EE 604
Digital Image Processing

IIT KANPUR
Indian Institute of Technology, Kanpur

Tanaya Guha
Aug - Nov 2017

Lecture outline

- Image denoising
    - Bilateral filtering
    - Non-local means
    - The diffusion perspective

Definitions

$\mathcal{S}$ = spatial domain (set of pixel locations)

$\mathcal{R}$ = range domain (set of all possible pixel values)

$\mathbf{p} = [\mathrm{p}_x, \mathrm{p}_y]$ 2D location of pixel p

$v(\mathbf{p})$ = intensity at pixel p

$$\underset{\substack{\Big\downarrow \\ \text{observed}}}{v(\mathbf{p})} = \underset{\substack{\Big\downarrow \\ \text{original}}}{u(\mathbf{p})} + \underset{\substack{\Big\downarrow \\ \text{noise}}}{n(\mathbf{p})}$$

Gaussian filtering

$$\hat{u}_G(\mathbf{p}) = \sum_{\mathbf{q}\in\mathcal{S}} \underbrace{\boxed{w(\mathbf{p},\mathbf{q})}}_{\text{weights}} v(\mathbf{q})$$

where $$w(\mathbf{p},\mathbf{q}) = G_\sigma(\|\mathbf{p}-\mathbf{q}\|)$$

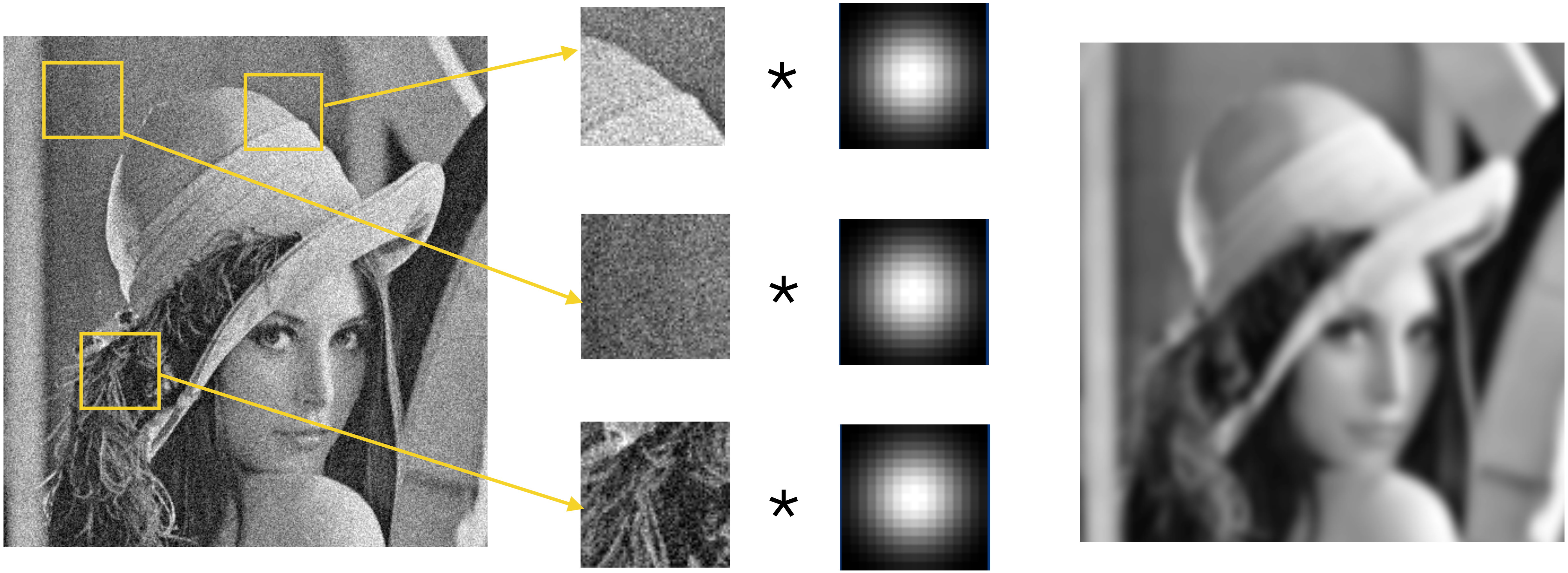
$$G_\sigma(x) = \frac{1}{2\pi\sigma^2}\mathrm{e}^{-\frac{x^2}{2\sigma^2}}$$

- Clearly, weights do not depend on image content.
- Only location information matters.

