DATA SCIENCE TRAINING

Assignment : 2

Matrix Multiplication

Method ① : Dot Product

① $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times3} \times \begin{bmatrix} m & n \\ o & p \\ q & r \end{bmatrix}_{3\times2} = \begin{bmatrix} (am+bo+cq) & (an+bp+cr) \\ (dm+eo+fq) & (dn+ep+fr) \\ (gm+ho+iq) & (gn+hp+ir) \end{bmatrix}_{3\times2}$

A(col) = B(row)

output (3×2) Matrix

② $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}_{3\times2} = \begin{bmatrix} [(1\times7)+(2\times9)+(3\times11)] & [(1\times8)+(2\times10)+(3\times12)] \\ [(4\times7)+(5\times9)+(6\times11)] & [(4\times8)+(5\times10)+(6\times12)] \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} [7+18+33] & [8+20+36] \\ [28+45+66] & [32+50+72] \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}_{2\times2}$

output (2×2) Matrix

③ $\begin{bmatrix} 8 & -1 \\ 3 & 4 \end{bmatrix}_{2\times2} \times \begin{bmatrix} -9 & 7 \\ 5 & 2 \end{bmatrix}_{2\times2} = \begin{bmatrix} (8\times-9)+(-1\times5) & (8\times7)+(-1\times2) \\ (3\times-9)+(4\times5) & (3\times7)+(4\times2) \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} -77 & 54 \\ -7 & 29 \end{bmatrix}_{2\times2}$

output 2×2 Matrix

④ $\begin{bmatrix} 3 & 4 & 2 \end{bmatrix}_{1\times3} \times \begin{bmatrix} 13 & 9 & 7 & 15 \\ 8 & 7 & 4 & 6 \\ 6 & 4 & 0 & 3 \end{bmatrix}_{3\times4} = \begin{bmatrix} [(3\times13)+(4\times8)+(2\times6)] \\ [(3\times9)+(4\times7)+(2\times4)] \\ [(3\times7)+(4\times4)+(2\times0)] \\ [(3\times15)+(4\times6)+(2\times3)] \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} 83 & 63 & 37 & 75 \end{bmatrix}_{1\times4}$

output (1×4) matrix

⑤ $\begin{bmatrix} -1 & 0 & 4 \\ 2 & 0 & 0 \end{bmatrix}$ (2×3) $\times$ $\begin{bmatrix} -1 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$ (3×2)

OUTPUT 2×2 Matrix

$= \begin{bmatrix} [(-1\times-1)+(0\times-1)+(4\times2)] & [(-1\times1)+(0\times3)+(4\times4)] \\ [(2\times-1)+(0\times-1)+(0\times2)] & [(2\times1)+(0\times3)+(0\times4)] \end{bmatrix}$

$= \begin{bmatrix} 9 & 15 \\ -2 & 2 \end{bmatrix}_{2\times2}$

Method ② :- Column Product

① $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ (3×3) $\times$ $\begin{bmatrix} m & n \\ o & p \\ q & r \end{bmatrix}$ (3×2)

output (3×2) matrix

$= \begin{bmatrix} ma+ob+qc & na+pb+rc \\ md+oe+qf & nd+pe+rf \\ mg+oh+qi & ng+ph+ri \end{bmatrix}_{3\times2}$

$Col_1 = m\begin{bmatrix} a \\ d \\ g \end{bmatrix} + o\begin{bmatrix} b \\ e \\ h \end{bmatrix} + q\begin{bmatrix} c \\ f \\ i \end{bmatrix}$

$= \begin{bmatrix} ma \\ md \\ mg \end{bmatrix} + \begin{bmatrix} ob \\ oe \\ oh \end{bmatrix} + \begin{bmatrix} qc \\ qf \\ qi \end{bmatrix}$

$= \begin{bmatrix} ma+ob+qc \\ md+oe+qf \\ mg+oh+qi \end{bmatrix}$

$Col_2 = n\begin{bmatrix} a \\ d \\ g \end{bmatrix} + p\begin{bmatrix} b \\ e \\ h \end{bmatrix} + r\begin{bmatrix} c \\ f \\ i \end{bmatrix}$

