# Chapter 3

## Continuous System Simulation

### Introduction

- A continuous system simulation is one, in which predominant activities of the system cause smooth changes in the attributes of the system entities.
- When such system is modeled mathematically, the variable of the model representing the attributes are controlled by continuous functions.
- In general, in continuous, the relationship describes the rate at which attributes changes, so that the model consists of differential equations.
- If a system can be represented using simple differential equation model then it is often possible to solve the model without the use of simulation otherwise we use simulation to solve those models which are complex to solve analytically.

### Differential Equation

- We can use differential equation to represent the behavior of continuous system.
- An example of linear differential equation with constant coefficient is one that describes the wheel suspension of an automobile.
- The equation is:

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

Where, $\ddot{x}$ = acceleration

$\dot{x}$ = velocity

$x$ = displacement

$K$ = stiffness of spring

$D$ = measure of viscosity (thickness) of shock absorber

$F(t)$ = input of system depends on independent variable t

- A **linear differential equation** with constant coefficients is always of this form, although derivatives of any order may other form, such as being raised to a power, or are combined in any way- for example, by being multiplied together, the differential equation is said to be **non-linear.**
- Linear just means that the variable in an equation appears only with a power of one.

- x is linear but $x^2$ is non-linear.
- The theory for solving linear equations is very well developed because linear equations are simple enough to be solvable.
- Non-linear equations can usually not be solved exactly and are the subject of much on-going research

$x'' + x = 0$ is linear

$x'' + 2x' + x = 0$ is linear

$x' + 1/x = 0$ is non-linear because 1/x is not a first power

$x' + x^2 = 0$ is non-linear because $x^2$ is not a first power

- When more than one independent variable occurs in a differential equation, the equation is said to be **partial differential equation.**
- It can involve the derivatives of same dependent variable with respect to each of the independent variables.
- An example is an equation describing the flow of heat in a three dimensional body. There are four independent variable representing the three dimensions and time, and one dependent variable, representing temperature.
- **Differential equation occurs repeatedly in scientific and engineering studies. The reason for this prominence is that most physical and chemical process involves rates of change, which require differential equations for their mathematical description.**
- Since a differential coefficient can also represent a growth rate, continuous models can also be applied to a problem of a social or economic nature where there is a need to understand the general effect of growth trends.

## Analog Computers

- Analog computers are generally used to solve continuous model but sometimes are also used to solve static models.
- Some device whose behavior is equivalent to a mathematical operation such as adder or integrator etc. is combined together in a manner specified by a mathematical model of a system to allow the system to be simulated.

- That combination which is used in the simulation of a continuous system is referred as **analog computer** or when they are used to solve differential equation they are referred to as **differential analyzer**.
- The most widely used form of analog computers is the electronics analog computers based on the **operational amplifiers.**
- Voltages in the computers are equated to mathematical variables and the operational amplifiers can add, integrate etc. the voltage.
- With appropriate circuits, an operational amplifier can be made to add several input voltages, each representing a variable of model, to produce a voltage representing the sum of the input variables.
- Different scale factors can be used on the input to represent the coefficient of the model equations. Such amplifiers are called summer.
- Another circuit arrangement produces an integrator for which the output is the integral with respect to time of single input voltage or the sum of several input voltages.
- All voltages can be positive or negative to correspond to the sign of the variable represented.
- To satisfy the equation of the model, it is sometime necessary to use a sign inverter.

**Advantage of Analog Simulation:**

1. Analog computers have higher speed of solution than that of digital simulation.
2. Analog simulations have direct access to an immediate display of computer result.
3. Analog simulation is more natural in sense so, can reflect system structure and interpret the results.

**Disadvantage of Analog Simulation:**

- Though analog computers came into existence much before the digital computers, and have played a major role in the simulation of continuous dynamic system, but they are giving way to digital computers at the fast pace.
- Following are some of the important disadvantages of digital computers.

a) Limited Accuracy: The result obtained from analog simulation has limited accuracy, while high accuracy in the result can be obtained by employing digital simulation. Thus, where, high accuracy in results is required, as in space vehicles, guided missiles and fusion, the analog method cannot be used.

b) Magnitude scaling: in analog simulation, values of various variables are represented by voltages, which have fixed and limited range. The variables must remain within the limited range, otherwise the result become inaccurate. Thus, all program variables have to be scaled, so that none exceeds the voltage range. This is a very difficult task, especially when the number of variables is large, and the range of their variation is not known. On the other hand, in digital computers, the problem of magnitude scaling does not raise, as they have a very large range. With floating point arithmetic, they have a very large precision. Hence, no magnitude scaling is normally required in digital simulation.

c) Hardware set up required: in analog simulation, hardware elements have to combine to simulate the system, they have to be tested and calibrated while no such set up is required in digital simulation. Switching from one simulation to other takes time in analog simulation, while it requires no time in digital simulation. A simulation program on a digital computer can be easily stored, for use.

d) The output of analog simulation is not understood by general people.

e) The analog computer is usually dedicated to one application at a time hence is not flexible.

f) It has lack of memory.

