-1}$ i.e. $1 \le 3.87$ true so accepted.

$2 \le \sqrt{16-2}$ i.e. $2 \le 3.46$ true so accepted.

$3 \le \sqrt{16-3}$ i.e. $3 \le 2.45$ false so rejected.

$4 \le \sqrt{16-16}$ i.e. $4 \le 0$ " " "

Since the accepted value ~~lines~~ lies inside the curve and rejected values lies outside the curve, we get:

$$x = 3 \text{ and } N = 5$$

Now,

$$\frac{\pi}{4} = \frac{x}{N}$$

i.e.

$$\pi = \frac{m}{N} \times 4$$

$$= \frac{3}{5} \times 4$$

$$= 2.4$$

$\pi = 3.14$

$$\text{Error} = \left| \frac{3.14 - 2.4}{3.14} \right| \times 100\%$$

$$= \frac{0.74}{3.14} \times 100\%$$

$$= 23.14\ \%$$

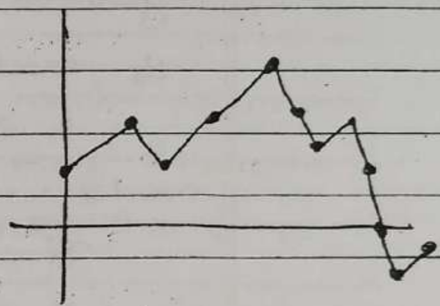
Random Walk Problem.

Let 'N' be the steps of equal length be taken along the time. Let 'p' be the probablity of taking steps to right and 'q' be the prob (taking steps to left)

n_1 = no. of steps to right
n_2 = no. of steps to left.

The quantity (p, q, n_1, n_2) can be related by

$$p + q = 1$$
$$n_1 + n_2 = N$$



ex:- Flip a coin and take steps

1. flip coin, if Head move right.
   , if tail move left.

DRUNKARD WALK PROBLEM

It's a type of random walk problem, the algorithm states

1. pick a random points in grid and mark it empty.
2. choose a random direction, North, East, South west.
3. move in that direction and mark it empty.
4. Repeat step 2 & 3 until you have emptied as many grid as desired.
5. steps drunkart took in direction North, East, East, South, East, East

Types of System Simulation.

We distinguish between continuous and discrete system as being in which smooth or sudden changes occur. The distinction between continuous and discrete model is not made during the classification of model. Distinction was not made ~~during the~~ because this distinction whether mathematical or analytical technique will applied to the model. However this distinction become important when it is decided to simulation. The general computational tehniques used with two kind of model differ significantly.

1. Numerical computational technique for discrete model.

To illustrate the general computation of simulation in discrete model consider the below example.

An office person begins his day, work with the pile of document to be processed, He works the pile, beginning each document as soon as he finishes previous one. Except that he takes 5 min break. If at the time he finishes ~~previous~~ ~~Except that he takes 5 min break. If at~~ document it is an hour or more, since the beginning to work or since he had a break. Suppose there are five documents each with processing time, 45, 16, 5, 29, 33. Explain how simulation takes place to calculate the total time required to finish the entire processing.

⇒ The numeric computation ~~cab b~~ can be organized as below.

Doc no	Start time	Processing Time	Finish time	Commutative time	Break flag	Remaining Job
0	0	45	43	45	0	4
1	43	16	61	61	1	3
2	66	5	71	5	0	2
3	71	29	100	34	0	1
4	100	33	133	67	1	0

⇒ In the above model diffi, attributes included are document number - indicate documents. Currently being processed start time - beinging time of processing.

Processing time:- Time require to work on each document.
Finish time:- Time require to finish each document.
Commutative time:- Sum of time since work started or last break.
Break flag:- value '1' denote, office person should break before processing next document otherwise no.

Remaining no. of job:- Initially hold the maximum no. of job or document to be processed and decremented with each document processing completes until zero.

So from the above solution we can conclude that the total time required to finish processing of five documents required is 133 mins/ time units.

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Numerical Computation Technique for continuous models

To illustrate this technique, Let us consider below example.

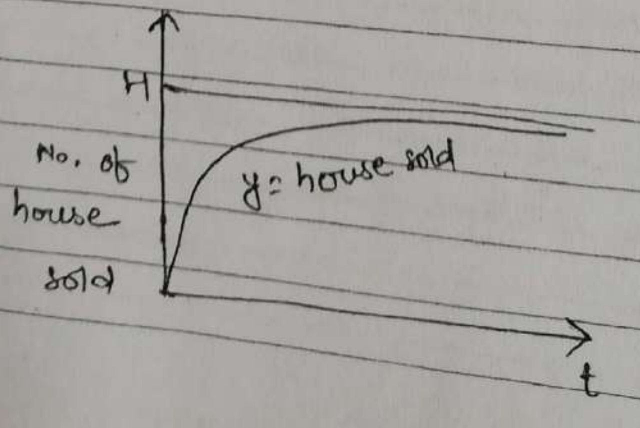
A builder observe that the rate at which he sell house depends directly upon the no. of families

In the above figure, Let 'H' be the potential no. of families with of house sold and 'y' be the no. of families with house. The horizontal line 'H' represent that potential market for house and 'y' rate at which 'y' increases i.e. slope of curve decreases as the distance 'H-y' gets less. So mathematically the trend can be expressed as $\frac{dy}{dt} \propto H-y$

$$y = k_1(H-y) \quad \text{——} \quad (1)$$

To solve, the problem using simulation we follow step by step procedure ; Suppose computation is made at uniform interval of time (Δt) and calculation be proceded to 'Hi'.

So rate of change of 'y' can be interpreted as amount of change per unit time.



i.e. rate of change of 'y' = $\frac{\Delta y_i}{\Delta t}$

can be interpreted as amount of change per unit time

i.e. rate of change of 'y' = $\frac{\Delta y_i}{\Delta t}$

So from ① we get

$$y' = k_1(H - y)$$

$$\frac{\Delta y_i}{\Delta t} = k_1(H - y) \longleftarrow (ii)$$

So, Repitition of calculation for each new value of 'y' produces the output at the end of next interval. So simulation output is the series of line segment approximately the curve

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Comparision of Simulation and Analytical method.

Analytical Method	Simulation Method.
1. If the model is simple, it may be possible to work with relationship and quantities to get an exact analytical method.	1. Most real world system are too complex to allow to be evaluated analytically. In this case we study the model by means of simulation.
2. Analytical method are expensive and time consuming.	2. The model uses computer to evaluate a model numerically i.e. data are gathered in order to estimate the desired true property.
3. Analytical method are expensive and time consuming than analytical method.	3. Simulation give result fa
4 Analytical method gives general solution.	4. Simulation gives specific solution.
5. Analytical method provides good accuracy.	5. Simulations model is just an approximation where we compromise accuracy.
6 There is limited problem that can be solved. analytically.	6. Simulation is an extension of analytical method

Experimental Nature of Simulation

⇒ Simulation technique make no specific attempt to isolate the relationship between any particular variables instead it observe the way in which all variable of the model changes with the time. The relationship between the variable must be ~~derived from~~ derivable from the observation. Many simulation runs have to be made to understand the relationship between the variable must be derived from the relationship between the variable must be derived from the observation. Many simulation runs have to be made to understand the relationship involved in the system. So the use of simulation in between 'o', 'k' and 'm' would had to be discovered in vehicle suspension system by observing the value of result in the motion being non oscillatory.

Steps in Simulation.

Step 1, Define the problem.

1.1 An initial set up is to describe problem to be solved in concized manner. The description must be enough to answer the question asked and what need to be taken in order to answer the question.

Step 2, Define the model

⇒ Based on the problem definition, a model must be defined.

Step 3, Simulate

After defining a model we have to decide

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either to use simulation or analytical method to solve the problem. If the problem can be solved analytically then solve problem using analytically method and stop. If the problem can not be solved analytically redefine the model and solve numerically i.e. perform simulation.

Step 4, Plan the study.

When it is decided to simulate we must plan the study by deciding the major parameters to be varied, the no. ~~the~~ of cases to be conducted and the order in which the runs are to be made.

Step 5, Write a program

Simulation is done by digital computer, hence program must be written.

Step 6, Validate model;

The model must be valid before the begining of major sets of runs.

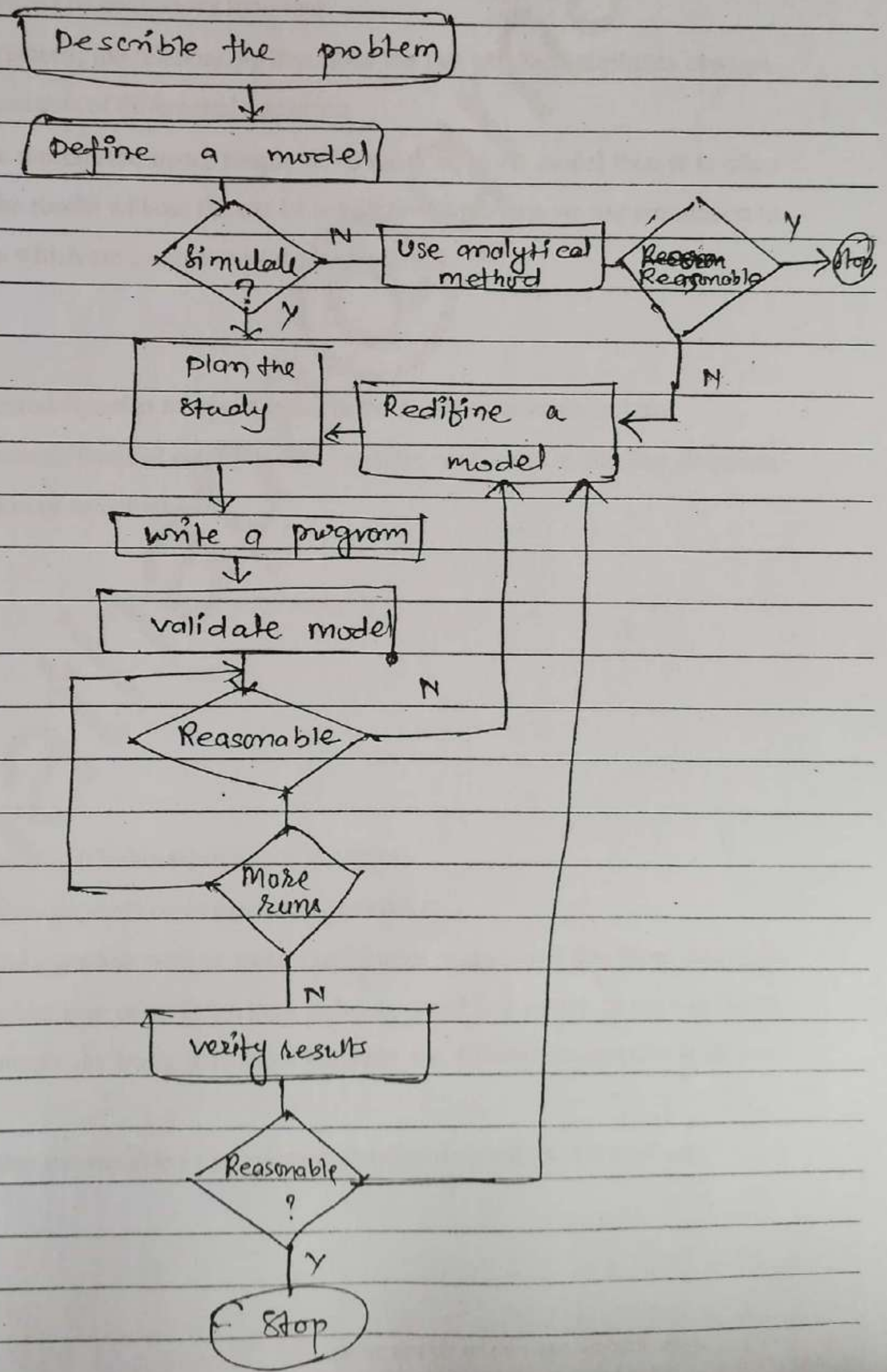
Step 7, Run the model.

The stage will then move into the stage of executing a series of runs according to study plan. It is essential to repeat the run with different set of random number so that more than one sample result is available.

Step 8, verify result.

After making multiple runs, we get various samples and those samples must be verified. If the approximation is found, stop the process otherwise redifine the model and repeat the entire process.

Steps in simulation



Chapter 3

Continuous System Simulation

Introduction

- A continuous system simulation is one, in which predominant activities of the system cause smooth changes in the attributes of the system entities.
- When such system is modeled mathematically, the variable of the model representing the attributes are controlled by continuous functions.
- In general, in continuous, the relationship describes the rate at which attributes changes, so that the model consists of differential equations.
- If a system can be represented using simple differential equation model then it is often possible to solve the model without the use of simulation, otherwise we use simulation to solve those models which are complex to solve analytically.

Differential Equation

- We can use differential equation to represent the behavior of continuous system.
- An example of linear differential equation with constant coefficient is one that describes the wheel suspension of an automobile.
- The equation is:

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

Where, $\ddot{x}$ = acceleration

$\dot{x}$ = velocity

x = displacement

K = stiffness of spring

D = measure of viscosity (thickness) of shock absorber

$F(t)$ = input of system depends on independent variable t

- A **linear differential equation** with constant coefficients is always of this form, although derivatives of any order may other form, such as being raised to a power, or are combined in any way- for example, by being multiplied together, the differential equation is said to be **non-linear.**
- Linear just means that the variable in an equation appears only with a power of one.

- x is linear but x^2 is non-linear.
- The theory for solving linear equations is very well developed because linear equations are simple enough to be solvable.
- Non-linear equations can usually not be solved exactly and are the subject of much on-going research

$x'' + x = 0$ is linear

$x'' + 2x' + x = 0$ is linear

$x' + 1/x = 0$ is non-linear because $1/x$ is not a first power

$x' + x^2 = 0$ is non-linear because x^2 is not a first power

- When more than one independent variable occurs in a differential equation, the equation is said to be **partial differential equation**.
- It can involve the derivatives of same dependent variable with respect to each of the independent variables.
- An example is an equation describing the flow of heat in a three dimensional body. There are four independent variable representing the three dimensions and time, and one dependent variable, representing temperature.
- **Differential equation occurs repeatedly in scientific and engineering studies. The reason for this prominence is that most physical and chemical process involves rates of change, which require differential equations for their mathematical description.**
- Since a differential coefficient can also represent a growth rate, continuous models can also be applied to a problem of a social or economic nature where there is a need to understand the general effect of growth trends.

Analog Computers

- Analog computers are generally used to solve continuous model but sometimes are also used to solve static models.
- Some device whose behavior is equivalent to a mathematical operation such as adder or integrator etc. is combined together in a manner specified by a mathematical model of a system to allow the system to be simulated.

- That combination which is used in the simulation of a continuous system is referred as **analog computer** or when they are used to solve differential equation they are referred to as **differential analyzer**.
- The most widely used form of analog computers is the electronics analog computers based on the **operational amplifiers.**
- Voltages in the computers are equated to mathematical variables and the operational amplifiers can add, integrate etc. the voltage.
- With appropriate circuits, an operational amplifier can be made to add several input voltages, each representing a variable of model, to produce a voltage representing the sum of the input variables.
- Different scale factors can be used on the input to represent the coefficient of the model equations. Such amplifiers are called summer.
- Another circuit arrangement produces an integrator for which the output is the integral with respect to time of single input voltage or the sum of several input voltages.
- All voltages can be positive or negative to correspond to the sign of the variable represented.
- To satisfy the equation of the model, it is sometime necessary to use a sign inverter.

Advantage of Analog Simulation:

1. Analog computers have higher speed of solution than that of digital simulation.
2. Analog simulations have direct access to an immediate display of computer result.
3. Analog simulation is more natural in sense so, can reflect system structure and interpret the results.

Disadvantage of Analog Simulation:

- Though analog computers came into existence much before the digital computers, and have played a major role in the simulation of continuous dynamic system, but they are giving way to digital computers at the fast pace.
- Following are some of the important disadvantages of digital computers.

a) Limited Accuracy: The result obtained from analog simulation has limited accuracy, while high accuracy in the result can be obtained by employing digital simulation. Thus, where, high accuracy in results is required, as in space vehicles, guided missiles and fusion, the analog method cannot be used.

b) Magnitude scaling: in analog simulation, values of various variables are represented by voltages, which have fixed and limited range. The variables must remain within the limited range, otherwise the result become inaccurate. Thus, all progtam variables have to be scaled, so that none exceeds the voltage range. This is a very difficult task, especially when the number of variables is large, and the range of their variation is not known. On the other hand, in digital computers, the problem of magnitude scaling does not raise, as they have a very large range. With floating point arithmetic, they have a very large precision. Hence, no magnitude scaling is normally required in digital simulation.

c) Hardware set up required: in analog simulation, hardware elements have to combine to simulate the system, they have to be tested and calibrated while no such set up is required in digital simulation. Switching from one simulation to other takes time in analog simulation, while it requires no time in digital simulation. A simulation program on a digital computer can be easily stored, for use.

d) The output of analog simulation is not understood by general people.

e) The analog computer is usually dedicated to one application at a time hence is not flexible.

f) It has lack of memory.

Components of Analog Computer/Analog Methods:

The general methods by which analog computer are applied can be demonstrated using the second order differential equation given as:

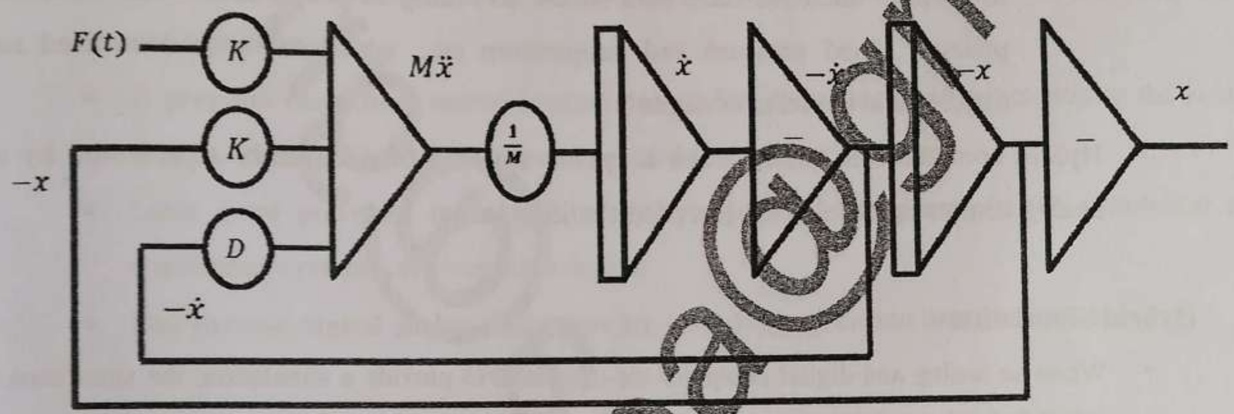
$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

Solving the equation for the highest order derivate gives

$$M\ddot{x} = KF(t) - D\dot{x} - Kx$$

Suppose a variable representing the input $F(t)$ is supplied, and assume for the time being that there exist variables representing $-x$ and $\dot{x}$. These three variables can be scaled and added with a summer to produce a voltage representing $M\ddot{x}$. Integrating this variable with a scale factor of $1/M$ produce $\dot{x}$. Changing the sign produce $-\dot{x}$, which supplies one of the variables initially assumed; and further integration produces $-x$, which was other assumed variables. For convenience, a further sign inverter is included to produce $+x$ as an output.

A block diagram to solve the problem in this manner is shown below. The symbols used in the figure are standard symbols for drawing block diagrams representing analog computer arrangements. The circle indicates scale factors applied on the variable. The triangular symbol at the left of the figure represents the operating of the adding variables. The triangular symbol with vertical bar represents integration, and the containing a minus sign is a sign changer.



Note: See class not for more examples

Hybrid Computers:

- The scope of analog computers has been considerably extended by developments of solid-logic electronic device (electronic equipment using semiconductor devices such as semiconductor diodes, transistors, and integrated circuits (ICs).
- Analog computers always used a few non-linear devices (like multiplier), which were expensive to make.
- So solid-logic device has made such non-linear cheaper & easier to obtain and also improves the design and performance.
- Digital operations are storing values, switching, and performing logical operation.
- Hybrid computer is a digital computer that accepts analog signal, converts them to digital and process them in digital form.

- Hybrid computer may be used to simulate system that are mainly continuous and also have some digital elements i.e. Hybrid computers are used mainly in specialized applications where both discrete and continuous data need to be processed
- For Example:
  - An artificial satellite for which both the continuous equation of motion and the digital controls signal must be simulated.
  - A petrol pump contains a processor that converts fuel flow measurements into quantity and price values.
  - In hospital Intensive Care Unit (ICU), an analog device is used which measures patient's blood pressure and temperature etc., which are then converted and displayed in the form of digits
- Hybrid computer are useful when a system that can be adequately represented by an analog computer model is subject of a repetitive study.

Hybrid Simulation

- When an analog and digital computer are combined to provide a simulation, the simulation is **called hybrid simulation.**
- System can be simulated as interconnection of continuous and discrete subsystem which can be best modeled by an analog and digital computer being linked together.
- In this approach to a continuous simulation clock is maintained and updates to the model are event-driven i.e. each event is represented by an occurrence in simulated time and must be scheduled to occur when a proper set of conditions exist.
- High speed converters are needed to transform signals from one form of representation to another.
- It is generally reserved for the case in which functionally distinct analog and digital computers are linked together for the purpose of simulation.
- Hybrid simulation may be used to simulate systems that are continuous, but don, in fact, have some digital elements.

Example:

- A petrol pump contains a processor that converts fuel flow measurements into quantity and price values.

- **Analog-Digital Simulators**
  - To avoid the disadvantage of analog computers, many digital computer programming languages have written to produce digital analog simulator.
  - These allow a continuous model to be programmed on a digital computer in a same way as it is solved on an analog computer.
  - These languages contain macro instructions that carry out the action of adders, integrators and sign chargers.
  - A program uses these macro-instructions to link them together in essentially the same way as operational amplifiers are connected in analog computers.
  - Later more powerful techniques of applying digital computers to the simulation of continuous system have been developed.
  - Due to these digital analog simulators are not now in common uses.

CSSL

- Confining a digital computer to routines that represents a function of analog or hybrid computer, as is done with digital analog simulator, this is clearly a restriction.
- To overcome the restriction, a number of continuous system simulation languages (CSSL) have been developed.
- They use the familiar statement type of input for digital computer, allowing a problem to be programmed directly from the equation broken into functional elements.
- A CSSL can include macros or subroutine which performs function of specific analog element.
- It also includes variety of algebraic and logical expression to describe the relationship between variable.
- Example, Continuous System Modeling Program (CSMP-3).

CSMP III

- This program is constructed from three general type of statement:

1) Structural Statement
   - This statement defines the model which consists of FORTRAN like statements and functional block designed for operation that frequently occur in a model definition.
   - This statement can make the use of operation of addition, subtraction, multiplication, division and exponent.

For example:

$X = 6Y / 10 + (Z-2)^2$

Fortran statement: 6.0 * Y / 10.0 + (Z – 2.0) * * 2.0

Real constants have decimal notations. Exponential notations can also be used.

Example: 1.2e-4

$= 1.2 * 10^{-4}$

=0.00012

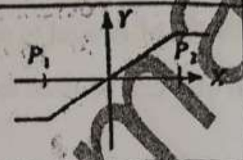
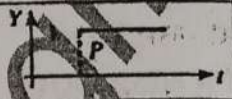
i.e. 1.2e-4 represents 0.00012

Fixed value need declaration, variable name must be six character or less.

Example:

PI=3.14159

Functions used in CSMP

GENERAL FORM	FUNCTION
Y = INTGRL (IC, X) Y (0) = IC INTEGRATOR	$Y = \int_0^t X dt + IC$
Y = LIMIT (P_1, P_2, X) LIMITER	$Y = P_1 \quad X < P_1$ $Y = P_2 \quad X > P_2$ $Y = X \quad P_1 \leq X \leq P_2$ 
Y = STEP (P) STEP FUNCTION	$Y = 0 \quad t < P$ $Y = 1 \quad t \geq P$ 
Y = EXP (X) EXPONENTIAL	$Y = e^X$
Y = ALOG (X) NATURAL LOGARITHM	$Y = \ln(X)$
Y = SIN (X) TRIGONOMETRIC SINE	$Y = \sin(X)$
Y = COS (X) TRIGONOMETRIC COSINE	$Y = \cos(X)$
Y = SQRT (X) SQUARE ROOT	$Y = X^{1/2}$
Y = ABS (X) ABSOLUTE VALUE (REAL ARGUMENT AND OUTPUT)	$Y = \|X\|$
Y = AMAX1 (X_1, X_2 . . . X_n) LARGEST VALUE (REAL ARGUMENTS AND OUTPUT)	$Y = \max(X_1, X_2, \ldots, X_n)$
Y = AMIN1 (X_1, X_2 . . . X_n) SMALLEST VALUE (REAL ARGUMENTS AND OUTPUT)	$Y = \min(X_1, X_2, \ldots, X_n)$

2) Data Statement

- These statement designs the numerical values to parameters, constant and initial conditions.
- ICON can be used to set the initial value of the integration block function.
- CONST can be used to set values for constant.
- PARAM is used to set parameters for individual.
- It also can be used to set series of numeric values for one parameter.

Example:

CONST A =0.5, XDOT = 1.25, YDOT=6.22

PARAM D=(0.25, 0.50, 0.75, 1.0)

3) Control Statement

- This statement specifies the option in the execution of the program and the choice of the output.
- TIMER is used to specify the timer interval.
- Example:

  TIMER DELT = 0.005, FINTIM=1.5, PRDEL = 0.1, OUTDEL = 0.1

  Where DELT= Integration interval

  FINTIM = Finish time

  PRDEL= Interval at which print results

  OUTDEL= Interval at which to print plot

- If printed or print-plotted output are required then control statement with the words PRINT and PRTPLT are used, followed by the name of variables to form the output.
- TITLE and LABEL can be used to put headings on the output.

- CSMP contain basic three segments
  1. Initial: executed at beginning
  2. Terminal: executed at end of simulation

3. Dynamic: heart of program where integration takes place

Example:

1. CSMP program for automobile suspension system, for given M=2.0, F=1 and K =400 and there will be different runs with different value of D as specified

Solution:

The model for automobile suspension system is represented by differential equation

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

The above equation can be written as

$$M\ddot{x} = KF(t) - D\dot{x} - Kx$$

$$\ddot{x} = \frac{KF(t) - D\dot{x} - Kx}{M}$$

CSMP program

```
TITILE AUTOMOBILE SUSPENSION SYSTEM
CONST M=2.0, F=1.0, K=400.0
PARAM D = (5.656, 16.968, 39.592, 56.56)        //initial
X2DOT=(1.0 / M) * (K * F – K * X – D * XDOT)
XDOT=INTGRL (0. 0, X2DOT)                       //dynamic
X=INTGRL (0. 0, XDOT)

TIMER DELT=0.005, FINTIM=1.5, PRDEL=0.05     //terminal
PRINT X, XDOT, X2DOT
LABEL Displacement vs time
END
STOP
```

i. Write a CSMP program for solving the differential equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t)$$

Soln:

The above eqn can be written as

$$\frac{d^2q}{dt^2} = \frac{1}{L}\left(E(t) - R\frac{dq}{dt} - \frac{q}{C}\right)$$

CSMP Program

```
TITLE Electrical System
CONST L=35.0, R=5.0, C=2.0, E=1.0
Q2DOT =(1.0/L)*(E-R*Q1DOT-1.0/C*Q)
Q1DOT = INTGRL (0.0, Q2DOT)
Q = INTGRL(0.0, Q1DOT)

TIMER DELT=0.005, FINTIM=1.5, PRDEL=0.5
                           OUTDEL=0.5
PRINT Q, Q1DOT, Q2DOT
PRTPLT Q
LABEL charge vs time
END
STOP
```

ii. Write CSMP program to solve the following differential equation.