Issues with Gaussian filtering

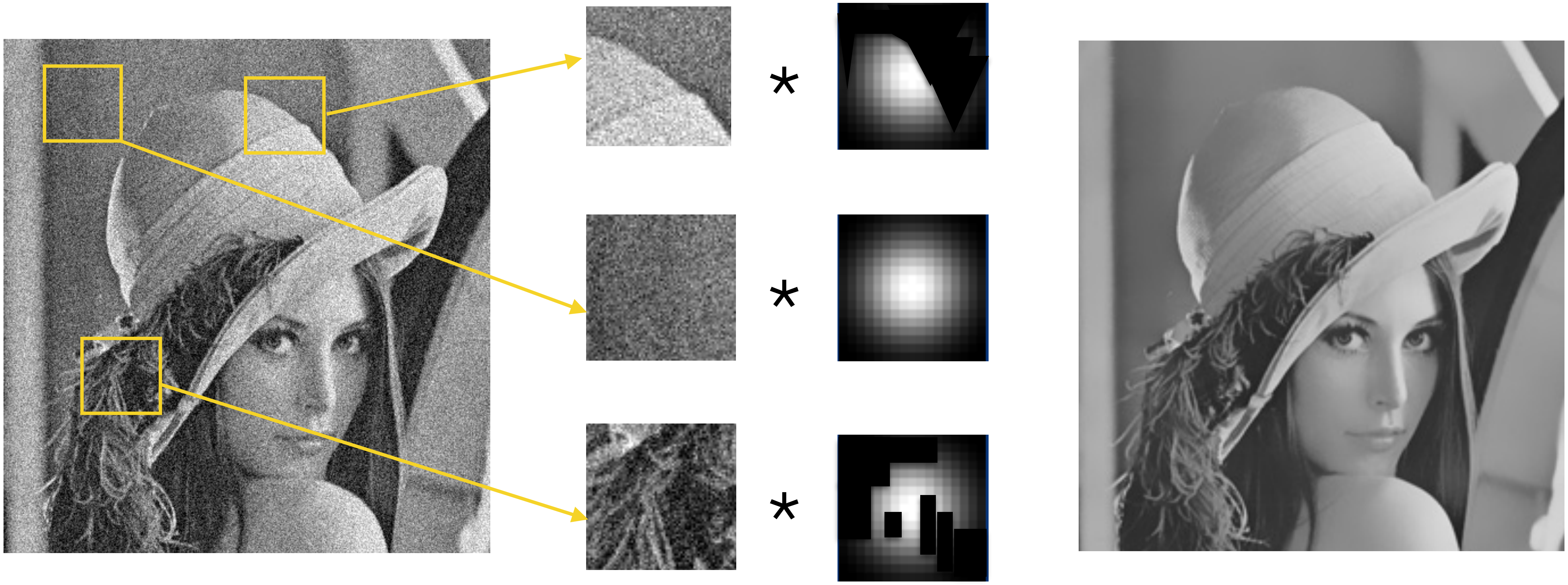
- Loss of edge information (high freq component)

- Same kernel everywhere



How to preserve edges?

- No blurring across edges
- Kernel should change with image content



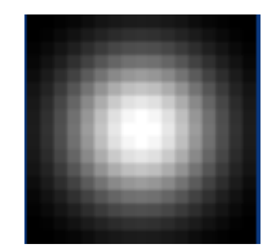
Lecture outline

- Image denoising
  - Bilateral filtering
  - Non-local means
- The diffusion perspective