**Components of Analog Computer/Analog Methods:**

The general methods by which analog computer are applied can be demonstrated using the second order differential equation given as:

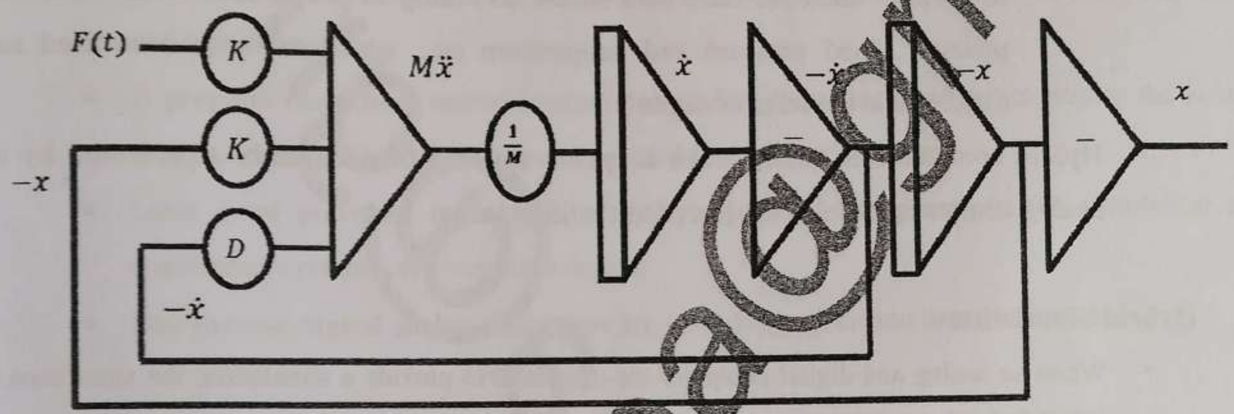
$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

Solving the equation for the highest order derivate gives

$$M\ddot{x} = KF(t) - D\dot{x} - Kx$$

Suppose a variable representing the input $F(t)$ is supplied, and assume for the time being that there exist variables representing $-x$ and $\dot{x}$. These three variables can be scaled and added with a summer to produce a voltage representing $M\ddot{x}$. Integrating this variable with a scale factor of $1/M$ produce $\dot{x}$. Changing the sign produce $-\dot{x}$, which supplies one of the variables initially assumed; and further integration produces $-x$, which was other assumed variables. For convenience, a further sign inverter is included to produce $+x$ as an output.

A block diagram to solve the problem in this manner is shown below. The symbols used in the figure are standard symbols for drawing block diagrams representing analog computer arrangements. The circle indicates scale factors applied on the variable. The triangular symbol at the left of the figure represents the operating of the adding variables. The triangular symbol with vertical bar represents integration, and the containing a minus sign is a sign changer.




**Note: See class not for more examples**

**Hybrid Computers:**

- The scope of analog computers has been considerably extended by developments of solid-logic electronic device (electronic equipment using semiconductor devices such as semiconductor diodes, transistors, and integrated circuits (ICs).
- Analog computers always used a few non-linear devices (like multiplier), which were expensive to make.
- So solid-logic device has made such non-linear cheaper & easier to obtain and also improves the design and performance.
- Digital operations are storing values, switching, and performing logical operation.
- Hybrid computer is a digital computer that accepts analog signal, converts them to digital and process them in digital form.

- Hybrid computer may be used to simulate system that are mainly continuous and also have some digital elements i.e. Hybrid computers are used mainly in specialized applications where both discrete and continuous data need to be processed
- For Example:
  - An artificial satellite for which both the continuous equation of motion and the digital controls signal must be simulated.
  - A petrol pump contains a processor that converts fuel flow measurements into quantity and price values.
  - In hospital Intensive Care Unit (ICU), an analog device is used which measures patient's blood pressure and temperature etc., which are then converted and displayed in the form of digits
- Hybrid computer are useful when a system that can be adequately represented by an analog computer model is subject of a repetitive study.

