Linear Regression Theory

Linear regression is a supervised machine learning method that is used to model the relationship between a dependent variable or a target variable and one or more independent variables. Linear regression assumes the linear relationship between the variables, and its main goal is to find the line of best fit.

Core Concepts of Linear Regression

Linear regression models the relationship between a dependent variable (target) and one or more independent variables (predictors) by fitting a linear equation to the observed data. The central assumption is that this relationship can be approximated by a straight line (or hyperplane in multiple dimensions).

Simple Linear Regression

In simple linear regression, we have one independent variable (x) and one dependent variable (y). The model takes the form:

$y = \beta_0 + \beta_1 x + \varepsilon$

Where:

- y is the dependent variable we're trying to predict
- x is the independent variable (predictor)
- β_0 is the y-intercept (the value of y when x = 0)
- β_1 is the slope coefficient (how much y changes when x increases by 1 unit)
- ε represents the error term (the part of y that can't be explained by the model)

Multiple Linear Regression

When dealing with multiple independent variables ($x_1, x_2, ..., x_n$), the equation extends to:

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n + \varepsilon$

Each coefficient (β) represents the change in y associated with a one-unit change in the corresponding predictor variable, while holding all other predictors constant.

Finding the Line of Best Fit

The "best fit" line minimizes the difference between the observed values and the values predicted by the model. The most common method for finding this line is Ordinary Least Squares (OLS), which minimizes the sum of squared residuals:

Minimize: $\Sigma(y_i - \hat{y}_i)^2$

Where:

- y_i is the actual observed value
- $\hat{y}_i$ is the predicted value from the model

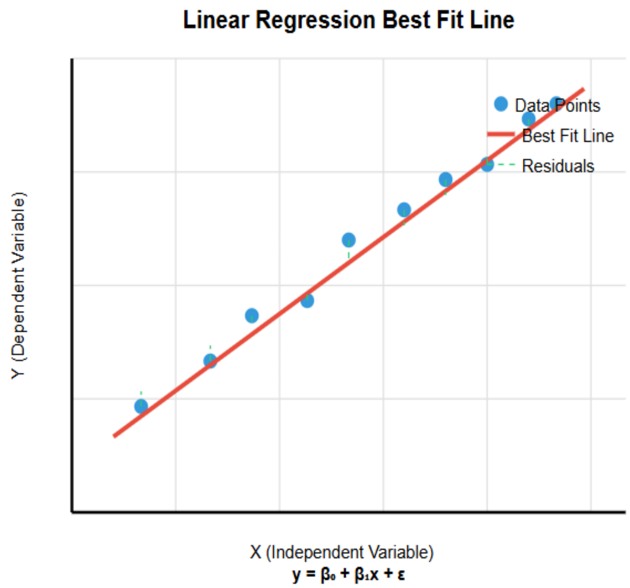

Fig: Linear Regression Best Fit Line Image

Model Evaluation

Several metrics help evaluate the performance of a linear regression model:

1. **R-squared (Coefficient of Determination)**: Measures the proportion of variance in the dependent variable explained by the independent variables (ranges from 0 to 1).
2. **Adjusted R-squared**: Adjusts the R-squared value based on the number of predictors, penalizing excessive complexity.
3. **Mean Squared Error (MSE)**: Average of the squared differences between predicted and actual values.
4. **Root Mean Squared Error (RMSE)**: Square root of MSE, providing an error measure in the same units as the dependent variable.

In **linear regression**, we use a **cost function** (often based on the **loss function**) to measure how well our model's predictions match the actual data. Here's a breakdown of the formulas:

Hypothesis Function (Prediction):

The linear regression model predicts:

$\hat{y}=h\theta(x)=\theta_0+\theta_1 x$

Loss Function (per sample):

This measures the error for **one data point**. In linear regression, we typically use **squared error**:

Loss= $(y- \hat{y})^2 = (y-h_\theta(x))^2$

Cost Function (over all data):

The cost function is the **average loss** over all mmm training examples. This is also called the **Mean Squared Error (MSE) cost function**:

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Applications of Linear Regression

Linear regression is widely used across various domains:

- Economics: Forecasting trends and analyzing relationships between economic variables
- Finance: Stock price prediction and risk assessment
- Marketing: Analyzing the impact of marketing campaigns on sales
- Medical research: Studying relationships between health variables
- Social sciences: Analyzing behavioral patterns and trends

Limitations

Despite its usefulness, linear regression has several limitations:

- It assumes a linear relationship, which may not accurately represent complex real-world relationships
- It's sensitive to outliers
- It can't capture non-linear patterns without transformation.
- It assumes that features are independent, which is often not the case.