$2x'' + 3x' - 15x = 60$

Soln:

The above equation can be written as

$2x'' = 60 - 3x' + 15x$

or $x'' = \frac{1}{2}(60 - 3x' + 15x)$

CSMP program:

```
TITLE   csmp demo
x2DOT = (1.0/2.0) * (60.0 - 3.0 * x1DOT + 15.0*x)
x1DOT = INTGRL(0.0, x2DOT)
x = INTGRL(0.0, x1DOT)

TIMER   DELT 0.005, FINTIM = 1.5,
        PRDEL = 0.05, OUTDEL = 0.05

PRINT   x, x1DOT, x2DOT
PRTPLT  x
LABEL   output
END
STOP
```

Feedback System:

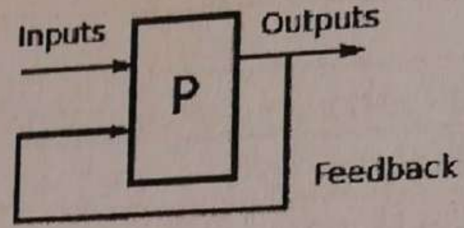


Fig: Feedback System

- A significant factor in the performance of many systems is that a coupling occurs between the input and output of the system.
- Feedback describes the situation when output from (or information about the result of) an event or phenomenon in the past will influence an occurrence or occurrences of the same (i.e. same defined) event / phenomenon in the present or future.
- Feedback occurs when outputs of a system are routed back as inputs as part of a chain of cause-and-effect that forms a circuit or loop.
- The system can then be said to feed back into itself.
- The notion of cause-and-effect has to be handled carefully when applied to feedback systems:
- A home heating system controlled by a thermostat(sensor) is a simple example of a feedback system. The system has a furnace(heater) whose purpose is to heat a room, and the output of the system can be measured as room temperature. Depending upon whether the temperature is below or above the thermostat (regulator) setting, the furnace (heater) will be turned on or off, so that information is being fed back from the output to the input. In this case, there are only two states, either the furnace will on or off.
- Another example of Feedback system **is Automatic Aircraft System** (Auto Pilot System)

Continuous simulation Example:

1. **Predator- Prey Model:**

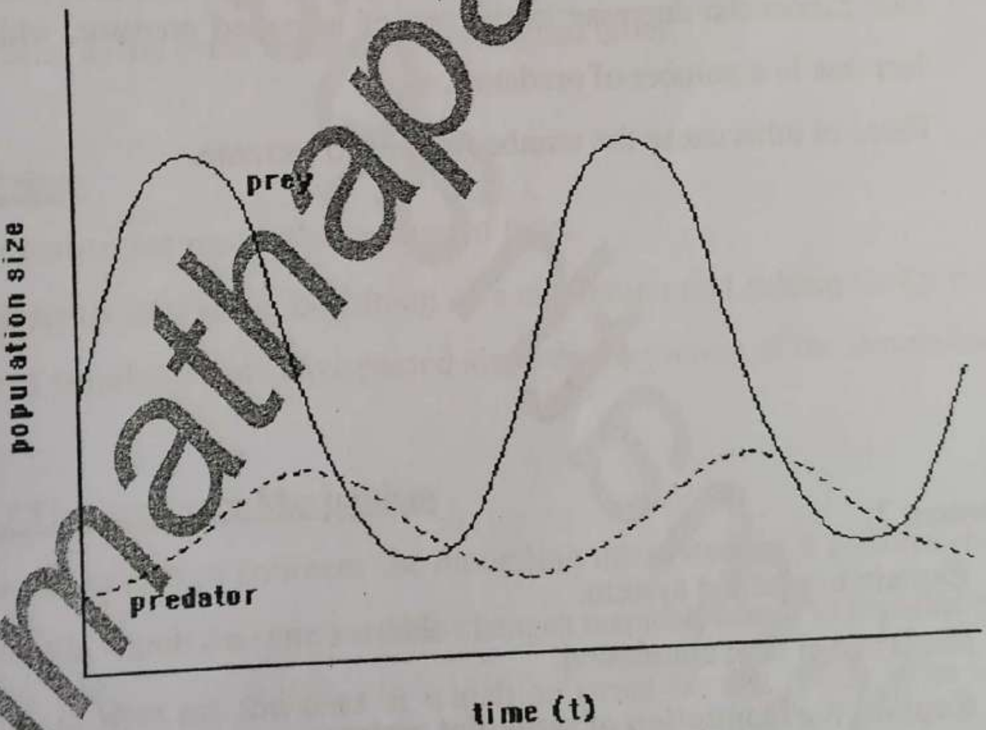
    - It is also called parasite-host model.
    - An environment consists of 2 populations, predator & prey.
    - Predator –Prey model is also a mathematical model.
    - The Prey is passive but the Predator depends on the Prey for their source of food.

    Let

    $x(t)$ = no. of prey population at time t.

    $y(t)$ = no. of predator population at time t.

    $rx(t)$ = rate of growth of prey for some positive 'r' where 'r' = natural birth rate and natural death rate.



- Because of the interaction between predator and prey, it will be reasonable to assume that the death rate of prey is proportional to the product of two population size $x(t).y(t)$ or death rate of prey is $a.x(t).y(t)$.
- Therefore the overall rate of change of prey population, dx/dt, is given by:

$$\frac{dx}{dt} = rx(t) - ax(t)y(t)$$

where a is positive constant of proportionality.

- Also the predators depend on their prey for their existence; the rate of change of predators in the absence of prey is $-sy(t)$ for some positive s.
- The interaction between the two populations causes the predator population to increase at a rate that is proportional to $x(t).y(t)$.
- Thus, the overall change of the predator population, dy/dt, is:

$$\frac{dy}{dt} = -sy(t) + bx(t)y(t)$$

where b is positive constant.

As the predator population increases, the prey population decreases.

This causes the decrease in the rate of increased predator, which eventually results in decrease in a number of predators.

These in turns cause the number of prey to increase.

Assignment 3:

1. Explain interactive system.
2. Explain real time simulation.
3. **Explain the simulation of autopilot system.**

Chapter 4

Discrete System Simulation

Discrete Events

- A simulation process will take system model through a series of state.
- We can describe the state of system at a given instant, since the model of system has a set of number to represent the state of a system.
- A number that is used to represent some aspect of some system state is called a state descriptor.
- As the simulation precedes the state descriptor changes their own values.
- Discrete event can be defined as a set of circumstances that cause an instantaneous change in one or more state descriptor of the system.
- Discrete event simulation concerns the modelling of the system as it evolves over time by a representation in which state variable changes instantaneously or separate point in time.
- It is also necessary that changes must be made sequentially.
- So a system simulation must contain a number, representing time.
- The simulation proceeds by executing all the changes to the system descriptors associated with each event, as the event occur in chronological order.

Representation of time

- It refers to number that record the passage of time.
- It is usually set to zero at the beginning of a simulation and subsequently indicates how many units of simulated time have passed since the beginning of the simulation.

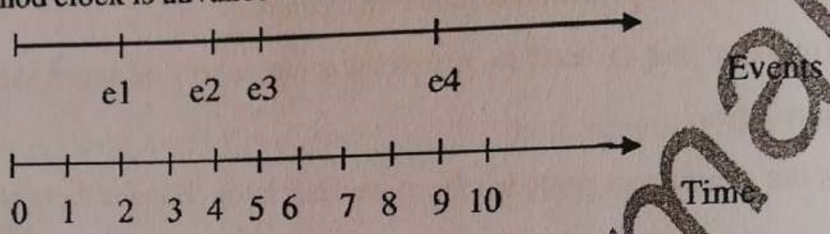
Simulation Time / Time advance Mechanism

- Discrete event simulation concerns the modelling the system as it evolves over time by a representation in which the state variable changes instantaneously at separate point in time.
- These points in time are the ones at which an event occurs. Event is an instantaneous occurrence that may change the state of system.
- It is necessary to keep track of current value of the simulated time as the simulation proceeds and also we need a mechanism to advance simulated time from one value to another.

- This is represented through simulation clock.
- Simulation clock refers to the integrated clock time. Note that it is not the time that a computer has taken to carry out the simulation.
- There are two basic methods used to update clock time

i) Event Oriented Method [Next Time Advance Mechanism]

- In this method clock is advance to the time at which the next event likely due to occur.

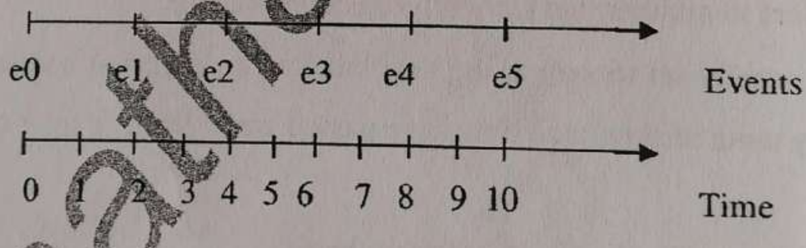


In the above figure we see that event e1, e2, e3, e4 occurs at time 2, 4, 5 and 9 time Units respectively.

- With the next time advance mechanism, the simulation clock is initialized at zero initially.
- A simulation clock is then advance to the time of occurrence of near happening events at which point the state of the system is updated.
- Clocks jump from one event time to next and doesn't exist for time between successive events.
- Discrete system simulation is usually carried out by using the event oriented method.

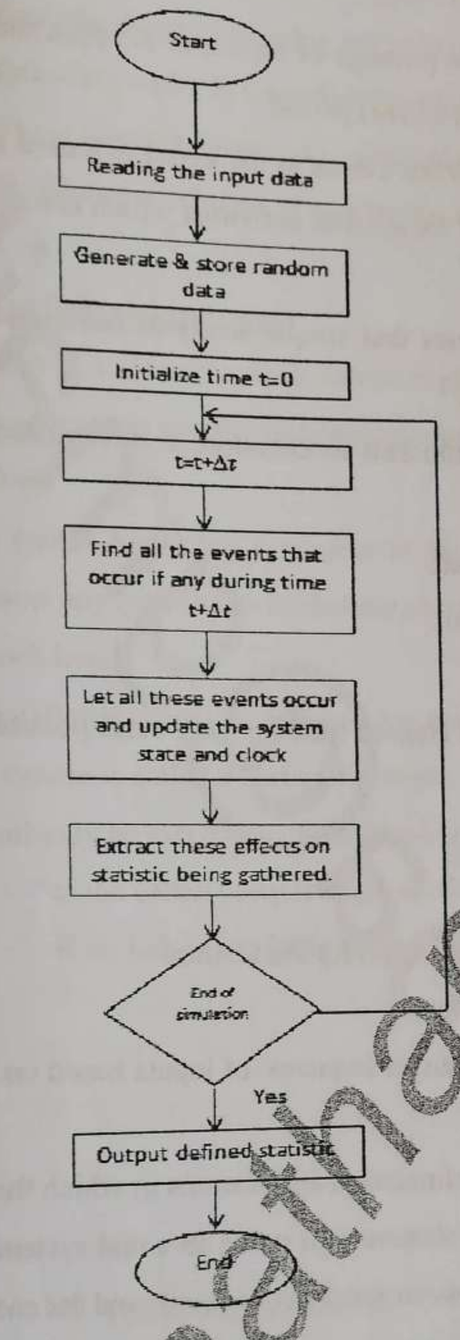
ii) Interval Oriented Method [Fixed Time Advance Mechanism]

- In this method, the clock is advance by small uniform intervals of time and determine at each interval whether an event is due to occur at that time.

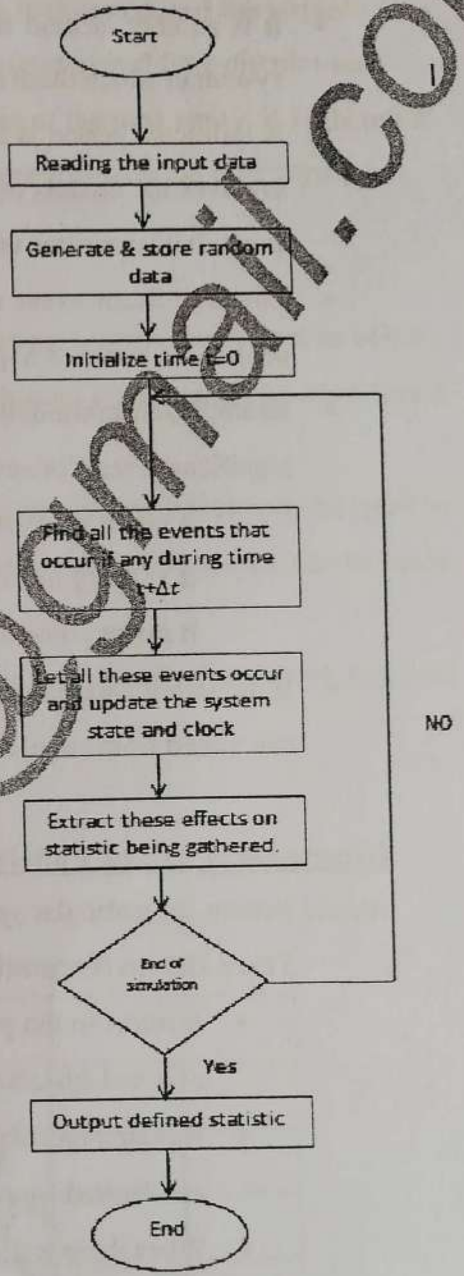


In the above figure we see that event e1, e2, e3, e4 occurs at time 0, 2, 4, 6, 8 and 10 time units respectively.

- With fixed time interval mechanism, simulation clock is incremented with small interval Δt (here 2), whether the event is occurred or not, we look at events at Δt to update the state of system.
- It is very simple method to implement but not an accurate realization of occurrence of events.
- Continuous system simulation normally uses the interval oriented method.



Flow chart for fixed time advance mechanism



Flow chart for next time advance mechanism

- An event oriented program will detect discrete changes and can therefore simulate discrete system.
- An interval oriented program can be made to follow continuous changes by artificially introducing events that occurs at regular time interval and can therefore simulate continuous system.

Significant Event Simulation

- It is another method **to represent the passage of time** and is applicable to continuous system in which there is quiescent (not active) period.
- A quiescent period is the interval between events in the **event oriented approach** but it involves the models representation of the **system activities** which create **a notice** of the event that terminates the interval.
- The significant event approach assumes that simple **analytic function** can be used to project the **value** of a quiescent period.
- Example, an automobile travelling at constant acceleration, it movement might result in a significant event for several reasons:
  - It might reach at the end the road.
  - It velocity most reach some limit.
  - It might come to rest.
- If the initial conditions are known, the elapsed time for each these possible events can be calculated from simple formula.

Generation of Arrival Patterns

Arrival pattern for particular system must be specified for simulation.

i. **Trace Driven Simulation**

- It refers to the process of gathering a sequence of inputs based on the observation of a real system.
- It is an important tool in many simulation applications in which the model's inputs are derived from a sequence of observations made on a real system
- When there is no interaction between exogenous arrivals and the endogenous events of a system, it is permissible to create a sequence of arrival in preparation for the simulation.
- Usually, the simulation proceeds by creating new arrivals as they are needed.

ii. **Bootstrapping:**

- Bootstrapping usually refers to a self-starting process that is supposed to proceed without external input.
- It is the process of making one entity creates its successor.

- Here the arrival time of next entity is recorded, and when the clock reaches this event time, the event of entering the entity into the system is executed, and the arrival of the following entity is immediately calculated from the inter-arrival time distribution.
- These methods require keeping only the arrival time of the next entity, it is therefore the preferred method of generating arrivals for computer simulation programs.

Simulation of Telephone System

The simulation of a discrete system can be explained by simulating a telephone system as below:

- The system has number of telephone (here only 8 are shown), connected to a switch board by line.
- The switch board has a number of links(here only 3 are shown) which can be used to connect any two line, provided the condition that only one connection at time can be made to each line.
- Any call that cannot be connected at the time it arrives is immediately abandoned, and then the system is called **a lost call system**.
- A call may be lost due to the following reasons:
  - If the called party is engaged (**busy**)
  - If no link is available (**a blocked call**)

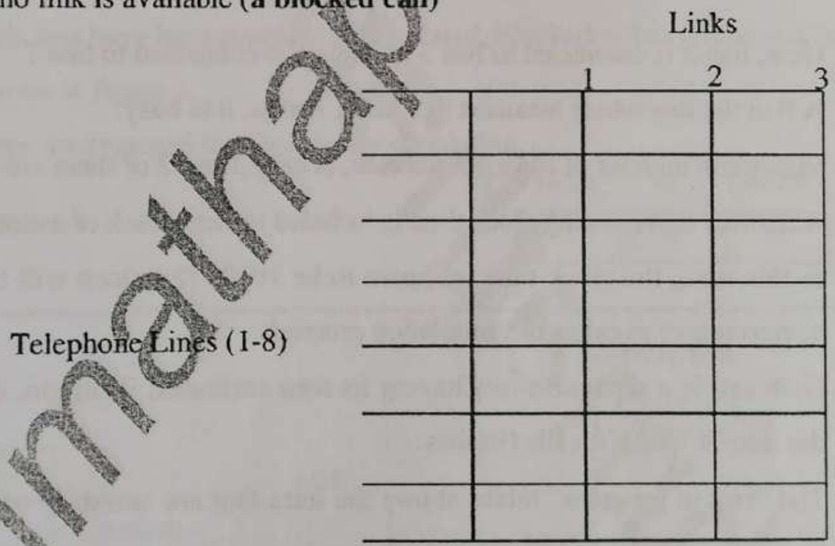


Figure 1: Simple Telephone System.

Object of Simulation:

- To process a given number of calls and determine what portion are successfully completed, blocked or found to be busy calls.

- Let us consider the current state of the system as below:

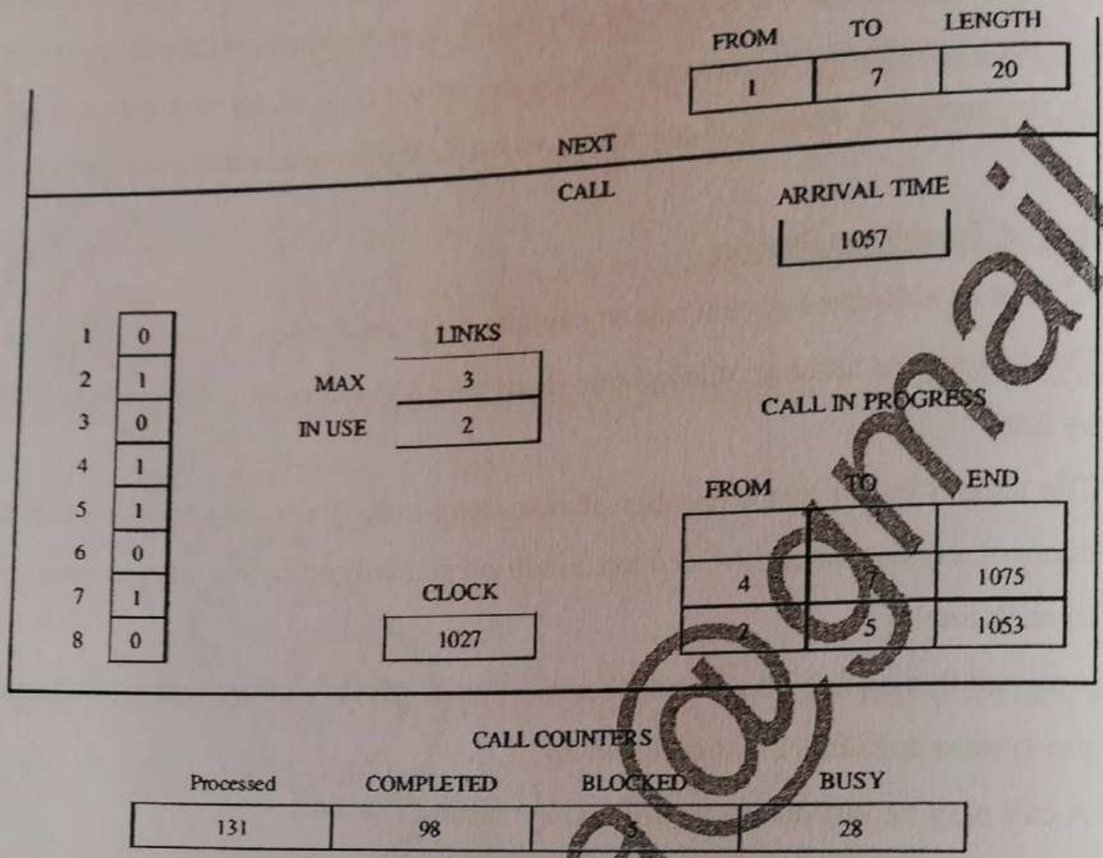


Figure: 2: Telephone System Initial State

- Here, line 2 is connected to line 5 and line 4 is connected to line 7.
- A 0 in the line tables means is free and 1 means, it is busy.
- Maximum number of links, in this case, is only 3 and 2 of them are in used.
- A number representing clock time is included to keep track of events.
- In this state, the clock time is shown to be 1027. The clock will be updated to the next occurrence of event as the simulation proceeds.
- Each call is a separate entity having its own attributes; its origin, destination, length and the time at which it calls finishes.
- The "call in progress" tables shows the lines that are currently connected and the finish time.
- Arrival time and detail of next call are also shown to generate the arrival of calls; here bootstrap method can be used to generate the arrival of the call.

- Two possible activities can make change to occur in the system state; a new call can arrive or an existing call can be finished. From the figure 2, it can be seen that three events can occur in the figure:

i) Call between 2 & 5 will finish at 1053

ii) A new call will arrive at 1057

iii) Call between 4 & 7 will finish at 1075

The simulation proceeds by executing a cycle below state to simulate each event:

a) Scans the events to determine which the next potential event is. Here, it is at 1053. The clock is updated then.

b) Select the activity that is to cause the event. Here, the activity is to disconnect a call between lines 2 & 5.

c) Test whether the potential event can be executed. Here, however, there are no conditions to disconnect a call.

d) Change the record to reflect the effect of the event. Here the call is shown to be disconnected by setting to zero in the line table for line 2 & 5, reducing the number of links in used by 1 and removing the finish call from the "call in progress" table.

e) Gather some statistics for the simulation output. Here calls counter are changed to record the number of calls that have been processed, completed, blocked or busy. The state of the system appears as shown in figure 3.

f) The above steps are repeated to continue the simulation.

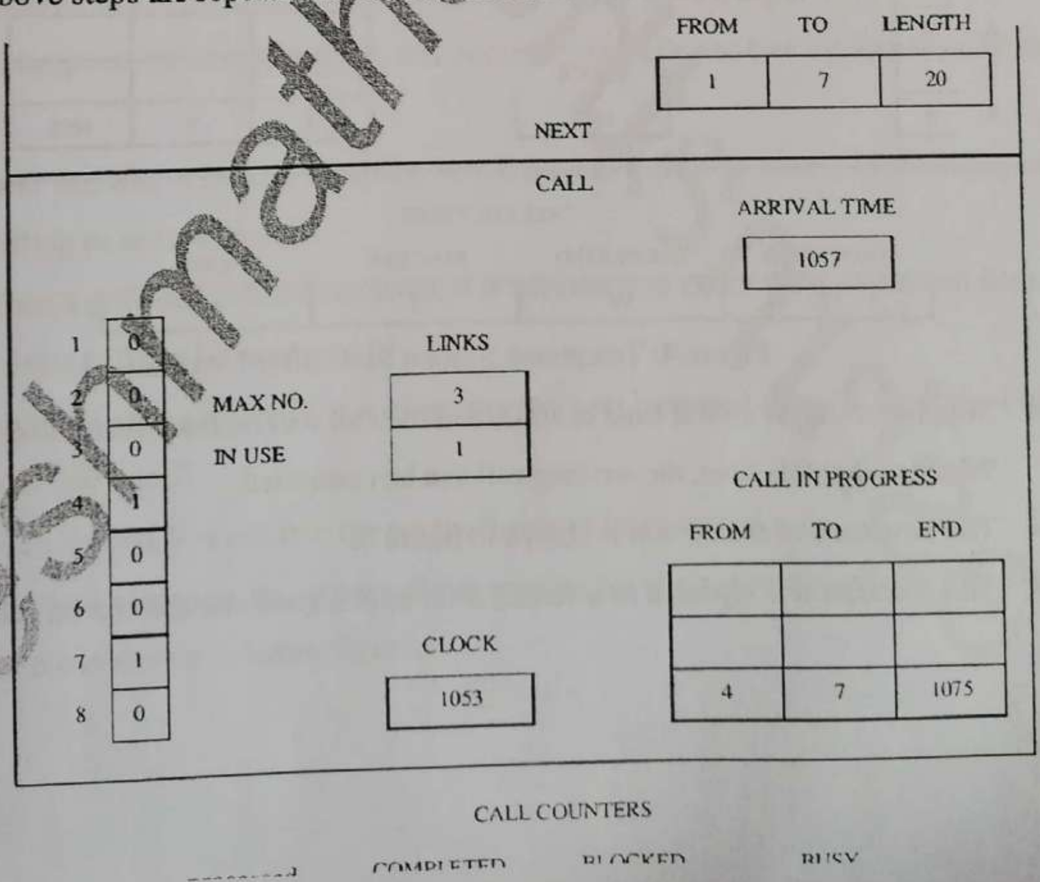


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Figure 3 : Telephone System State after Successful Call

- In figure, it can be seen that the next potential event is the arrival of a call at time 1057 sec.
- The clock is updated to 1057 and the attributes of new arrival is generated.
- Since the selected activity is to connect a call, it is necessary to test; to find whether a link is available and to find whether the party is busy or not.
- In this case, the called party, the line 7 is busy so the call is lost.
- Both the processed call and the busy call counter are increased by one.
- A new arrival is generated. These are shown in figure 4.

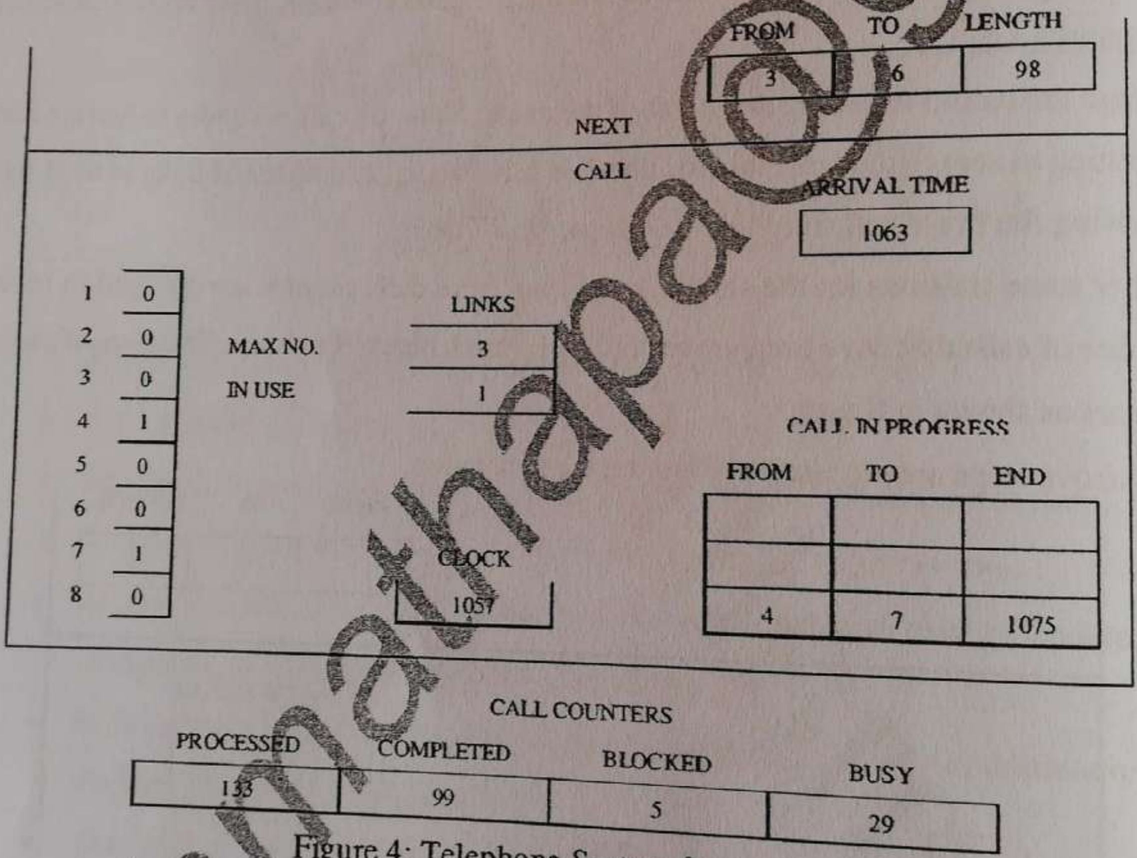


Figure 4: Telephone System State after Lost Call

- Suppose the next arrival time is 1063, and the call will be from line 3 to 6 and will spend 98 sec and at this time, the arriving call can be connected.
- The new state of the system is shown in figure 5.
- The procedural is repeated to a certain limit until a good statistic can be gathered.

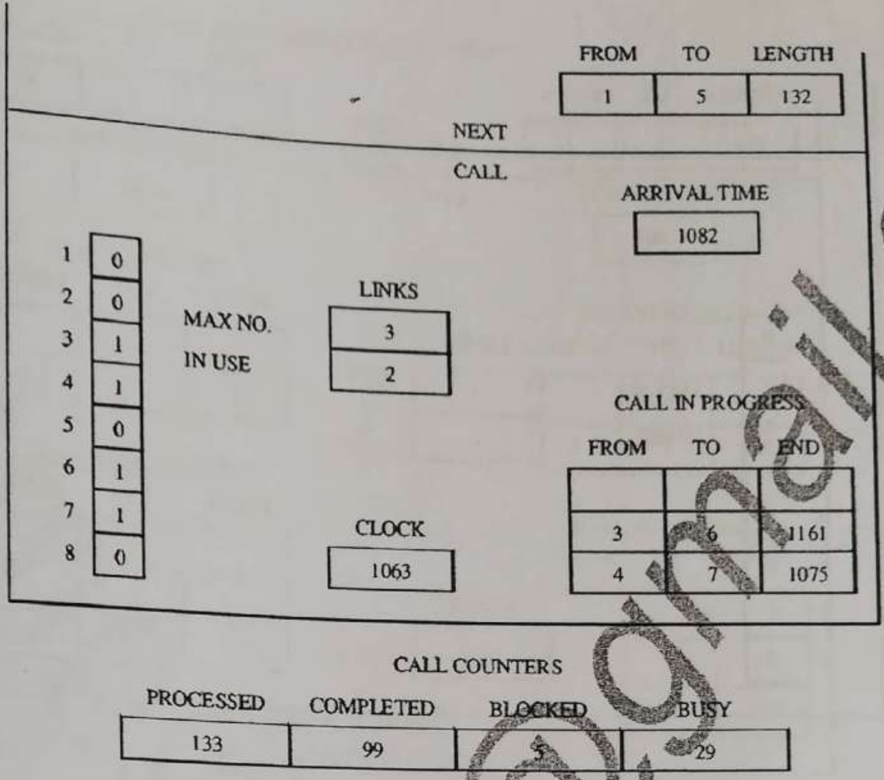


Figure 5: System State

Delayed Call

- When the telephone system is modified so that calls that cannot be connected are not lost, and then they will wait until they can be connected later.
- Such calls are referred to as delayed call.
- We know that it is not possible in the case a real time telephone system but possible to message in a switching system that has store and forward capability.
- To keep record of delayed call, it is necessary to build another list like the call in progress list.
- After arriving a call, if it cannot be connected, then it is placed in the delayed call list, waiting to next time.
- When a previous call is completed, it is necessary to check the delayed call list to find if a waiting call can be connected.
- From the above figure 3, it is clear that the next potential event is the arrival of next call from line 1 to 7.
- Since the line 7 is not free the next call cannot be executed.
- It is then placed in the delay call list waiting for the next time and the new stated of the system is shown in below figure.

