Question bank for CIE-I

Module 1:

A: (03 Marks)

1. Let X be a random variable giving the number of heads minus the number of tails in three tosses of a coin. Find the probability distribution, mean and variance.

2. A random variable X has the following probability distribution.

x	-2	-1	0	1	2	3
$f(x)$	0.1	k	0.2	$2k$	0.3	k

Find (i) k; (ii) $F(2)$; (iii)$P(-2 < X < 2)$; (iv) $P(-1 < X \leq 2)$.

3. A shoe keeper in a temple returns the shoes of three friends A, B, C and each receives one pair of shoes randomly. List the sample points of possible orders of the returning shoes to A, B, C, and find probability distribution of the X which denotes the number of persons received correctly, and also find mean.

4. A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good and 3 defective component. A sample of 3 is taken by the inspector. Find the expected value of good component in this sample.

5. A salespersons for a medical company has a two appointments on a given day. At the first appointment, he has 70% chance to make the deal, and if he succeed he can earn 2000 Rs commission. But he has only 40% chance to make the deal with second appointment, for which if successful he can make 3000Rs. If the appointment results are independent, find his expected commission.

6. A random variable X has density function $f(x) = \begin{cases} kx^2, & 0 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$.
Evaluate k , and find (i) $P(x \leq 1)$ (ii) $P(1 \leq x \leq 2)$.

7. Suppose that the error in the reaction, for a controlled laboratory experiment is a continuous random variable having the p.d.f , $f(x) = \begin{cases} \frac{x^2}{3}, & -1 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$.
Find c.d.f. $F(x)$ and use it to evaluate $P\ (0 < X \leq 1)$.

8. Let X be the random variable that denotes the life in hours of a certain electronic device.
If the probability density function is $f(x) = \begin{cases} \frac{20{,}000}{x^3}, & 100 < x \\ 0, & \text{elsewhere} \end{cases}$. Find the average life of the device.

9. Total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is continuous random variable X, and p.d.f. is $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & \text{elsewhere} \end{cases}$.
Find the probability that over a period of one year, a family runs their vacuum cleaner
i) less than 120 hours ii) between 50 to 100 hours.

10. Show that $f(x) = \begin{cases} \frac{1}{2}, & 1 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$ is p.d.f of a continuous random variable X. Find mean and variance.

B: (03 Marks)

1. Derive the expression for mean and variance for the Binomial distribution.

2. Derive the expression for mean and variance for the Poisson distribution.

3. If a car agency sells 30% of its inventory of certain foreign car equipped with side airbags, find the probability distribution of the number of cars with side airbags among the next four cars sold by the agency, and also find mean.

4. The chance that a bomb dropped from an airplane will strike a target is 0.4. 6 bombs are dropped from the airplane. Find the probability that (i) exactly 2 bombs strike the target? (ii) At least 2 strikes the target. (iii) At the most 5 strikes the target.

5. A die is thrown 8 times. Find the probability that 3 or 4 falls,
   (i) Exactly 2 times (ii) at least once (iii) at the most 7 times.

6. The probability that an individual suffers a bad reaction from a certain injection is 0.002. Determine the probability that out of 1000 individuals (i) exactly 3 (ii) more than 2 will suffer a bad reaction.

7. Find the binomial distribution which has mean 2 and variance $\frac{4}{3}$.

8. In sampling a large number of parts are manufactured by a machine, the mean number of defective in a sample of 20 is 2,out of 1000 such samples, how many would be expected to contain at least 3 defective parts ?

9. In 800 families with 5 children each, how many families would be expected to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys and (iv) at most 2 girls by assuming probabilities of births of boys and girls to be equal.

10. A car hire –firm has two cars which it hires out on a day to day basis. The number of demands for a car is known to be Poisson distribution with mean1.5. Find the probability of day on which (i) There is no demand for the car and (ii) The demand is rejected.

C: (04 Marks)

1. The duration of telephone conservation has been found to have an exponential distribution with mean 2 minutes. Find the probabilities that the conservation may last (i) more than 3 minutes, (ii) less than 6 minutes and (iii) between 3 and 9 minutes.
2. In a town, the duration of a rain is exponentially distributed with mean equal to 5 minutes. What is the probability that (i) the rain will last not more than 10 minutes (ii) less than 15 minutes and (iii) between 5 and 10 minutes?
3. Suppose that a system contains one particular component, whose failure time (life) is exponentially distributed with mean 5 years. If 5 of these component is installed in five different system, what is the probability that at the end of 8 years at least two are still functioning?

4. Time taken to serve food in a cafeteria is exponentially distributed with mean 4 minutes. Find the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days.

5. In a test on 1000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2000 hours and standard deviation of 50 hours. Estimate the number of bulbs likely to burn for (a) more than 2100 hours, (b) less than 1950 hours and (c) more than 1950 hours and but less than 2150 hours.

6. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and Standard deviation given that $A(0.5) = 0.19$, and $A(1.4) = 0.42$.

