

- Find equation of bezier curve which passes through points (0,0) and (-2,1) and is controlled through points (7,5) and (2,0).

(i) Let $P_0 = (0,0)$; $P_1 = (7, 5)$; $P_2 = (2,0)$; $P_3 = (-2, 1)$.

The corresponding cubic Bezier curve is given by

$$\begin{aligned}
 P(t) &= [t^3 \ t^2 \ t \ 1] \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 0 \\ -2 & 1 \end{pmatrix} \quad (0 \leq t \leq 1) \\
 &= [t^3 \ t^2 \ t \ 1] \begin{pmatrix} 13 & 16 \\ -36 & -30 \\ 21 & 15 \\ 0 & 0 \end{pmatrix} \\
 &= [(13t^3 - 36t^2 + 21t) \ (16t^3 - 30t^2 + 15t)]
 \end{aligned}$$

(ii) If $P_0 = (0, 0)$, $P_1 = (2,0)$, $P_2 = (7, 5)$ and $P_3 = (-2, 1)$ then the resulting cubic Bezier curve is given by,

$$\begin{aligned}
 P(t) &= [t^3 \ t^2 \ t \ 1] \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 2 & 0 \\ 7 & 5 \\ -2 & 1 \end{pmatrix} \quad (0 \leq t \leq 1) \\
 &= [t^3 \ t^2 \ t \ 1] \begin{pmatrix} -17 & -14 \\ 9 & 15 \\ 6 & 0 \\ 0 & 0 \end{pmatrix} \\
 &= [(-17t^3 + 9t^2 + 6t) \ (-14t^3 + 15t^2)] \quad (ii)
 \end{aligned}$$