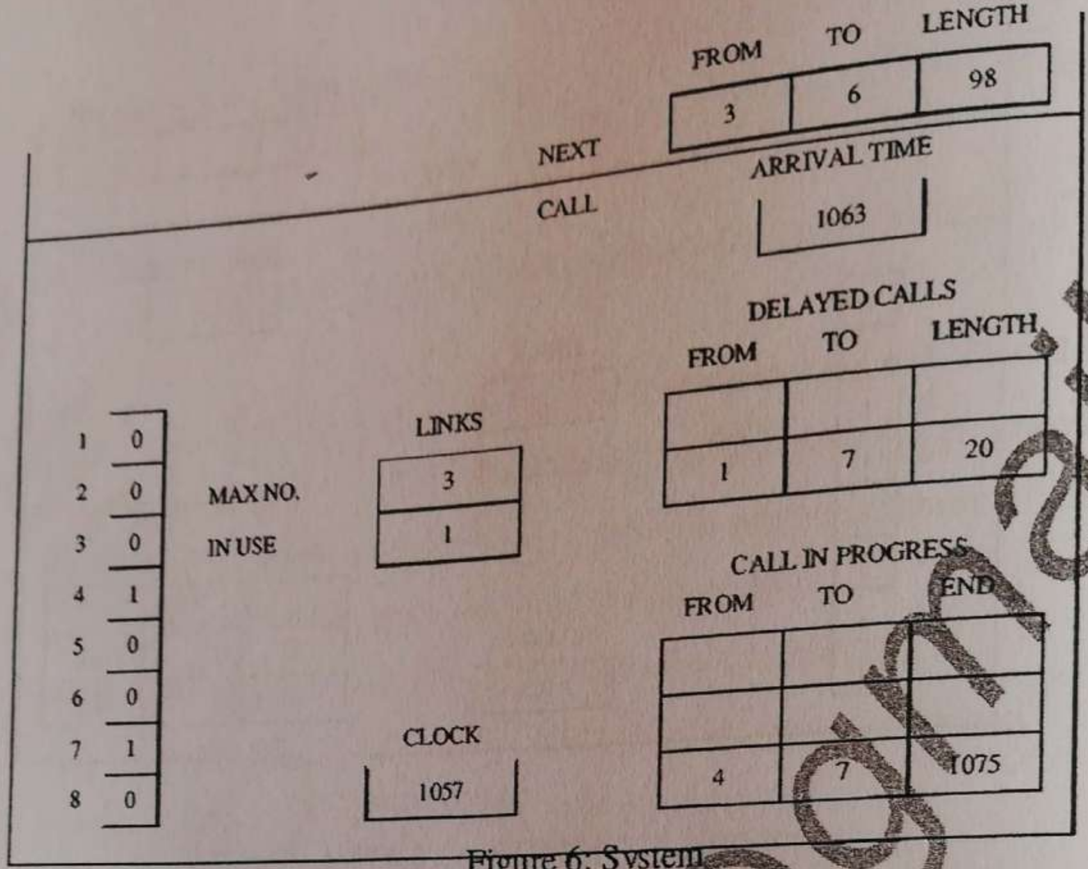


Figure 6: System

CALL COUNTERS

CLOCK	COMPLETED	BLOCKED	BUSY
132	99	5	29

- Now when the call is completed, it is necessary to check the delayed call list to see if a waiting call can be connected.
- The next event here is the arrival of call at 1063, from line 3 to 6. But for now, it is still not possible to connect call from 1 to 7 as line 7 is still busy.
- Hence it continues to wait.

- The system goes to state as shown in figure below.

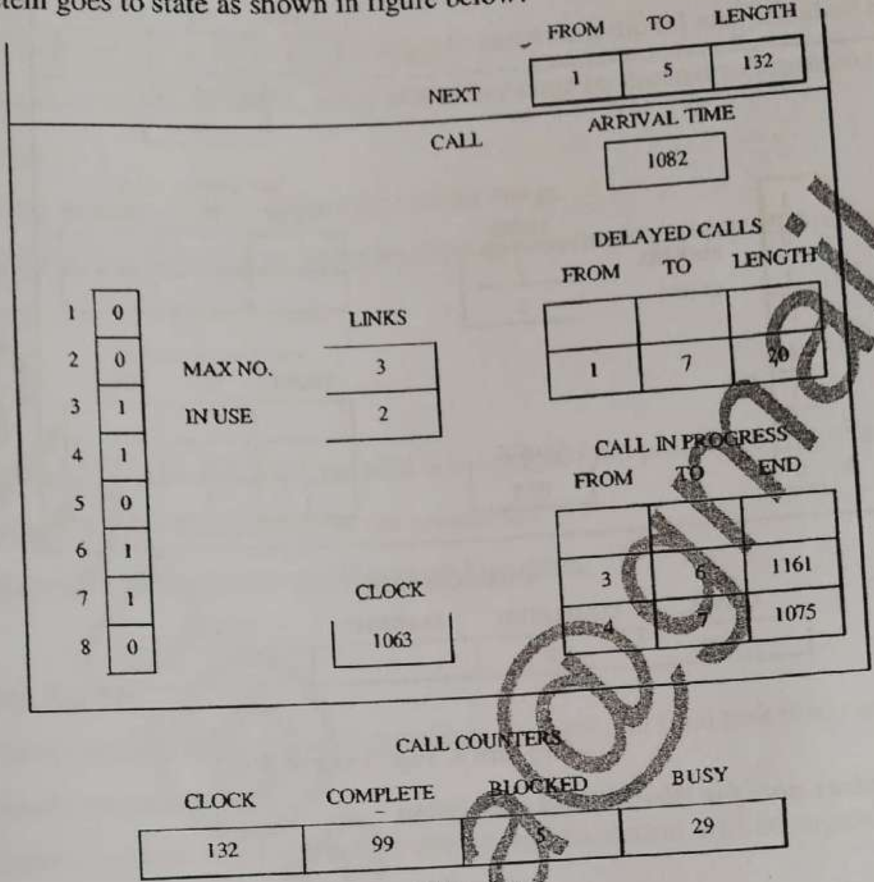


Figure 7

- Now the next potential event to be occurred is at 1075.
- So, when the call from line 4 and 7 completed at time 1075, then for next event, the delayed call list is checked first.
- At these time, the lines 1 and 7 can be connected which is shown in below figure.

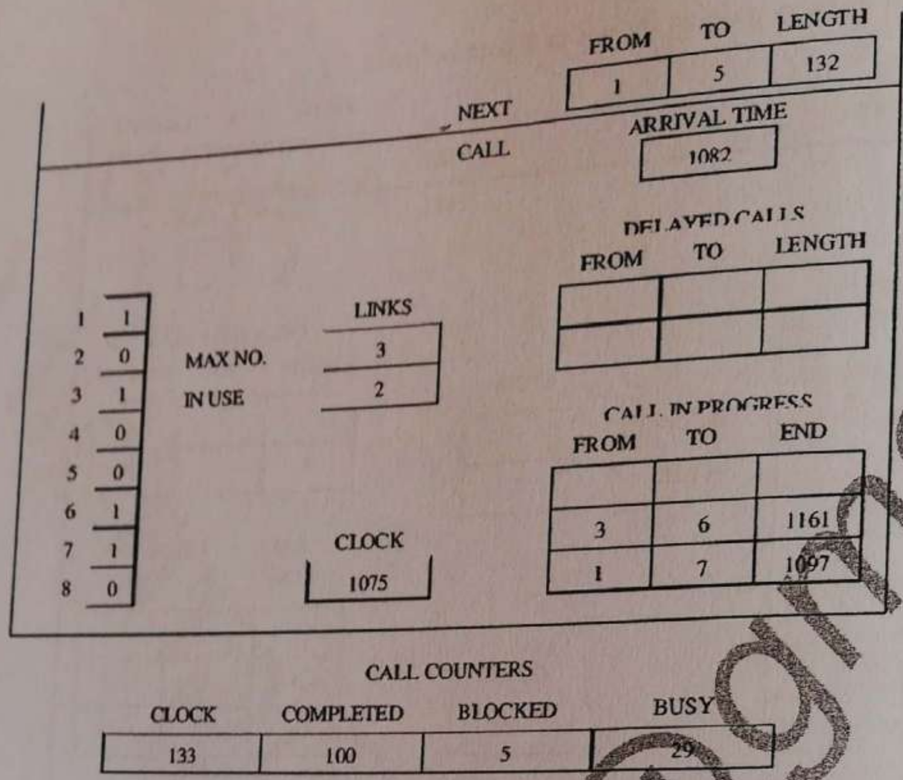


Figure 8: Fig: System State

See your class note for Blocked call simulation

Simulation Programming Task

- There are mainly three tasks to be performed in simulation programming. They are:

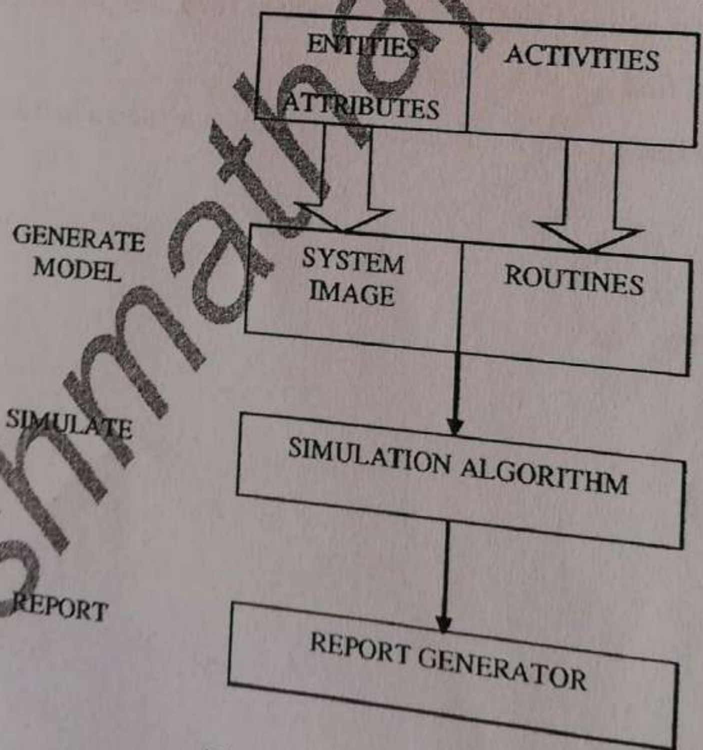


Fig: Simulation programming task

i) **Generating a Model**

- ➤ The first task in simulation programming is to generate a model and initialize it.
- ➤ From the description of system, a set of number must be created to represent the state of the system.
- ➤ Thus, these set of number is called the system image.
- ➤ The activities of a system must be represented as a routine that create discrete event making the changes to the system image.

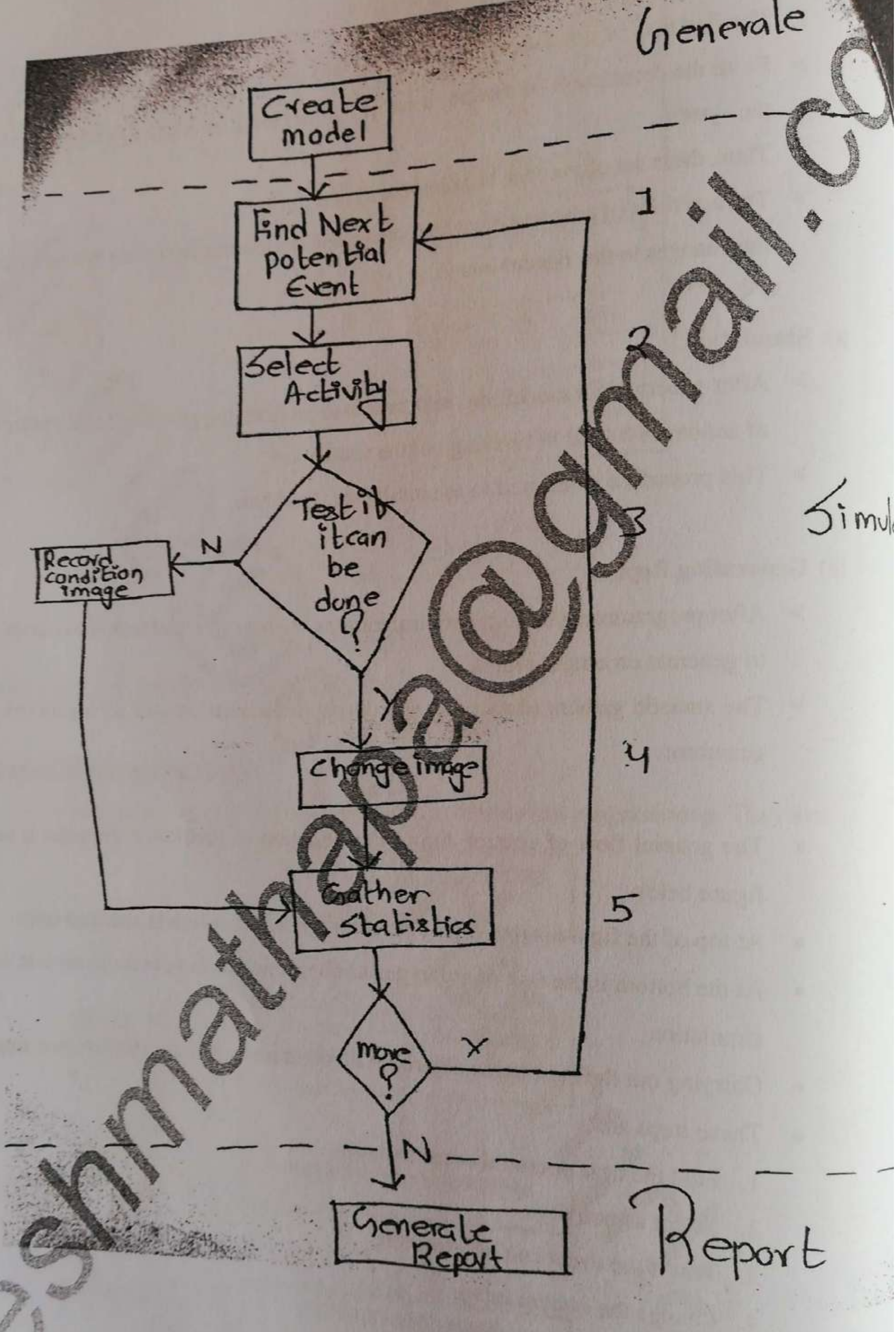
ii) **Simulation**

- ➤ After generating a model, the next task is to program the procedure that execute the cycle of actions involved in carrying out the simulation.
- ➤ This procedure is referred to as simulation algorithm.

iii) **Generating Report**

- ➤ After programming the simulation algorithm, the next and final task is to run the simulation to generate on output report.
- ➤ The statistic gathers (data collected) during the simulation will be organized by a report generator.

- The general flow of control during the execution of simulation program is shown in the figure below.
- At top of the figure is the task of generating a model, which is checked once.
- At the bottom is the task of report generation, which is also executed once at the end of the simulation.
- Carrying out the simulation algorithm involves repeated execution of five steps.
- These steps are
  1. Find the next potential event to occur.
  2. Select an activity.
  3. Test if the event can be executed.
  4. Change the system image.
  5. Gather statistic.


Generale
Create model
1
Find Next potential Event
2
Select Activity
3
Test if it can be done ?
N
Record condition image
Y
Change image
4
Gather Statistics
5
more ?
Y
N
Simul
Generale Report
Report

Gathering Statistics:

- To print out the statistics gathered during the run, simulation programming system include a report generator.
- The exact statistics required from a model depend upon the study being performed.
- Some of the commonly required statistic which is usually included in the output during the simulation is as follows:

a) Counts

➢ This gives the number of entities of a particular type or the number of times some events occurs.

b) Summary measures

➢ This includes measuring some quantities such as extreme values, mean values and standard deviation.

c) Utilization

➢ This measure the time in fraction or percentage, that some entity is engaged.

d) Occupancy

➢ These give the percentage of a group of entities in use on the average.

e) Distribution

➢ These record the distribution of important variables such as queue length or waiting time.

f) Transit time

➢ These records the time taken from an entity to move from one part of the system to some other part.

- When there are stochastic effects operating in the system, all these measures will fluctuate as a simulation proceeds, and the particular values reached at the simulation are taken as estimates of the true values they are designed to measure.

Counter and Summary Statistics

- Counter which are the basic for most statistics are used to accumulate totals and to record current values of some level in the system.
- For example, in the simulation of telephone system, counters are used to record the total number of lost call, busy call, and to keep track of how many link were in use at any time.

- Whenever a new value of a count is established, it is compared with the record of the current maximum or minimum, and the record is changed when necessary.
- The accumulated sum of observation must be kept to derive the mean value and standard deviation as below:
- The mean of set of N observation $X_i (i = 1,2,3, \dots \dots \dots . n)$ is given by

$$\text{Mean, } M = \frac{1}{N} \sum_{i=1}^{N} X_i$$

- A mean is derived by accumulating the total value of the observation, and also accumulating a count of the number of observations.
- And, Standard Deviation, $\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (M - X_i)^2}$
- The accumulated sum of the observations must be kept to derive the mean value, and so the additional record needed to derive a standard deviation is the sum of the squares.

Measuring Utilization & Occupancy

- The common requirement of simulation is measuring the load on some entity such as an item of equipment.
- To measure the load on some entity, the simplest way is to determine what fraction/percentage of time, the item as engaged during the simulation.
- Measuring those statistic is referred to as the utilization of that equipment.

The time history of an equipment usage might appear as shown in figure below:

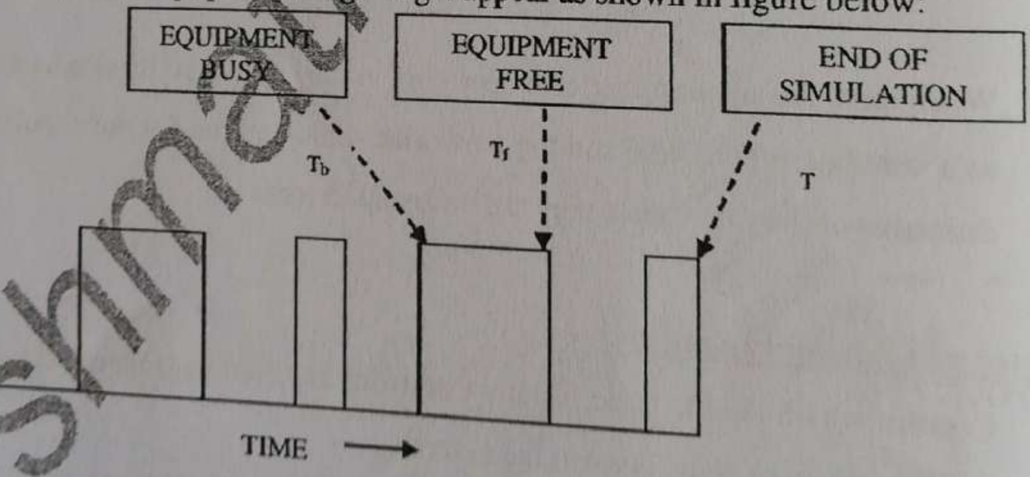


Figure: Utilization of equipment.

Let, t_b= the time at which item last become busy.

T_f= the time at which the item last become free.

T =total time in simulation run.

- To measure utilization, it is necessary to keep record of time Tb at which the last item become busy.
- When the item become free at Tf, the interval (Tf-Tb) is derived and added to the counter.
- At the end of the simulation, the utilization U is derived by dividing the accumulated total by the total time T, so that if the entity is used N times, then the utilization u is given by:

$$u = \frac{1}{T}\sum_{i=1}^{N}(t_f - t_b)_i$$

- In dealing with group of entities, rather than individual items, the calculation is similar, requiring that information about the number of entities involved also kept.
- The below figure represents as a function of time and the number of links in telephone system that are busy.
- To find average number of links in use, a record must be kept of the number of links currently in use and the time at which the last change occurred.
- If the number changes at time t_r to the value n_r, then, at the time of the next change t_{r+1}, the quantity $n_r(t_{r+1}-t_r)$ must be calculated and added to an accumulated total.
- The average number in use during the simulation run, A, is then calculated at the end of the run by dividing the total by the total simulation time T, so that:

$$A = \frac{1}{T} n_i(t_{r+1} - t_r)$$

- the below figure shows the time history of busy telephone links

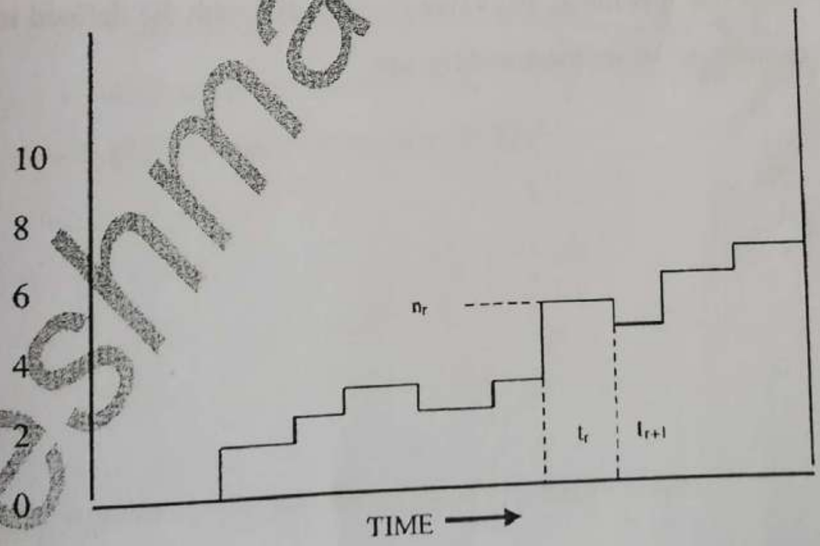


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Figure: Time history of busy telephone links.

- If there is an upper limit on the number of entities, as there was a limit on a number of links in the telephone system then the occupancy is defined as the ratio of the average number in use to the maximum number.

$$\text{Occupancy} = \frac{\text{Average no is use}}{\text{Maximum number}}$$

- If M= total number of links in a telephone System
- N_r = number of busy link in the interval t_r and t_{r+1} then the average occupancy, assuming the number n_r changes N times is given by:

$$\text{Occupancy (B)} = \frac{1}{NM}\sum_{r=1}^{N} n_r (t_{r+1} - t_r)$$

So the basic difference between utilization and occupancy is that timing information must be kept for each individual entity for utilization while in case of occupancy only require keeping count of a class of entities and recording the last time the count was changed.
Another is that, if the number of active entities is large, it can cost a more space and a time to record utilization than occupancy.

Recording Distribution & Transit Time

- To determine the distribution of a variable it requires counting how many times the value of the variable fall within specific interval.
- For this purpose, a table with location where we can define the interval and accumulate the count has to be maintained.
- When any new observation is made, the value is compared with the defined intervals and the appropriate count must be incremented by one.

- The definition of a distribution table requires specification for lower limit of the tabulation, the interval size and the number of intervals. Normally, the tabulation interval sizes are unifor

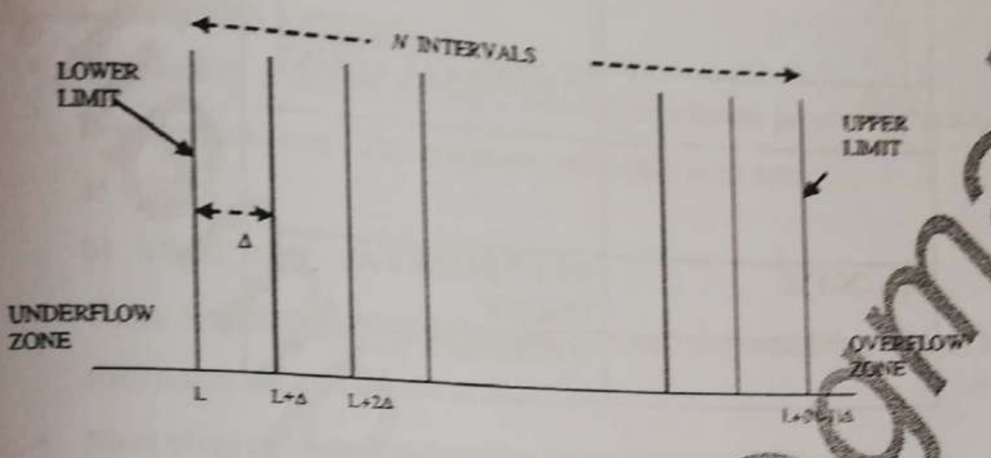


Figure: Distribution Table

- Generally, it is necessary to count how many times an observation falls below lower limit and beyond upper limit.
- It is also necessary to accumulate the number of observations and the sum of the square, in order to calculate the mean value and the standard derivation, at the same time the distribution is being derived.
- To determine the mean and standard deviation, it will be required to accumulate number of observation (X_i) and the sum of squares $\sum(X_i)^2$.
- For each observation X_i,
  - One is added to appropriate counter
  - X_i is added to the sum, $\sum(X_i)$
  - $(X_i)^2$ is added to the sum, $\sum(X_i)^2$.

The space required for the tabulation shown in figure below:

	L, LOWER LIMIT	UNDERFLOW COUNT
i=1	L+Δ	1st INTERVAL COUNT
	L+iΔ	ith INTERVAL COUNT
i=N-1	L + (N-1)Δ	(N-1)th INTERVAL COUNT
i=N		OVERFLOW COUNT

NUMBER OF ENTRIES
TOTAL OF ENTRIES
SUM OF SQUARES

Fig: Space reserved for distribution table

- Since values of observation are matched to an interval, the derived distribution is an approximation.
- However, note that the mean and standard deviation will be accurate within the accuracy limit of the computer even if some observations fall outside the table limits.
- The instances when the observations are made are determined by the native of random variable being measured.
- To understand this, consider the following two examples:
  - ➢ To measure the mean waiting time for a service, an observation must be taken as each entity starts to receive a service so that the times at which the observations are tabulated are randomly spaced.
  - ➢ To measure the distribution of the number of entities waiting, observation should be taken at uniform interval time.
- The final output weight also expresses the data in other commercial form as required.
- For example, cumulative distribution may be given or the distribution may be resolved to express the counts as percentage of total observation.
- These all results are calculated at the end of simulation.
- A clock is used in the manner of time stamp to measure **transit time.**
- When an entity reaches a point from which a measured of transit time is to start.
- A note of the time of arrival is made.

- Later when the entity reaches the point, at which, the measured ends, a note of the clock time upon arrival is made and these two clock time noted are used to compute the elapsed time.

Discrete Simulation Language

- To simplify a task of writing a discrete simulation program, a number of programming languages are sued. The two most commonly used are:
  a) GPSS
  b) SIMSCRIPT
- These languages are used describe a system and establish a system image and execute a simulation algorithm.
- Most programming also provides report generators.
- These languages also offer many convenient facilities such as,
  - Automatic generators of streams of pseudo random numbers for any desired statistical destination.
  - Automatic data collection
  - Their statistical analysis and report generations
  - Automatic handling of queues, etc.
- In addition, a good simulation also provides a nodded builder with the view of the world that next nodded building easier.
- Every discrete system simulation language must provide the concept and statement for:
  a) Representing the state of a system at a single point in time (static modeling)
  b) Moving a system from state to state (Dynamic modeling), &
  c) Performing relevant task such as random number of generation, data analysis and report generation.
- Discrete system simulation languages can be classified into three main categories:
  a) Event oriented languages
  b) Activity oriented languages
  c) Process oriented languages.