**Hybrid Simulation**

- When an analog and digital computer are combined to provide a simulation, the simulation is **called hybrid simulation.**
- System can be simulated as interconnection of continuous and discrete subsystem which can be best modeled by an analog and digital computer being linked together.
- In this approach to a continuous simulation clock is maintained and updates to the model are event-driven i.e. each event is represented by an occurrence in simulated time and must be scheduled to occur when a proper set of conditions exist.
- High speed converters are needed to transform signals from one form of representation to another.
- It is generally reserved for the case in which functionally distinct analog and digital computers are linked together for the purpose of simulation.
- Hybrid simulation may be used to simulate systems that are continuous, but don, in fact, have some digital elements.

Example:

- A petrol pump contains a processor that converts fuel flow measurements into quantity and price values.

- **Analog-Digital Simulators**
  - To avoid the disadvantage of analog computers, many digital computer programming languages have written to produce digital analog simulator.
  - These allow a continuous model to be programmed on a digital computer in a same way as it is solved on an analog computer.
  - These languages contain macro instructions that carry out the action of adders, integrators and sign chargers.
  - A program uses these macro-instructions to link them together in essentially the same way as operational amplifiers are connected in analog computers.
  - Later more powerful techniques of applying digital computers to the simulation of continuous system have been developed.
  - Due to these digital analog simulators are not now in common uses.

**CSSL**

- Confining a digital computer to routines that represents a function of analog or hybrid computer, as is done with digital analog simulator, this is clearly a restriction.
- To overcome the restriction, a number of continuous system simulation languages (CSSL) have been developed.
- They use the familiar statement type of input for digital computer, allowing a problem to be programmed directly from the equation broken into functional elements.
- A CSSL can include macros or subroutine which performs function of specific analog element.
- It also includes variety of algebraic and logical expression to describe the relationship between variable.
- Example, Continuous System Modeling Program (CSMP-3).

**CSMP III**

- This program is constructed from three general type of statement:

1) Structural Statement

   - This statement defines the model which consists of FORTRAN like statements and functional block designed for operation that frequently occur in a model definition.
   - This statement can make the use of operation of addition, subtraction, multiplication, division and exponent.

For example:

$X = 6\,Y / 10 + (Z-2)^2$

Fortran statement: 6.0 * Y / 10.0 + (Z – 2.0) * * 2.0

Real constants have decimal notations. Exponential notations can also be used.

Example: 1.2e-4

$= 1.2 * 10^{-4}$

=0.00012

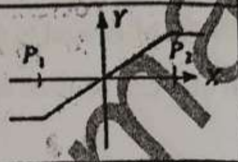
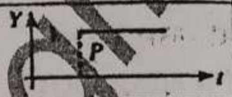
i.e. 1.2e-4 represents 0.00012

Fixed value need declaration, variable name must be six character or less.

Example:

PI=3.14159

## Functions used in CSMP

| GENERAL FORM | FUNCTION |
|---|---|
| Y = INTGRL (IC, X)<br>Y (0) = IC<br>INTEGRATOR | $Y = \int_0^t X\,dt + IC$ |
| Y = LIMIT ($P_1$, $P_2$, X)<br>LIMITER | $Y = P_1 \quad X < P_1$<br>$Y = P_2 \quad X > P_2$<br>$Y = X \quad P_1 \leq X \leq P_2$<br> |
| Y = STEP (P)<br>STEP FUNCTION | $Y = 0 \quad t < P$<br>$Y = 1 \quad t \geq P$<br> |
| Y = EXP (X)<br>EXPONENTIAL | $Y = e^X$ |
| Y = ALOG (X)<br>NATURAL LOGARITHM | $Y = \ln(X)$ |
| Y = SIN (X)<br>TRIGONOMETRIC SINE | $Y = \sin(X)$ |
| Y = COS (X)<br>TRIGONOMETRIC COSINE | $Y = \cos(X)$ |
| Y = SQRT (X)<br>SQUARE ROOT | $Y = X^{1/2}$ |
| Y = ABS (X)<br>ABSOLUTE VALUE<br>(REAL ARGUMENT AND OUTPUT) | $Y = \lvert X \rvert$ |
| Y = AMAX1 ($X_1$, $X_2$ . . . $X_n$)<br>LARGEST VALUE<br>(REAL ARGUMENTS AND OUTPUT) | $Y = \max(X_1, X_2, \ldots, X_n)$ |
| Y = AMIN1 ($X_1$, $X_2$ . . . $X_n$)<br>SMALLEST VALUE<br>(REAL ARGUMENTS AND OUTPUT) | $Y = \min(X_1, X_2, \ldots, X_n)$ |

2) Data Statement

- These statement designs the numerical values to parameters, constant and initial conditions.
- ICON can be used to set the initial value of the integration block function.
- CONST can be used to set values for constant.
- PARAM is used to set parameters for individual.
- It also can be used to set series of numeric values for one parameter.

