

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 3 & 1 & 3 \\ 4 & -1 & 1 \end{bmatrix} \quad \begin{array}{l} R_2' \rightarrow \frac{2}{3} R_2 - R_1 \\ R_3' \rightarrow \frac{1}{2} R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 2/3 & -1 \\ 0 & -1/2 & -5/2 \end{bmatrix} \quad R_3'' \rightarrow \frac{4}{3} R_3 + R_2$$

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 2/3 & -1 \\ 0 & 0 & -13/3 \end{bmatrix} \quad \text{Rank 3}$$

$$E_{21} = \frac{2}{3} R_2' - R_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_{31} = \frac{1}{2} R_3' - R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1/2 \end{bmatrix}$$

$$E_{31}' = \frac{4}{3} R_3 + R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 4/3 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 4/3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1/2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4/3 & 1 & 2/3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2/3 & 0 \\ -4/3 & 2/3 & 2/3 \end{bmatrix}$$

$$GA = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 2/3 & 0 \\ -4/3 & 2/3 & 2/3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 3 \\ 3 & 1 & 3 \\ 4 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2/3 & -1 \\ 0 & 0 & -13/3 \end{bmatrix}$$

(UTM)