Chapter 5

Probability Concept and Random Number Generation

Probability concepts in Simulation-Stochastic variable

Probability theory is applied to situations where uncertainty exists. In our daily life, we have many situations where uncertainty plays a vital role. The chance of rain tomorrow is 40%. This means, the probability of rain 0.4. The description of activities can be of two type: Deterministic and Stochastic. The process in which the outcome of an activity can be described completely in terms of its input is deterministic. On the other hand, when the outcome of an activity is random i.e. there can be various possible outcomes, the activity is said to be stochastic. In case of an automatic machine, the output per hour is deterministic, while in a repair shop, the number of machine repaired will vary from hour to hour in random fashion. The term random and stochastic are interchangeable.

Discrete Probability Function

If a random variable is discrete variable, its probability distribution is called discrete probability distribution. If a random variable X can take X_i (i= 1, 2.....................N) countably infinite no of values with the probability of value X_i being $P(X_i)$, the set of numbers $P(X_i)$ is said to be a probability distribution or probability mass function of random variable X. The number $P(X_i)$ must satisfy the following two conditions.

1. $P(X_i) \geq 0$ for all values of i
2. $\sum_{X=1}^{n} P(Xi) = 1$

Example:

If we flip a coin two times then we can have four possible outcomes/ Sample Space: HH, HT, TH, TT. Let X be a random variable that represents the number of heads in above experiment. The random variable X can only take values 0, 1 or 2. So it is discrete random variable. The probability distribution for this statistical experiment appears below.

Number of Head: X	0	1	2
P(X)	0.25	0.5	0.25

Continuous Probability Function

If a random variable is a continuous variable, its probability distributions is called continuous probability distribution. If a random variable is continuous and not limited to a discrete value, it will have infinite number of values in any interval. Such variable is defined by a function F(X) called probability density function(PDF). The probability that a variable X falls in the range x1 and x2 is given by

$$\int_{x1}^{x2} F(X)dx \quad F(X) \geq 0$$

Random variable and Random Number

A random variable is a function that assigns a real number to each outcome of the experiment.

Example:

Consider the experiment of rolling a pair of dice. Then S= {(1,1), (1,2), (1,3).....................(6,6)}

If x is random variable corresponding to the sum of the two dice, then X assigns the value 2, 3, 4, …………12 to the above outcome of experiment.

There are two types of random variable: Discrete and Continuous.

A random variable X is said to be discrete if the number of possible values of X is finite or countably infinite i.e. possible values of x may be in the range X1, X2……………… X_N. A random variable X is said to be continuous if its range space is an interval or a collection of interval. A continuous variable can assume any value over a continuous range.

Random number is simply a value taken on by a random variable. **Random numbers are numbers** that occur in a sequence such that two conditions are met: (1) the values are uniformly distributed over a defined interval or set, and (2) it is impossible to predict future values based on past or present ones.

Properties of random number

- **The generated random numbers should satisfy the property of uniformity.**
- **The generated random numbers should satisfy the property of independence.**
- The random number should be replicable.
- It should take a long time before the number starts to repeat.
- The routine should be fast
- The routine should not require a lot of storage.

Pseudo Random number

Pseudo means false but her pseudo means that the random number are generated by using some known arithmetic operation and known mathematical formula. Since the arithmetic operation/ formula is known and the sequence of random numbers can be repeatedly obtained, the numbers can't be truly random. Truly random number are generated by physical generators. For example, tossing a coin. However, the pseudo random numbers generated by many computer routines very closely fulfilled the requirement of desire randomness.

If the method of random number generation i.e. random number generator, is defective, the generated pseudo random numbers,

- May not be uniformly distributed.
- May not be continuous.
- The mean of generated numbers may be too high or too low
- The variance may be too high or too low.

Generation of Random number

Random number can be generated by the following methods.

1. Random numbers may be drawn from the random number tables stored in the computer memory. It is very slow process and the number considerably occupy space of computer memory.
2. An electronic device may be constructed as part of a digital computer to generate truly random number. This is however considered as expensive.
3. Pseudo random numbers may be generated by using mathematical formulas and arithmetic operation. This method commonly specifies a procedure, where starting with an initial number, the second number is generated and from that third number and so on. A number of recursive procedure are used for generating random number.
   One of the method for generating PRN is Mid square method. It starts with a fixed initial value, say 4-digit integer, called seed. The number is squared and the middle four digit of this square

become the second number. The middle digit of this second number are then squared again to generate third random number and so on. We may also have to add zero to make the digit's length eight if necessary. Finally, we get realization from the uniform (0,1) distribution after placement of decimal points i.e. after division by 10000.
Example: if we take seed $Z_0 = 1234$, then we will get the sequence of numbers as 0.1234, 0.5227, 0.3215, 0.3362, 0.3030, 0.1809........................

Generate the random number sequence of number for $Z_0 = 2100$ using mid square method.

Quality of an efficient random number Generator

- It should have a sufficiently long cycle i.e. it should be sufficiently long sequence of random numbers before beginning to repeat the sequence.
- The random numbers generated should be replicable i.e. by specifying a starting condition, it should be possible to obtain the same set of random numbers. Many time common random numbers are required for the comparison of two systems.
- The generated random number should fulfil the requirement of uniformity and independence.
- The random number generator should be fast and cost effective.
- It should be portable to different computer and ideally to different programming language.

Testing number for randomness

A sequence of random number is considered to be random if

1. The number are uniformly distributed i.e. every number has an equal chance of occurrence.
2. The number are not serially auto-correlated i.e. there is no correlation between adjacent pair or number, or the appearance of one number doesn't influence the appearance of next number.

There are a number of test, which are used to ensure that random numbers are uniformly distributed and are not serially auto-correlated.

Uniformity Test(Frequency Test)

The test of uniformity or frequency test is a basic test that should always be performed to validate a random number generator. The uniformity test counts how often numbers in a given range occur in the sequence to ensure that the number are uniformly distributed. Two frequency test are available and they are

1. Kolmogorov-Smirnov test i.e. K-S test
2. Chi-Square test

Both of these test compare the generated random number with the theoretical uniform distribution. The algorithms of testing a random number generator are based on some statistics theory, i.e. testing the hypotheses. The basic ideas are the following, using testing of uniformity as an example.

We have two hypotheses one says the random number generator is indeed uniformly distributed. We call this H_o, known in statistics as *null hypothesis*. The other hypothesis says the random number generator is not uniformly distributed. We call this H_1, known in statistics as *alternative hypothesis*. We are interested in testing result of H0, reject it, or fail to reject it.

K-S Test

This test compares the cdf (Continuous distributed function) of uniform distribution $F(x)$ to the empirical cdf, $S_N(X)$ of the sample of N observations. The largest deviation between F(X) and $S_N(X)$ is determined and is compared with the critical value, which is available as function of N.

Procedure of applying K-S uniformity test

1. Rank the data from smallest to largest

$$R_{(1)}, \leq R_{(2)}, \leq \dots \leq R_{(N)}$$

2. Number are computed from the empirical distribution $S_N(X)$ i.e. i/N
3. Compute the deviations

$$D^{+} = \max_{1 \leq i \leq N} \left\{ \frac{i}{N} - R_{(i)} \right\}$$

$$D^{-} = \max_{1 \leq i \leq N} \left\{ R_{(i)} - \frac{i-1}{N} \right\}$$

4. Compute the largest deviation i.e. D is also know as sample distribution

$$D = \max(D^{+}, D^{-})$$

1. Determine the critical value, D_α, from Table below for the specified significance level α and the given sample size N.

TABLE 8.3-13: CRITICAL VALUES $d_{\alpha;n}$ OF THE MAXIMUM ABSOLUTE DIFFERENCE BETWEEN SAMPLE AND POPULATION RELIABILITY FUNCTIONS

Sample Size, N	Level of Significance, α				
	0.20	0.15	0.10	0.05	0.01
3	0.565	0.597	0.642	0.708	0.828
4	0.494	0.525	0.564	0.624	0.733
5	0.446	0.474	0.474	0.565	0.669
10	0.322	0.342	0.368	0.410	0.490
15	0.266	0.283	0.304	0.338	0.404
20	0.231	0.246	0.264	0.294	0.356
25	0.21	0.22	0.24	0.27	0.32
30	0.19	0.20	0.22	0.24	0.29
35	0.18	0.19	0.21	0.23	0.27
40	0.17	0.18	0.19	0.21	0.25
45	0.16	0.17	0.18	0.20	0.24
50	0.15	0.16	0.17	0.19	0.23
over 50	$\frac{1.07}{\sqrt{N}}$	$\frac{1.14}{\sqrt{N}}$	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

2. If the sample statistic D is greater than the critical value D_α , the null hypothesis that the sample data is from a uniform distribution is rejected; if $D \leq D_\alpha$, then there is no evidence to reject it.

Note: the level α is the probability of rejecting the H_0 null while H_0 null is true.

Example:

The sequence of number 0.24, 0.89,0.11, 0.61, 0.23, 0.86, 0.41, 0.64, 0.50, 0.65 has been generated. Use the K-S test with a level of significance $\alpha = 0.05$ and $D_{0.05} = 0.410$ to determine if the hypothesis that the number are uniformly distributed on the interval [0,1] can be rejected.

R_i	0.11	0.23	0.24	0.41	0.50	0.61	0.64	0.65	0.86	0.89
i/N	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1
$(I/N) - R_i$	-	-	0.06	-	0	-	0.06	0.15	0.04	0.11
$R_i - (i-1)/N$	0.11	0.13	0.04	0.11	0.10	0.11	0.04	-	0.06	-

From the above table we get,
$D^+ = Max(i/N - R_i) = 0.15$
$D^- = Max(R_i - (i-1)/N) = 0.13$
$D = Max(D^+, D^-) = 0.15$
The critical value $D_{0.05}$ is 0.410 (i.e. the critical value of D for $\alpha = 0.05$ and N=10 is 0.410).
Since $D < D_\alpha$, there is no chance to reject the hypothesis that the given random numbers are uniform. (or, since the computed value is less than critical value, the given random numbers are uniform at 95% level of significance)

The sequence of number 0.54, 0.73, 0.98, 0.11 and 0.68 has been generated. Use the K-S test with $\alpha = 0.05$ to determine if the hypothesis that the number are uniformly distributed on the interval [0,1] can be rejected.

R_i	0.11	0.54	0.68	0.73	0.98
i/N	0.2	0.4	0.6	0.8	1
$(I/N) - R_i$	0.09	-	-	0.07	0.02
$R_i - (i-1)/N$	0.11	0.34	0.28	0.13	0.18

From the above table we get,
$D^+ = Max(i/N - R_i) = 0.09$
$D^- = Max(R_i - (i-1)/N) = 0.34$
$D = Max(D^+, D^-) = 0.34$
The critical value $D_{0.05}$ is 0.565 (i.e. the critical value of D for $\alpha = 0.05$ and N=5 is 0.565).
Since $D < D_\alpha$, there is no chance to reject the hypothesis that the given random numbers are uniform. (or, since the computed value is less than critical value, the given random numbers are uniform)

Chi-Squared Test

The Chi-Squared test is very important and useful statistical test to determine how often certain observed data fit the theoretical expected data.

$$\text{Chi-Square} = X_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

Where,
n= Number of class
O_i = Number (frequency) of times the observed data flalls in each class i for i=1,2,3……..n
E_i = Expected number of occurrence in each class i
For the uniform distribution, E_i, the expected number in each class is given by $E_i = N/n$ where N = total number of observation. The sampling distribution of X_0^2 is approximately the Chi- squared distribution with n-1 degree of freedom. Chi-square is a characteristics of distribution which is a measure of its randomness. The statistics chi-square is computed by subtracting the number of the random number in each

class from the expected number, squaring the difference, adding the squares and dividing the sum by the expectation.

Example:
If we have a sequence of 4000 (i.e. N=4000), 3 digits random number from 000 to 999. Then we can have n=10 classes in the range 00-99, 100-199, ...900-999. Then the expected number of occurrence in each class is given by $E_i = N/n = 4000/10 = 400$. Now we have to measure how far the observed frequency deviates from the expected value i.e. if deviation from 400 is too much then we would suspect about non uniformity. So by how much deviation is accepted in sequence to be uniformly distributed is given by **Chi-square test.**

The Chi^2 statistics can be calculated as follows.

i	Class i	O_i (Number of observed occurrence) *(Assumed frequency)*	E_i (Number of expected occurrence)	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	00-99	425	400	25	625	1.5625
2	100-199	372	400	-28	784	1.96
3	200-299	395	400	-5	25	0.0625
4	300-399	415	400	15	225	0.5625
5	400-499	340	400	-60	3600	9
6	500-599	370	400	-30	900	2.25
7	600-699	410	400	10	100	0.25
8	700-799	382	400	-18	324	0.81
9	800-899	365	400	-35	1225	3.0625
10	900-999	394	400	-6	36	0.09

Total = 19.61

We know,

$$\text{Chi-Square} = X_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= 19.61$$

Now to conclude how small or large a computed Chi-square value can we accept for verifying the uniformity, we have to use statistical table which gives critical value of Chi-square. We need degree of freedom(d_f) to read Chi-square table where $d_f = n-1$.
Here df=10-1=9.
Now use the Chi^2 table and find the row of entries for 9.

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2							
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92

Let level of significance $\alpha = 0.05$
So from the above Chi^2 table, at 95% confidence level the acceptable value for 9 degree of freedom is 16.92 which is less than 19.61 i.e. $(X^2_0 > X^2_{\alpha, n-1})$. hence the given set of random number is **not acceptable** so far as it's uniformity in distribution is concerned i.e. not uniformly distributed

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016						
2	0.020	0.103	0.211	0.102	0.455	1.32	2.71	3.84	6.63
3	0.115	0.352	0.584	0.575	1.386	2.77	4.61	5.99	9.21
4	0.297	0.711	1.064	1.212	2.366	4.11	6.25	7.81	11.34
5	0.554	1.145	1.610	1.923	3.357	5.39	7.78	9.49	13.28
6	0.872	1.635	2.204	2.675	4.351	6.63	9.24	11.07	15.09
7	1.239	2.167	2.833	3.455	5.348	7.84	10.64	12.59	16.81
8	1.647	2.733	3.490	4.255	6.346	9.04	12.02	14.07	18.48
9	2.088	3.325	4.168	5.071	7.344	10.22	13.36	15.51	20.09
10	2.558	3.940	4.865	5.899	8.343	11.39	14.68	16.92	21.67
11	3.053	4.575	5.578	6.737	9.342	12.55	15.99	18.31	23.21
12	3.571	5.226	6.304	7.584	10.341	13.70	17.28	19.68	24.72
13	4.107	5.892	7.042	8.438	11.340	14.85	18.55	21.03	26.22
14	4.660	6.571	7.790	9.299	12.340	15.98	19.81	22.36	27.69
15	5.229	7.261	8.547	10.165	13.339	17.12	21.06	23.68	29.14
16	5.812	7.962	9.312	11.037	14.339	18.25	22.31	25.00	30.58
17	6.408	8.672	10.085	11.912	15.338	19.37	23.54	26.30	32.00
18	7.015	9.390	10.865	12.792	16.338	20.49	24.77	27.59	33.41
19	7.633	10.117	11.651	13.675	17.338	21.60	25.99	28.87	34.80
20	8.260	10.851	12.443	14.562	18.338	22.72	27.20	30.14	36.19
22	9.542	12.338	14.041	15.452	19.337	23.83	28.41	31.41	37.57
24	10.856	13.848	15.659	17.240	21.337	26.04	30.81	33.92	40.29
26	12.198	15.379	17.292	19.037	23.337	28.24	33.20	36.42	42.98
28	13.565	16.928	18.939	20.843	25.336	30.43	35.56	38.89	45.64
30	14.953	18.493	20.599	22.657	27.336	32.62	37.92	41.34	48.28
40	22.164	26.509	29.051	24.478	29.336	34.80	40.26	43.77	50.89
50	27.707	34.764	37.689	33.660	39.335	45.62	51.80	55.76	63.69
60	37.485	43.188	46.459	42.942	49.335	56.33	63.17	67.50	76.15
				52.294	59.335	66.98	74.40	79.08	88.38

Consider the following 100 two-digits value in table below. Use the Chi-square test with $\alpha = 0.05$ to test whether the data shown below are uniformly distributed, where $X_{0.05, 9} = 16.9$.

0.34	0.90	0.25	0.89	0.87	0.44	0.12	.21	.46	.67
.83	.76	.79	.64	.70	.81	.94	.74	.22	.74
.96	.99	.77	.67	.56	.41	.52	.73	.99	0.02
.97	.30	.17	.82	.56	.05	.45	.31	.78	.05
.79	.71	.23	.19	.82	.93	.65	.37	.39	.42
.99	.17	.99	.46	.05	.66	.10	.42	.18	.49
.37	.51	.54	.01	.81	.28	.69	.34	.75	.49
.72	.43	.56	.97	.30	.94	.96	.58	.73	.05
.06	.39	.84	.24	.40	.64	.40	.19	.79	.62
.18	.26	.97	.88	.64	.47	.60	.11	.29	.78

Solution:

Defining the intervals of equal length as 0 - 0.1, 0.1 - 0.2,......................0.9 – 1.0, we will get n=10 interval classes.

We have observed from the table that frequency of occurrence of 0.0 to 0.1 range is 8, 0.1 to 0.2 range is 8 and so on.

(Note: the interval is closed so in first interval the permitted values are from 0 to 0.1, similarly for second interval the permitted values are from 0.11 to 0.2 and so on)

Now Expected no of occurrences for each class i, $E_i = N/n = 100/10 = 10$

Class i	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
0 - 0.1	8	10	-2	4	0.4
0.1 – 0.2	8	10	-2	4	0.4
0.2 – 0.3	10	10	0	0	0
0.3 – 0.4	9	10	-1	1	0.1
0.4 – 0.5	12	10	2	4	.4
0.5 – 0.6	8	10	-2	4	.4
0.6 – 0.7	10	10	0	0	0
0.7 – 0.8	14	10	4	16	1.6
0.8 – 0.9	10	10	0	0	0
0.9 - 1	11	10	1	1	0.1
					Total=3.4

So from the above table,

$$\text{Chi-Square} = X_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$
$$= 3.4$$

Degree of freedom, $d_f = n-1 = 10-1 = 9$

The critical value of X_0^2 obtained from Chi^2 table for $\alpha = 0.05$ and 9 degree of freedom is 16.9. since $X_0^2 \leq X^2_{0.05,9}$ the null hypothesis of a uniform distribution is not rejected i.e. it can be accepted.

#Determine the Chi-square. Is it acceptable at 95% confidence level?
36, 91, 51, 2, 54, 6, 58, 6, 58, 2, 54, 1, 48, 97, 43, 22, 83, 25, 79, 95, 42, 57, 73, 17, 2, 42, 95, 38, 79, 29, 65, 9, 55, 97, 39, 83, 31, 77, 17, 62, 3, 49, 90, 37, 13, 17, 58, 11, 51, 92, 33, 78, 21, 66, 9, 54, 49, 90, 35, 84, 26, 74, 22, 62, 12, 90, 36, 83, 32, 75, 31, 94, 34, 87, 40, 7, 58, 5, 56, 22, 58, 77, 71, 10, 73, 23, 57, 13, 36, 89, 22, 68, 2, 44, 99, 27, 81, 26, 85, 62

Note: use $\alpha = 0.05$
Interval =(0-10), (10-20), ………(90-100)

Independence Test

Sometime the random number generator passes the K-S test and Chi-square tests for uniformity, but the number generated my not be independent. Hence there is need of independence test. There are many methods to check the independence of generators. Two among them are

1. Auto correlation test
2. Run test

Auto Correlation test

The uniformity test of random number is only a necessary test for randomness but not sufficient one. A sequence of numbers may be perfectly uniform but still not random. For example, the sequence 1,2,3,4,5,6,7,8,9,1,2,3,4……..and so on would give a perfectly uniform distribution with Chi-square value as zero. But also the sequence can't be regarded as random because the number are not independent as the occurrence of one number decides the occurrence of next i.e. 3 determine 4. This defect is called serial auto correlation of adjacent pair of random numbers.

The chi square test for serial auto correlation uses a 10 * 10 matrix. The 10 classes described in the uniformity test are represented both along the rows and the column giving a large set of groups. Thus to reduce the number of groups, instead of 10, random numbers are divided into smaller number of classes like three or four. Example, three classes well be

i. Less than or equal to 0.33
ii. Less than or equal to 0.66
iii. Less than or equal to 1.0

With three classes in a row and three classes in column there will be 9 groups.

Let us consider the following random numbers
49, 95, 82, 19, 41, 31, 12, 53, 62, 40, 87, 83, 26, 01, 91, 55, 38, 75, 90, 35, 71, 57, 27, 85, 52, 08, 35, 57, 88, 38, 77, 86, 29, 18, 09, 96, 58, 22, 08, 93, 85, 45, 79, 68, 20, 11, 78, 93, 21, 13, 06, 32, 63, 79, 54, 67, 35, 18, 81, 14, 62, 13, 76, 74, 76, 45, 29, 36, 80, 78, 95, 25, 52
Determine whether the hypothesis of independence can be rejected, where $\alpha = 0.05$
Solution,
these N = 73 random numbers, giving (N-1)=72 pairs, are grouped in 9 classes with expectation $E_i = N/n = 72/9 \sim 8$

Class	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
R1≤ 0.33 & R2≤ 0.33					
R1≤ 0.67 & R2≤ 0.33					
R1≤ 1.0 & R2≤ 0.33					
R1≤ 0.33 & R2≤ 0.67					
R1≤ 0.67 & R2≤ 0.67					
R1≤ 1.0 & R2≤ 0.67					
R1≤ 0.33 & R2≤ 1.0					
R1≤ 0.67 & R2≤ 1.0					
R1≤ 1.0 & R2≤ 1.0					

Total=?

From the above table,

$$\text{Chi-Square} = X_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$= ?$

The observation O_i in different class is determined by taking the pair of random numbers. Pair 0.49 and 0.95 falls in class R1≤0.67 & R2≤1.0, similarly next pair 0.95 and o.82 falls in range R1≤1.0 &R≤1.0 and so on. Since the total number of pair is 72.
Since there are two random variables, the degree of freedom d_f = n-2 = 9-2 = 7. The critical value of Chi square for 7 degree of freedom at α=0.05 level of significance is 14.07.
since $X_0^2 \leq X^2_{0.05,7}$ the null hypothesis of independence is not rejected i.e. it can be accepted.

Run Test

Tests the run up and down or the runs above and below the mean by comparing the actual value to expected value. The run test examines the arrangements of numbers in a sequence to test the hypothesis of independence.
A run is defined as a succession of similar events proceeded and followed by a different event.
Example: in a sequence of tosses of a coin, we have H, T, T, H, H, T, T, T, H, T. we get six runs as 1, 2, 2, 3, 1, 1
An up run is a sequence of numbers each of which is succeeded by a larger number. A down run is a sequence of numbers each of which is succeeded by a smaller number. If a sequence of number of too few runs, it is not likely a random sequence. Example, 0.08, 0.18, 0.23, 0.36, 0.42, ..., the sequence has only

one run, i.e. an up run, so not likely a random sequence. Also, if a sequence has too many runs, it is not likely to be random. Example, 0.08, 0.93, 0.15, 0.96, 0.26, 0.84, 0.28, 0.79, 0.36, 0.57. it has total 9 runs, five up and four down, so it is not likely a random sequence.

Example:
Based on the runs up and runs down, determine whether the following sequence of 40 numbers is such that the hypothesis of independence can be rejected where $\alpha = 0.05$

.41	.68	.89	.94	.74	.91	.55	.62	.36	.27
.19	.72	.76	.08	.54	.02	.01	.36	.16	.28
.18	.01	.95	.69	.18	.47	.23	.32	.82	.53
.31	.42	.73	.04	.83	.45	.13	.57	.63	.29

Solution,
Assigning a "+" or "-" in the number based on whether they are followed by a larger or smaller number. The last number is followed by no events so it will neither get a "+ "nor a "-". The sequence of runs up and runs down is below.

+++-+-+---++-+--+-+--+--+-++--++-+--++-

Total number of runs in this sequence, a=26
For N=40, the mean an variance is given by

$$\mu_a = \frac{2N-1}{3}$$

$$= \frac{2*40-1}{3}$$

$$=26.33$$

And

$$\sigma_a^2 = \frac{16N-29}{90}$$

$$= \frac{16*40-29}{90}$$

$$= 6.79$$

Then, converting it to a standard normal distribution

$$Z_0 = \frac{a - \mu_a}{\sigma_a}$$

$$= \frac{26-26.33}{\sqrt{6.79}}$$

$$= -0.13$$

Now, the critical value , $Z_{\alpha/2} = Z_{0.025} = 1.96$ (from below z score table). since $Z_0 >= -Z_{\alpha/2}$, and $Z_0 <= Z_{\alpha/2}$, so the independence of the numbers can't be rejected on the basis of this test. i.e. null hypothesis of independence can't be rejected.

Z-Value	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357

Fig: Z value

#Consider the 50 two-digit values.

0.34	0.9	0.25	.89	.87
.83	.76	.79	.64	.7
.96	.99	.77	.67	.56
.47	.3	.17	.82	.56
.79	.71	.23	.19	.82
.99	.17	.99	.46	.05
.37	.51	.54	.01	.81
.72	.43	.56	.97	.3
.06	.39	.84	.24	.4
.18	.26	.97	.88	.64

Based upon run up and down, determine whether the hypothesis of independence can be rejected, where $\alpha = 0.05$

Chapter 6
Simulation Languages

Introduction

- A computer **simulation language** describes the operation of a simulation on a computer.
- Most languages also have a graphical interface and at least simple statistical gathering capability for the analysis of the results.
- In a sense, these languages are similar to language like Fortran, java or vb.net but also include specific features to facilitate the modelling process.
- Some example of modern simulation language is GPSS, SIMSCRIPT etc.

Merits of Simulation Language

- Since most the features to be programmed are in-built simulation language like **comparatively less programming time and effort.**
- Since the simulation language consists blocks, specially constructed to simulate the common features, they provide a **natural framework** for simulation modeling.
- The simulation models coded in simulation languages can **easily be changed and modified.**
- The **error detection and analysis is done automatically** is simulation language.
- The simulation models developed in simulation languages, especially the specific application packages, called simulators, **are very easy to use.**
- Random number generator and related set of tools
- Provides tools **for reporting result.**
- **Simulation clock** or mechanism for advancing simulated time.

Comparisons / Advantage of using simulation packages over programming language

1. Simulation package automatically provide features needed to build a simulation model, resulting significant decrease in programming time and project cost.
2. It provides natural framework for simulation modelling.
3. Simulation models are generally easier to modify and maintain when written in a simulation package.
4. They provide better error detection because many potential types of error are checked for automatically.

However, Simulation model are still written in programming language because it offers following advantages.

1. Analysis modeler already know a programming language this is often not the case with the simulation package.
2. Simulation model efficiently written in C or C++ may require less execution time then a model developed by simulation package.
3. A simulation package is designed to address a wide variety of systems with one set of modelling constructs however, a C program may be more closely tailored to a particular application.
4. Programming language may sometime allow greater programming than certain simulation package.

Types of Simulation Language

. There are following types of simulation languages:

1. Continuous system simulation language (CSSLS)
2. Discrete system simulation language (DSSLS)
3. Hybrid system simulation language (HSL)

Continuous System simulation language

- Before digital computers come into common use, analog computers were being used for simulating continuous system.
- The system in an analog computer was represented by the block of electronic devices such as operational amplifier to carry out the required operation.
- As soon as the digital computer came, it provides many advantages over analog computer like greater accuracy, freedom for scaling, easily stored for reuse etc.

- Therefore, a special program package were implemented on digital computer to make a digital computer app[illegible] like analog computer.
- This is called a digital analog simulators.
- Example: DEPI, DEPI-4, DAS, MIDAS, IBM-1130, CSMP etc.

Types:

1. Block structured continuous simulation language
    - These are designed to simulate only those system that could be represented as analog block diagrams thus their application are is limited.
    - It requires user to prepare a analog computer type block diagram and then input in this diagram.

2. Expression based languge
    - A block oriented continuous system simulation languge is limited because of its small speciali[illegible] vocabulary.
    - Since, the system must be represented first in analog block diagram so this languge are suited only analog system and not for those who model the system as set of equations.
    - Expresssion based language or statement based language overcome this drawback.
    - Expression based language do not follow block structure diagram but directly implements the differen[illegible] equation of the model.

Discrete System simulation language

- These languages are used to simulate discrete system and provide the following facilities.
    - Automatic generation of random no.
    - Automatic data collection
    - Statistical analyses of data
    - Reporter generators, etc.
- The other classification of discrete simulation languages is based on the general world view inherent in the langu[illegible]

Types

1. Event oriented language
    - In an event oriented language each event is represented by an instantaneous occurrence in simulated t[illegible] and must be scheduled to occur in advance when a proper set of conditions exists.
    - The system image changes i.e. state of system changes at the occurrence of event. Example, SIMSCR[illegible] GASP

2. Activity oriented language
    - In an activity oriented language, the discrete occurrences are not scheduled in advance.
    - They are created by a program which contains description of the conditions under which an activity take place.
    - The activity oriented language have two components: Test component and action component.
    - Here the event occurrence must be controlled by a cyclic scanning activity program. Example, MILITR[illegible]

3. Process oriented languages
    - Here a single process routine, composed of number of segments describe a sequence of activities.
    - Each segment behaves has an independently controlled program.
    - On receiving control, only the statement composing the segments are executed, and then control is retur[illegible] Example, ASPOL, SIMUFOR

4. Transaction flow oriented languages.
    - These languages are the sub category of process oriented language where flow of activities passes thro[illegible] specially defined block.
    - System model is represented by a flow chart consisting of language.

