

Oder 3 set b E I

Answer:

$$P_1 = P_3 = (6, 4)$$

Step-by-step explanation:

Bezier curves can be generated under the control of other points.

Approximate tangents by using control points are used to generate curve.

The Bezier curve can be represented mathematically as –

$$\sum_{k=0}^n P_i B_i^n(t)$$

Where P_i is the set of points and $B_i^n(t)$ represents the Bernstein polynomials which are given by –

$$B_i^n(t) = \binom{n}{i} (1-t)^{n-i} t^i$$

Where n is the polynomial degree, i is the index, and t is the variable.

The equation for the Bezier curve is given

as:-

$$P(U) = (1-U)^3 P_1 + 3U(1-U)^2 P_2 + 3U^2(1-U) P_3 + U^3 P_4$$

for $0 \leq u \leq 1$ where $P(U)$

is the point on the curve P_1, P_2, P_3, P_4

Let us take $U = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$

Therefore, $P(0) = (1, 1)$

$$P(1/4) = (1 - 1/4)x^3 P_1 + 3\frac{1}{4}(1 - 1/4)x^2 P_2 + 3(1/4)x^2(1 - 1/4)P_3 + (1/4)x^3 P_4$$

$$\Rightarrow \frac{27}{64}(1, 1) + \frac{27}{64}(2, 3) + \frac{9}{64}(4, 3) + \frac{1}{64}(6, 4)$$

$$\begin{aligned}
P(1/4) &= (1 - 1/4)x^3P_1 + 3\frac{1}{4}(1 - 1/4)x^2P_2 + 3(1/4)x^2(1 - 1/4)P_3 + (1/4)x^3P_4 \\
&\Rightarrow \frac{27}{6^2}(1, 1) + \frac{27}{64}(2, 3) + \frac{9}{64}(4, 3) + \frac{1}{64}(6, 4) \\
&\Rightarrow \left[\frac{27}{64} \times 1 + \frac{27}{64} \times 2 + \frac{9}{64} \times 6, \frac{27}{64} \times 1 + \frac{27}{64} \times 2 + \frac{9}{64} \times 3 + \frac{1}{64} \times 4 \right] \\
&\Rightarrow \left[\frac{123}{64}, \frac{139}{64} \right] \\
&= (1.9218, 2.1718)
\end{aligned}$$

$$\begin{aligned}
P(\frac{1}{2}) &= (1 - \frac{1}{2})^3P_1 + 3\frac{1}{2}(1 - \frac{1}{2})^2P_2 + 3\frac{1}{2}^4(1 - \frac{1}{2})P_3 + \frac{1}{2}^3P_4 \\
&\Rightarrow \frac{1}{8}(1, 1) + \frac{3}{8}(2, 3) + \frac{3}{8}(4, 3) + \frac{1}{8}(6, 4) \\
&\Rightarrow \left[\frac{1}{8} \times 1 + \frac{3}{8} \times 2 + \frac{3}{8} \times 4 + \frac{1}{8} \times 6, \frac{1}{8} \times 1 + \frac{3}{8} \times 3 + \frac{3}{8} \times 3 + \frac{1}{8} \times 4 \right] \\
&\Rightarrow \left[\frac{25}{8}, \frac{23}{8} \right] \\
&= (3.125, 2.375)
\end{aligned}$$

$$\begin{aligned}
\text{Therefore, } P(\frac{3}{4}) &= (1 - \frac{3}{4})^3P_1 + 3\frac{3}{4}(1 - \frac{3}{4})^2P_2 + 3(\frac{3}{4})^2(1 - \frac{3}{4})P_3 + (\frac{3}{4})^2P_4 \\
&\Rightarrow \frac{1}{64}P_1 + \frac{9}{64}P_2 + \frac{27}{64}P_3 + \frac{27}{64}P_4 \\
&\Rightarrow \frac{1}{64}(1, 1) + \frac{9}{64}(2, 3) + \frac{27}{64}(4, 3) + \frac{27}{64}(6, 4) \\
&\Rightarrow \left[\frac{1}{64} \times 1 + \frac{9}{64} \times 2 + \frac{27}{64} \times 4 + \frac{27}{64} \times 6, \frac{1}{64} \times 1 + \frac{9}{64} \times 3 + \frac{27}{64} \times 3 + \frac{27}{64} \times 4 \right] \\
&\Rightarrow \left[\frac{289}{64}, \frac{217}{64} \right] \\
&= (4.5156, 3.3906)
\end{aligned}$$

$$P_1 = P_3 = (6, 4)$$