Example:

CONST A =0.5, XDOT = 1.25, YDOT=6.22

PARAM D=(0.25, 0.50, 0.75, 1.0)

3) Control Statement

- This statement specifies the option in the execution of the program and the choice of the output.
- TIMER is used to specify the timer interval.
- Example:

  TIMER DELT = 0.005, FINTIM=1.5, PRDEL = 0.1, OUTDEL = 0.1

  Where DELT= Integration interval

  FINTIM = Finish time

  PRDEL= Interval at which print results

  OUTDEL= Interval at which to print plot

- If printed or print-plotted output are required then control statement with the words PRINT and PRTPLT are used, followed by the name of variables to form the output.
- TITLE and LABEL can be used to put headings on the output.

- CSMP contain basic three segments
  1. Initial: executed at beginning
  2. Terminal: executed at end of simulation

3. Dynamic: heart of program where integration takes place

Example:

1. CSMP program for automobile suspension system, for given M=2.0, F=1 and K = 400 and there will be different runs with different value of D as specified

   Solution:

   The model for automobile suspension system is represented by differential equation

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

The above equation can be written as

$$M\ddot{x} = KF(t) - D\dot{x} - Kx$$

$$\ddot{x} = \frac{KF(t) - D\dot{x} - Kx}{M}$$

**CSMP program**

```
TITILE AUTOMOBILE SUSPENSION SYSTEM
CONST M=2.0, F=1.0, K=400.0
PARAM D = (5.656, 16.968, 39.592, 56.56)        //initial
X2DOT=(1.0 / M) * (K * F – K * X – D * XDOT)
XDOT=INTGRL (0. 0, X2DOT)                        //dynamic
X=INTGRL (0. 0, XDOT)

TIMER DELT=0.005, FINTIM=1.5, PRDEL=0.05     //terminal
PRINT X, XDOT, X2DOT
LABEL Displacement vs time
END
STOP
```

i. Write a CSMP program for solving the differential equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t)$$

Soln:

The above eqn can be written as

$$\frac{d^2q}{dt^2} = \frac{1}{L}\left(E(t) - R\frac{dq}{dt} - \frac{q}{C}\right)$$

CSMP Program

```
TITLE Electrical System
CONST L = 35.0, R = 5.0, C = 2.0, E = 1.0
Q2DOT = (1.0/L)*(E-R*Q1DOT-1.0/C*Q)
Q1DOT = INTGRL (0.0, Q2DOT)
Q = INTGRL(0.0, Q1DOT)

TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.5
                                   OUTDEL = 0.5
PRINT Q, Q1DOT, Q2DOT
PRTPLT Q
LABEL charge vs. time
END
STOP
```

ii. Write CSMP program to solve the following differential equation.

$2x'' + 3x' - 15x = 60$

Soln:

The above equation can be written as

$2x'' = 60 - 3x' + 15x$

or $x'' = \frac{1}{2}(60 - 3x' + 15x)$

CSMP program:

```
TITLE  csmp demo
x2DOT = (1.0/2.0)*(60.0 - 3.0*x1DOT + 15.0*x)
x1DOT = INTGRL(0.0, x2DOT)
x = INTGRL(0.0, x1DOT)

TIMER  DELT = 0.005, FINTIM=1.5,
       PRDEL= 0.05, OUTDEL=0.05

PRINT  x, x1DOT, x2DOT
PRTPLT x
LABEL  output
END
STOP
```

**Feedback System:**

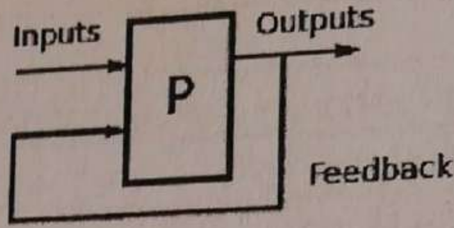



**Fig: Feedback System**

- A significant factor in the performance of many systems is that a coupling occurs between the input and output of the system.
- Feedback describes the situation when output from (or information about the result of) an event or phenomenon in the past will influence an occurrence or occurrences of the same (i.e. same defined) event / phenomenon in the present or future.
- Feedback occurs when outputs of a system are routed back as inputs as part of a chain of cause-and-effect that forms a circuit or loop.
- The system can then be said to feed back into itself.
- The notion of cause-and-effect has to be handled carefully when applied to feedback systems:
- A home heating system controlled by a thermostat(sensor) is a simple example of a feedback system. The system has a furnace(heater) whose purpose is to heat a room, and the output of the system can be measured as room temperature. Depending upon whether the temperature is below or above the thermostat (regulator) setting, the furnace (heater) will be turned on or off, so that information is being fed back from the output to the input. In this case, there are only two states, either the furnace will on or off.
- Another example of Feedback system is **Automatic Aircraft System** (Auto Pilot System)