$= \begin{bmatrix} na \\ nd \\ ng \end{bmatrix} + \begin{bmatrix} pb \\ pe \\ ph \end{bmatrix} + \begin{bmatrix} rc \\ rf \\ ri \end{bmatrix}$

$= \begin{bmatrix} na+pb+rc \\ nd+pe+rf \\ ng+ph+ri \end{bmatrix}$

Answer $= \begin{bmatrix} \begin{pmatrix} ma+ob+qc \\ md+oe+qf \\ mg+oh+qi \end{pmatrix} & \begin{pmatrix} na+pb+rc \\ nd+pe+rf \\ ng+ph+ri \end{pmatrix} \end{bmatrix}$

(2)

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$$

(2×3) (3×2)

output 2×2 matrix

$$= \begin{bmatrix} [(7\times1)+(9\times2)+(11\times3)] & [(8\times1)+(10\times2)+(11\times3)] \\ [(7\times4)+(9\times5)+(11\times6)] & [(8\times4)+(10\times5)+(12\times6)] \end{bmatrix}$$

$$= \begin{bmatrix} 58 & 64 \\ 139 & 134 \end{bmatrix}_{2\times2}$$

(3)

$$\begin{bmatrix} 8 & -1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} -9 & 7 \\ 5 & 2 \end{bmatrix}$$

(2×2) (2×2)

output 2×2 matrix

$$= \begin{bmatrix} [(-9\times8)+(5\times-1)] & [(7\times8)+(2\times-1)] \\ [(-9\times3)+(5\times4)] & [(7\times3)+(2\times4)] \end{bmatrix}$$

$$= \begin{bmatrix} -77 & 54 \\ -7 & 29 \end{bmatrix}_{2\times2}$$

(4)

$$\begin{bmatrix} 3 & 4 & 2 \end{bmatrix} \times \begin{bmatrix} 13 & 9 & 7 & 15 \\ 8 & 7 & 4 & 6 \\ 6 & 4 & 0 & 3 \end{bmatrix}$$

(1×3) (3×4)

Output (1×4) matrix

$$= \begin{bmatrix} [(3\times13)+(8\times4)+(6\times2)] & [(9\times3)+(9\times4)+(4\times2)] & [(7\times3)+(4\times4)+(2\times0)] & [(15\times3)+(6\times4)+(3\times2)] \end{bmatrix}$$

$$= \begin{bmatrix} 83 & 63 & 37 & 75 \end{bmatrix}_{1\times4}$$

⑤

$$\begin{bmatrix} -1 & 0 & 4 \\ 2 & 0 & 0 \end{bmatrix}_{2\times3} \times \begin{bmatrix} -1 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$$

3×2

output (2×2) matrix

$$= \begin{bmatrix} [(-1\times-1)+(-1\times0)+(2\times4)] & [(1\times-1)+(3\times0)+(4\times4)] \\ [(-1\times2)+(-1\times0]+(2\times0)] & [(1\times2)+(3\times0)+(4\times0)] \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 15 \\ -2 & 2 \end{bmatrix}$$

Method ③ :— Row Multiplication.

①

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}_{4\times4} \times \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} aw+bx+cy+dz \\ ew+fx+gy+hz \\ iw+jx+ky+lz \\ mw+nx+oy+pz \end{bmatrix}_{4\times1}$$

4×4 · 4×1

output (4×1) matrix

②

$$\begin{bmatrix} 15 \\ 26 \\ 37 \\ 48 \end{bmatrix}_{4\times2} \times \begin{bmatrix} 10 & 20 & 30 & 40 \\ 100 & 200 & 300 & 400 \end{bmatrix}_{2\times4}$$

4×2 · 2×4

output 4×4 matrix

$$= \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix} \begin{bmatrix} 1[10\ 20\ 30\ 40]+5[100\ 200\ 300\ 400] \\ 2[10\ 20\ 30\ 40]+6[100\ 200\ 300\ 400] \\ 3[10\ 20\ 30\ 40]+7[100\ 200\ 300\ 400] \\ 4[10\ 20\ 30\ 40]+8[100\ 200\ 300\ 400] \end{bmatrix}$$