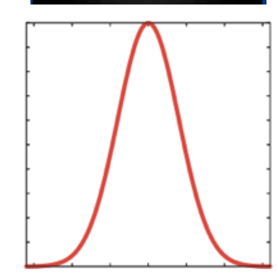
Bilateral filtering

$$\hat{u}_{BL}(\mathbf{p}) = \frac{1}{C}\sum_{\mathbf{q}\in S} \underbrace{w_s(\mathbf{p},\mathbf{q})}_{\text{space kernel}}\,\underbrace{w_r(v(\mathbf{p}),v(\mathbf{q}))}_{\text{range kernel}}\,v(\mathbf{q})$$

where, $w_s(\mathbf{p},\mathbf{q}) = G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)$



$$w_r(\mathbf{p},\mathbf{q}) = G_{\sigma_r}(|v(p)-v(q)|)$$



$$C = \sum_{\mathbf{q}\in \mathcal{S}} w_s(\mathbf{p},\mathbf{q})w_r(v(\mathbf{p}),v(\mathbf{q}))$$

normalizing factor

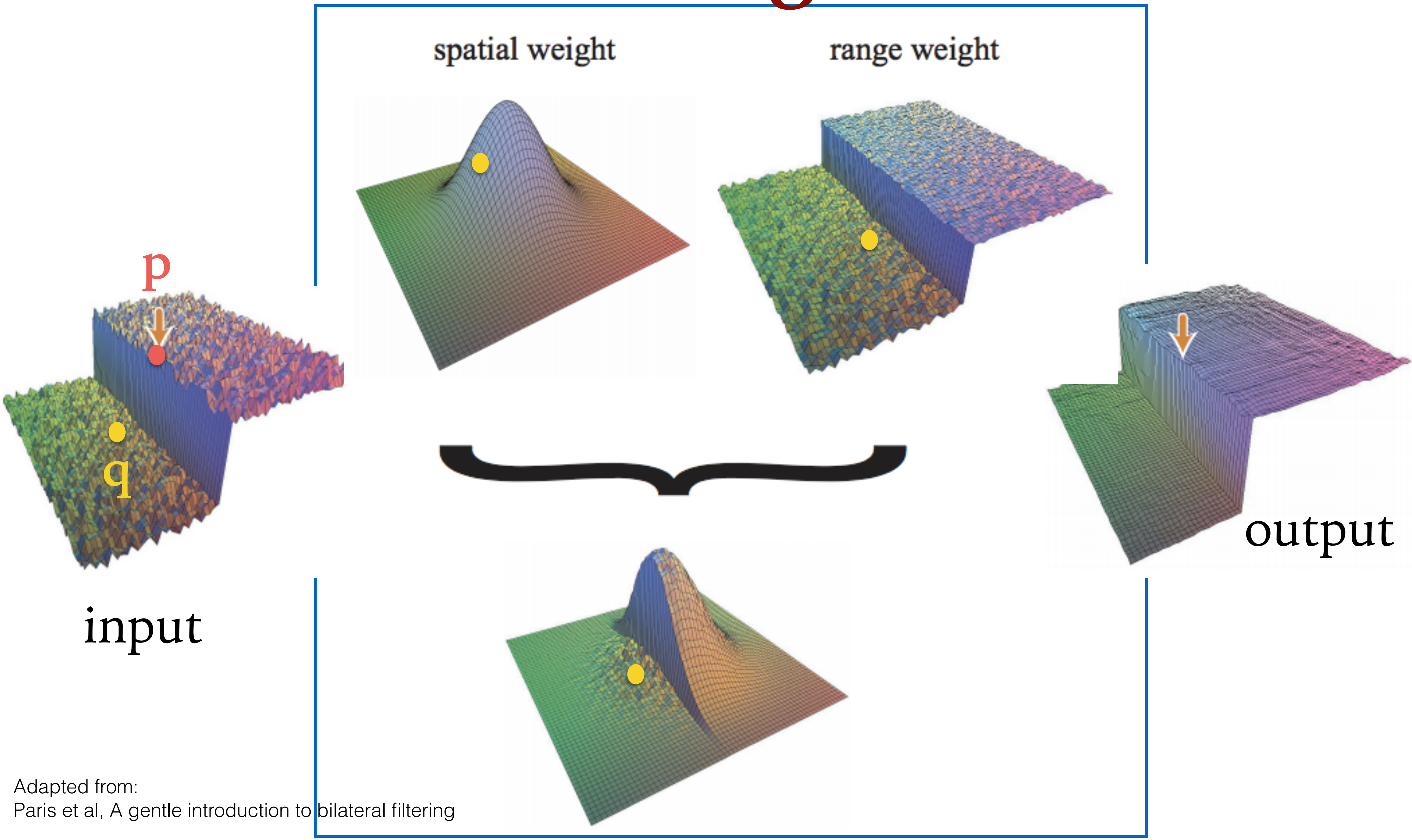
Bilateral filtering

Bilateral filter is controlled by two parameters:

- σ_s : spatial parameter, spatial extent of the kernel
- σ_r : range parameter - preserve edges.

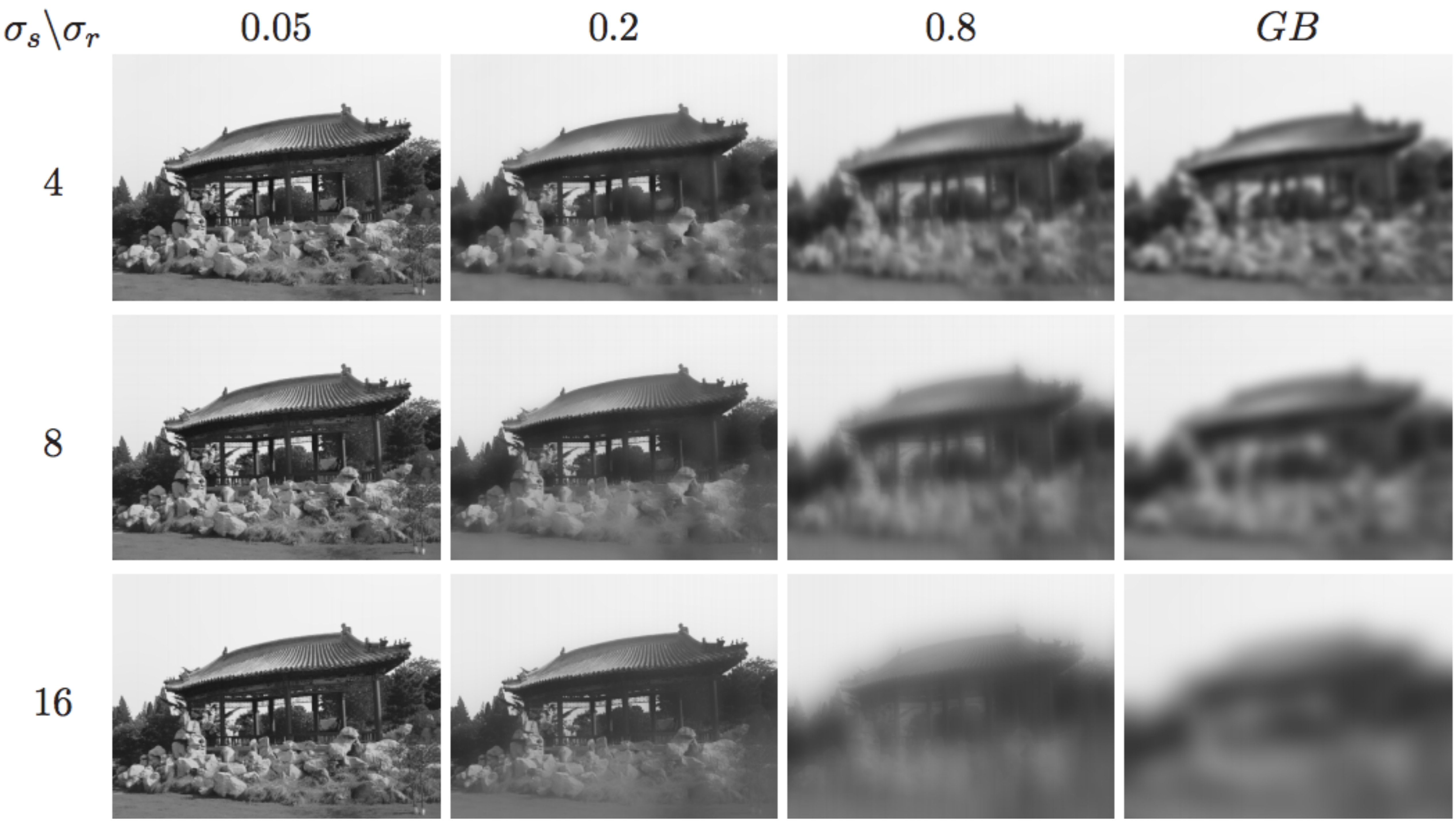
- If $\sigma_r \rightarrow \infty$?
  - It becomes a constant, so no edge preservation, similar to Gaussian filtering

