Determinants

A determinant is a square array of numbers (written within a pair of vertical lines) which represents a certain sum of products.

Below is an example of a 3 × 3 determinant (it has 3 rows and 3 columns).

$$\begin{vmatrix} 10 & 0 & -3 \\ -2 & -4 & 1 \\ 3 & 0 & 2 \end{vmatrix}$$

The result of multiplying out, then simplifying the elements of a determinant is a single number (a **scalar** quantity).

Calculating a 2 × 2 Determinant

In general, we find the value of a 2 × 2 determinant with elements a, b, c, d as follows:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

We multiply the diagonals (top left × bottom right first), then subtract.

Example 1

$$\begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= 4 \times 3 - 2 \times 1$$

$$= 12 - 2$$

$$= 10$$

The final result is a single **number**.

Using Determinants to Solve Systems of Equations

We can solve a system of equations using determinants, but it becomes very tedious for large systems. We will only do 2 × 2 and 3 × 3 systems using determinants.

Cramer's Rule

The solution (x, y) of the system

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

can be found using determinants:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Solve the system using Cramer's Rule:

$$x - 3y = 6$$

$$2x + 3y = 3$$

First we determine the values we will need for Cramer's Rule:

$a_1 = 1 \qquad b_1 = -3 \qquad c_1 = 6$

$a_2 = 2 \qquad b_2 = 3 \qquad c_2 = 3$

$$x = \frac{\begin{vmatrix} 6 & -3 \\ 3 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix}} = \frac{18+9}{3+6} = 3$$

$$y = \frac{\begin{vmatrix} 1 & 6 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix}} = \frac{3-12}{3+6} = \frac{-9}{9} = -1$$

So the solution is $(3, -1)$.

Check:

[1] $3 + 3 = 6$ OK

[2] $6 - 3 = 3$ OK

3×3 Determinants

A 3 × 3 determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

can be evaluated in various ways.

We will use the method called "expansion by minors". But first, we need a definition.

Cofactors

The 2 × 2 determinant

$$\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

is called the **cofactor** of a_1 for the 3 × 3 determinant:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The cofactor is formed from the elements that are not in the same row as a_1 and not in the same column as a_1.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Similarly, the determinant

$$\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}$$

is called the **cofactor** of a_2. It is formed from the elements not in the same row as a_2 and not in the same column as a_2.

We continue the pattern for the cofactor of a_3.

Expansion by Minors

We evaluate our 3 × 3 determinant using expansion by minors. This involves multiplying the **elements** in the first column of the determinant by the **cofactors** of those elements. We subtract the middle product and add the final product.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

Note that we are working down the first column and multiplying by the cofactor of each element.

Evaluate

$$\begin{vmatrix} -2 & 3 & -1 \\ 5 & -1 & 4 \\ 4 & -8 & 2 \end{vmatrix}$$

$$\begin{vmatrix} -2 & 3 & -1 \\ 5 & -1 & 4 \\ 4 & -8 & 2 \end{vmatrix} = -2 \begin{vmatrix} -1 & 4 \\ -8 & 2 \end{vmatrix} -5 \begin{vmatrix} 3 & -1 \\ -8 & 2 \end{vmatrix} +4 \begin{vmatrix} 3 & -1 \\ -1 & 4 \end{vmatrix}$$

$$= -2[(-1)(2) - (-8)(4)] - 5[(3)(2) - (-8)(-1)] + 4[(3)(4) - (-1)(-1)]$$

$$= -2(30) - 5(-2) + 4(11)$$

$$= -60 + 10 + 44$$

$$= -6$$

Here, we are **expanding by the first column.** We can do the expansion by using the first row and we will get the same result.

Cramer's Rule to Solve 3 × 3 Systems of Linear Equations

We can solve the general system of equations,

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

by using the determinants:

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\Delta}$$

$$y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\Delta}$$

$$z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\Delta}$$

where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Solve, using Cramer's Rule:

$$2x + 3y + z = 2$$

$$-x + 2y + 3z = -1$$

$$-3x - 3y + z = 0$$

$$x = \frac{\begin{vmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 0 & -3 & 1 \end{vmatrix}}{\Delta}$$

$$y = \frac{\begin{vmatrix} 2 & 2 & 1 \\ -1 & -1 & 3 \\ -3 & 0 & 1 \end{vmatrix}}{\Delta}$$

$$z = \frac{\begin{vmatrix} 2 & 3 & 2 \\ -1 & 2 & -1 \\ -3 & -3 & 0 \end{vmatrix}}{\Delta}$$

where

$$\Delta = \begin{vmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ -3 & -3 & 1 \end{vmatrix} = 2(11) + 1(6) - 3(7) = 7$$