- The program creates transaction(entities), executes them in the blocks and moves them along the flowchart.
- These languages are flowchart oriented; the best known example is GPPS.

Hybrid System Simulation Language

- Some language has been developed which are suitable for both discrete as well as continuous models.
- This type of language is called hybrid system simulation language or combined simulation language.
- These are written particularly for system models in which some of the variable changes continuously and other variable changes discretely.
- Example, GASP IV

GPSS (General Purpose Simulation System)

- GPPS, one of the earliest DSL was developed by Geoffrey Gordon (1961- 1962).
- The first release of this language was implemented on the IBM 704, 709 and 7090 computers.
- Later on improved and more powerful versions have been developed and implemented, including GPSS 2ND, GPSS 3RD (1965), GPSS 1360 (1967) and GPSS V (The latest version).
- GPSS was designed especially for those analyses who weren't necessary computer programmer because they do not write programmed in the logic as the SIMCRIPT programmer does.
- Instead he constructs a block diagram – an n/w of inter connected blocks, each performing a special simulation oriented function.
- GPSS is particularly suited for traffic and queuing system.

Characteristics of GPSS

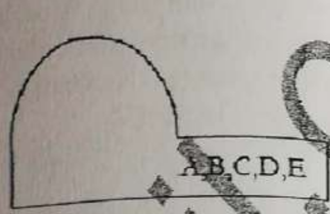
- Designed for analyst not programmers
- GPSS is described as block diagram in which **the block represents the activities**, and the lines joining the block indicates the sequence in which the activities can be executed.
- GPSS V is a set of 48 different block types which perform some specific task.
- Machine efficiency is poor because GPSS is interprets system.
- Restricted to simple queuing problems.

GPSS Block Diagram

- The development of a simulation model in GPSS is a block-by-block construction.
- A set of standard block is arranged in the form of a block diagram that represents the flow of entities(transaction) through the various paths of the system.
- Each block represents a step in the action of the system and links, joining the blocks, represent the sequence of events that can occur.
- To build a block diagram, it is essential to have a completed description of the system.
- The meanings of the blocks used in the system must be clearly defined.
- Each block must be assigned the block time, i.e. the time which the execution of the block will take.
- A set of block types have been designed, which can be used in the construction of a block diagram.
- Each block type can be used any number of times in a block diagram, but the total number of blocks should not exceed 2047.
- On the completion of the block diagram, each block is assigned a number, between 1 and 2047, automatically called the block number or block identification number or location.
- The system to be simulated in GPSS is described as a block diagram in which block represents activities and lines joining the block indicate the sequence in which the activity can be executed.
- Each block performs a simulated oriented function.
- GPSS V provides a set of 48 different blocks, each of which can be used repeatedly.
- Each block has a name and specific task to perform.
- Each block type has a no. of data field such as A, B, C, and so on.
- The entities of the system being simulated are called as transaction.
- Egg; costumer in a queuing system, message in communication system.

Typical blocks are;

1. GENERATE: Create transaction and place on future event chain



- GENERATE A,B,C,D,E
  - create Xact for future entry into system
  - birth block
    - A: mean value of interarrival time
    - B: half-range of interarrival time
      - $B \leq A$
    - C: start delay time
    - D: creation limit
    - E: priority
- interarrival time uniformly distributed in $A \pm B$

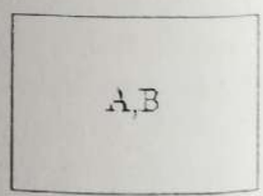
2. TERMINATE: Removes transaction form the system.

TERMINATE A

- Destroys an Xact
- Death block
- A: Termination count increment
- when Xact move into block , transaction count(tc) value is incremented by A and continues until it re some predefined simulation steps.

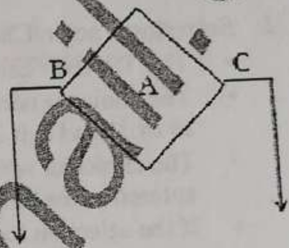
3. Advance: The block type 'ADVANCED' is concerned with representing the expenditure of time. The prog computes an interval of time called an action time for each transaction as it enters an 'ADVANCED' blocks transaction remain at this block up to that action time.

- ADVANCE A,B
  - delay movement
    - A: mean value for delay amount
    - B: half range for delay amount
  - delay amount uniformly distributed in $A \pm B$



4. Transfer: The 'TRANSFER' block allows some location other than the next sequential location to be selected i.e. when there is choice of activities, we use this block by stating the condition for the choice. The choice is normally between 2 blocks refer to as next block 'A' and 'B'.

- TRANSFER A,B,C
  - jump to new location
  - A: probability or mode
    - determines which block to choose
    - probability shows transfer to location C
    - if empty: unconditional transfer to location B
    - Both: chooses empty block (B first if both empty)
  - B: block location
  - C: block location



Note: A is selection factor, B and C fields are exit1 and exit2 respectively.
Example: TRANSFER 0.1, ACC, REJ means 10% go to location REJ(exit2) and 90% to ACC(exit1)

Characteristics of the Blocks

1. **Transactions(X_{acts})**
   - In each system represented by a block diagram, some entities pass through the system.
   - In petrol pump system, they may be vehicles, in a production system they may be parts and in supermarket system they may be costumer.
   - Those entities are referred to as Transactions.
   - In simulation the transaction are created, which move through the block diagram in the same way, as the entities pass through the actual system being simulated.
   - There can be many transactions simultaneously moving through the block diagram.

2. **Block / Action Time:**
   - The block time also called action time is an integer giving the number of units required to execute the action represented by the block.
   - The program computes an action time for each transaction entering a block to represent the time taken by the system action simulated by the block.
   - The program compute and interval of time called action time for each transaction as it enters and ADVANCE block, and the transaction remains at the block for this interval of simulated time before attempting to proceed.
   - The action time may be a fixed interval or a random variable.
   - An action time is defined by giving a mean and modifier. Example, T=A±B where A is the mean and B is the modifier.

3. **Succession of Events**
   - The program maintains records of when each transaction in the system is due to move.
   - It proceeds by completing all movements that are scheduled for execution at a particular instant of time and that can be logically performed.
   - When there is more than one transaction due to move, the program processes transactions in the order of priority or FCFS basis.
   - Normally, a transaction spends no time at a block other than ADVANCE block.

- Once the program begun moving a transaction, it continuous to move the transaction through the block diagram until one of the following circumstances occur.
  i. The transaction enters ADVANCE block with non zero action time then it wakes up other transaction and return to itself when action time is expended
  ii. The transaction enters TERMINATE block then the transaction is removed from simulation
  iii. The transaction is blocked because some action the transaction is attempting to perform in particular block can't be perform currently(may be lack of resources). However, the program automatically detects when the blocking condition has been removed then the transaction will start to move.

4. **Selection Factor/Choice of path**
   - The TRANSFER block allow some location other than the next sequential location to be selected.
   - The choice is normally between two blocks referred to as blocks B and C as shown in above figure (also called as exit1 and exit 2).
   - The choice of which exits to be followed is given by a number called the selection factors, S ($0<S<1$) which is entered in the block. S is the probability of selecting exit 2 and $1-S$ is the probability of selecting exit 1.
   - If the selection factor is empty, then an unconditional branch is made i.e. it choose block B.
   - Also if the selection factor is BOTH, it first chooses empty block B i.e. exit1 and then exit2.

5. **Items of Equipment**
   - In each system, there are physical equipment, which perform some operation on the transactions.
   - Machine tools in a production shop perform machining operations on work pieces.
   - Example, in transportation system, a toll booth on a road is equipment, while vehicles are transactions.
   - In Factory Manufacturing system, an inspector is equipment, while parts are transaction, etc.
   - The item of equipment may operate the transactions individually or may handle groups.

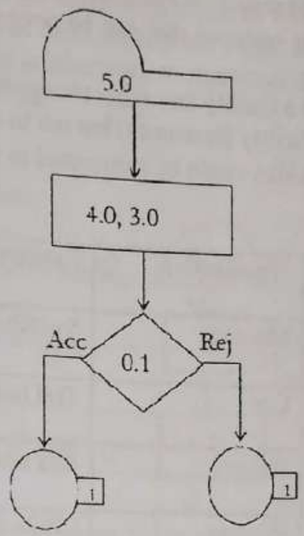
6. **Storage and Facilities**
   - Item of equipment can be classified into storage and facilities, depending upon the capacity for handling the transactions.
   - An item of equipment, which can handle only one transaction at a time, is called a facility while the items of equipment, which can handle a large number of transactions at the same time is called storeage
   - Example: In communication system, message is transaction, switch (pass only one message) is facility while trunk (collection of wire carrying several messages simultaneously) is storage.

GPSS PROGRAM Example

Example 1: Simulation of manufacturing shop

In a manufacturing shop, a machine tools turns out parts at the rate of one every five minutes. As they are finished the parts go to inspector, who takes 4±3 minutes to examine each part and rejects 10% of parts. Each part will be represented by one transaction and time unit selected for problem will be one minute. Simulate for 100 parts to leave the system.

A GPSS block diagram for this system is as follows:



GPSS coding of the above block diagram of the manufacturing shop:

```
GERNERATE              5
ADVANCE                4,3
TRANSFER               0.1, ACC, REJ
ACC   TERMINATE        1
REJ   TERMINATE        1
START                  1000
```

Figure Description;

A GENERATE block is used to represent the output of machine by creating one transaction every five minutes of time. The ADVANCE blocks with a mean of four and modifier of three is used to represent inspection will therefore be anyone of values 1,2,3,4,5,6,7. After completion of the inspection transaction go to or a TRANSFER block with a selection factor of 0.1, so that 90% of the parts go to next location i.e. exit1 call ACC and 10% go to another location i.e. exit2 called REJ. Since there is no further interest both location reached from the transfer blocks are terminate blocks. The program runs until a certain **count** is reached as a result of transactions terminating. Field A of terminate block carries a number indicating by how much the termination count should be incremented when transaction terminates at that block. The control statement START has a field A that indicates a value the terminating counter should reach to end the simulation. When the simulation is completed, the program automatically generates a report, in a prearranged format.

Note:

i. In the case exit1(i.e.ACC) is the next sequential block so it is allowed to omit the name ACC in both the Transfer Field B and the location field of first terminate block. The GPSS code becomes like this TRANSFER 0.1, , REJ. Both comma must be given to indicate B is missing.

ii. If transfer block is used as unconditional mode, then we need not have to specify field A. however comma must be given to indicate A is missing. The GPSS code become like this TRANSFER , REJ

7

Facilities and storage;

- A facility is defined as an entity or resources that can be engaged by a single transaction at a time.
- Storage is defined as an entity or resource that can be occupied by much transaction at a time up to some p determined limit. -
- Once a transaction seizes (holds) a facility any other transaction trying to seize the same facility is delayed u the first transaction releases the facility (resources) but not in case of storage until storage limit reaches.
- 1Some example of the system entities might be interpreted in different system are

Types of system	Transaction	Facility	Storage
Communication	Message	Switch	Trunk
Transportation	Car	Toll both	Road
Data processing	Record	Key stroke	Computer memory
Computer	Process	CPU	memory

There are additional four block types concerned with using facilities and storages

1. SEIZE: The SEIZE block allow a transaction to engage a facility if it is available.
2. RELEASE: The RELEASE block allows the transaction to disengage the facility
3. ENTER: The ENTER block allows a transaction to occupy space in storage, if it is available
4. LEAVE: The LEAVE block allows a transaction to give up the occupied space.

SEIZE
Capture facility A

RELEASE
Release Facility A

ENTER
Capture B unit of Storage A

LEAVE
Release B unit of Storage

Field A in each case indicates which facility or storage in intended, and the choice is usually marked in flag attached to the symbols of the block. If the field B in ENTER and LEAVE blocks are blank, the storage contents are changed by 1. there is a numeric (≥1), then the content changes by that value.

Example:

Let us consider a situation as below;

```
SEIZE      CPU
ADVANCE    7
RELEASE    CPU
```

Here CPU is a facility and it seems that a transaction needs to use the resource for 7 times units. Any other transacti arriving at the block 'SEIZE' is refused to enter until the former transaction has entered RELEASE block.

Resources which can be shared by several transactions are modeled using storage.
Suppose we want to model a computer system which has 64kb of memory then we might declare 'MEMORY STORAG 64' and then a request for 16kb memory might be represented by the sequences of blocks.

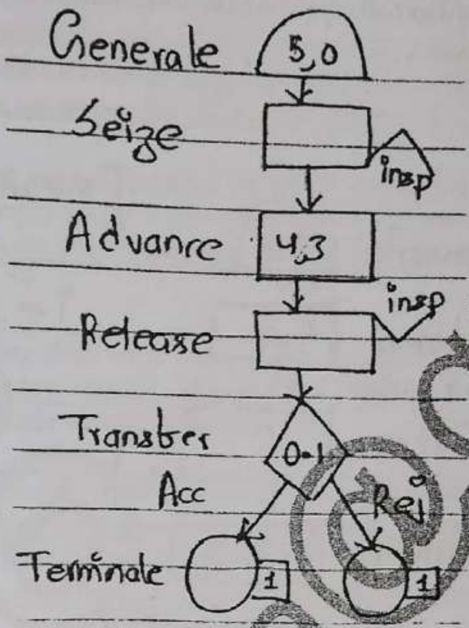
ENTER MEMORY 16
LEAVE MEMORY 16

As the facilities, a transaction arriving at ENTER block at a time when it is used by other transactions, is delayed until th previous transaction release the necessary memory with. A transaction controlling and facility can be interrupted o preempted by other transactions.

Example

- In a manufacturing shop, a machine tools turns out parts at the rate of one every five minutes. As they are finished the parts go to an inspector, who takes 4±3 minutes to examine each part and rejects 10% of parts. Simulate for 100 parts to leave the system assuming that there is only one inspector.
  Note: since there is only one inspector, it is necessary to represent the inspector by a **facility**, to simulate the fact that only one part at a time can be inspected.

A GPSS block diagram for this system is as follows:



GPSS coding of the above block diagram of the manufacturing shop:

```
GENERATE 5              Create parts
SEIZE insp              Get Inspector
ADVANCE 4,3             Inspect
RELEASE insp            Free Inspector
TRANSFER 0.1,Acc,Rej    Select reject
Acc TERMINATE 1         Accepted Parts
Rej TERMINATE 1         Rejected Parts
START 100               Run 100 parts
```

Example(*top*)

- In a manufacturing shop, a machine tools turns out parts at the rate of one every five minutes. As they are finished the posts go to inspector, who takes 4±3 minutes to examine each part and rejects 10% of parts. Simulate for 100 parts to leave the system assuming that there are three inspectors
  Note: if more than one inspector is available, they can be represented as a group by **Storage** with a capacity equal to the number of inspectors.

GPSS code:

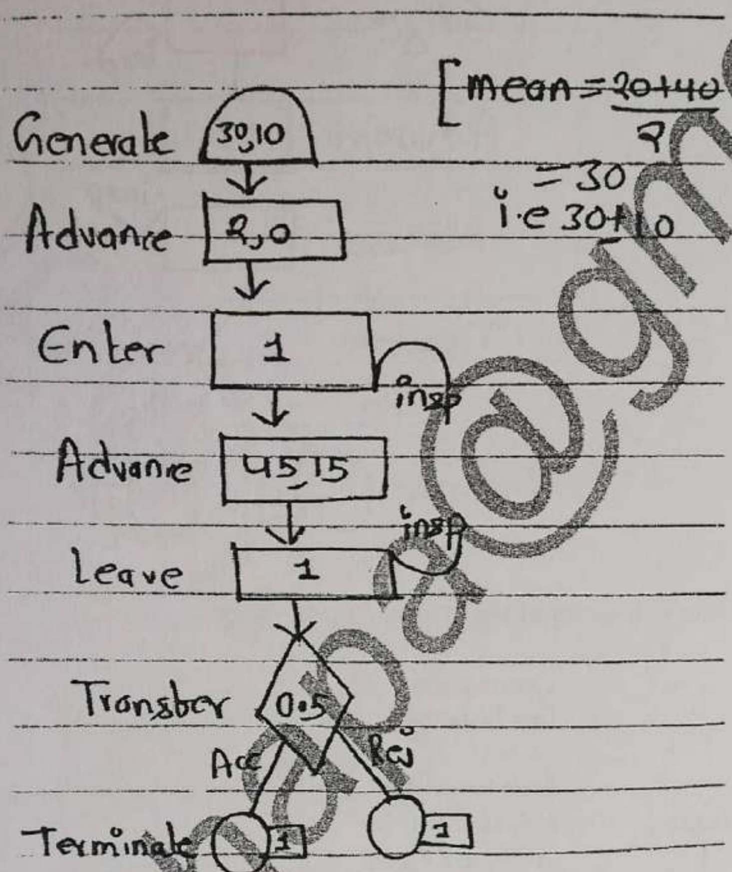
```
GENERATE 5              create parts
ENTER insp              get an inspector
ADVANCE 4,3             inspect
LEAVE insp              free inspector
TRANSFER 0.1,Acc,Rej    select reject
Acc TERMINATE 1         accepted part
```

```
Rej TERMINATE 1        rejected part
insp STORAGE 3         Numbers of inspector
START 100              Run 100 parts
```

Note: STORAGE is a control statement which assigns the capacity of storage.

- Consider a factory that manufactures football taking 20 to 40 minutes. The ball is moved from the generation to th inspection machine taking a 2 minutes. There are 3 inspection machine at one place and need 30 to 60 minutes fo inspection and reject 30% of football. Simulate for 1000 transaction Draw GPSS block diagram to simulate thi system.

GPSS Block Diagram

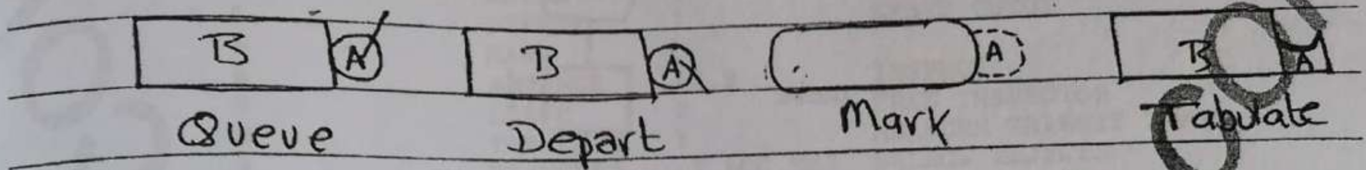


GPSS code

```
GENERATE 30,10
ADVANCE 2, 0
ENTER insp
ADVANCE 45, 15
LEAVE insp
TRANSFER 0.5, Acc, Rej
Acc TERMINATE 1
Rej TERMINATE 1
insp STORAGE 3
START 1000
```

Gathering Statistics

Block types such as QUEUE, DEPART, MARK and TABULATE are used to gather statistic about the system performance but not to represent the system action.



When the condition for advancing a transaction is not satisfied several transactions may be kept waiting at a block and due to this the QUEUE block increase and DEPART block decreases the number in field A i.e. queue block increases the queue number while depart block decreases the queue number. A is queue number, B is the quantity by which the queue number is being increased

The MARK and TABULATE blocks are used to measure the length of time taken by transaction to move the system or parts of the system. The MARK block simply notes the **arrival time** on the transaction and the TABULATE blocks subtracts the time noted by MARKED block from the time of arrival at the TABULATE block.

Example

Consider the example of manufacturing shop with 3 inspectors. Parts will now accumulate on the inspectors works bench if inspection doesn't finish quickly enough. Simulate the situation for 1000 parts to measure how long the parts take to be inspected, excluding their waiting time in queue.

Note:

i. Up to the above examples, the properties of generate block is such that if a transaction is unable to leave a block at the time it is created, no further transactions are generated until the block is cleared so if all the inspectors are busy, the machining of further parts stops until the machine is cleared.

ii. So if all inspectors are busy then queue is used to accumulate incoming parts/ transactions.

GPSS block Diagram:

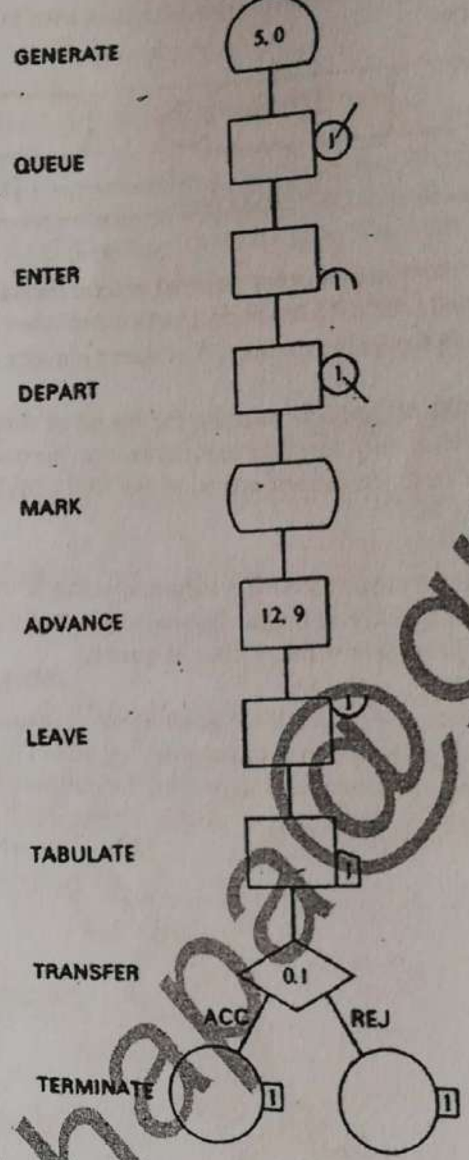


Figure 9-8. Manufacturing shop - model 4.

GPSS Code

- In the above solution, A QUEUE block using queue number 1 is placed immediately before ENTER block and a DEPART block is placed immediately after the ENTER block to remove the part from the queue when inspection begins.
- If some inspectors are free for inspection, then transaction doesn't have to wait in queue otherwise they have to wait. The program will automatically measure the length of stay in queue.
- The MARK and TABULATE block will measure how long the parts take to be inspected, excluding their waiting time in the queue.
- Note another control statement TABLE, the first field A i.e. M1, indicates the quantity to be tabulated/measured, second field B i.e. 5 indicates the lower limit of table, third field C i.e. 5 indicates the tabulation interval size and fourth field D i.e. 10 indicates the number of intervals respectively.
- If MARK block is omitted, the tabulated time is the time since the transaction first entered the system.

```
*
           GENERATE   5              CREATE PARTS
           QUEUE      1              QUEUE FOR AN INSPECTOR
           ENTER      1              GET AN INSPECTOR
           DEPART     1              LEAVE QUEUE
           MARK
           ADVANCE    12,9           INSPECT
           LEAVE      1              FREE INSPECTOR
           TABULATE   1              MEASURE TRANSIT TIME
           TRANSFER   .1,ACC,REJ     SELECT REJECTS
ACC        TERMINATE  1              ACCEPTED PARTS
REJ        TERMINATE  1              REJECTED PARTS
*
1          STORAGE    3              NUMBER OF INSPECTORS
1          TABLE      M1,5,5,10      TABULATION INTERVALS
*
           START      1000           RUN FOR 1000 PARTS
```

Example

Simulate one day of operation of a barber shop for 200 customers. Customers arrive in a barber shop at the rate of 18±6 minutes , enter the shop, queue if the barber is busy, get their hair cut which take 16±4 minutes on a first-come first-served basis, and then leave the shop.

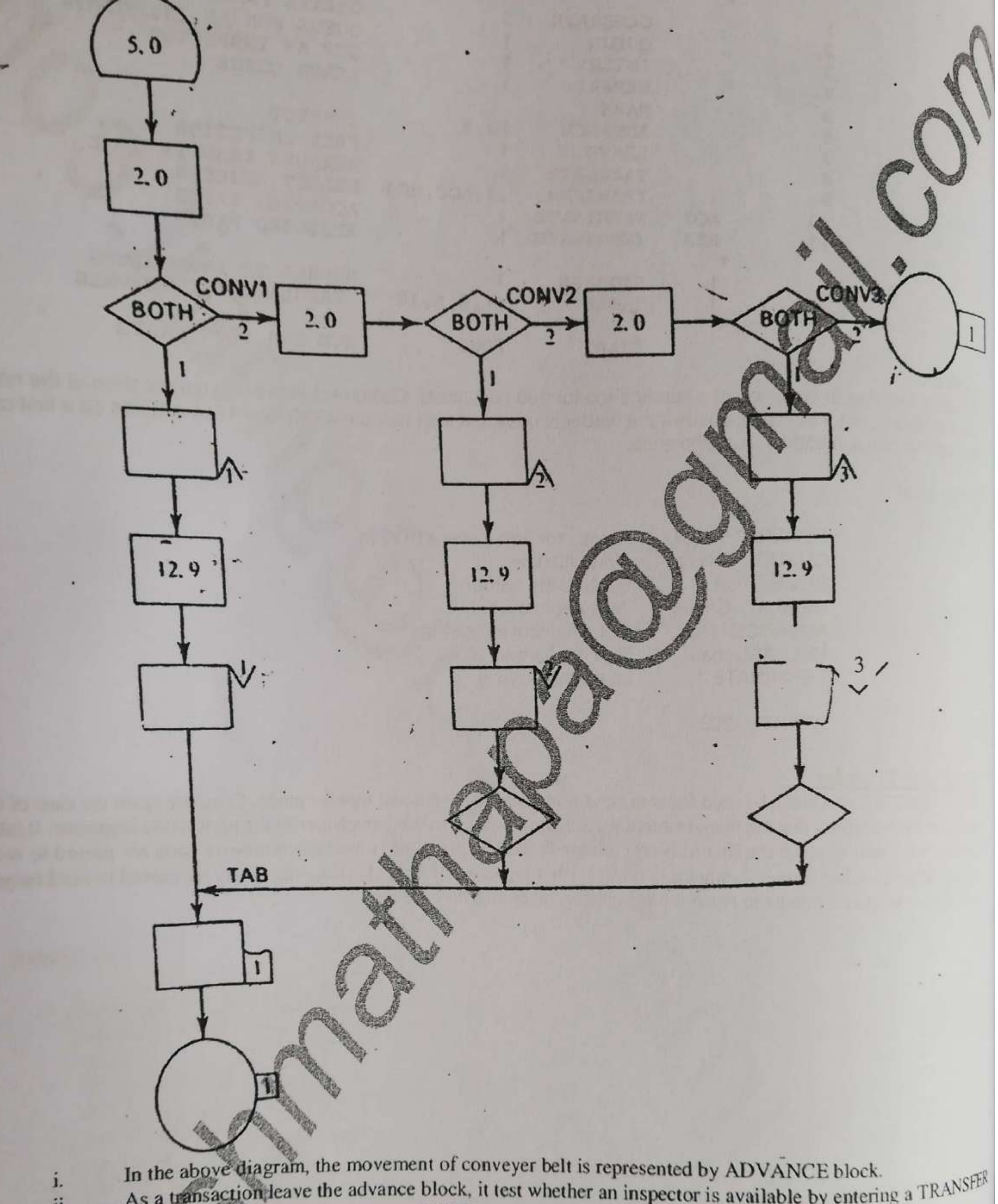
GPSS code

```
GENERATE 18,6      Customer arrive every 18±6 m
QUEUE    Chairs    Enter the line
SEIZE    hari      Capture the barber
DEPART   Chairs    Away from the line
ADVANCE  16,4      Get a hair cut in 16±4 mn
RELEASE  hari      Free the barber
TERMINATE 1        Leave the shop

START    200
```

Condition Transfer

The TRANSFER block can be used for both conditional and unconditional transfer mode. Consider again the case of three inspectors but suppose that the manufactured parts are put on a conveyor, which carries the parts to the inspector. It takes 2 minutes for a part to reach the first inspector who will take the part if he is not busy otherwise parts are passed to second inspector which takes further 2 minutes to reach second inspector. If he is also busy then parts are passed to third inspector which take another 2 minute to reach third inspector, otherwise they are lost.



i. In the above diagram, the movement of conveyer belt is represented by ADVANCE block.

ii. As a transaction leave the advance block, it test whether an inspector is available by entering a TRANSFER block with a selection factor set to BOTH.

iii. In the TRANSFER block the selection mode BOTH implies A i.e. exit1 first if both empty.

iv. TRANSFER block lead to SEIZE block that represent 1 inspector here 1, 2 and 3.(NOTE : we also can give name like ins1, ins2, ins3)

v. When parts finish inspection, the transaction goes to single a single TABULATE block where the transit time is recorded.

vi. Because of the rule by which transactions normally pass from one block to next higher numbered block, only one of the three release block that complete the inspection is able to pass transactions directly to the TABULATE block.