$R_1 = [10 \quad 20 \quad 30 \quad 40] + [500 \quad 1000 \quad 1500 \quad 2000]$ Pg 5

$R_2 = [20 \quad 40 \quad 60 \quad 80] + [600 \quad 1200 \quad 1800 \quad 2400]$

$R_3 = [30 \quad 60 \quad 90 \quad 120] + [700 \quad 1400 \quad 2100 \quad 2800]$

$R_4 = [40 \quad 80 \quad 120 \quad 160] + [800 \quad 1600 \quad 2400 \quad 3200]$

$$= \begin{bmatrix} 510 & 1020 & 1530 & 2040 \\ 620 & 1240 & 1860 & 2480 \\ 730 & 1460 & 2190 & 2920 \\ 840 & 1680 & 2520 & 3360 \end{bmatrix}$$

4×4

(3)

$$\begin{bmatrix} 4 & 4 \\ 1 & 0 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 3 \\ 0 & 1 \end{bmatrix} =$$

$(3 \times 2) \times (2 \times 2)$

output (3×2) matrix

$R_1 = 4[0\ 3] + 4[0\ 1] = [0\ 12] + [0\ 4]$

$R_2 = 1[0\ 3] + 0[0\ 1] = [0\ 3] + [0\ 0]$

$R_3 = -2[0\ 3] + 1[0\ 1] = [0\ -6] + [0\ 1]$

$R_1 = [0 \quad 16]$

$R_2 = [0 \quad 3]$

$R_3 = [0 \quad -5]$

$$= \begin{bmatrix} 0 & 16 \\ 0 & 3 \\ 0 & -5 \end{bmatrix}$$

(4) $\begin{bmatrix} 8 & 1 & 2 \\ -5 & 6 & 7 \end{bmatrix} \times \begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix} =$

2×3 × 3×2 → output (2×2) matrix

$$R_1 = \left[8[-5\ 1] + 1[0\ 2] + 2[-11\ \ 7] \right]$$

$$R_2 = \left[-5[-5\ 1] + 6[0\ 2] + 7[-11\ 7] \right]$$

$$R_1 = \left[[-40\ \ 8] + [0\ \ 2] + [-22\ \ 14] \right]$$

$$R_2 = \left[[+25\ \ -5] + [0\ \ 12] + [-77\ \ 49] \right]$$

Answer $R_{2\times 2} = \begin{bmatrix} -62 & 24 \\ -52 & 56 \end{bmatrix}$

(5) $\begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} -5[1\ -2] + 1[-3\ \ 4] \\ 0[1\ -2] + 2[-3\ \ 4] \\ -11[1\ -2] + 7[-3\ \ 4] \end{bmatrix}$

3×2 × 2×2 → output (3×2) matrix

$$= \begin{bmatrix} [-5\ +10] + [-3\ \ 4] \\ [0\ \ 0] + [-6\ \ 8] \\ [-11\ \ 22] + [-21\ \ 28] \end{bmatrix}$$

Answer $= \begin{bmatrix} -8 & 14 \\ -6 & 8 \\ -32 & 50 \end{bmatrix}$

Method (4) :- Outer Product

(1) $\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \times \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$ (4×4) × (4×1) → output (4×1) matrix

$= \begin{bmatrix} a \\ e \\ i \\ m \end{bmatrix} \times [w] + \begin{bmatrix} b \\ f \\ j \\ n \end{bmatrix} \times [x] + \begin{bmatrix} c \\ g \\ k \\ o \end{bmatrix} \times [y] + \begin{bmatrix} d \\ h \\ l \\ p \end{bmatrix} \times [z]$

$= \begin{bmatrix} \begin{bmatrix} aw \\ ew \\ iw \\ mw \end{bmatrix} + \begin{bmatrix} bx \\ fx \\ jx \\ nx \end{bmatrix} + \begin{bmatrix} cy \\ gy \\ ky \\ oy \end{bmatrix} + \begin{bmatrix} dz \\ hz \\ lz \\ pz \end{bmatrix} \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix} \times \begin{bmatrix} 10 & 20 & 30 & 40 \\ 100 & 200 & 300 & 400 \end{bmatrix}$ (4×2) × (2×4) → output (4×4) matrix

