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Course Code: **BCS301**

DEPARTMENT OF SCIENCE & HUMANITIES, CANARA ENGINEERING COLLEGE

Programme: **B.E.** Semester: **III** **CIE: TEST - 1** Date: **08-01-2024**

Course Title: **MATHEMATICS FOR CSE**

Duration: **1½ Hour** Max. Marks: **25**

Note: *Answer one full question from each module.*

MODULE-I			**Marks**	**CO**	**RBTL**
1	(a)	A lot containing 8 components is sampled by a quality inspector; the lot contains 5 good and 3 defective component. A sample of 3 is taken by the inspector. Find the expected value of good component in this sample.	3M	1	3
	(b)	Find the binomial distribution which has mean 3 and variance $\frac{6}{5}$.	3M	1	2
	(c)	In a normal distribution, 69% of the items are over 45 and 92% are under 64. Find the mean and Standard deviation, given that $A(0.5) = 0.19$, and $A(1.4) = 0.42$.	4M	1	3
OR					
2	(a)	Derive the expression for mean and variance for the Poisson distribution.	3M	1	2
	(b)	Let X be a random variable giving the number of heads minus the number of tails in three tosses of a coin. Find the probability distribution, mean and variance.	3M	1	3
	(c)	Suppose that a system contains one particular component, whose life time is exponentially distributed with mean 5 years. If 5 of these components are installed in five different system, what is the probability that at the end of 8 years at least one is still functioning?	4M	1	3
MODULE-II					
3	(a)	If X and Y are two independent variables, X take the values 1 and 2 with probabilities 0.6, 0.4 , and Y take the values $-2, 5, 8$ with probabilities $0.3, 0.3, 0.4$ respectively. Find the joint probability distribution of X and Y and $Cov(x, y)$.	3M	2	3
	(b)	Define Stochastic Process and classify the stochastic process with example.	3M	2	2
	(c)	A salesman's territory consists of 3 cities A, B and C. he never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However if he sells in either B or C, then the next day he is twice as likely to sell in city A as in other city. In long run, how often does he sell in each of the cities?	4M	2	3
OR					
4	(a)	From 5 boys and 3 girls a committee of 4 is to be formed. Let X be the number of boys, Y be the number of girls in the committee, find the joint distribution and $E(X+Y)$.	3M	2	3
	(b)	Define Stochastic matrix, regular stochastic matrix and Marko-Chain with example.	3M	2	2
	(c)	Three boys A, B, C are throwing a ball to each other. A always throws ball to B, B always throws the ball to C, but C is just as likely to throw the ball to B as to A.	4M	2	3

		If B was the first person to throw the ball, find the probability that after Four throws i) A has the ball, ii) B has the ball, iii) C has the ball.			
MODULE-III					
5	(a)	Explain the terms; i) Statistic ii) Type II error.	2M	3	1
	(b)	In a sample of 500 people from a state 265 take tea, and rest take coffee. Can we assume that tea and coffee are equally popular in the state at 5% level of significance?	3M	3	3
OR					
6	(a)	Explain the terms; i) Parameters ii) Type I error.	2M	3	1
	(b)	A die was thrown 9000 times and a throw of 5 or 6 was obtained 3100 times. On the assumption of random throwing, do the data indicate an unbiased die?	3M	3	3

Signature of Course Instructor/Paper Setter with Date	**Signature of Course Coordinator with Date**
Questions, CO, RBTL, Coverage and difficulty level is appropriate. **APPROVED**	
Signature of Senior Faculty/Expert with Date	**Signature of Senior Faculty/Expert with Date**
HEAD OF THE DEPARTMENT/CHAIRMAN – MODERATION COMMITTEE	

Course Code: **BCS301**

DEPARTMENT OF SCIENCE & HUMANITIES, CANARA ENGINEERING COLLEGE

Programme: **B.E.** Semester: **III** **CIE: TEST - I** Date: **08-01-2024**

Course Title: **MATHEMATICS FOR CSE**

Duration: **1½ Hour** **SCHEME OF VALUATION** Max. Marks: **25**

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1. a) A lot containing 8 components is sampled by a quality inspector; the lot contains 5 good and 3 defective component. A sample of 3 is taken by the inspector. Find the expected value of good component in this sample.

Solution: Let x denote the number of good component in the sample of 3.