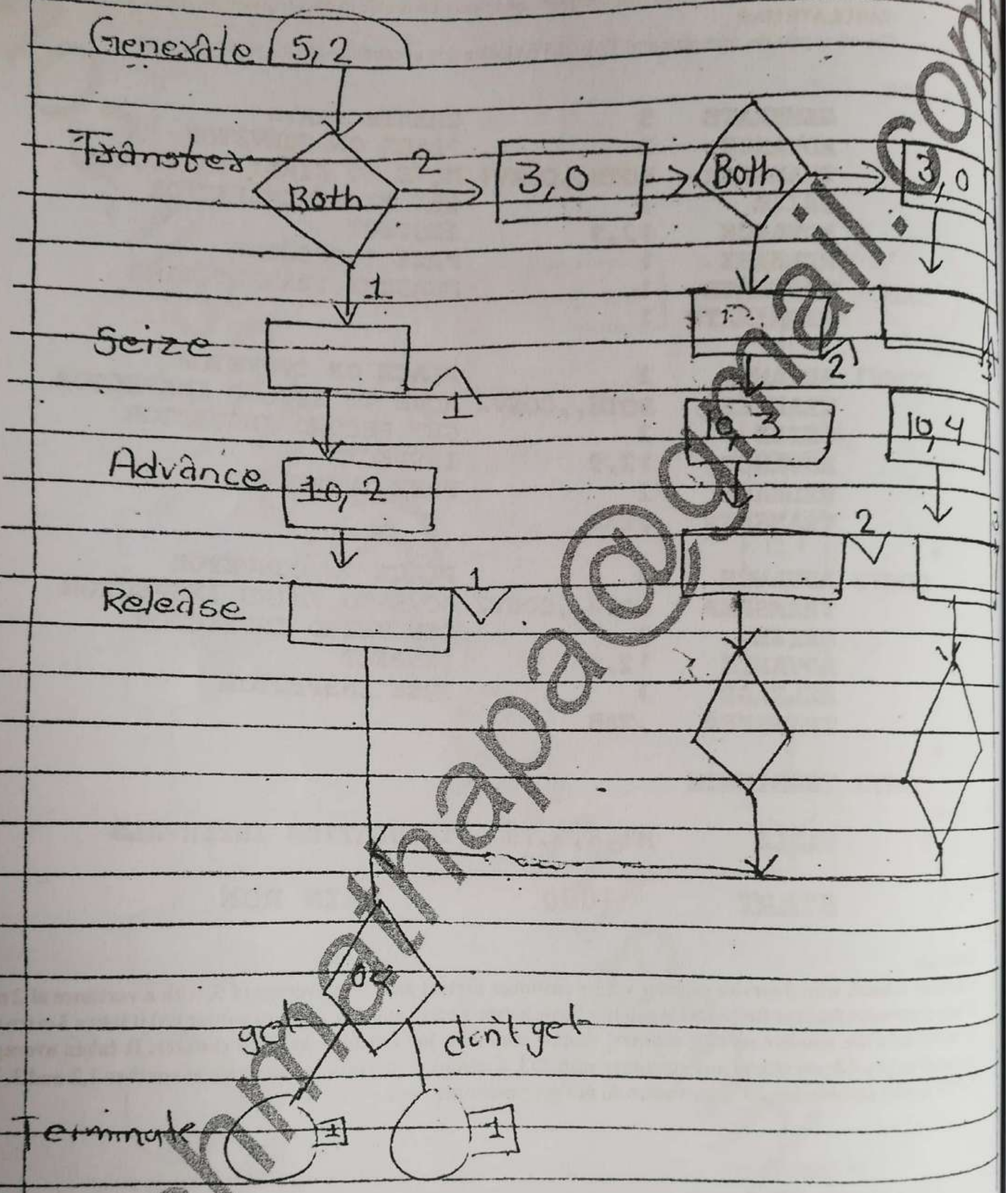
vii. The other pass the transaction to TABULTE block using unconditional TRANSFER Block.

GPSS CODE:

```
        GENERATE    5              CREATE PARTS
        ADVANCE     2              PLACE ON CONVEYOR
        TRANSFER    BOTH,,CONV1    MOVE TO FIRST INSPECTOR
        SEIZE       1              GET FIRST INSPECTOR
        ADVANCE     12,9           INSPECT
        RELEASE     1              FREE INSPECTOR
TAB     TABULATE    1              MEASURE TRANSIT TIME
        TERMINATE   1
*
CONV1   ADVANCE     2              PLACE ON CONVEYOR
        TRANSFER    BOTH,,CONV2    MOVE TO SECOND INSPECTOR
        SEIZE       2              GET SECOND INSPECTOR
        ADVANCE     12,9           INSPECT
        RELEASE     2              FREE INSPECTOR
        TRANSFER    ,TAB
*
CONV2   ADVANCE     2              PLACE ON CONVEYOR
        TRANSFER    BOTH,,CONV2    MOVE TO THIRD INSPECTOR
        SEIZE       3              GET THIRD INSPECTOR
        ADVANCE     12,9           INSPECT
        RELEASE     3              FREE INSPECTOR
        TRANSFER    ,TAB
*
CONV3   TERMINATE
*
1       TABLE       M1,5,5,10      TABULATION INTERVALS
*
        START       1000           MAIN RUN
```

Example

Consider a bank with 3 service counter where customer arrival time is in average of 5, with a variance of 2 minutes. If any customers find the first service counter busy, he/she goes to another service counter but it takes 3 extra minutes to move into the another service counter, similar condition for reaching to third counter. It takes average of 10 minutes to provide service to any customer with 2, 3, 4 minutes' variance respectively at counter 1,2 and 3. Develop GPSS model considering 20 % customer do not get proper services.



Generate
5, 2
Transfer
Both
2
3, 0
Both
3, 0
1
Seize
1
10, 3
10, 4
Advance
10, 2
2
Release
1
or
got
don't get
Terminate
1
1

Assignment

1. **People arrive at the rate of one every 12±6 minutes to make a call from a public telephone booth. If the telephone is busy, 50% of the people come back 10 minutes later to try again. The rest give up. Assuming a call take 5±2, count how many people have given up by the time 100 calls have been completed. Give GPSS block diagram.**
2. **There are 48 people in an institution studying programming. They all started writing a program at the same time. It take each student 15±3 minutes to write a program. Only 35 % of the program will run correctly for the first time. When a program has an error it take 4±2.5 minutes to do the debugging. After debugging only 35% of the program will run successfully. How long does it take for whole class to finish every ones program correctly? Simulate the given program with block diagram and GPSS code.**
3. **Workers come to a supply at the rate of one every 5±2 minutes. Their requisitions are processed by one of two clerks who take 8±4 minutes for each requisition. The requisition are then passed to a single storekeeper who fills them one at a time, taking 4±3 minute for each request. Simulate queue of workers and measure the distribution of time taken for 1000 requisitions to be filled.**
4. **Simulation of Supermarket (See page no. 228 of Gordon)**

Program Control Statement

- Every GPSS program must contain a control **statement SIMULATE** statement. Although it need not be the first statement in the program, it usually is. Most programmers always start their programs with the SIMULATE statement. The general form of it is: SIMULATE n The operand n is optional. If used, it limits that amount of time in minutes which the program will run. This can be handy to use in the de-bugging stage to avoid infinite loops. For example,SIMULATE 2.5 would limit the program to 2.5 minutes running time. When a GPSS simulation run is finished the program does not immediately destroy the model. Instead it looks for more i/p following the START statement keeping the model exactly it was at completion of the run.
- I/P following **the START** statement can change the model. For example, A storage capacity would be redefine by increasing a 'storage' statement for assigning a new value.
- The model could also be modifying by changing existing blocks.
- Certain control statement like **RESET** can be included between START statement will wipe out all the statistics gathered so far, but will learn the system loaded with transaction.
- Another control statement, **CLEAR**, will not only wine wipe out the statistics of preceding run but will also wipe out transaction in the system so that the simulation started from the beginning at the rerun.
- **The END** statement is used to terminate the run.

SIMSCRIPT Program

- The SIMSCRIPT programming system is especially designed to facilitate the writing simulation program.
- SIMSCRIPT is a very widely used language for simulating discrete system.
- This language is very FORTRAN in appearance.
- SIMSCRIPT II.5 can be view as a general programming language with extra features for discrete event simulation.
- Because of this general power and FORTRAN based, SIMSCRIPT has been widely implemented and used discrete simulation language.
- The language can be considered more than just a simulation language since it can be applied to general programming problems.

SIMSCRIPT System Concept

- The system to be simulated is considered to consist of entities having attributes that interact with activities. The interaction causes event that change the state of the system. In describing the system, SIMSCRIPT uses the terms, entities attributes, sets, event routine. When an entity is created it can be interpreted as a generic definition for a class of entities. Individual entities have values all attributes which define particular state of the entity attributes are named not a number. For example, we may define employee to be an entity and AGE, SALARY are attributes.

Entities can be a 2 type

1) Permanent

Entities that are created and remains during the run

2) Temporary

Entities that are created and destroyed during the execution of simulation.

A special emphasis is placed on the manner in which temporary entities form sets. SIMSCRIPT uses pointer to chain together entities that are members of sets. Commands are available to manipulate these sets. The user can define sets and facilities are provided for entering and removing entities into and prompt sets.

Activities are considered as extending over time with their beginning and their end being mark as events occurring instantaneously. Each type of event is described by an event routine, each of which is given a name and programmed as a separate closed sub routine. Events may be

1. Endogenous event(internal)
   - Results from action within the system
   - An endogenous event is caused by scheduling statement in some event routine. The event marking the beginning of an activity will usually schedule the event that marks the end of the activity. If the beginning or ending of an activity implies the beginning of some activity, then the event routing marking the beginning or endo of the first activity will schedule the beginning of second.

2. Exogenous event(External)
   - Results from action in the system environment.
   - Exogenous event requires the reading of data supplied by the users.
   - There can be many data sets representing different sets of external event such as time at which the event occurs.
   - Event routine are needed to execute the changes that results when an external event becomes due for execution
   - In practice, external event such as arrival are often generated using bootstrap method.

Structure / Organization of Simscript program

Since event routines are closed routines some means must be provided for transferring control between them. It is done by the use of event notice. These event notices are created when it is determined that an event is scheduled. An event notice exists for every endogenous events schedule to occur.

An event notice records the time the event is due to occur and the event routine that is to execute the event. If the event is to involve one of the temporary entities, the event notice will usually identify which one is involved. The event notices are filled in chronological order.

The general manner in which simulation proceed is illustrated in below fig.

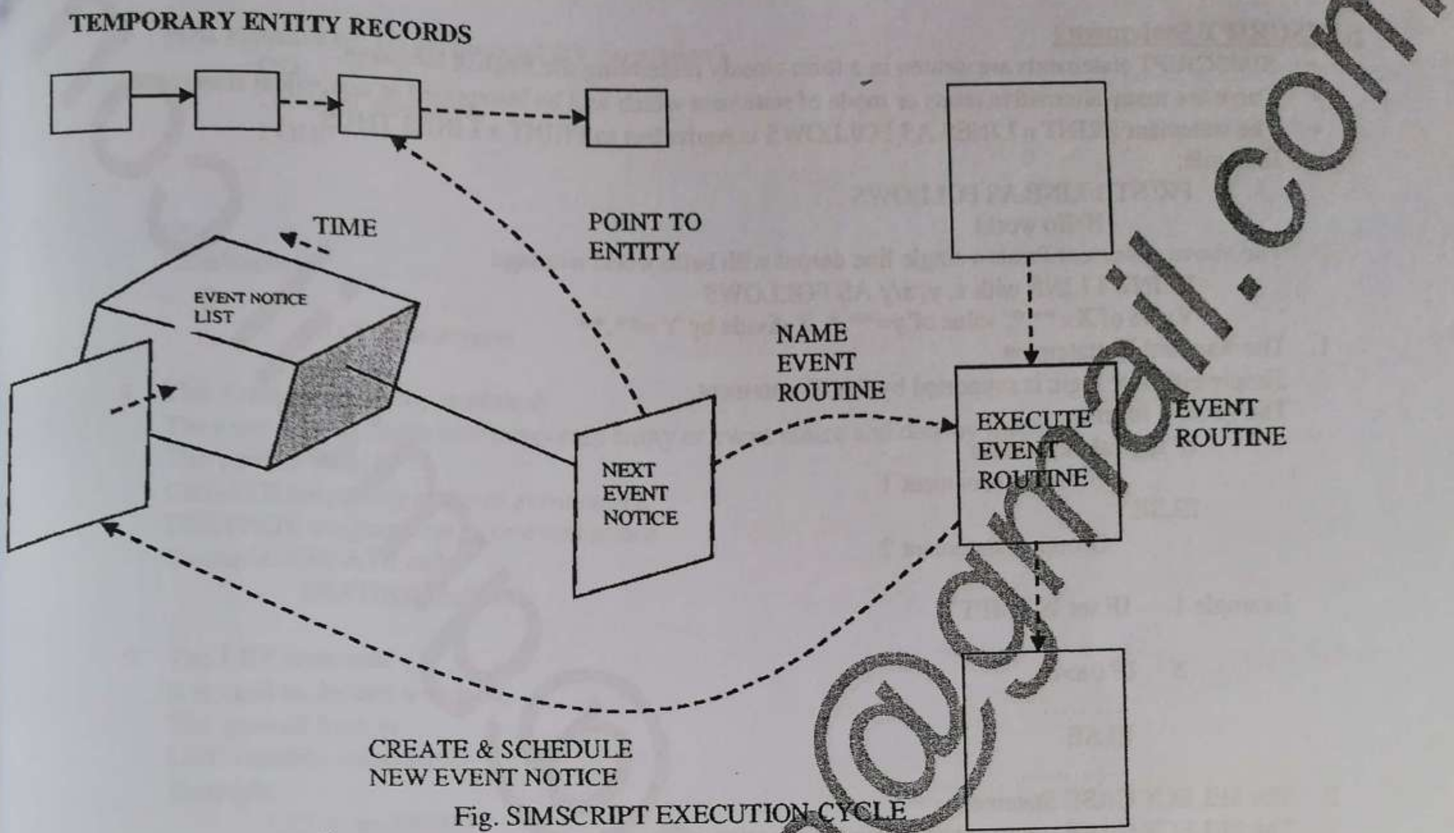


Fig. SIMSCRIPT EXECUTION CYCLE

When all events (that can be executed at a particular time) have been processed, the clock is update to the time next event notice and the control is passed to the event routine as identified by the notice. These actions are automatic and doesn't need to be programmed.

If the event executed by a routine results another events, the routine must create a new event notice and filed it with the other notice.

In case of exogenous events a series of exogenous event statements are created. For each event this statements are similar to event notices are also filed into chronological order and are read by the program.

Names and Label

- User must describe each entities by giving a name and its attributes.
- Name may consist of any combination of letters and digits(atleast one letter).
- Reserved name can't be used.
- For example, if an entity type is to represent a person, it could be named PERSON. If the attributes of PERSONG are age and education then they can be named as AGE and EDUCATION or PERSONG.AGE and PERSON.EDUCATION respectively.
-
- Labels are used for identifying programming statements.
- Labels are identified by being enclosed between single quotation mark.

```
      ' top' PRINT ..........
      ...
      ...
      ...
            GOTO top
```

SIMSCRIPT Statements

- SIMSCRIPT statements are written in a form closely resembling the English language.
- There are many alternative terms or mode of statement which will be interpreted as equivalent statement
- The statement PRINT n LINES AS FOLLOWS is equivalent to PRINT n LINES THUS

  Example:

```
        PRINT 1 LINE AS FOLLOWS
                Hello world
```

  The above statement Prints a single line output with hello world message

```
        PRINT 1 LINE with x, y, x/y AS FOLLOWS
        Value of X=**.*, value of y=**.*, X divide by Y =**.**
```

1. The standard IF statement

   Simple either or logic is supported by the IF statement.

   The general form is

```
        IF logical expression
                Group of statement 1
        ELSE
                Group of statement 2
```

```
Example 1.   IF set IS EMPTY
             ......................
         2   IF ( a>b)
             ..........
             ELSE
             ............
```

2. The SELECT CASE Statement

   The SELECT CASE statement provides for choosing one among many

   The general form is

```
        SELECT CASE expression
        CASE constantlist1
                Group of statement1
        CASE constantlist2

                Group of statement2
        CASE constantlistn
                Group of statement
        DEFAULT
                Group of Statements
        END SELECT
```

3. Looping statement

   These statements serves for iteration purpose.

   The general form is

```
WHILE expression
        DO
        Group of statements
        LOOP

Or

UNTIL expression
        DO
        Group of statements
        LOOP
```

Or

FOR expression1 TO expression2 BY expression3
DO
Group of statements
LOOP

Or
'labelname' ……………..
……………………………..
GO TO labelname

4. The create and destroy statement
They are used to create new temporary entity or event notice and destroy theme
The general form is
CREATE *temporary entity or event notice*
DESTROY temporay entity or event notice
Example: CREATE call
DESTROY call

5. The LET statement
It is used to declare a variable
The general form is
LET variable=expression
Example:
LET pop=400000
LET SUM = (A+B)

1. A simple SIMSCRIPT II program two read three number and display their sum

```
READ X, Y AND Z
ADD X + Y TO Z
PRINT 1 LINE WITH Z AS FOLLOWS
   X + Y + Z = ****
STOP
```

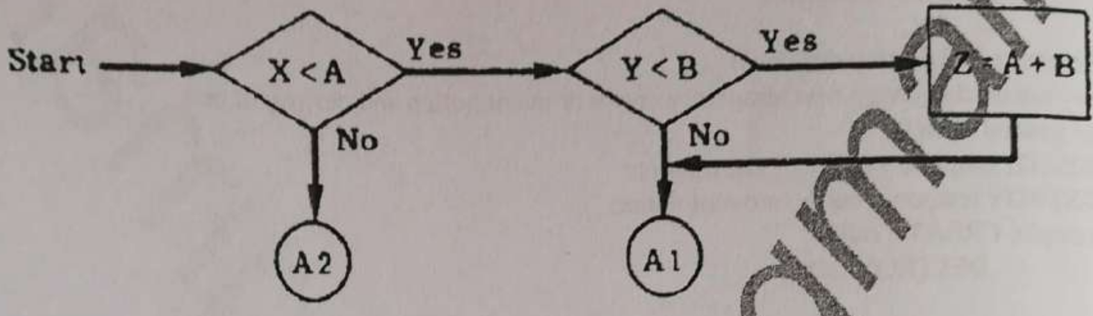
The above program displays the sum of three number with label X+Y+Z=, and 4 digit integer value as sum

2. A SIMSCRIPT II program to read a set of 100 numbers and add together those number which are greater than 5.

```
'READ'     ADD 1 TO COUNT
           IF COUNT > 100 GO TO FINISH
           ELSE READ N
           IF N IS GREATER THAN 5   ADD N TO SUM
           REGARDLESS GO TO READ.
'FINISH'   ''CONTINUE PROGRAM''
```

Note: if a pair of single quote is enclosed then it means comment.
In addition to else, we can use REGARDLESS, OTHERWISE, ALWAYS as needed as shown in below example

```
IF X IS LESS THAN A
     IF Y IS LESS THAN B
          LET Z = A + B
     REGARDLESS
          GO TO 'A1'
OTHERWISE
GO TO 'A2'
```



Defining the SYSTEM

- A SIMSCRIPT program begins with a preamble section that define the structure and establishes the condition under which simulation will run.
- Names must be given to every entity and its attributes.
- Items in the SIMSCRIPT list can be separated by commas or the word AND.
- Variables can be real, integer or alphabetical
- If the mode is other than the normal mode(INTEGER), Define statement list the variable and declares their mod
  Example: DECLARE name1, name2 AS REAL VARIABLES
- Events are represented by individual routine which consists of local variable so they need not be defined in preamble i.e. beginning. Note only global variable are defined in the preamble.
- **The permanent entities temporary entities and the event notices are defined in** the list, following the statement PERMANENT ENTITIES, TEMPORARY ENTITIES and EVENT NOTICES respectively. Then it is followed by the statement of the form
  EVERY entity HAS A attr1, AND attr2……………….AND attrn
- In telephone simulation for lost call, telephone lines are permanent entities and has attributes to indicate whether the line is busy or not. So the name TLINE is used to represent the telephone line and the attribute will be called STATE. STATE is simply an integer variable which take value 0 to indicate free and 1 to indicate busy state.
- Calls are represented by temporary entities, with two attributes for carrying the origin and destination.
- The temporary entities has two attributes for recording the origin and destination of the call.

```
1   ''    TELEPHONE SYSTEM - MODEL 1
2   PREAMBLE
3   NORMALLY,MODE IS INTEGER
4   EVENT_NOTICES INCLUDE ARRIVAL AND CLOSING
5         EVERY DISCONNECT HAS A DIS.CALL
6   TEMPORARY ENTITIES
7         EVERY CALL HAS AN ORIGIN AND A DESTINATION
8   DEFINE LINKS.IN.USE, MAX.LINKS, BLOCKED, BUSY, FINISHED
9         AND STOP.TIME AS VARIABLES
10  DEFINE INTER.ARRIVAL.TIME AND MEAN.LENGTH AS REAL VARIABLES
11  PERMANENT ENTITIES
12        EVERY TLINE HAS A STATE
13  .     DEFINE SECS TO MEAN /60 MINUTES
14  END
```

Fig: Telephone System(lost call)

In the above figure, three events routines is given, so there will be three event notice. Line 4 and 5 define the event notice. The event notice name and event routine name must be same. Here in this case, the routine are going to be called ARRIVAL, DISCONNECT and CLOSING. Because the ARRIVAL and CLOSING event notices do not need attribute / **parameter**, their definition is included in the event notice header. The DISCONNECT routine however, will need to know which particular call is to be disconnected, so the DISCONNECT event notice needs an attribute / **parameter** with which to identify a call.

Referencing Variable

- Single variables are simply referenced by using their name.
- Permanent entities such as telephone line, are represented by arrays, with an array of attribute, identified by the attribute name. It's because they are fixed/static in nature.
- Temporary entities and event notices are created and destroyed as simulation proceeds so they can't be kept in fixed table.
- For example, CREATE A CALL and DESTROY A CALL, for each creation, the program refers to data structure given in preamble and creates a block of words. Since the number of block will fluctuate, they can't be kept in fixed table.
- So every type of temporary entity, or event notice, the system automatically reserves a location by holding a pointer that has the same name as the entity or notice. When a CREATE statement is used, the pointer to the entity or notice being created is placed at that location. For example, the command CREATE CALL, puts the pointer to the created record in a location called CALL.
- If during the current execution of a routine, it is necessary to create a second coy of an entity or notice, the second creation will write over the pointer to the first unless another form of CREATE statement is used. This take the form CREATE A CALL CALLED name. the pointer to the created record then goes to location called name.
- Also, SIMSCRIPT allow the formation of set linking groups of temporary entities having a common property. The set are organized as list structure. Every individual temporary entity that might belong to the set has two word pus aside as pointer for identifying its predecessor and successor in the set.

Assignment

1. **Write the main routine in simscript program for telephone system.**

chapter- 7

ANALYSIS OF SIMULATION OUTPUT

By: Prof. S. Shakya

ANALYSIS OF SIMULATION OUTPUT

- **Estimation Methods**
- **Simulation Run Statistics**
- **Replications of Runs**
- **Elimination of Initial Bias**

Estimation Methods

- Estimates the range for the random variable so that the desired output can be achieved.
- Infinite population has a stationary probability distribution with a finite mean μ and finite variance σ^2.
- Sample variable and time does not affect population distribution .
- Variables that meet all these conditions are called independently and identically distributed.

Estimation Methods

- Central limit theorem must be invoked to rely upon normal distribution of infinite population.
- Only then we can apply estimation method to that variable taken from infinite population.

Estimation Methods

- A random variable is drawn from an infinite population that has a stationary probability distribution with a finite mean, μ, and finite variance, σ^2.

- Random variables that meet all these conditions are said to be independently and identically distributed, usually abbreviated to i.i.d. for which the central limit theorem can be applied.

- The theorem states that the sum of n i.i.d. variables, drawn from a population that has a mean of μ and a variance of σ^2, is approximately distributed as a normal variable with a mean of nμ and a variance of $n\sigma^2$.

5

Estimation Methods

- Let x_i (i=1,2,...,n) be the n i.i.d. random variables. Then normal variate:

$$z = \frac{\sum_{i=1}^{n} x_i - n\mu}{\sqrt{n}\,\sigma}$$

6

Estimation Methods

- In terms of sample mean $\bar{x}$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Where,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Estimation Methods

- The probability density function of the standard normal variate is shown in the figure.

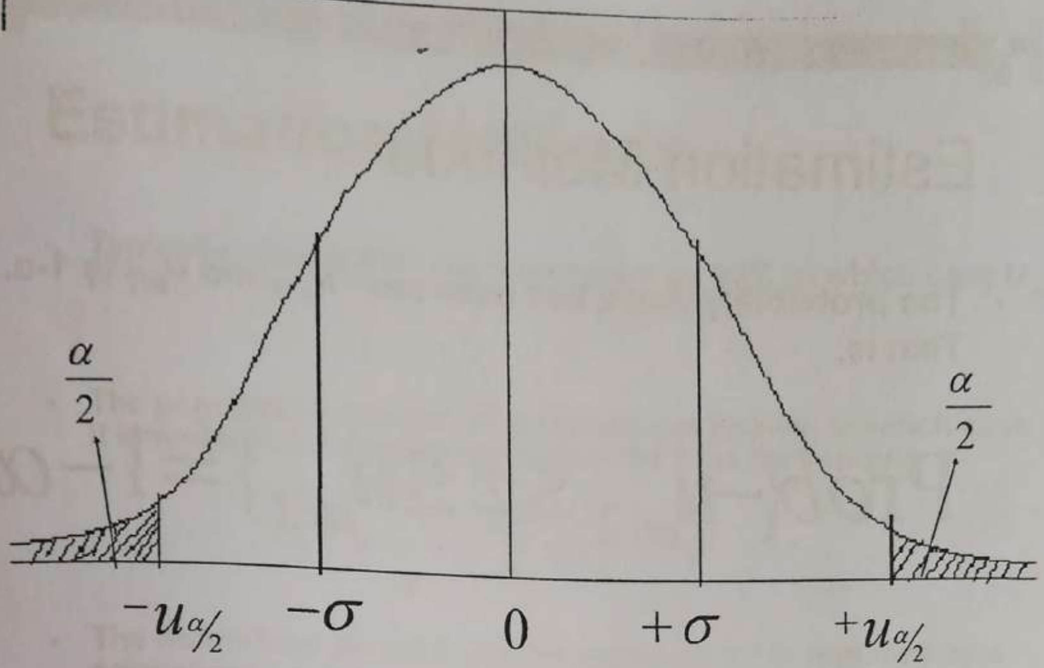


Fig:Probability density function of standard normal variate

Estimation Methods

- The integral from -∞ to a value μ is the probability that z is less than or equal to μ. The integral is denoted by $\phi(u)$
- Suppose the value of μ is chosen so that $\phi(u)=1-\alpha/2$ where α is some constant less than 1, and denote this value of u by $u_{\alpha/2}$.
- The normal distribution is symmetric about its mean, so the probability that z is less than $-u_{\alpha/2}$ is also α/2.

Estimation Methods

- The probability that z lies between $-u_{\alpha/2}$ and $u_{\alpha/2}$ is 1-α. That is,

$$\mathrm{Prob}\{-u_{\alpha/2} \le z \le u_{\alpha/2}\} = 1-\alpha$$

Estimation Methods

- In terms of sample mean, this probability statement can be written as:

$$\mathrm{Prob}\{\bar{x} + \frac{\sigma}{\sqrt{n}} u_{\alpha/2} \ge \mu \ge \bar{x} - \frac{\sigma}{\sqrt{n}} u_{\alpha/2}\} = 1-\alpha$$

- The constant 1- α is the confidence level and the confidence interval is

$$\bar{x} \pm \frac{\sigma}{\sqrt{n}} u_{\alpha/2}$$

Estimation Methods

- **Typically, the confidence level might be 90% in which case $u_{\alpha/2}$ is 1.65.**

- **The population variance σ^2 is usually not known; in which case it is replaced by an estimate calculated from the formula**

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- **The normalized random variable based on σ^2 is replaced by a normalized random variable based on s^2. This has a Student-t distribution, with n-1 degrees of freedom.**

Student-t Distribution

	Confidence Probability			
v	.80	.90	.96	.98
1	3.078	6.314	15.895	31.821
2	1.886	2.920	4.849	6.965
3	1.638	2.353	3.482	4.541
4	1.533	2.132	2.999	3.747
5	1.476	2.015	2.757	3.365
6	1.440	1.943	2.612	3.143
7	1.415	1.895	2.517	2.998
8	1.397	1.860	2.449	2.896
9	1.383	1.833	2.398	2.821
10	1.372	1.812	2.359	2.764
11	1.363	1.796	2.328	2.718
12	1.356	1.782	2.303	2.681
13	1.350	1.771	2.282	2.650
14	1.345	1.761	2.264	2.624
15	1.341	1.753	2.249	2.602
16	1.337	1.746	2.235	2.583
17	1.333	1.740	2.224	2.567
18	1.330	1.734	2.214	2.552
19	1.328	1.729	2.205	2.539
20	1.325	1.725	2.197	2.528
25	1.316	1.708	2.167	2.485
30	1.310	1.697	2.147	2.457
40	1.303	1.684	2.123	2.423
50	1.299	1.676	2.109	2.403
75	1.293	1.665	2.090	2.377
100	1.290	1.660	2.081	2.364
∞	1.282	1.645	2.054	2.326

Estimation Methods

- The quantity $u_{\alpha/2}$ used in the definition of a confidence interval given above, is replaced by a similar quantity, $t_{n-1,\alpha/2}$ based on the Student-t distribution.
  (which tables are also readily available)
- The Student-t distribution is strictly accurate only when the population from which the samples are drawn is normally distributed.
- Expressed in terms of the estimated variance s^2, the confidence interval for $\underline{x}$ is defined by

$$\overline{x} \pm \frac{s}{\sqrt{n}} t_{n-1,\alpha/2}$$

CONCLUSION

- Hence the estimation method gives the desired range of the sample variable taken from infinite population.

