

**COURSE: VECTOR SPACES, SAMPLING THEORY & OPTIMIZATION**

**COURSE CODE: 21MAT31A**

**MODULE - 2: EIGEN VALUES AND EIGEN VECTORS**

<b>Q. No.</b>	<b>Questions</b>	<b>Marks</b>	<b>COs</b>	<b>BLs</b>
<b>1</b>	Find the characteristic polynomial and the eigenvalue and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ .	<b>6</b>	<b>5</b>	<b>1</b>
<b>2</b>	Find the characteristic polynomial and the eigenvalue and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ .	<b>6</b>	<b>5</b>	<b>1</b>
<b>3</b>	Test whether $\lambda = 1$ an eigenvalue of matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ ? If so, find one corresponding eigenvector.	<b>6</b>	<b>5</b>	<b>4</b>
<b>4</b>	Find a basis for the eigenspace corresponding to eigenvalue $\lambda = 2$ for the matrix $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$	<b>6</b>	<b>4</b>	<b>1</b>
<b>5</b>	Show that $\lambda^{-1}$ is an eigenvalue of $A^{-1}$ , If $\lambda$ be an eigenvalue of an invertible matrix $A$	<b>6</b>	<b>5</b>	<b>1</b>
<b>6</b>	Find the characteristic polynomial and the eigenvalue of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ . Also, verify that eigenvalues of $A^2$ are squares of those of eigenvalues of matrix $A$ .	<b>7</b>	<b>5</b>	<b>5</b>
<b>7</b>	Find the eigenspace of the matrix $A = \begin{bmatrix} 16 & -4 & -2 \\ 3 & 3 & -6 \\ 2 & -8 & 11 \end{bmatrix}$ for $\lambda = 5$ .	<b>6</b>	<b>4</b>	<b>1</b>
<b>8</b>	Find the value of $h$ in the matrix $A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ such that the eigenspace for $\lambda = 5$ is two dimensional.	<b>6</b>	<b>4</b>	<b>1</b>
<b>9</b>	Test whether the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is diagonalizable. If so, find $P$ such that $P^{-1}AP$ is a diagonal matrix.	<b>6</b>	<b>4</b>	<b>4</b>
<b>10</b>	Test whether the matrix $A = \begin{bmatrix} 0 & i & i \\ i & 0 & i \\ i & i & 0 \end{bmatrix}$ is diagonalizable.	<b>7</b>	<b>4</b>	<b>4</b>

<b>11</b>	Find the matrix $A$ , if the eigenvectors of a $3 \times 3$ matrix $A$ corresponding to eigenvalues 1,1,3 are $[1,0,-1]^T$ , $[0,1,-1]^T$ and $[1,1,0]^T$ respectively.	<b>7</b>	<b>5</b>	<b>3</b>
<b>12</b>	Find a formula for $A^n$ , given that $A = PDP^{-1}$ , where $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$ , $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$ .	<b>7</b>	<b>2</b>	<b>3</b>
<b>13</b>	Determine $A^4$ , where $A = \begin{bmatrix} -3 & 12 \\ -2 & 7 \end{bmatrix}$ .	<b>7</b>	<b>4</b>	<b>5</b>
<b>14</b>	Examine that product of two orthogonal matrix of the same order is also an orthogonal matrix.	<b>7</b>	<b>4</b>	<b>4</b>
<b>15</b>	Examine that $ A  = \pm 1$ , if $A$ is an orthogonal matrix.	<b>7</b>	<b>4</b>	<b>4</b>
<b>16</b>	Compute the orthogonal transform which transforms the quadratic form $Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_1x_3 - 2x_1x_2$ to canonical form.	<b>7</b>	<b>4</b>	<b>3</b>
<b>17</b>	Compute the orthogonal transform which transforms the quadratic form $Q = x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to canonical form.	<b>7</b>	<b>4</b>	<b>3</b>
<b>18</b>	Compute the canonical form which transforms the quadratic form $Q = 17x_1^2 + 17x_2^2 - 30x_1x_2$ .	<b>7</b>	<b>4</b>	<b>5</b>
<b>19</b>	Compute the canonical form which transforms the quadratic form $Q = 3x_1^2 + 3x_3^2 + 4x_1x_2 + 8x_1x_3 + 8x_2x_3$ .	<b>7</b>	<b>4</b>	<b>3</b>
<b>20</b>	Compute the canonical form which transforms the quadratic form $Q = 2x_1^2 + 2x_2^2 + 2x_1x_2$ .	<b>7</b>	<b>4</b>	<b>3</b>