Number of selection of 3 from 8 is ${}^{8}C_3 = 56.$

In the sample of 3, x may be 0, 1, 2 or 3 (number of good components)

Number of samples with $x = 0$ is ${}^{5}C_0 \times {}^{3}C_3 = 1$, with $x = 1$, is ${}^{5}C_1 \times {}^{3}C_2 = 15$, $with\, x = 2$, is ${}^{5}C_2 \times {}^{3}C_1 = 30$ and with $x = 3$, is ${}^{5}C_3 \times {}^{3}C_0 = 10.$ 1

Probability distribution is 1

x	0	1	2	3
$f(x)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

Expected value of good component= $E(X) = \sum x f(x) = 1.875.$ 1

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1. b) Find the binomial distribution which has mean 3 and variance $\frac{6}{5}$.

Solution: Given that, $\mu = np = 3, \quad and \;\; V = npq = \frac{6}{5}.$

$$q = \frac{V}{\mu} = \frac{2}{5} \quad \Longrightarrow p = \frac{3}{5} \quad \text{and} \quad n = 5\,.$$ 1

$$f(x) = \binom{n}{x} p^x q^{n-x} = \binom{5}{x} \frac{3^x \times 2^{5-x}}{5^5}\,.$$ 1

Distribution

x	0	1	2	3	4	5
$f(x)$	0.0102	0.0768	0.2304	0.3456	0.2592	0.0778

1

1.c) In a normal distribution, 69% of the items are over 45 and 92% are under 64. Find the mean and Standard deviation, given that $A(0.5) = 0.19$, and $A(1.4) = 0.42$.

Solution: Given that $P(x > 45) = 0.69$ and $P(x < 64) = 0.92$

$$\Longrightarrow \quad P\left(z > \frac{45-\mu}{\sigma}\right) = 0.69 \quad \text{and} \quad P\left(z < \frac{64-\mu}{\sigma}\right) = 0.92$$ 1

$$\Longrightarrow \quad 0.5 + A\left(\frac{\mu-45}{\sigma}\right) = 0.69 \quad \text{and} \quad 0.5 + A\left(\frac{64-\mu}{\sigma}\right) = 0.92$$ 1

$$\Longrightarrow \quad A\left(\frac{\mu-45}{\sigma}\right) = 0.19 \quad \text{and} \quad A\left(\frac{64-\mu}{\sigma}\right) = 0.42$$

$$\therefore \quad \frac{\mu-45}{\sigma} = 0.5 \quad and \quad \frac{64-\mu}{\sigma} = 1.4$$ 1

On solving we get, $\mu = 50$, and $\sigma = 10.$ 1

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2. a) Derive the expression for mean and variance for the Poisson distribution.

Mean$= \mu = \sum_{x \in X} x f(x) = \sum_{x=0}^{\infty} x \frac{m^x e^{-m}}{x!}$

$$= me^{-m}\sum_{x=1}^{\infty}\frac{m^{x-1}}{(x-1)!}$$

$$= me^{-m}\left\{1+m+\frac{m^2}{2!}+\cdots\cdots\right\} = me^{-m}e^{m} = m.$$ 1

Variance $= V = \sum_{x\in X} x^2 f(x) - \mu^2 = \sum_{x\in X}[x(x-1)+x]f(x) - \mu^2$

$$= \sum_{x\in X}[x(x-1)]f(x) + \mu - \mu^2$$

$$= \sum_{x=0}^{\infty}[x(x-1)]\frac{m^x e^{-m}}{x!} + m - m^2$$

$$= m^2 e^{-m}\sum_{x=2}^{\infty}\frac{m^{x-2}}{(x-2)!} + m - m^2$$

$$= m^2 e^{-m}\left\{1+m+\frac{m^2}{2!}+\cdots\cdots\right\} + m - m^2 = m^2 e^{-m}e^{m} + m - m^2 = m.$$ 2

2. b) Let X be a random variable giving the number of heads minus the number of tails in three tosses of a coin. Find the probability distribution, mean and variance.

Solution: Probability distribution is

x	-3	-1	1	3
$f(x)$	0.125	0.375	0.375	0.125

1

Mean$= \mu = \sum_{x\in X} xf(x) = 0,$ 1

Variance $= V = \sum_{x\in X} x^2 f(x) - \mu^2 = 3$ 1

2. c) Suppose that a system contains one particular component, whose life time is exponentially distributed with mean 5 years. If 5 of these components are installed in five different system, what is the probability that at the end of 8 years at least one is still functioning?