Simulation Run Statistics

- Consider a single-server system in which the arrivals occur with a Poisson distribution and the service time has an exponential distribution.
- Suppose the study objective is to measure the mean waiting time, defined as the time entities spend waiting to receive service and excluding the service time itself.
- This system is commonly denoted by M/M/1 which indicates; first , that the inter-arrival time is distributed exponentially; second that the service time is distributed exponentially; and, third, that there is one server. The M stands for Markovian, which implies an exponential distribution.

Simulation Run Statistics

- In a simulation run, the simplest approach is to estimate the mean waiting time by accumulating the waiting time of n successive entities and dividing by n.
- This measure, the sample mean, is denoted by $\bar{x}(n)$ to emphasize the fact that its value depends upon the number of observations taken.
- If x_i (i=1,2,….,n) are the individual waiting times (including the value 0 for those entities that do not have to wait), then

$$\bar{x}(n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Simulation Run Statistics

- Whenever a waiting line forms, the waiting time of each entity on the line clearly depends upon the waiting time of its predecessors.

- Any series of data that has this property of having one value affect other values is said to be autocorrelated.

- The sample mean of autocorrelated data can be shown to approximate a normal distribution as the sample size increases.

Simulation Run Statistics

- The equation $\bar{x}(n) = \frac{1}{n}\sum_{i=1}^{n} x_i$ remains a satisfactory estimate for the mean of autocorrelated data.

- A simulation run is started with the system in some initial state, frequently the idle state, in which no service is being given and no entities are waiting.

- The early arrivals then have a more than normal probability of obtaining service quickly, so a sample mean that includes the early arrivals will be biased.

Simulation Run Statistics

- For a given sample size starting from a given initial condition, the sample mean distribution is stationary; but , if the distributions could be compared for different sample sizes, the distribution would be slightly different.

- The following figure is based on theoretical results, which shows how the expected value of sample mean depends upon the sample length, for the M/M/1 system, starting from an initial empty state, with a server utilization of 0.9.

Simulation Run Statistics

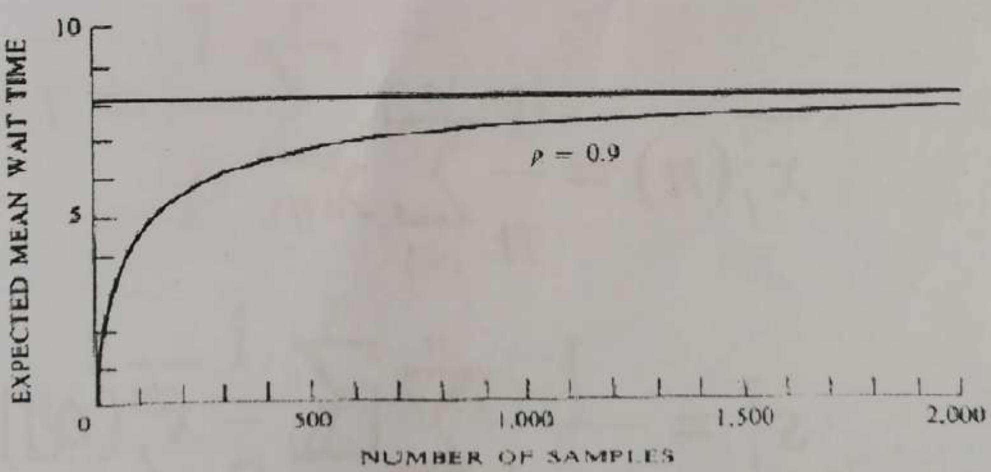


Figure 14.2. Mean wait time in M/M/1 system for different sample sizes.

Replications of Runs

- The precision of results of a dynamic stochastic can be increased by repeating the experiment with different random numbers strings.
- For each replication of a small sample size, the sample mean is determined.
- The sample means of the independent runs can be further used to estimate the variance of distribution. Let X_{ij} be the i^{th} observation in j^{th} run, then the sample mean and variance for the j^{th} run are:

Replications of Runs

$$\overline{x_j}(n) = \frac{1}{n}\sum_{i=1}^{n} x_{ij}$$

$$s_j^2 = \frac{1}{n-1}\sum_{i=1}^{n}[x_{ij} - \overline{x_j}(n)]^2$$

Replications of Runs

- When we have similar means and variances for m independent measurements, then by combining them, the mean and variance for the population can be obtained as:

Replications of Runs

$$\bar{x} = \frac{1}{p}\sum_{j=1}^{p}\overline{x_j(n)}$$

$$s^2 = \frac{1}{p}\sum_{j=1}^{p}s_j^2(n)$$

Replications of Runs

- The following figure shows the result of applying the procedure to experimental results for the **M/M/1** system.

Replications of Runs

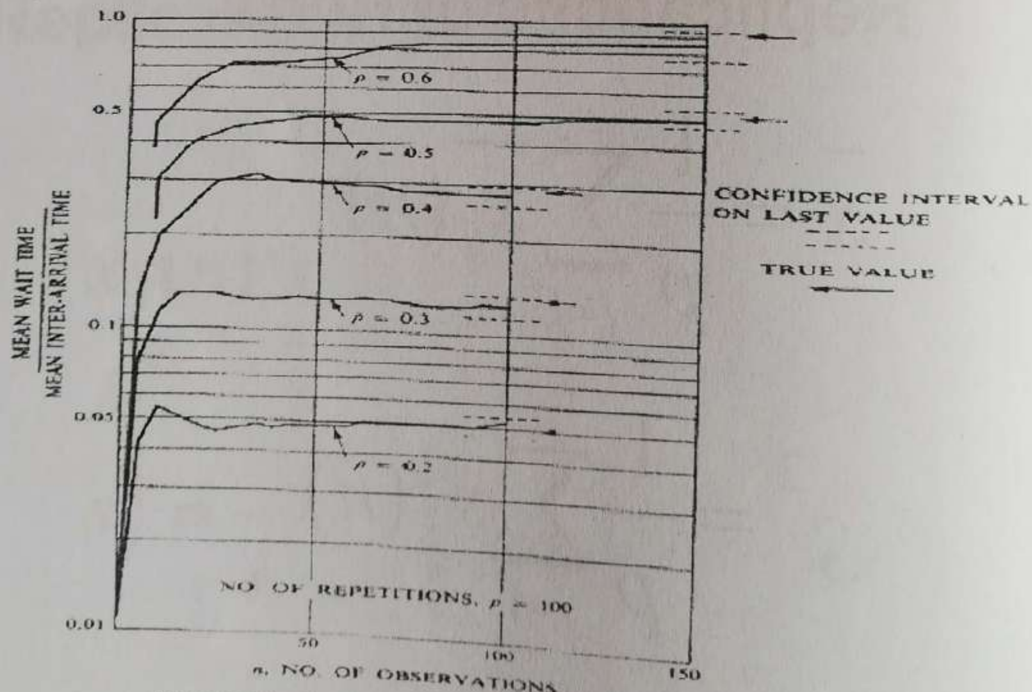


Figure 14-3. Experimentally measured wait time in M/M/1 system for different sample sizes.

Replications of Runs

- This variance can further be used to establish the confidence interval for p-1 degrees of freedom.
- The length of run of replications is so selected that all combined it comes to the sample size N. i.e. p.n=N.
- By increasing the number of replications and shortening their length of run, the confidence interval can be narrowed.
- But due to shortening of length of replication the effect of starting conditions will increase.
- The results obtained will not be accurate, especially when the initiazation of the runs is not proper.
- Thus, a compromise has to be made.
- There is no established procedure of dividing the sample size N into replications.
- However, it is suggested that the number of replications should not be very large, and that the sample means should approximate a normal distribution.

Elimination of Initial Bias

- Two general approaches can be taken to remove the bias: the system can be started in a more representative state that the empty state, or the first part of the simulation can be ignored.
- The ideal situation is to know the steady state distribution for the system, and select the initial condition from that distribution.
- In the study previously discussed, repeated the experiments on the M/M/1 system, supplying an initial waiting line for each run, selected at random from the known steady state distribution of waiting line.

Elimination of Initial Bias

- The case of 40 repetitions of 320 samples, which previously resulted in a coverage of only 9% was improved to coverage of 88%.
- The more common approach to removing the initial bias is to eliminate an initial section of the run.
- The run is started from an idle state and stopped after a certain period of time.

Elimination of Initial Bias

- The run is then restarted with statistics being gathered from the point of restart.
- It is usual to program the simulation so that statistics are gathered from the beginning, and simply wipe out the statistics gathered up to the point of restart.
- No simple rules can be given to decide how long an interval should be eliminated.

Elimination of Initial Bias

- The disadvantage of eliminating the first part of a simulation run is that the estimate of the variance, needed to establish a confidence limit, must be based on less information.

- The reduction in bias, therefore, is obtained at the price of increasing the confidence interval size.

Reference

- Geoffrey Gordon, System Simulation, Chapter 14, analysis of simulation output

2016 spring

2.(a) Why simulation is essential in depicting real world problem. Differentiate continous and discrete system.

Simulation is a representation of real life system by another system, which inherits the important characteristics of real system and allows experimentation on it. simulation is used when conducting experiments on a real system.

Simulation is essential in depicting real world because it may be too difficult or expensive to observe a real operational system, parts of system may not be observable (e.g internals of a ~~system~~ silicon chips.

1. Analyze the system before they build
2. Reduce risk: how do you know they are right changes?
3. Minimize your spending: cash is tight. You cannot afford to waste a single dollars
4. Reduces no. of design mistakes
5. Optimize design
6. Analyze operational system
7. Create virtual environment for training

Continous and Discrete system

→ System such as aircraft, in which the changes are ~~peri~~ predominantly smooth, are called continous system

→ System like the factory, in which the changes are predominantly discontinous, called discrete systems.

→ Few systems are wholly continous or discrete

→ aircraft may make discrete adjustment to to its trim as altitude changes. while in factory, machining proceeds continously, even though start and finish of a job are discrete changes.

→ The complete aircraft system might even be regarded as a discrete system.

→ In factory system, if the number of parts is sufficiently large, there may be no point in treating the no. as a discrete variable.

→ Instead, the no. of parts might be represented by a continous variable with the machining activity controlling the rate at which parts flow from one state to another.

→ System that are intrinsically continous but information about them is only available at discrete points in time.

→ General programming methods used to simulate continous system and discrete system differently.

Qn. 21a) What are the steps used in simulation study? Explain with neat block.

There are four phases in simulation study:

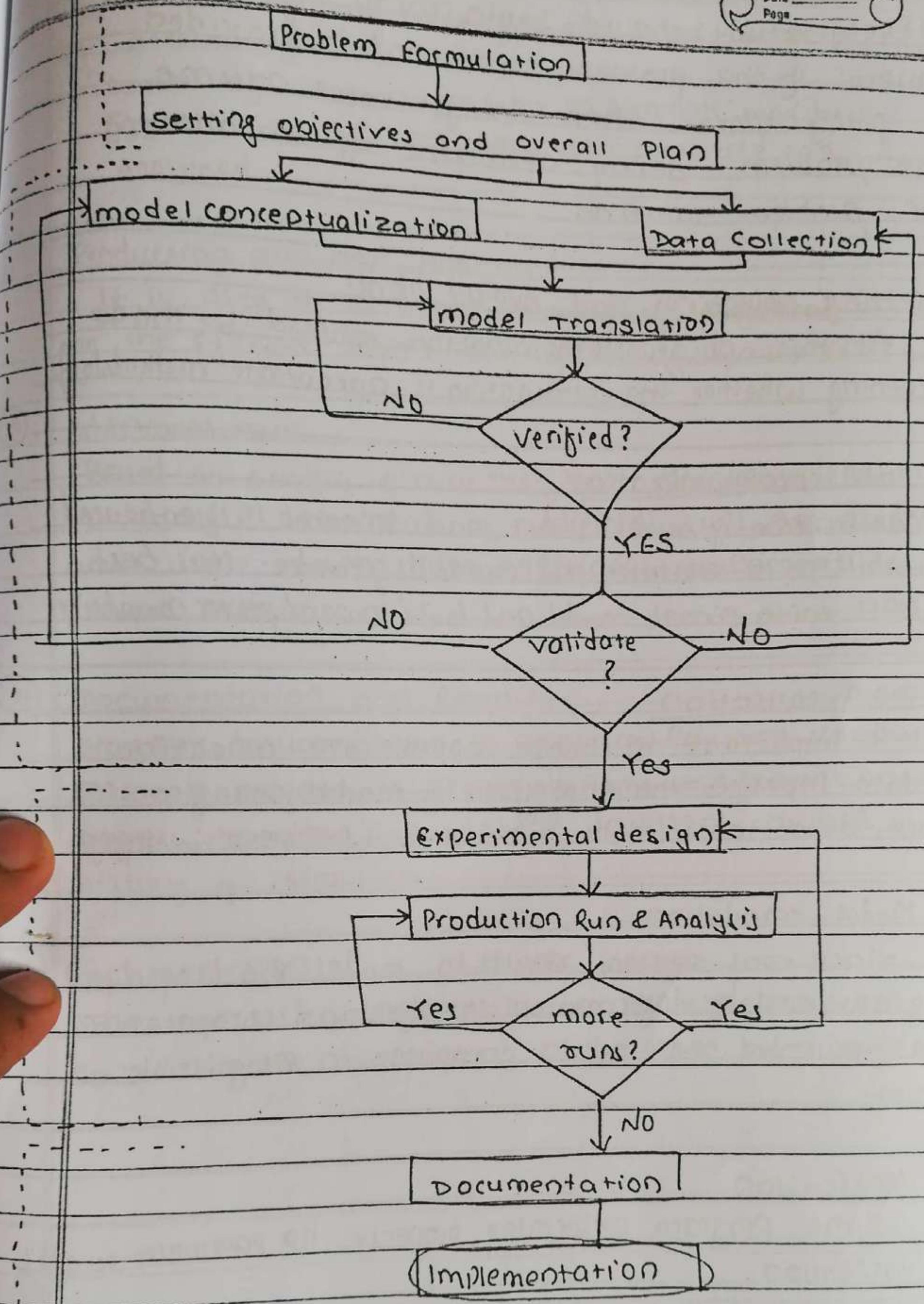
Phase I
- Problem formulation
- Setting objectives and overall plan

Phase II
- Model Conceptualization
- Data collection
- Model translation
- verification / validation

Phase III — Experimental design
- Production runs and Analysis
- Additional runs

Phase IV — (i) Documentation and reporting (ii) implementation.

Scanned by CamScanner


Problem formulation
setting objectives and overall Plan
model conceptualization
Data Collection
model Translation
verified?
No
YES
validate ?
NO
No
Yes
Experimental design
Production Run & Analysis
more runs?
Yes
Yes
NO
Documentation
Implementation

(i) problem formulation:

Every study should begin with 'statement of problem'. If the problem statement is provided by policy maker, then analyst must assume that problem being developed by the analysis for ~~problem~~ solution

(ii) setting objectives and overall plans.

At this point, determination should be made concerning whether the simulation is appropriate methodology.

(iii) Model conceptualization

Construction of model is art and science. It is enhanced by ability to abstract the essential feature. It is best to start with simple model and build toward greater complexity.

(iv) Data collection

It is important to choose appropriate collection of data input as the complexity of model changes. since, data takes more time and is important in early.

(v) Model translation

since, real system results in model that require a great deal of information storage and computation and must be entered in computer in recognizable format.

(vi) verification

is the program performing properly. i/p parameter are right?

(vii) validation

determination that model is an accurate representation of real system usually.

(viii) experimental design:

- the real system may be simulated correctly by determining alternative.
- concerning with alternative to simulate may be a function of runs that have been completed and Analyzed.

(ix) Production run and Analysis:

It is used to estimate measure of performance for the system design that are being simulated

(x) Additional runs

Based on analysis of runs that have been completed the analyst determines if additional runs are needed and what design those additional experiment should follow.

(xi) Documentation and Reporting

program documentation is necessary for the various reason if program is re-used by same or different analyst progress reporting provides the important written history of simulation.

(xii) Implementation

The success of implementation phase depends on how well the previous steps have been performed.

2016 Fall

11a) Diff bet^n continous and discrete system in detail. What do you mean by verification process.

First part done.

It is concerned with building the model right. It is utilized in the comparision of conceptual model to the computer representation that implements that conception

Verification is the process of determining that a model implementation ~~that a model imp~~ and its associated data accurately represent the developer's conceptual description and specification

validation is the process of determining the degree to which a model and its associated data are an accurate representation of the real world from the ~~ver~~ perspective of the intended use of the model.

verification: system ले हामीले सोचे जस्तो काम गर्छ?
validation: system ले real world ले जस्तो काम गर्छ?

verification phase focus on comparing the element of a simulation a model of the system with the description of what the requirements and capabilities of model were to be. Verification is a iterative process aimed at determining whether the difference are acceptable given the intented use of the model.

The Technique of Simulation

2016 Spring

(1b)

$$I = \int_0^5 \frac{x^3}{x^4+16}$$

$\rightarrow$ let $y = \int_0^5 \frac{x^3}{x^4+16}$

$$f(x) = \frac{x^3}{x^4+16}$$

The To find max^m value of y.

$$y'(x) = 0$$

$$\frac{d}{dx}\left(\frac{x^3}{x^4+16}\right) = 0$$

$$\left[\frac{d(u/v)}{dx} = \frac{v\,du - u\,dv}{v^2}\right]$$

$$\frac{(x^4+16)\cdot 3x^2 - x^3 \cdot 4x^3}{(x^4+16)^2}$$

$$3x^6 + 48x^2 - 4x^6 = 0$$

$$x^2(48 - x^4) = 0$$

$\therefore x = 0$ and $x^4 = 48$

$$x = \sqrt[4]{48}$$

C is maximum optimum at $x = \sqrt[4]{48}$

$$\therefore C = f(x) = \frac{x^3}{x^4+16} = 0.2849$$

$b = 5$ by $9n$)
$a = 0$

By monte carlo.

$$\frac{n}{N} = \frac{\int_a^b f(x)\,dx}{c * (b-a)}$$

$$\therefore \int_a^b f(x)\,dx = \frac{n}{N} * c * (b-a)$$

$$= \frac{n}{N} * 0.2859 * (5-0)$$

$$= \frac{n}{N} * 1.4295$$

~~Suppose, let~~ choose N = total no. of sample
n = sample falls under the $f(x)$

S.N	N	n	(S) standard	(O) Observe	error = $\left(\frac{S-O}{S}\right) * 100\%$
1	1000	500	0.9296	0.7147	23.11 %
2	1000	700	0.9296	1.00	7.64 %
3	1000	600	0.9296	0.8577	7.73 %

Extra:

Use calculator, to find observe value
ie Observe = 0.9296

trick, ~~multiply~~ divide it by (1.4295) we get 0.6502

$$\left[\frac{n}{N} * 1.4295\right]$$

multiply by 1000, 650

Choose n around 650

Why monte-carlo method is best for computing static model? Derive the value of pi using monte-carlo simulation method.

Monte-carlo simulation ~~or~~ probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management and cost.

In general, Montecarlo method are used in mathematics to solve various problem by generating suitable random numbers and observing that fraction of numbers that obey some properties.

For static model:

Useful when we need to estimate the value of function: complicated to deal with mathematically.

In static, we deal with repeated random trials

Code for pi in Python

```
from random import random
from math import sqrt
#Number of random points
N = 10000
# Counter of points inside
I = 0
for i in range(N):
    #Generate random point
    # in the 1x1 square:
    x = random()
    y = random()
    #Is it inside the circle?
     r = sqrt(x**2 + y**2)
     If r<1: I += 1
#calculate Pi:
print(4*(I/N))
```

Output:

3.136
3.04
3.1768
3.1496

2(b) Explain Analog Computer. Draw suitable analog computer model for following set of diff. eq?

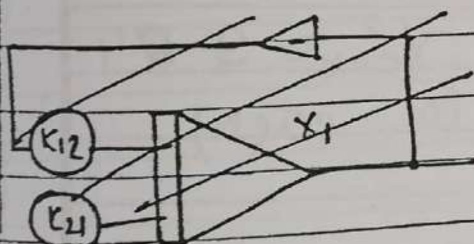
x' $$\frac{dx_1}{dt} = -k_{12}x_1 + k_{21}x_2$$

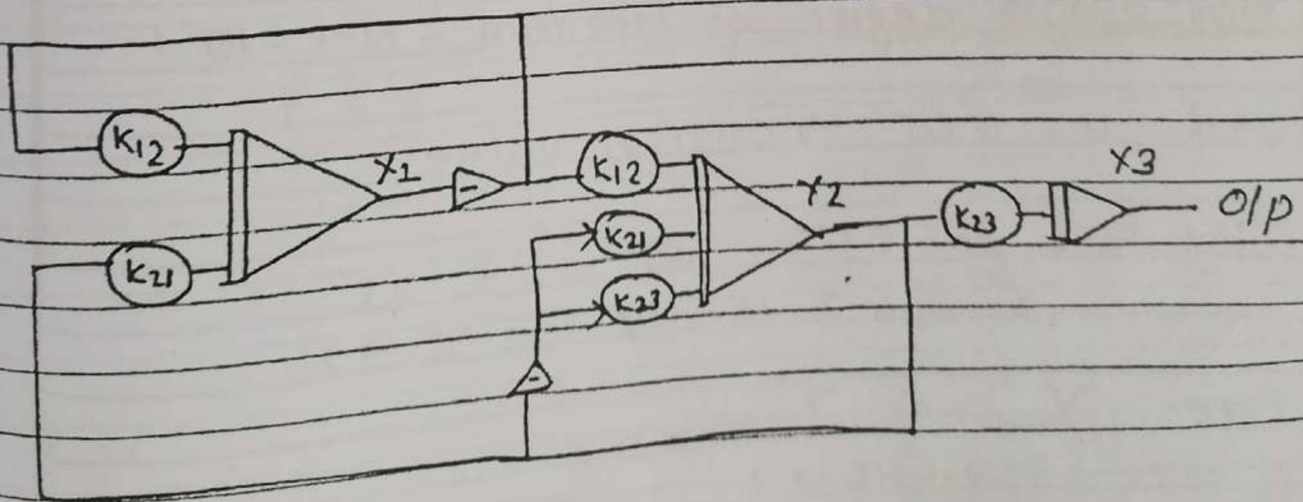
x'' $$\frac{dx_2}{dt} = -k_{12}x_1 - (k_{21}+k_{23})x_2$$

x''' $$\frac{dx_3}{dt} = k_{23}x_2$$

→ An analog computer is a computer which is used to process analog data. Analog data are stored in continous form of physical quantities and perform calculation with the help of measures. Analog computer are excellent for situations which require data to be measured directly without converting into numerals or codes.

Advantage: real time operation
simultaneous computation





Use Chi-Square test with $\alpha = 0.05$ to test whether data shown below are uniformly distributed Critical value for $n=10$ is 16.9

$$X_{calc} = \sum \frac{O_i - e_i}{e_i}$$

S.N	interval	Observed $(O_i$	expected E_i	$(O_i - e_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	0 - 0.10	8	10	4	0.4
2	0.11 - 0.20	8	10	4	0.4
3	0.21 - 0.30	10	10	0	0
4	0.31 - 0.40	9	10	1	0.1
5	0.41 - 0.50	12	10	4	0.4
6	0.51 - 0.60	8	10	4	0.4
7	0.61 - 0.70	10	10	0	0
8	0.71 - 0.80	14	10	16	1.6
9	0.81 - 0.90	10	10	0	0
10	0.91 - 1.00	11	10	1	0.1

$X^2_{calc} = 3.4$

$e_i = \frac{\sum O_i}{n} = \frac{100}{10} = 10$ (For each)

degree of freedom $= N-1 = 10-1 = 9$

and $\alpha = 0.05 \rightarrow$ critical value

Using table for 5% at $N=9$ is

$X^2_{table} = 16.919 \approx 16.92$

Here $X^2_0 < X^2_{0.5}$

$3.8 < 16.92$

$\therefore$ value is accepted [yes distributed]

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How to get Observed — count each & mark in range.

0 – 0.10	|||||||||	8
0.11 – 0.2	|||||||||	8
0.21 – 0.3	||||||||||	10
0.31 – 0.4	|||||||||	9
0.41 – 0.5	||||||||||||	12
0.51 – 0.6	||||||||	8
0.61 – 0.7	||||||||||	10
0.71 – 0.8	||||||||||||||	14
0.81 – 0.9	||||||||||	10
0.91 – 1/00	|||||||||||	11
		100

(~~correct~~ if no. of data)

2015 spring

Draw cobweb model (in graph) for:

(i) fluctuation of market price

(ii) cobweb model for market economy graph from given data.

$D = 12.4 - 1.2P$ Demand

$S = 8.0 - 0.6P_{-1}$ Supply

$D = S$

$P_0 = 1.0$ market price

Cobweb model

A particularly simple, but nevertheless useful, distributed lag model can be constructed from the static market model. ~~the~~ To be more realistic, the supply should be dependent upon the price from the previous marketing period since that is only figure available to the supplier at the time of making future plans. The demand however, will respond to current price

$D = 12.4 - 1.2P$

$S = 8.0 - 0.6P_{-1}$

$D = S$

Given an intial value of Price P_0, the value of S at the end of the first interval can be derived. This determines the value of D, since the market is cleared, and from this the new value of P can be derived. This becomes the value of P_{-1} used to calculate the values for the second interval and so on.