So

$$x = \frac{2(11) + 1(6) + 0}{7} = \frac{28}{7} = 4$$

$$y = \frac{2(-1) + 1(2) - 3(7)}{7} = -\frac{21}{7} = -3$$

$$z = \frac{2(-3) + 1(6) - 3(-7)}{7} = \frac{21}{7} = 3$$

Checking solutions:

[1] $2(4) + 3(\text{-}3) + 3 = 2$ OK

[2] $-(4) + 2(\text{-}3) + 3(3) = \text{-}1$ OK

[3] $-3(4) - 3(-3) + 3 = 0$ OK

So the solution is $(4, -3, 3)$.

1. Evaluate by expansion of minors:

$$\begin{vmatrix} 10 & 0 & -3 \\ -2 & -4 & 1 \\ 3 & 0 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 10 & 0 & -3 \\ -2 & -4 & 1 \\ 3 & 0 & 2 \end{vmatrix} = 10\begin{vmatrix} -4 & 1 \\ 0 & 2 \end{vmatrix} -(-2)\begin{vmatrix} 0 & -3 \\ 0 & 2 \end{vmatrix} +3\begin{vmatrix} 0 & -3 \\ -4 & 1 \end{vmatrix}$$

$$= 10[(-4)(2) - (0)(1)] + 2[(0)(2) - (0)(-3)] + 3[(0)(1) - (-4)(-3)]$$

$$= 10(-8) + 2(0) + 3(-12)$$

$$= -80 - 36$$

$$= -116$$

Solve the system by use of determinants:

$x + 3y + z = 4$

$2x - 6y - 3z = 10$

$4x - 9y + 3z = 4$

$$x = \frac{\begin{vmatrix} 4 & 3 & 1 \\ 10 & -6 & -3 \\ 4 & -9 & 3 \end{vmatrix}}{\Delta}$$

$$y = \frac{\begin{vmatrix} 1 & 4 & 1 \\ 2 & 10 & -3 \\ 4 & 4 & 3 \end{vmatrix}}{\Delta}$$

where

$$\Delta = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -6 & -3 \\ 4 & -9 & 3 \end{vmatrix}$$

$= 1(-45) - 2(18) + 4(-3)$

$= -93$

Note: Once we have x and y, we can find z without using Cramer's Rule.

So

$$x = \frac{4(-45) - 10(18) + 4(-3)}{-93} = \frac{-372}{-93} = 4$$

$$y = \frac{1(42) - 2(8) + 4(-22)}{-93} = \frac{-62}{-93} = \frac{2}{3}$$

Using these two results, we can easily find that $z =$ -2.

Checking the solution:

[1] $(4) + 3\left(\frac{2}{3}\right) + -2 = 4$

[2] $2(4) - 6\left(\frac{2}{3}\right) - 3(-2) = 10$

[3] $4(4) - 9\left(\frac{2}{3}\right) + 3(-2) = 4$

So the solution is $\left(4, \frac{2}{3}, -2\right)$.

Definition of eigenvalues and eigenvectors of a matrix

Let A be any square matrix. A non-zero vector **v is an eigenvector** of A if

$$Av = \lambda v$$

for some number **λ, called the corresponding eigenvalue**.

NOTE: The German word "eigen" roughly translates as "own" or "belonging to". Eigenvalues and eigenvectors correspond to each other (are paired) for any particular matrix A.

The solved examples below give some insight into what these concepts mean. First, a summary of what we're going to do:

How to find the eigenvalues and eigenvectors of a 2x2 matrix

1. Set up the characteristic equation, using $|A - \lambda I| = 0$
2. Solve the characteristic equation, giving us the eigenvalues (2 eigenvalues for a 2x2 system)
3. Substitute the eigenvalues into the two equations given by $A - \lambda I$
4. Choose a convenient value for x1, then find x2
5. The resulting values form the corresponding eigenvectors of A (2 eigenvectors for a 2x2 system)

There is no single eigenvector formula as such - it's more of a set of steps that we need to go through to find the eigenvalues and eigenvectors.