7. Assume that the reduction of a person's oxygen consumption during a period of T.M. is a continuous random variable X normally distributed with mean 37.6 cc/min and S.D. 4.6 cc/min. Determine the probability that during a period of T.M. a person's oxygen consumption will be reduced by (a) at least 44.5 cc/min (b) at most 35.0 cc/min and (c) anywhere from 30.0 to 40.0 cc/min.

8. An analog signal received at a detector (measured in micro volts) may be modeled as a Gaussian random variable N (200, 256) at a fixed point in time. What is the probability that the signal will exceed 240 micro volts? What is the probability that the signal is less than 240 micro volts, given that it is larger than 210 micro volts? (N (200, 256) means $\mu = 200$, $\sigma^2 = 256$.

9. In an examination 7% of the students score less than 35% marks and 89% of the students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed.

10. Steel rods are manufactured to be 3 cms in diameter. But they are acceptable if they are between 2.99 cms and 3.01 cms. It is observed that 5% are rejected as oversized. If the diameters are normally distributed find the standard deviation of the distribution.

Module 2:

A: (03 Marks)

1. Joint distribution of X and Y is given, Find μ_x , μ_y and $E(XY^2)$.

X \ Y	1	3	4
2	0.1	0.2	0.1
4	0.15	0.3	0.15

2. A fair coin is tossed 3 times. Let X denote 0 or 1 according as a head or tail occurs on the first toss. Let Y denote the number of heads which occur. Find the joint distribution and marginal distribution of X and Y . Also find $Cov\,(X,\ Y)$.

3. The joint probability distribution for two random variables X and Y as follows

$Y \backslash X$	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine i) marginal distribution of X and Y ii) $Cov\ (X,\ Y)$ iii) Correlation of X and Y

4. The distribution of two independent variables X and Y are as follows

x	0	1	2
$f(x)$	0.3	0.3	0.4

y	1	2
$g(y)$	0.3	0.7

Determine the joint distribution, verify that $Cov\ (X,\ Y) = 0$.

5. If X and Y are two independent variables, X take the values 1 and 2 with probabilities $0.7,\ 0.3$ and Y take the values $-2, 5, 8$ with probabilities $0.3, 0.5, 0.2$ respectively. Find the joint probability distribution of X and Y and $Cov(x, y)$.

6. The joint distribution of two discrete random variables X and Y is given by $P(x,\ y) = k(2x + y)$ Where x and y are integers such that $0 \leq x \leq 2, \quad 0 \leq y \leq 3$.
Determine i) k ii) marginal distribution of X and Y . iii) Verify that X and Y are dependent.

7. Two pens are selected at random from a box that contains 3 blue pens, 2 red pens and 3 green pens. If X is the number of blue pens selected, and Y is the number of red pens selected,
then find i) Joint probability distribution ii) $P(x + y \leq 1)$.

8. Two card are selected at random from a box contains five cards numbered 1, 1, 2, 2, 3. If X denotes sum of numbers on the card and Y the maximum of two numbers drawn. Find the joint distribution, covariance and coefficient of correlation.

9. Joint distribution of X and Y is given below.

$X \backslash Y$	0	1	2
1	0.08	0.12	0.2
2	0.12	0.18	0.3

Verify that X and Y are independent. Find the probability distribution of $Z = XY$.

10. From 5 boys and 3 girls a committee of 4 is to be formed. Let X be the number of boys, Y be the number of girls in the committee, find the joint distribution and $E(X + Y)$.

B: (03 Marks)

1. Define Stochastic Process and classify the stochastic process with example.
2. Define Stochastic matrix, regular stochastic matrix and Marko-Chain with example.
3. Define Absorbing state, Recurrent state and Transient state of a Marko-chain.
4. Define regular Stochastic matrix with example, prove that $\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$ is not regular.
5. Show that $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is regular and find the unique fixed probability vector.
6. Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$ is regular and find the associated unique fixed probability vector.
7. Show that $P = \begin{bmatrix} 0 & 1 \\ 0.3 & 0.7 \end{bmatrix}$ is regular and find the associated unique fixed probability vector.
8. Define regular stochastic matrix and show that $P = \begin{bmatrix} 0.25 & 0.75 \\ 0 & 1 \end{bmatrix}$ is not regular.
9. Show that $v = (p,\ q)$ is the fixed probability vector of $\begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}$.
10. Find the unique fixed probability vector for $\begin{bmatrix} 0.3 & 0.45 & 0.25 \\ 0.5 & 0.3 & 0.2 \\ 0.8 & 0.1 & 0.1 \end{bmatrix}$.

C: (04 Marks)

1. Every year, a man trades his car for a new car. If he has a Maruti, he trades it for a Ford, If he has a Ford, he trades it for Santro. However, if he has a Santro, he is just as likely to trade it for a new Santro as to trade it for a Maruti or a Ford. In 2014 he bought his first car, which was a Santro.
   Find the probability that he has a i) 2015 Santro, ii) 2016 Maruti , iii) 2017 Ford.