**Continuous simulation Example:**

1. **Predator- Prey Model:**

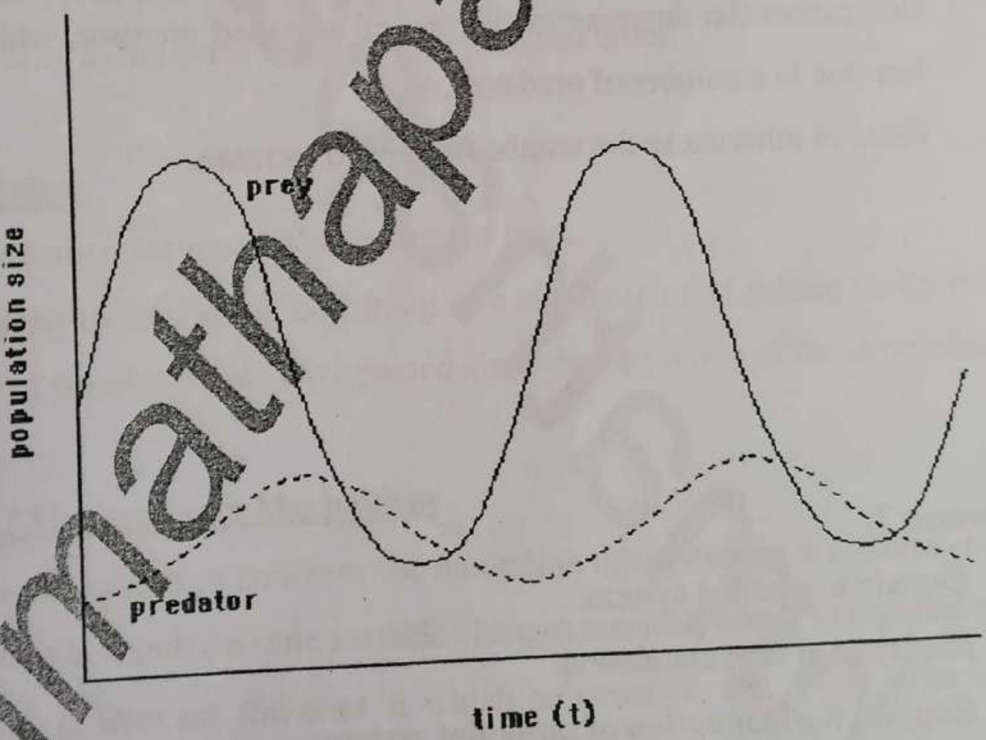
- It is also called parasite-host model.
- An environment consists of 2 populations, predator & prey.
- Predator –Prey model is also a mathematical model.
- The Prey is passive but the Predator depends on the Prey for their source of food.

Let

$x(t)$ = no. of prey population at time t.

$y(t)$ = no. of predator population at time t.

$rx(t)$ = rate of growth of prey for some positive 'r' where 'r' = natural birth rate and natural death rate.



- Because of the interaction between predator and prey, it will be reasonable to assume that the death rate of prey is proportional to the product of two population size $x(t).y(t)$ or death rate of prey is $a.x(t).y(t)$.
- Therefore the overall rate of change of prey population, $dx/dt$, is given by:

$$\frac{dx}{dt} = rx(t) - ax(t)y(t)$$

where a is positive constant of proportionality.

- Also the predators depend on their prey for their existence; the rate of change of predators in the absence of prey is $-sy(t)$ for some positive s.
- The interaction between the two populations causes the predator population to increase at a rate that is proportional to $x(t).y(t)$.
- Thus, the overall change of the predator population, $dy/dt$, is:

$$\frac{dy}{dt} = -sy(t) + bx(t)y(t)$$

where b is positive constant.

As the predator population increases, the prey population decreases.

This causes the decrease in the rate of increased predator, which eventually results in decrease in a number of predators.

These in turns cause the number of prey to increase.

Assignment 3:

1. Explain interactive system.
2. Explain real time simulation.
3. **Explain the simulation of autopilot system.**