$= \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 10 & 20 & 30 & 40 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \begin{bmatrix} 100 & 200 & 300 & 400 \end{bmatrix}$

(4×1) × (1×4) → 4×4 ; (4×1) × (1×4) → 4×4

$= \begin{bmatrix} 10 & 20 & 30 & 40 \\ 20 & 40 & 60 & 80 \\ 30 & 60 & 90 & 120 \\ 40 & 80 & 120 & 160 \end{bmatrix} + \begin{bmatrix} 500 & 1000 & 1500 & 2000 \\ 600 & 1200 & 1800 & 2400 \\ 700 & 1400 & 2100 & 2800 \\ 800 & 1600 & 2400 & 3200 \end{bmatrix}$

$= \begin{bmatrix} 510 & 1020 & 1530 & 2040 \\ 620 & 1240 & 1860 & 2480 \\ 730 & 1460 & 2190 & 2920 \\ 840 & 1680 & 2520 & 3360 \end{bmatrix}$

(3)

$$\begin{bmatrix} 4 & 4 \\ 1 & 0 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4[0\ 3] + 4[0\ 1] \\ 1[0\ 3] + 0[0\ 1] \\ -2[0\ 3] + 1[0\ 1] \end{bmatrix}$$

3×2 · 2×2 → output 3×2 matrix

(3)

$$\begin{bmatrix} 4 & 4 \\ 1 & 0 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix} \times [0\ 3] + \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} \times [0\ 1]$$

3×2 · 2×2 → output 3×2 matrix

3×1 · 1×2 → 3×2 ; 3×1 · 1×2 → 3×2

DOTPOINT

$$= \begin{bmatrix} (4\times0) & (4\times3) \\ (1\times0) & (1\times3) \\ (-2\times0) & (-2\times3) \end{bmatrix} + \begin{bmatrix} (4\times0) & (4\times1) \\ (0\times0) & (0\times1) \\ (1\times0) & (1\times1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 12 \\ 0 & 3 \\ 0 & -6 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 16 \\ 0 & 3 \\ 0 & -5 \end{bmatrix}$$

(4)

$$\begin{bmatrix} 8 & 1 & 2 \\ -5 & 6 & 7 \end{bmatrix} \times \begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \end{bmatrix} \times [-5\ 1] + \begin{bmatrix} 1 \\ 6 \end{bmatrix} \times [0\ 2] + \begin{bmatrix} 2 \\ 7 \end{bmatrix} \times [-11\ 7]$$

2×3 · 3×2 → output (2×2) matrix

2×1 · 1×2 → 2×2 ; 2×1 · 1×2 → 2×2

Row Multiplication.

$$\begin{matrix} R_1 \\ = \\ R_2 \end{matrix} \begin{matrix} 8(-5\ 1) \\ -5(-5\ 1) \end{matrix} + \begin{matrix} 1(0\ 2) \\ 6(0\ 2) \end{matrix} + \begin{matrix} 2(-11\ 7) \\ 7(-11\ 7) \end{matrix}$$

$$= \begin{bmatrix} -40 & 8 \\ +25 & -5 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 0 & 12 \end{bmatrix} + \begin{bmatrix} -22 & 14 \\ -77 & 49 \end{bmatrix}$$

$$= \begin{bmatrix} -62 & 24 \\ -52 & 56 \end{bmatrix}$$

Method ⑤ :- Block Multiplication.