2. There are 2 white marbles in box A and 4 red marbles in box B. At each step of the process a marble is selected from each box and the two marbles selected are interchanged. Let the state a_i of the system is number of i red marbles in box A. a) Find the transition probability matrix. b) What is the probability that there are 2 red marbles in box A after 3 steps? c) In long run what is the probability that there are 2 red marbles in box A?

3. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure to study the next night. If he does not study Monday night what is the probability that he studies on Thursday night. In long run how often does he study?

4. Three boys A, B, C are throwing a ball to each other. A always throws ball to B, B always throws the ball to C, but C is just as likely to throw the ball to B as to A. If A was the first person to throw the ball, find the probability that after Five throws i) A has the ball, ii) B has the ball, iii) C has the ball.

5. Explain Absorbing state, Transient state and Recurrent state of a Marko-chain with examples.

6. Two boys B_1, B_2 and two girls G_1, G_2 throwing ball from one to another. Each boy throws the ball to other boy with probability $\frac{1}{2}$, and to each girl with probability $\frac{1}{4}$. On the other hand each girl throws the ball to each boy

with probability $\frac{1}{2}$, and never to the other girl. If ball is with G_1 find the probability that after three throws each receives the ball.

7. A player has Rs. 300. At each play of a game, he losses Rs. 100 with probability $\frac{3}{4}$, but wins Rs. 200 with probability $\frac{1}{4}$. He stops playing if he lost his Rs 300 or he has won Rs. 300. Find transition probability matrix. Identify the absorbing states.

8. Find the unique fixed probability vector of $P = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 \\ 0.5 & 0.25 & 0.25 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix}$.

9. A salesman's territory consists of 3 cities A, B and C. he never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However if he sells in either B or C, then the next day he is twice as likely to sell in city A as in other city. In long run, how often does he sell in each of the cities.

10. A player has Rs. 200. He bets Rs. 100 at a time and wins s. 100 with probability $\frac{1}{2}$. He stops playing if he loses the Rs. 200 or wins Rs. 400. Find the probability that the game lasts more than 4 plays.

Module 3:

A: (02 Marks)

1. Explain the terms, i) Null Hypothesis ii) Significant level.
2. Explain the terms; i) Sampling ii) Parameters.
3. Explain the terms; i) Statistic ii) Standard error.
4. Explain the terms; i) Statistical hypothesis ii) Critical region.
5. Explain the terms; i) Type I error ii) Type II error.
6. Explain the terms; i) Critical region ii) Level of significance.
7. Explain the terms; i) Parameters ii) Type I error.
8. Explain the terms; i) Statistic ii) Type II error.

B: (03 Marks)

1. A coin was tossed 400 times and the head turned up 222 times. Test the hypothesis that the coin is unbiased at 1% and 5% level of significance.
2. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3110 times. On the assumption of random throwing, do the data indicate an unbiased die?
3. In a locality containing 18000 families, a sample of 800 families was selected at random. Of these 800 Families, 200 families were found to have a monthly income of Rs 3000 or less. It is desired to estimate how many out of 18,000 families have a monthly income of Rs 3000 or less. Whiten what limits would you place your estimate in 1% level of significance?
4. 12 dice are thrown 3086 times and a throw of 2, 3, 4 is reckoned as a success. Suppose that 19142 throws of 2, 3, 4 have been made out. Do you think that this observed value deviates from the expected value? If so, can the deviation from the expected value be due to fluctuations of simple sampling?
5. In a group of 50 first cousins there were found to be 27 males and 23 females. Ascertain if the observed

proportions are inconsistent with the hypothesis that the sexes should be in equal proportion.

6. A random sample of 500 apples was taken from a large consignment and 65 were found to be bad. Estimate the proportion of the bad apples in the consignment and standard error of the estimate. Deduce that the percentage of bad apples in the consignment is between 8.5 and 17.5 .
7. 400 children are chosen in an industrial town and 150 are found to be underweight. Assuming the conditions of simple sampling, estimate the percentage of children who are underweight in the industrial town and assign limits within which the percentage probably lies.
8. In a sample of 500 people from a state 280 take tea, and rest take coffee. Can we assume that tea and coffee are equally popular in the state at 5% level of significance?

Question paper pattern

Part1 (Module 1)-10 Marks		
1 a. (3 Marks) b. (3 Marks) c. (4 Marks)	Or	2 a. (3 Marks) b. (3 Marks) c. (4 Marks)
Part2 (Module 2)-10 Marks		
3 a. (3 Marks) b. (3 Marks) c. (4 Marks)	Or	4 a. (3 Marks) b. (3 Marks) c. (4 Marks)
Part3 (Module 3)-05 Marks		
5 a. (2 Marks) b. (3 Marks)	Or	6 a. (2 Marks) b. (3 Marks)