We start with a system of two equations, as follows:

$$y_1 = -5x_1 + 2x_2$$

$$y_2 = -9x_1 + 6x_2$$

We can write those equations in matrix form as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

In general we can write the above matrices as:

$$\mathbf{y} = \mathbf{A}\mathbf{v}$$

where

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}, \text{ and}$$

$$\mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Step 1. Set up the characteristic equation, using |A − λI| = 0

Our task is to find the eigenvalues λ, and eigenvectors v, such that: y = λv

We are looking for scalar values λ (numbers, not matrices) that can replace the matrix A in the expression y = Av.

That is, we want to find λ such that :

$$-5x_1 + 2x_2 = \lambda x_1$$

$$-9x_1 + 6x_2 = \lambda x_2$$

Rearranging gives:

$$\begin{aligned} -(5 - \lambda)x_1 + 2x_2 = 0 \\ -9x_1 + (6 - \lambda)x_2 = 0 \end{aligned} \qquad (1)$$

This can be written using matrix notation with the identity matrix $\mathbf{I}$ as:

$(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = 0$, that is:

$$\left(\mathbf{A} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\mathbf{v} = 0$$

$$\left(\mathbf{A} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\mathbf{v} = 0$$

Clearly, we have a trivial solution $\mathbf{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, but in order to find any non-trivial solutions, we apply a result following from Cramer's Rule, that this equation will have a non-trivial (that is, non-zero) solution $\mathbf{v}$ if its coefficient determinant has value 0.

The resulting equation, using determinants, $|\mathbf{A} - \lambda\mathbf{I}| = 0$ is called the **characteristic equation**.

Step 2. Solve the characteristic equation, giving us the eigenvalues (2 eigenvalues for a 2x2 system)

In this example, the coefficient determinant from equations (1) is:

$$\begin{aligned} |\mathbf{A} - \lambda\mathbf{I}| &= \begin{vmatrix} -5 - \lambda & 2 \\ -9 & 6 - \lambda \end{vmatrix} \\ &= (-5 - \lambda)(6 - \lambda) - (-9)(2) \\ &= -30 - \lambda + \lambda^2 + 18 \\ &= \lambda^2 - \lambda - 12 \\ &= (\lambda + 3)(\lambda - 4) \end{aligned}$$

Now this equals 0 when:

$$(\lambda + 3)(\lambda - 4) = 0$$

That is, when:

$$\lambda = -3 \quad \text{or} \quad 4.$$

These two values are the **eigenvalues** for this particular matrix $\mathbf{A}$.

Step 3. Substitute the eigenvalues into the two equations given by $\mathbf{A} - \lambda\mathbf{I}$

Case 1: $\lambda_1 = -3$

When $\lambda = \lambda_1 = -3$, equations (1) become:

$$[-5 - (-3)]x_1 + 2x_2 = 0$$

$$-9x_1 + [6 - (-3)]x_2 = 0$$

That is:

$$\begin{aligned} -2x_1 + 2x_2 &= 0 \\ -9x_1 + 9x_2 &= 0 \end{aligned} \qquad (2)$$

Dividing the first line of Equations (2) by -2 and the second line by -9 (not really necessary, but helps us see what is happening) gives us the identical equations:

$$x_1 - x_2 = 0$$

$$x_1 - x_2 = 0$$

Step 4. Choose a convenient value for x_1, then find x_2

There are infinite solutions of course, where $x_1 = x_2$. We choose a convenient value for x_1 of, say 1, giving $x_2 = 1$.

Step 5. The resulting values form the corresponding eigenvectors of A (2 eigenvectors for a 2x2 system)

So the corresponding eigenvector is:

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

NOTE: We could have easily chosen $x_1 = 3$, $x_2 = 3$, or for that matter, $x_1 = -100$, $x_2 = -100$. These values will still "work" in the matrix equation.

In general, we could have written our answer as "$x_1 = t$, $x_2 = t$, for any value t", however it's usually more meaningful to choose a convenient starting value (usually for x_1), and then derive the resulting remaining value(s).

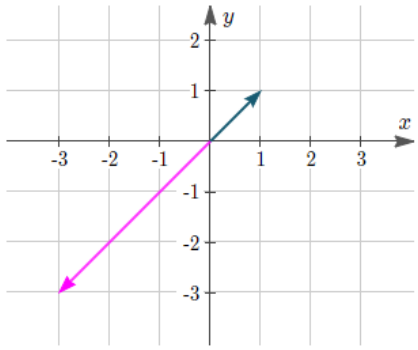
Is it correct?