Since $D = S$

$12.4 - 1.2P = 8.0 - 0.6P$

$12.4 - 8.0 = 1.2P - 0.6P$

$4.4 = 0.6P$

$P = \frac{4.4}{0.6}$

$P = 7.33$

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Practice

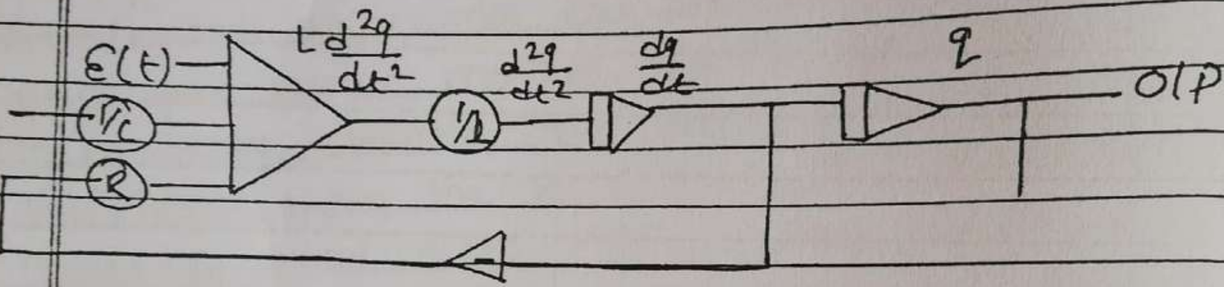
~~Digital~~ Analog computer

(i) $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + q/C = \varepsilon(t)$

Sol^n:

$L\frac{d^2q}{dt^2} = \varepsilon(t) + \left(-R\frac{dq}{dt}\right) + \left(-q/C\right)$

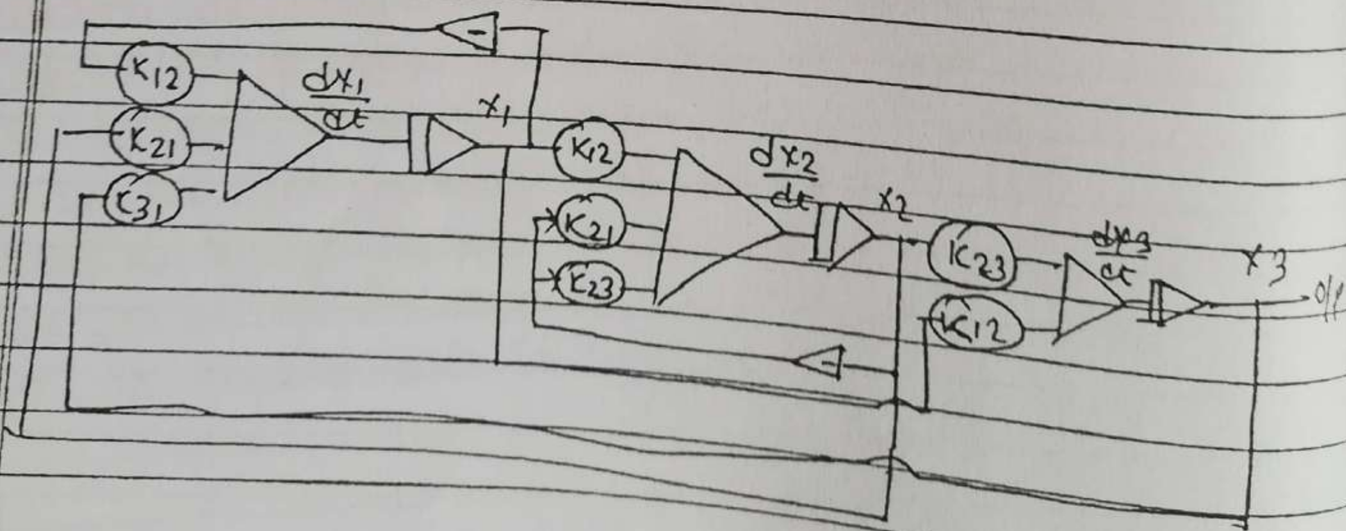
target: find q as output



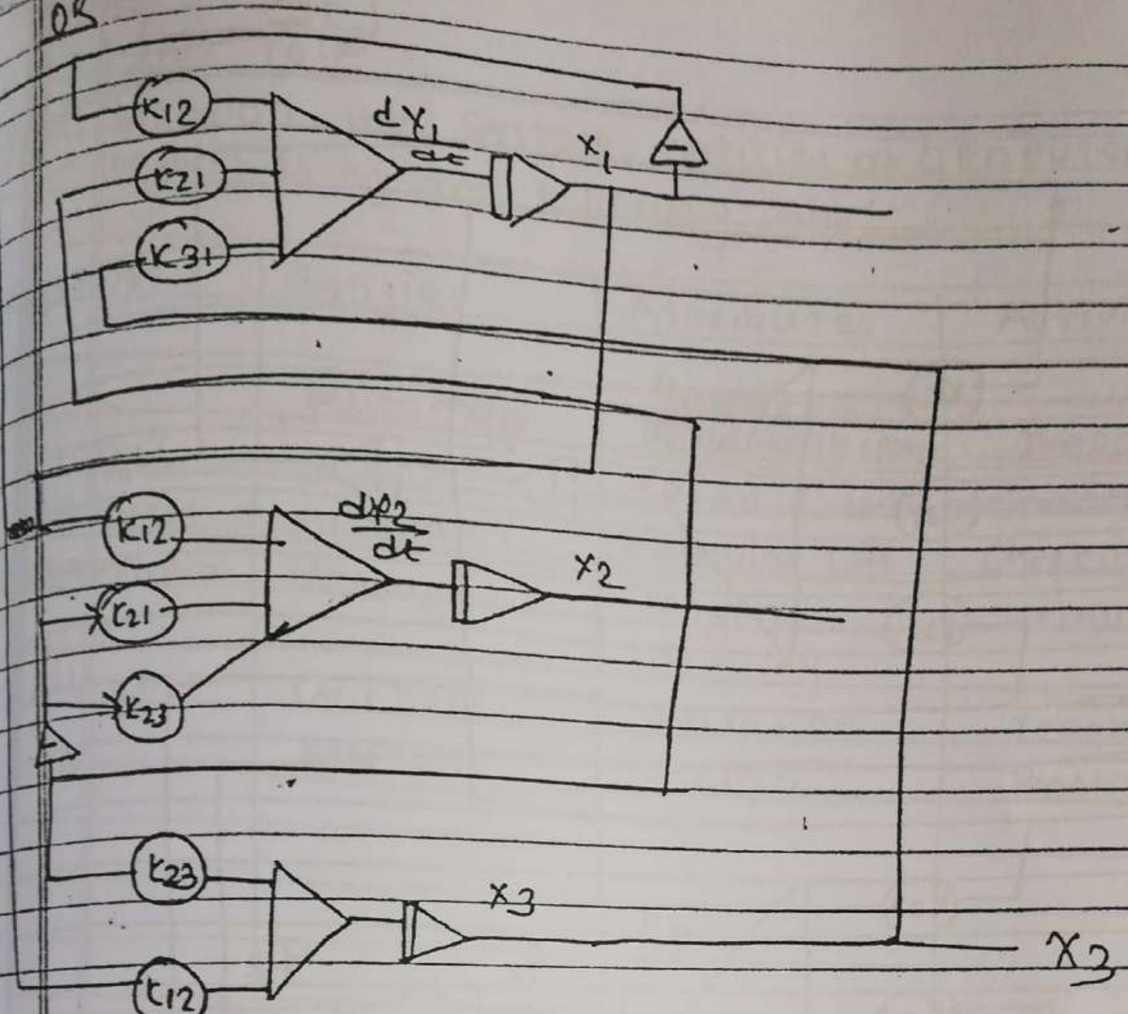
(ii) $\frac{dx_1}{dt} = -k_{12}x_1 + k_{21}x_2 + k_{31}x_3$

$\frac{dx_2}{dt} = k_{12}x_1 - (k_{21} + k_{23})x_2$

$\frac{dx_3}{dt} = k_{23}x_2 - k_{12}x_1$



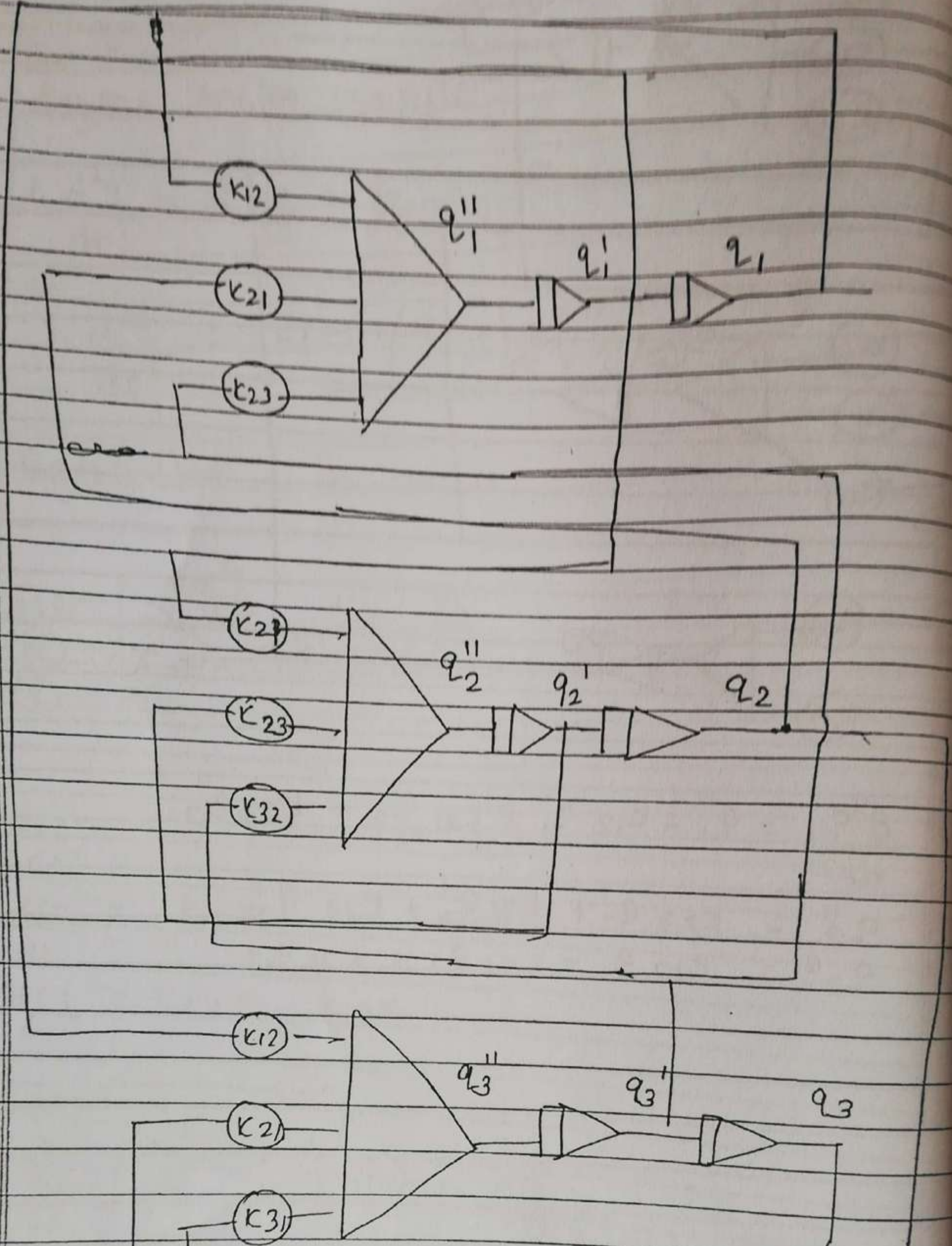
Q8.



(ii) $\frac{d^2q_1}{dt^2} = q_1'' = k_{12}\, q_1' + k_{21}\, q_2 + k_{23}\, q_3'$

$q_2'' = k_{23}\, q_2' + k_{21}\, q_1' + k_{32}\, q_3'$

$q_3'' = k_{12}\, q_1 + k_{21}\, q_2 + k_{31}\, q_3$


K12
K21
K23
q_1''
q_1'
q_1
K21
K23
K32
q_2''
q_2'
q_2
K12
K21
K31
q_3''
q_3'
q_3

2012 fall

(1b) Mention the System, entities ~~as~~ attributes and activities of the following table:

System	Entities	Attributes	Activities
Bank	customers	Balance, credit ~~Depositing~~ status	Depositing
Traffic	cars	speed, ~~distance~~	driving
Supermarket	customers	shopping List	Checking point
Communication	Messages	length Priority	transmitting
College	Teachers student	education cource	Teaching Practical

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find the value of π using Moore carlo method
let us consider a circle having radius 'r'
then area of quadrature of circle is given by

$\therefore$ Area of quadrature of circle (A) $= \frac{\pi r^2}{4}$ —— (i)

as shown in figure below

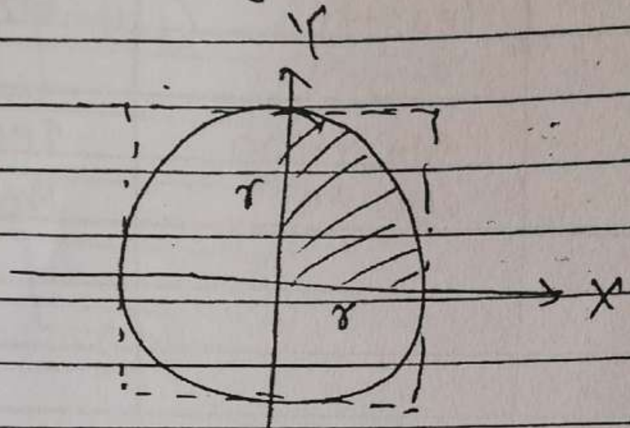

fig: determining the value of π using MC method

Now, the area square having length 'r' in fig above

$\therefore$ Area of square of quadrature (a) $= r^2$ —— (ii)

let 'N' be the total no. of points (sample) which falls with the area of square out of which 'n' be the no. of points falling within area of circle.
Then by moore-carlo

$$\frac{\text{Area of quadrature circle (A)}}{\text{area of quadrature square (a)}} = \frac{n}{N}$$

$$\frac{\left(\frac{\pi \cancel{r^2}}{4}\right)}{\cancel{r^2}} = \frac{n}{N}$$

$$\boxed{\pi = 4 * \frac{n}{N}}$$

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where n, N be the random sample points of corresponding circle and sequence

let $N = 12$ then $\pi = \frac{10}{12} * 4 = 3.33$
$n = 10$

$\boxed{\pi = 3.33}$

let $N = 100$
$n = 80$
then ~~$\pi = \frac{11}{12} * 4$~~ $\pi = \frac{80}{100} * 4 = 3.2$

$\boxed{\pi = 3.2}$

Hence the value of Pi (π) using monte carlo simulation method can be determined using sampling random points.
As the no. of sampling point increases, the accuracy of value for π will increase.

2012 fall

(2/b) Design analog computer from following electrical circuit system.

$$Mx'' + Dx' + Kx = f(t)$$

(target find x)

→

$$Mx'' + Dx' + Kx = f(t)$$

$$Mx'' = \underline{f(t)} + (-\underline{D}x') + (-\underline{K}x)$$

Constant: f(t), K, D

target: to find x as o/p

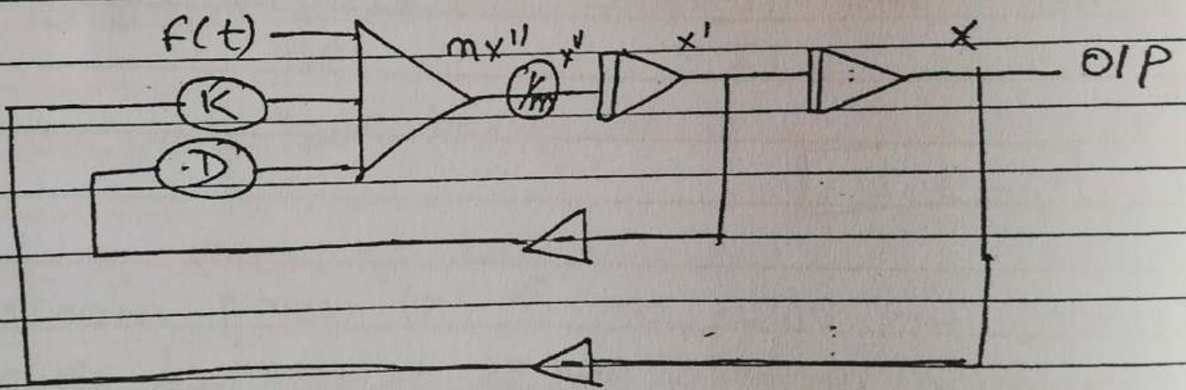


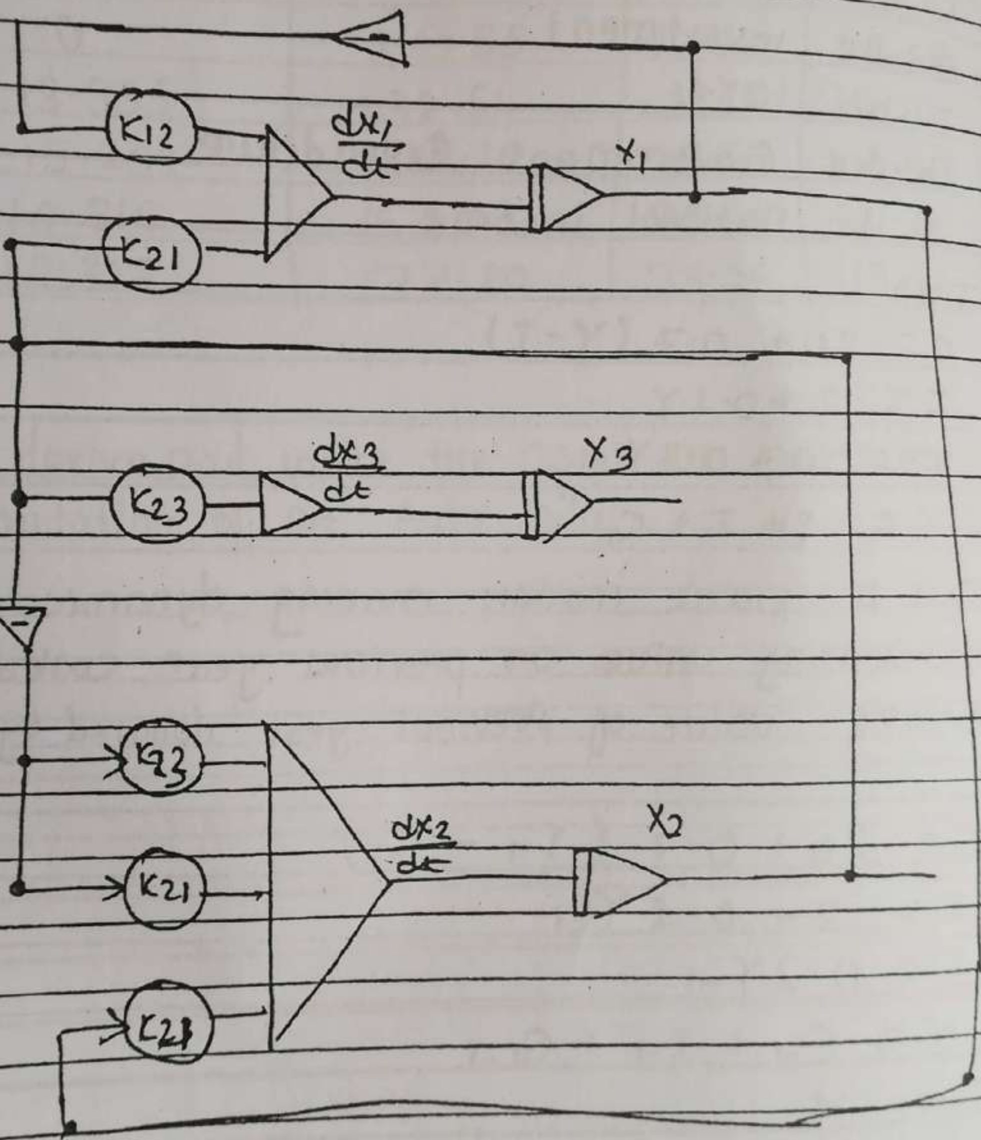
fig: analog computer model from given equation

2012 spring

2(b) $\frac{dx_1}{dt} = -k_{12}x_1 + k_{21}x_2$

$\frac{dx_3}{dt} = k_{23}x_2$

$\frac{dx_2}{dt} = k_{12}x_1 - (k_{21}+k_{23})x_2$



(iii) Find the growth in national consumption for five years using the model given. Assume the initial income Y_{-1} is 80 and take gov. exp. in 5 yrs to be follow

Year	1	2	3	4	5
G	20	25	30	35	40

C be consumption
I be investment
T be taxes
G be Government Expenditure
Y be national income

Then

$C = 20 + 0.7(Y-T)$

$I = 2 + 0.1Y$

$T = 0 + 0.2Y$

$Y = C + I + G$

This is static model, making dynamic by representing values on previous year called lagged variable. value of previous year denoted by suffix -1

$C = 20 + 0.7(Y_{-1} - T_{-1})$

$I = 2 + 0.1Y_{-1}$

$T = 0.2Y_{-1}$

$Y = C_{-1} + I_{-1} + G_{-1}$

Solve Y

$Y = \cancel{20} \; C + I + G$

$= 20 + 0.7(Y_{-1} - T_{-1}) + I + G$

$= 20 + 0.7(Y - 0.2Y) + I + G$

$Y = 20 + 0.56Y + I + G$

~~Thus, we have~~ $Y = 45.45 + 2.27(I + G)$

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AQ:

X! Y! A = 2·0 + 0·1Y : B = 45·45 + 2·27(Y + A) :

classmate B - c)

$I = 2.0 + 0.1 Y_{-1}$ $Y_0 = 80$

$Y = 45.45 + 2.27 (I + G)$

$T = 0.2Y$

$C = 20 + 0.7 (Y - T)$

Year	G	$I = 2.0 + 0.1Y_{-1}$	$Y = 45.45 + 2.27(I+G)$	$T = 0.2Y$	$C = 20 + 0.7(Y-T)$
1	20	10	113.55	22.71	83.588
2	25	13.355	132.51	26.5	94.20
3	30	15.251	148.16	29.63	102.975
4	35	16.816	163.07	32.61	111.32
5	40	18.307.	177.80	35.56	119.57

(2)(a) Explain, derive and write the CSMP - III program for simulation of an Auto - pilot.

2014 fall

Chi square for 95% accuracy

See data carefully, all data are in between

~~xo~~ 1000 – 1050

total no. of sample = 42

i	Class	observed o_i	expected e_i	$(o_i - e_i)^2$	$\frac{(o_i-e_i)^2}{e_i}$
1	1000–1009	4	8	$4^2 = 16$	2
2	1010–1019	8	8	$0^2 = 0$	0
3	1020–1029	16	8	$8^2 = 64$	8
4	1030–1039	4	8	$4^2 = 16$	2
5	1040–1049	10	8	$2^2 = 4$	0.5
					12.5

$$e_i = \frac{\Sigma o_i}{\text{no. of class}} = \frac{42}{5} = 8.4 \approx 8$$

degree of freedom $(N-1) = 5-1 = 4$

for $\alpha = 95\%$

From calculation $\chi^2_{calc} = 12.5$

from table $\chi^2_{table} = 0.711$

Here, $\chi^2_{calc} > \chi^2_{table}$

$12.5 > 0.711$

hence, not uniformly distributed.

1000 - 1010	||||||
1010 - 1020	||||||||
1021 - 1030	||||||||||||||||
1031 - 1040	||||
1041 - 1050	||||||||||

1000 - 1009	||||	4
1010 - 1019	||||||||	8
1020 - 1029	||||||||||||||||	16
1030 - 1039	||||	4
1040 - 1049	||||||||||	10

2013 fall

(1/0) Integrate MC: $\sin x \cos x \, dx$ $\quad 0 <= x <= \pi/2$

$$\int_0^{\pi/2} \sin x \cos x \, dx$$

Standard value:

$$= \frac{1}{2} \int_0^{\pi/2} 2 \sin x \cos x \, dx$$

$$= \frac{1}{2} \int_0^{\pi/2} \sin 2x \, dx$$

$$= \frac{1}{2} * - \left(\frac{\cos 2x}{2} \right)_0^{\pi/2}$$

$$= -\frac{1}{4} \left[\cos(2 * \pi/2) - \cos 0 \right]$$

$$= \frac{-1}{4} * (\cos \pi - \cos 0)$$

$$= \frac{-1}{4} * (-2)$$

$$= \frac{1}{2}$$

Or, use calculator 

By monte carlo method

$f(x) = \sin x \cos x$ $\qquad \frac{d(u,v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$b = \pi/2 \quad a = 0$

For optimum value of $f(x)$

$f'(x) = 0$

$\frac{d(\sin x \cos x)}{dx} = 0 \Rightarrow \sin x \cdot (-\sin x) + \cos x \cdot \cos x$

$\cos^2 x - \sin^2 x = 0$

$\sin x = \cos x$ OR $\cancel{\sin x = \cos x}(1 - \sin^2 x) - \sin^2 x$

$\therefore x = \sin^{-1}(1/\sqrt{2})$ $\qquad 1 - 2\sin^2 x = 0$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \frac{1}{\sqrt{2}}$$

$$x = \sin^{-1}(1/\sqrt{2})$$

$$x = 45^\circ \text{ or } \frac{\pi}{4}$$

$$\therefore C = f(x) = \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{4} = \frac{1}{2}$$

Now by monte carlo method, we have

$$\frac{n}{N} = \frac{\int_a^b f(x)\,dx}{C * (b-a)}$$

$$\int_a^b f(x)\,dx = \frac{n}{N} * C * (b-a)$$

$$= \frac{n}{N} * 0.5 * \left(\frac{\pi}{2} - 0\right)$$

$$= \frac{n}{N} * \frac{\pi}{4}$$

i	N	n	$\int_a^b f(x)\,dx$	error $= \left\lvert\frac{S_i - O_i}{S_i}\right\rvert \times 100\%$
1	1000	500	$\pi/8$	21.46 %
2	1000	600	$3\pi/20$	5.75 %
3	1000	650	$13\pi/80$	2.10 %
4	10,000	6880	0.540	8.07 %
5	1000	660	$33\pi/200$	3.67 %

2012 Spring

1 (1 b) find Y using MC

$$Y = \int_2^6 \frac{7x}{5}\, dx$$

$\rightarrow$ $f(x) = \dfrac{7x}{5}$

x	2	3	4	5	6
y	2.8	4.2	5.6	7	8.4

$b = 6 \quad a = 2$

Standard value :

$$\int_a^b f(x)\, dx = \int_2^6 \frac{7x}{5}\, dx$$

$$= \frac{7}{5}\left[\frac{x^2}{2}\right]_2^6$$

$$= \frac{7}{5 \times 2}\left[6^2 - 2^2\right]$$

$$= \frac{112}{5} \approx 22.4$$

By monte-carlo

$b = 6$

$a = 2$

To find c, for maximum value of $f(x)$ or y

$$f'(x) = 0$$

$$\frac{d}{dx}\left(\frac{7x}{5}\right) = 0$$

$$x = 0$$

$\therefore \; c = f(x) =$

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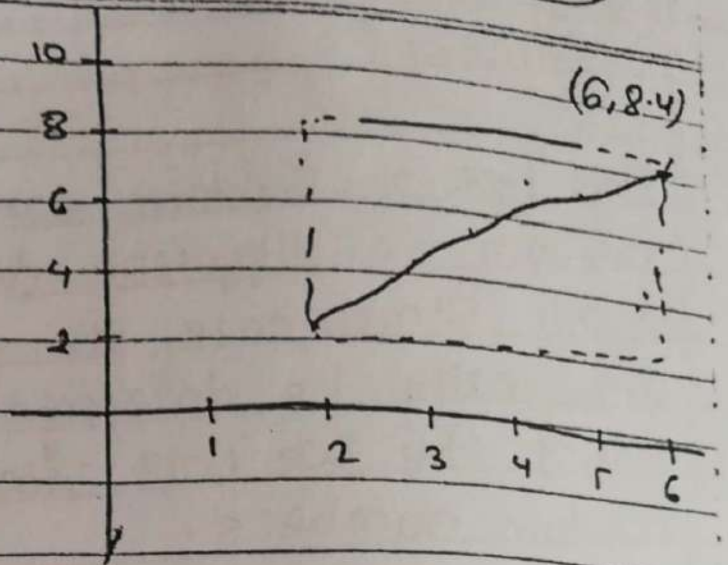
$$y = \int_2^6 \frac{7x}{5}\,dx$$

x	2	3	4	5	6
y	2.8	4.2	5.6	7	8.4



Area of rectangle $= l \times b$

$= (6-2) * (8.4 - 2.8)$

$= 4 * 5.6$

$= 22.4$

By monte carlo

$$\frac{n}{N} = \frac{\int_2^6 \frac{7x}{5}\,dx}{} = \frac{\text{Area of curve}}{\text{Area of rectangle}}$$

$$\frac{n}{N} = \frac{\int_2^6 \frac{7x}{5}\,dx}{22.4}$$

$$\int_2^6 \frac{7x}{5}\,dx = 22.4 * \frac{n}{N}$$

let $N = 1000$

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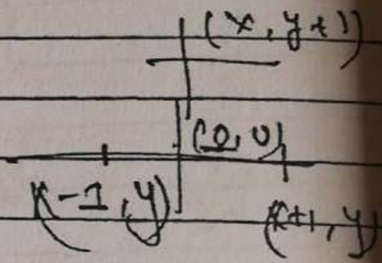
2011

1(b) A drunkard ......

This type of Problem, where a person walks at random in different direction is called random walk. In this simple case, the position of the drunkard can easily be determined by using the probability theory. But we will simulate the walk using random numbers.

Using simple digit random number, the random number can be allocated to steps in different direction as under

Direction		Probability	Random no.
Forward	(F)	0.5	0, 1, 2, 3, 4
Left	(L)	0.3	5, 6, 7
Right	(R)	0.2	8, 9



initially (0, 0)

Step	Random Number	Direction	Position X , Y
1	6	L	-1 , 0
2	2	F	-1 , 1
3	0	F	-1 2
4	6	L	-2 , 2
5	8	R	-1 , 2
6	5	L	-2 , 2
7	7	L	-3 , 2
8	7	L	-4 , 2
9	9	R	-3 , 2
10	8	R	-2 , 2

L 0.3 5, 6, 7 X - -
R 0.2 8, 9 Y + +

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Step	Random Number	Direction	Position (X, Y)
11	4	F	-2, 3
12	8	R	-1, 3
13	2	F	-1, 4
14	6	L	-2, 4
15	2	F	-2, 5
16	1	F	-2, 6
17	3	F	-2, 7
18	0	F	-2, 8
19	8	R	-1, 8
20	4	F	-1, 9
21	9	R	0, 9
22	0	F	0, 10
23	8	R	1, 10
24	6	L	0, 10
25	5	L	-1, 10
26	1	F	-1, 11
27	0	F	-1, 12
28	9	R	0, 12
29	9	R	1, 12
30	8	R	2, 12
31	7	L	1, 12
32	4	F	1, 13
33	3	F	1, 14
34	1	F	1, 15
35	8	R	2, 15
36	5	L	1, 15
37	2	F	1, 16
38	1	F	1, 17
39	0	F	1, 18
40	8	R	2, 18

STEP	Random Number	Direction	Position (X, Y)
41	6	L	1, 18
42	7	L	0, 18
43	8	R	1, 18
44	4	F	1, 19
45	9	R	2, 19
46	3	F	2, 20
47	0	F	2, 21
48	5	L	1, 21
49	1	F	1, 22
50	2	F	1, 23

s/He will be at (1, 23) cordinate position

Comparision of Simulation and Analytical method

Both Analytical and Simulation methods are modeling approaches which aim at providing an idea of system performance in different conditions

An analytical model is a mathematical abstraction that can be extended to address various ~~kind~~ working condition. It provides a generic way to get performance results in various conditions through a mathematical formulation. The Accuracy of the model is to be considered through the validity of the assumption to derive the mathematical formulation. Some uncertainties can be handled through a stochastic model to account for modeling and measurement model.

A simulation method such as a monte-carlo also make assumption of a model and some about the behaviour of the process. It is used when it is too complex for analytical formulation (ie - when size of model is too large
- When no exact solution can be derived)
Simulation model provides specific use case and should be run many times to counterbalance the effect of numerical calculation. A simulation model accepted when results are validated in a number of working condition under various input assumption.

When two approaches can be used, preference should given to analytical approach & Simulation used for validate the assumption.