Solution: Let x be the life of the component in years. Given that **Mean**$= \mu = \frac{1}{\alpha} = 5.$

Probability density function is $f(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}}, & for\ \ x \geq 0 \\ 0, & for\ \ x < 0 \end{cases}$. 1

Probability that life of a component is 8 years or more $P(X \geq 8) = \int_8^{\infty}\frac{1}{5}e^{-\frac{x}{5}}dx = -e^{-\frac{x}{5}}\Big|_8^{\infty} = 0.2019.$ 1

Let t be the number of components still functioning at the end of 8 years.

Then $p = 0.2019,\ q = 0.7981,\quad f(x) = \binom{5}{t}0.2019^t 0.7981^{5-t}.$ 1

probability that at the end of 8 years at least one is still functioning

$$= P(x \geq 1) = 1 - f(0) = 1 - 0.3238 = 0.6762\ .$$ 1

3. a) If X and Y are two independent variables, X take the values 1 and 2 with probabilities 0.6, 0.4 , and Y take the values $-2, 5, 8$ with probabilities $0.3, 0.3, 0.4$ respectively. Find the joint probability distribution of X and Y and $Cov(x, y)$.

Solution: If X and Y are independent then $P(x,\ y) = f(x)g(y)$ for all $x,\ y$.

Joint distribution is

X \ Y	-2	5	8
1	0.18	0.18	0.24
2	0.12	0.12	0.16

1

M $E(x) = E(X) = \sum xf(x) = 1.4,\ \ E(Y) = \sum yg(y) = 4.1,$ 1

$E(XY) = \sum_x\sum_y xyP(x,\ y) = 5.74$

$$Cov\ (X,\ Y) = E(XY) - E(X)E(Y) = 0.$$ 1

3. b) Define Stochastic Process and classify the stochastic process with example. Stochastic Process: The process $\{t,\ x_t\}$ is called stochastic Process. Where t is the parameter and x_t are the values of the random variables called states. 1. Discrete state discrete parameter process. Example: Number of telephone calls in different days in a telephone booth. 2. Discrete state continuous parameter process. Example: Number of telephone calls in different time intervals in a telephone booth. 3. Continuous state discrete parameter process. Example: Average duration of a telephone calls in different days in a telephone booth. 4. Continuous state continuous parameter process. Example: Average duration of telephone calls in different time intervals in a telephone booth.	1 1 1
3. c) A salesman's territory consists of 3 cities A, B and C. he never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However if he sells in either B or C, then the next day he is twice as likely to sell in city A as in other city. In long run, how often does he sell in each of the cities? Solution: Transition probability matrix is $P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} \end{array}$. Let $v = (x,\ y,\ z)$ be the fixed probability vector, then $vP = v$, that is $(x,\ y,\ z)\begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} = (x,\ y,\ z)$ $\begin{matrix} x+y+z=1 \\ \Rightarrow \frac{2y}{3}+\frac{2z}{3}=x \\ x+\frac{z}{3}=y \end{matrix} \quad \Rightarrow \begin{matrix} x+y+z=1 \\ -x+\frac{2y}{3}+\frac{2z}{3}=0 \\ x-y+\frac{z}{3}=0 \end{matrix} \Rightarrow \begin{matrix} x=0.40 \\ y=0.45\ . \\ z=0.15 \end{matrix}$ In long run, he sell in the cities A, B, C with probability 0.4, 0.45, and 0.15 respectively.	2 1 1
4. a) From 5 boys and 3 girls a committee of 4 is to be formed. Let X be the number of boys, Y be the number of girls in the committee, find the joint distribution and $E(X+Y)$. Solution: Possible number of committees is $^8C_4 = 70$. Among these 70, $^5C_4$ committees may contains 4 boys and 0 girl, $^5C_3 \times\ ^3C_1$ committees may contains 3 boys and 1 girl , $^5C_2 \times\ ^3C_2$ committees may contains 2boys and 2 girls or $^5C_1 \times\ ^3C_3$ committees may contains 1 boy and 3 girls. Therefore joint distribution is	2

X \ Y	0	1	2	3
1	0	0	0	$\frac{1}{14}$
2	0	0	$\frac{3}{7}$	0
3	0	$\frac{3}{7}$	0	0
4	$\frac{1}{14}$	0	0	0

$E(X+Y) = \sum_x \sum_y (x+y)P(x,\ y) = 4$. 1

4. b) Define Stochastic matrix, regular stochastic matrix and Marko-Chain with example.