We can check by substituting:

$$\begin{aligned} \mathbf{A}\mathbf{v}_1 &= \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -3 \\ -3 \end{bmatrix} \\ &= -3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \lambda_1 \mathbf{v}_1 \end{aligned}$$

We have found an **eigenvalue** $\lambda_1 = -3$ and an **eigenvector** $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for the matrix $\mathbf{A} = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ such that $\mathbf{A}\mathbf{v}_1 = \lambda_1 \mathbf{v}_1$.

Graphically, we can see that matrix $\mathbf{A} = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ acting on vector $\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is equivalent to multiplying $\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ by the scalar $\lambda_1 = -3$. The result is applying a scale of -3.



Graph indicating the transform $\mathbf{y_1} = \mathbf{A}\mathbf{v_1}$

Case 2: $\lambda_2 = 4$

When $\lambda = \lambda_2 = 4$, equations (1) become:

$$(-5 - (4))x_1 + 2x_2 = 0$$

$$-9x_1 + (6 - (4))x_2 = 0$$

That is:

$$-9x_1 + 2x_2 = 0$$

$$-9x_1 + 2x_2 = 0$$

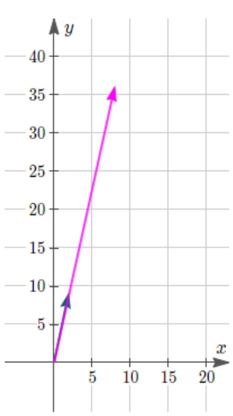
We choose a convenient value for x_1 of 2, giving $x_2 = 9$. So the corresponding eigenvector is:

$$\mathbf{v}_2 = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

We could check this by multiplying and concluding $\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 9 \end{bmatrix} = 4\begin{bmatrix} 2 \\ 9 \end{bmatrix}$, that is $\mathbf{A}\mathbf{v}_2 = \lambda_2 \mathbf{v}_2$.

We have found an **eigenvalue** $\lambda_2 = 4$ and an **eigenvector** $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$ for the matrix $\mathbf{A} = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ such that $\mathbf{A}\mathbf{v}_2 = \lambda_2 \mathbf{v}_2$.

Graphically, we can see that matrix $\mathbf{A} = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ acting on vector $\mathbf{v_2} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$ is equivalent to multiplying $\mathbf{v_2} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$ by the scalar $\lambda_2 = 4$. The result is applying a scale of 4.



Graph indicating the transform $\mathbf{y_2} = \mathbf{A}\mathbf{v_2} = \lambda_2 \mathbf{x_2}$

How many eigenvalues and eigenvectors?

In the above example, we were dealing with a 2×2 system, and we found 2 eigenvalues and 2 corresponding eigenvectors.

If we had a 3×3 system, we would have found 3 eigenvalues and 3 corresponding eigenvectors.

In general, a $n \times n$ system will produce n eigenvalues and n corresponding eigenvectors.

Find the eigenvalues and corresponding eigenvectors for the matrix $\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$.

The matrix $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$ corresponds to the linear equations:

$$y_1 = 2x_1 + 3x_2$$

$$y_2 = 2x_1 + x_2$$

We want to find λ such that :

$$2x_1 + 3x_2 = \lambda x_1$$

$$2x_1 + x_2 = \lambda x_2$$

Rearranging gives:

$$\begin{aligned} (2-\lambda)x_1 + 3x_2 &= 0 \\ 2x_1 + (1-\lambda)x_2 &= 0 \end{aligned} \qquad (3)$$

The **characterstic equation** $|\mathbf{A} - \lambda\mathbf{I}| = 0$ for this example is given by:

$$\begin{aligned} |\mathbf{A} - \lambda\mathbf{I}| &= \begin{vmatrix} 2-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} \\ &= 2 - 3\lambda + \lambda^2 - 6 \\ &= \lambda^2 - 3\lambda - 4 \\ &= 0 \end{aligned}$$

This has value 0 when $(\lambda - 4)(\lambda + 1) = 0$.