① $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ (A: 3×3) $\times$ $\begin{bmatrix} m & n \\ o & p \\ q & r \end{bmatrix}$ (B: 3×2) = output 3×2 matrix

$= AC + BD$

$= \begin{bmatrix} a & b \\ d & e \\ g & h \end{bmatrix}$ (3×2) $\times \begin{bmatrix} m & n \\ o & p \end{bmatrix}$ (2×2) $+ \begin{bmatrix} c \\ f \\ i \end{bmatrix}$ (3×1) $\begin{bmatrix} q & r \end{bmatrix}$ (1×2)

(3×2) (3×2)

Column Product method

$$= \begin{bmatrix} ma + oa & na + pb \\ md + od & nd + pe \\ mg + og & ng + ph \end{bmatrix} + \begin{bmatrix} qc & rc \\ qf & rf \\ qi & ri \end{bmatrix}$$

$$AC+BD = \begin{bmatrix} ma+oa+qc & na+pb+rc \\ md+od+qf & nd+pc+rf \\ mg+og+qi & ng+ph+ri \end{bmatrix}$$

3×2

(2) $\begin{bmatrix} -1 & 0 & 4 \\ 2 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix} = AC + BD.$

A: 2×3, B: 3×2

$\begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix} \times \begin{bmatrix} 2 & 4 \end{bmatrix}$

2×2 2×2 2×1 1×2

2×2 2×2

Dot Product

2×2 matrix

$AC = \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} (-1 \times -1) + (0 \times -1) & (-1 \times 1) + (0 \times 3) \\ (2 \times -1) + (0 \times -1) & (2 \times 1) + (0 \times 3) \end{bmatrix}$

$= \begin{bmatrix} 1+0 & -1+0 \\ -2+0 & 2+0 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

Dot Product

$BD = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \begin{bmatrix} 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} (4 \times 2) & (4 \times 4) \\ (0 \times 2) & (0 \times 4) \end{bmatrix}$

$= \begin{bmatrix} 8 & 16 \\ 0 & 0 \end{bmatrix}$

$AC + BD = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 16 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 9 & 15 \\ -2 & 2 \end{bmatrix}$

(3) $\begin{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}_A & \begin{bmatrix} 3 \\ 2 \end{bmatrix}_B \end{bmatrix} \quad \begin{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}_C \\ \begin{bmatrix} 2 & 1 \end{bmatrix}_D \end{bmatrix}$

$2\times\textcircled{3} \quad \textcircled{3}\times 2$

output 2×2 matrix

$AC + BD$

$= \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}_{2\times 2} \times \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}_{2\times 2} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}_{2\times 1} \times \begin{bmatrix} 2 & 1 \end{bmatrix}_{1\times 2}$

$= \begin{bmatrix} 1 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 \\ 12 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 10 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 11 & 9 \\ 21 & 20 \end{bmatrix}_{2\times 2}$

(4) $\begin{bmatrix} \begin{bmatrix} a & b \\ D & E \end{bmatrix}_A & \begin{bmatrix} c \\ F \end{bmatrix}_B \end{bmatrix} \quad \begin{bmatrix} \begin{bmatrix} x & 2 \\ y & 3 \end{bmatrix}_C \\ \begin{bmatrix} z & 4 \end{bmatrix}_D \end{bmatrix}$

$2\times\textcircled{3} \quad \textcircled{3}\times 2$

output 2×2

$AC + BD$

$= \begin{bmatrix} a & b \\ D & E \end{bmatrix} \begin{bmatrix} x & 2 \\ y & 3 \end{bmatrix} + \begin{bmatrix} c \\ F \end{bmatrix} \begin{bmatrix} z & 4 \end{bmatrix}$