Bilateral filtering



Adapted from:
Paris et al, A gentle introduction to bilateral filtering

Bilateral filtering



Bilateral filtering

How to set the values?

- σ_s : 2% of image diagonal
- σ_r : mean or median of image gradients

- Only pixels close in space and intensity will be considered

Lecture outline

- Image denoising
    - Bilateral filtering
    - Non-local means
    - The diffusion perspective

Non-local means

- A local neighborhood is specified by spatial position *only*
- A non-local neighborhood is specified by the image pixels (or patches) of similar grayscale values
- Non-local techniques rely on **redundancy** in natural images
- Reference:

  Buades et al., "A non-local algorithm for image denoising", CVPR 2005

Main idea - NL means

- Assumes non-local neighborhood based on grayscale similarity
- Weighted average of pixels with similar neighborhood
- Weights depend on the similarity between the neighborhoods
- Preserves edges better

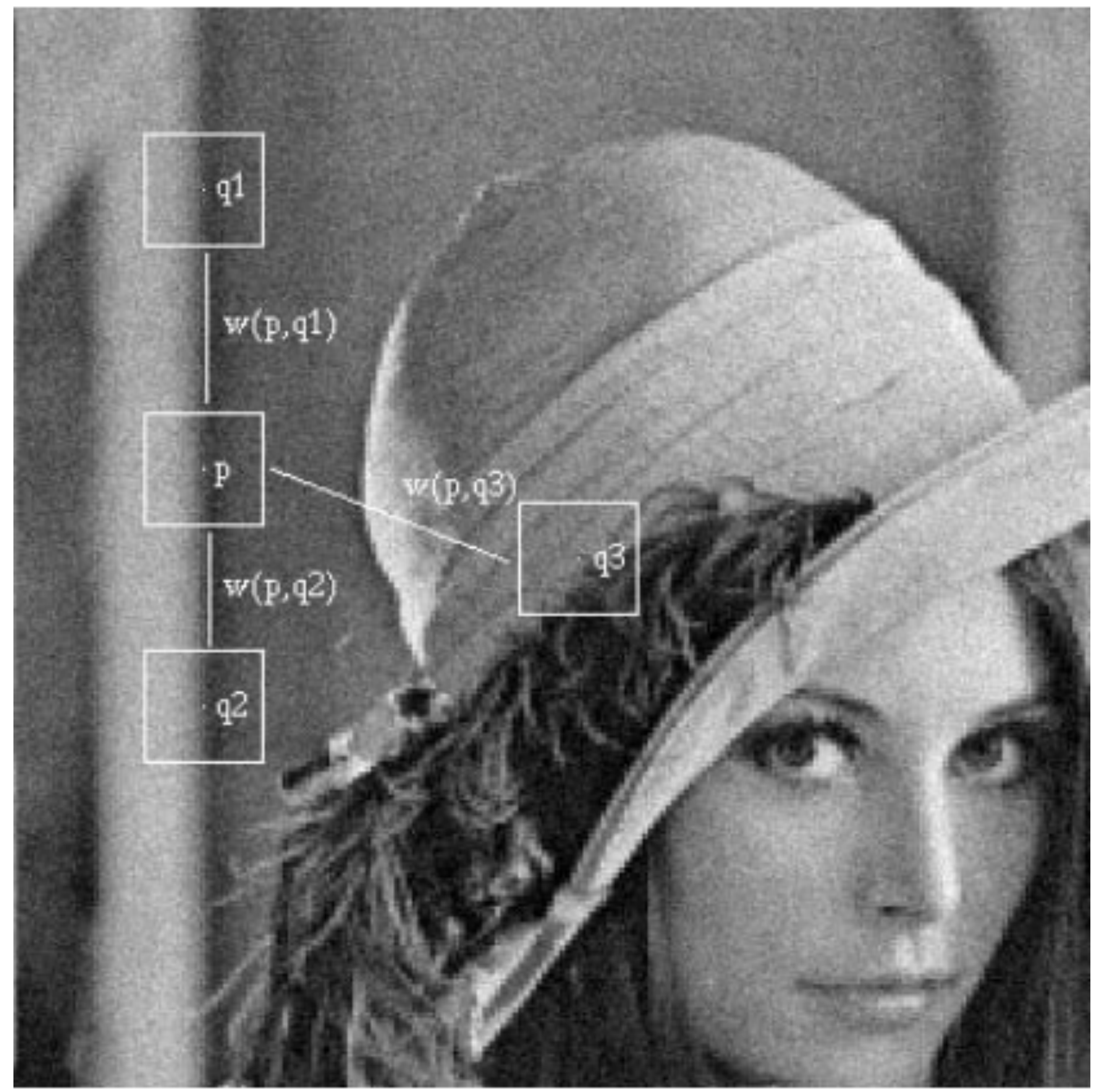


image source: Buades et al cvpr 2005

Non-local means

Consider a noisy image $\mathcal{I}$

Intensity at pixel $\mathbf{p}$ is $v(\mathbf{p}) = u(\mathbf{p}) + n(\mathbf{p})$

The NL-denoised estimate

$$\hat{u}_{NL}(\mathbf{p}) = \sum_{\mathbf{q} \in \mathcal{I}} w(\mathbf{p}, \mathbf{q}) v(\mathbf{q})$$