Case 1: $\lambda = 4$

With $\lambda_1 = 4$, equations (3) become:

$$(2-4)x_1 + 3x_2 = 0$$

$$2x_1 + (1-4)x_2 = 0$$

That is:

$$-2x_1 + 3x_2 = 0$$

$$2x_1 - 3x_2 = 0$$

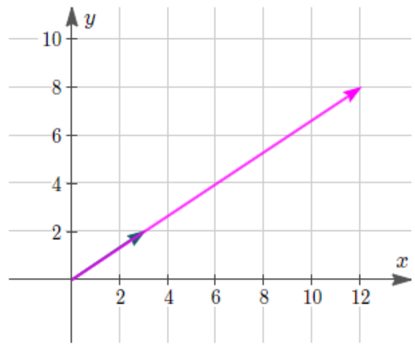
We choose a convenient value for x_1 of 3, giving $x_2 = 2$. So the corresponding eigenvector is:

$$\mathbf{v_1} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Multiplying to check our answer, we would find:

$$\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix} = 4\begin{bmatrix}3\\2\end{bmatrix}, \text{ that is } \mathbf{A}\mathbf{v}_1 = \lambda_1\mathbf{v}_1.$$

Graphically, we can see that matrix $\mathbf{A} = \begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}$ acting on vector $\mathbf{v_1} = \begin{bmatrix}3\\2\end{bmatrix}$ is equivalent to multiplying $\mathbf{v_1} = \begin{bmatrix}3\\2\end{bmatrix}$ by the scalar $\lambda_1 = 4$. The result is applying a scale of 4.



Graph indicating the transform $\mathbf{y_1} = \mathbf{A}\mathbf{v_1} = \lambda_1\mathbf{x_1}$

Case 2: $\lambda = -1$

With $\lambda_2 = -1$, equations (3) become:

$$(2+1)x_1 + 3x_2 = 0$$

$$2x_1 + (1+1)x_2 = 0$$

That is:

$$3x_1 + 3x_2 = 0$$

$$2x_1 + 2x_2 = 0$$

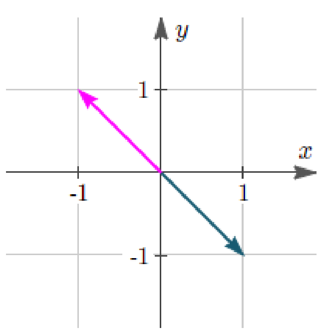
We choose a convenient value $x_1 = 1$, giving $x_2 = -1$. So the corresponding eigenvector is:

$$\mathbf{v_2} = \begin{bmatrix}1\\-1\end{bmatrix}$$

Multiplying to check our answer, we would find:

$$\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = -1\begin{bmatrix}1\\-1\end{bmatrix}, \text{ that is } \mathbf{A}\mathbf{v}_2 = \lambda_2\mathbf{v}_2.$$

Graphically, we can see that matrix $\mathbf{A} = \begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}$ acting on vector $\mathbf{v_2} = \begin{bmatrix}1\\-1\end{bmatrix}$ is equivalent to multiplying $\mathbf{v_2} = \begin{bmatrix}1\\-1\end{bmatrix}$ by the scalar $\lambda_2 = -1$. We are scaling vector $\mathbf{v_2}$ by -1.



Graph indicating the transform $\mathbf{y_2} = \mathbf{A}\mathbf{v_2} = \lambda_2\mathbf{x_2}$

In real life, we effectively use eigen vectors and eigen values on a daily basis though sub-consciously most of the time.

Example 1: If you eat pizza, French fries or any food you are typically translating their taste into sour, sweet, bitter, salty, hot, etc.

- Principal components of taste -- though in reality the way a food is prepared is formulated in terms of ingredients' ratios (sugar, flour, butter, etc. 10's of 100's of things that go into making a specific food)
- However our mind will transform all such information into the principal components of taste (eigen vector having sour, bitter, sweet, hot) automatically along with the food texture and smell.
- So we use eigen vectors every day in many situations without realizing that's how we learn about a system more effectively.
- Our brain simply transforms all the ingredients, cooking methods, final food product into some very effective eigen vector whose elements are taste sub parts, smell and visual appearance internally.
- All the ingredients and their quantities along with the cooking procedure represent some transformation matrix A and we can find some principal eigen vector(s) V with elements as taste + smell + appearance + touch having some linear transformation directly related.
- AV = wV,
    - where w represent eigen values scalar and V an eigen vector
    - top chicken fryer like KFC tasters probably have bigger taste + smell + appearance eigen vector
    - also with much bigger eigen values in each dimension.