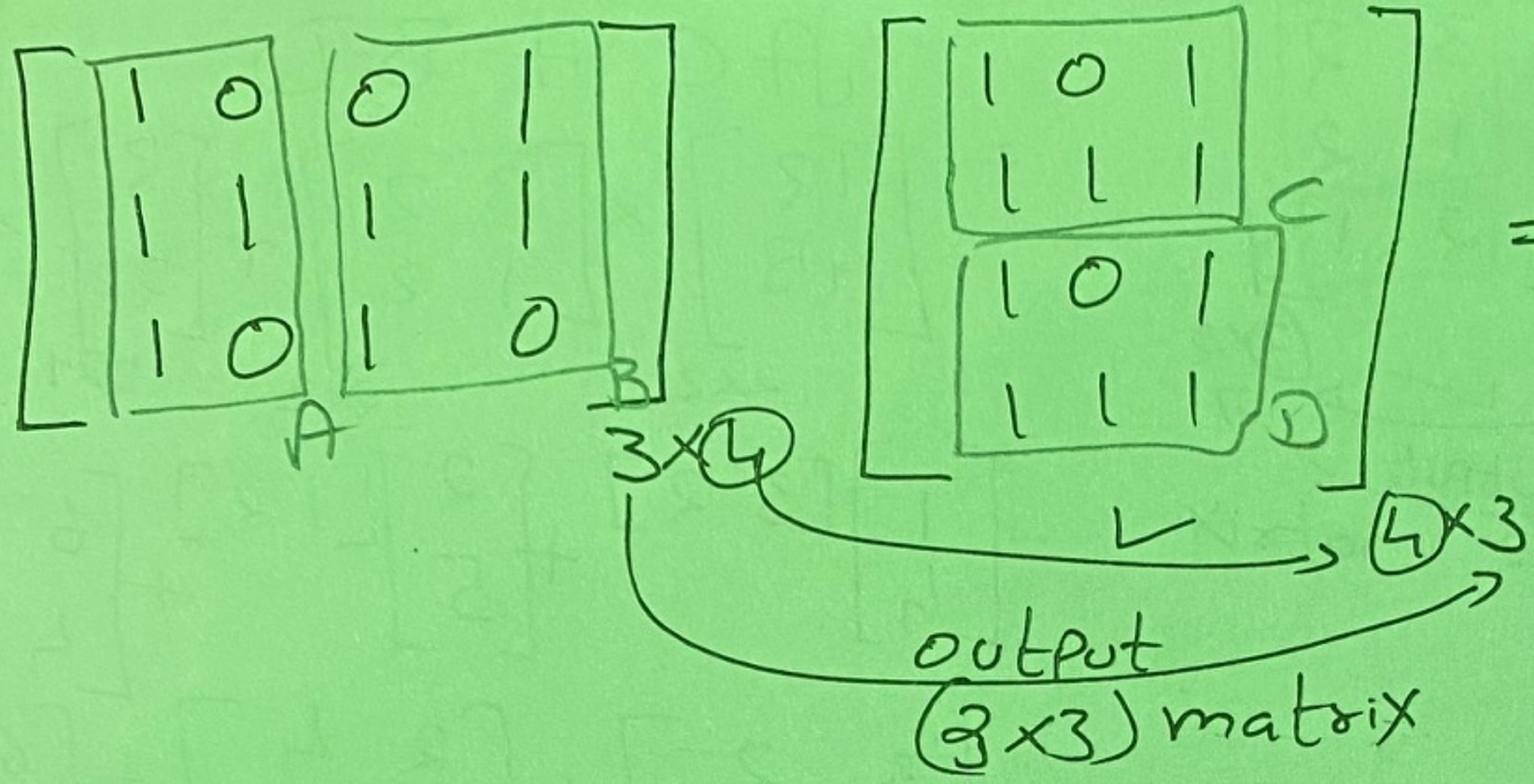
$= \begin{bmatrix} a \\ D \end{bmatrix} \begin{bmatrix} x & 2 \end{bmatrix} + \begin{bmatrix} b \\ E \end{bmatrix} \begin{bmatrix} y & 3 \end{bmatrix} + \begin{bmatrix} cz & c4 \\ Fz & F4 \end{bmatrix}$

$= \begin{bmatrix} ax & a2 \\ Dx & D2 \end{bmatrix} + \begin{bmatrix} by & b3 \\ Ey & E3 \end{bmatrix} + \begin{bmatrix} cz & c4 \\ Fz & F4 \end{bmatrix}$

$= \begin{bmatrix} ax+by+cz & a2+b3+c4 \\ Dx+Ey+Fz & D2+E3+F4 \end{bmatrix}$

$2\times 2.$

(5)



$= AC + BD$

$$AC+BD = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{2\times3} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{2\times3}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}_{2\times2} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times1} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 0 \end{bmatrix}_{3\times3}$$