Stochastic matrices: A square matrix P is said to be stochastic matrices if each row of P is a probability vector.

Examples: $\begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0 & 1 & 0 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$, $\begin{bmatrix} 0.3 & 0.7 \\ 1 & 0 \end{bmatrix}$. 1

Regular stochastic matrices: A stochastic matrices P is said to be regular if all the entries of P^k are nonzero for some positive integer k.

$P = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$ is regular. 1

Marko-chain: Marko-chain is a discrete state discrete parameter process in which state space is finite and the probability of any state depends at the most predecessor state.

Example: Three boys A, B, C are throwing a ball to each other. A always throws ball to B, B always throws the ball to C, but C is just as likely to throw the ball to B as to A. 1

4. c) Three boys A, B, C are throwing a ball to each other. A always throws ball to B, B always throws the ball to C, but C is just as likely to throw the ball to B as to A. If B was the first person to throw the ball, find the probability that after Four throws i) A has the ball, ii) B has the ball, iii) C has the ball.

Let a, b, c indicate the states ball is with A, B or C respectively.

Transition probability matrix is $P = \begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$. 1

Since B was the first person to throw the ball, $v_0 = (0,\ 1,\ 0)$

Then, after the first throw the probability vector is $v_1 = v_0 P = (0,\ 0,\ 1)$ 1

After the second throw the probability vector is $v_2 = v_1 P == (1/2\,,\ 1/2\,,\ 0)$,

After the third throw the probability vector is $v_3 = v_2 P = (0,\ 1/2\,,\ 1/2)$ 1

After the 4th throw the probability vector is $v_4 = v_3 P = (1/4\,,\ 1/4\,,\ 1/2)$.

Therefore after the 4th throw probabilities that A has the ball, B has the ball, and C has the ball are respectively $1/4\,,\ 1/4\,,\ 1/2$. 1

5. a) Explain the terms; i) Statistic ii) Type II error.

Statistic: The statistical constants for the sample drawn from the given population such as mean $(\overline{x})$, standard deviation (S) etc. are called the **Statistic**. 1

Type II error: If a hypothesis is accepted while it should have been rejected, then we say that type II error has been made. 1

5. b) In a sample of 500 people from a state 265 take tea, and rest take coffee. Can we assume that tea and coffee are equally popular in the state at 5% level of significance?

Solution: Assume that tea and coffee are equally popular in the state.

Then $p = 0.5,\ q = 0.5$. 1

Given that, $n = 500,\ x = 265 \quad \mu = np = 250$.

$\sigma = \sqrt{npq} = 11.18$.

$z = \frac{x-\mu}{\sigma} = \frac{15}{11.18} = 1.3416 < 1.96$. 1

Since the difference is not significance at 5 level. Hypothesis is accepted.	1
6. a) Explain the terms; i) Parameters ii) Type I error. **Parameters**: The statistical constants of the population such as mean (μ), standard deviation (σ) etc. are called the **parameters**. **Type I error:** If a hypothesis is rejected while it should have been accepted, then we say that type I error has been committed.	 1 1
6. b) A die was thrown 9000 times and a throw of 5 or 6 was obtained 3100 times. On the assumption of random throwing, do the data indicate an unbiased die? Solution: Suppose the die is unbiased. Then the probability of throwing 5 or 6 in each throw is $p = \frac{1}{3}$. Therefore expected number of successes is $\mu = np = \frac{9000}{3} = 3000.$ And the observed value of successes is $x = 3100.$ Since $\mu = 3000,\quad \sigma = \sqrt{npq} = \sqrt{2000} = 44.7214.\quad z = \frac{x-\mu}{\sigma} = \frac{100}{44.7214} = 2.24.$ Since $1.96 < z < 2.58$, difference is significant at 5% level of significance but not significant at 1% level. Hypothesis is rejected at 5% level and accepted at 1% level.	 1 1 1

Signature of Course Instructor/Paper Setter with Date	**Signature of Course Coordinator with Date**
Distribution of marks and contents/answer keys of scheme of evaluation is adequate for fair evaluation/assessment. **APPROVED**	
Signature of Senior Faculty/Expert with Date	**Signature of Senior Faculty/Expert with Date**
HEAD OF THE DEPARTMENT/CHAIRMAN – MODERATION COMMITTEE	