where $0 \leq w(\mathbf{p}, \mathbf{q}) \leq 1$ and $\sum_{\mathbf{q} \in \mathcal{I}} w(\mathbf{p}, \mathbf{q}) = 1$

$w(\mathbf{p}, \mathbf{q})$ depends on the similarity between $\mathbf{p}$ and $\mathbf{q}$

Non-local means

Define neighborhoods $\mathcal{N}(\mathbf{p})$ and $\mathcal{N}(\mathbf{q})$

distance between $\mathcal{N}(\mathbf{p})$ and $\mathcal{N}(\mathbf{q})$: $d(\mathcal{N}(\mathbf{p}), \mathcal{N}(\mathbf{q}))$

$d(\mathcal{N}(\mathbf{p}), \mathcal{N}(\mathbf{q})) \rightarrow$ Euclidean or <u>Weighted Euclidean</u>

$d(\mathcal{N}(\mathbf{p}), \mathcal{N}(\mathbf{q})) = \|v(\mathcal{N}(\mathbf{p})) - v(\mathcal{N}(\mathbf{q}))\|_{2,a}$

$$w(\mathbf{p}, \mathbf{q}) = \frac{1}{C(\mathbf{p})} e^{-\frac{d(\mathcal{N}(\mathbf{p}), \mathcal{N}(\mathbf{q}))^2}{h^2}}$$

where C is a normalizing constant, and h controls the decay of weight.

Non-local means

NL Means weights

$$w(\mathbf{p},\mathbf{q}) = \frac{1}{C(\mathbf{p})} e^{-\frac{d(\mathcal{N}(\mathbf{p}),\mathcal{N}(\mathbf{q}))^2}{h^2}}$$

$$= \frac{1}{C(\mathbf{p})} e^{-\frac{\|v(\mathcal{N}(\mathbf{p}))-v(\mathcal{N}(\mathbf{q}))\|_{2,a}^2}{h^2}}$$

Neighborhood filtering weights

$$w(\mathbf{p},\mathbf{q}) = \frac{1}{C(\mathbf{p})} e^{-\frac{|v(\mathbf{p})-v(\mathbf{q})|^2}{h^2}}$$

NL-means filter weights

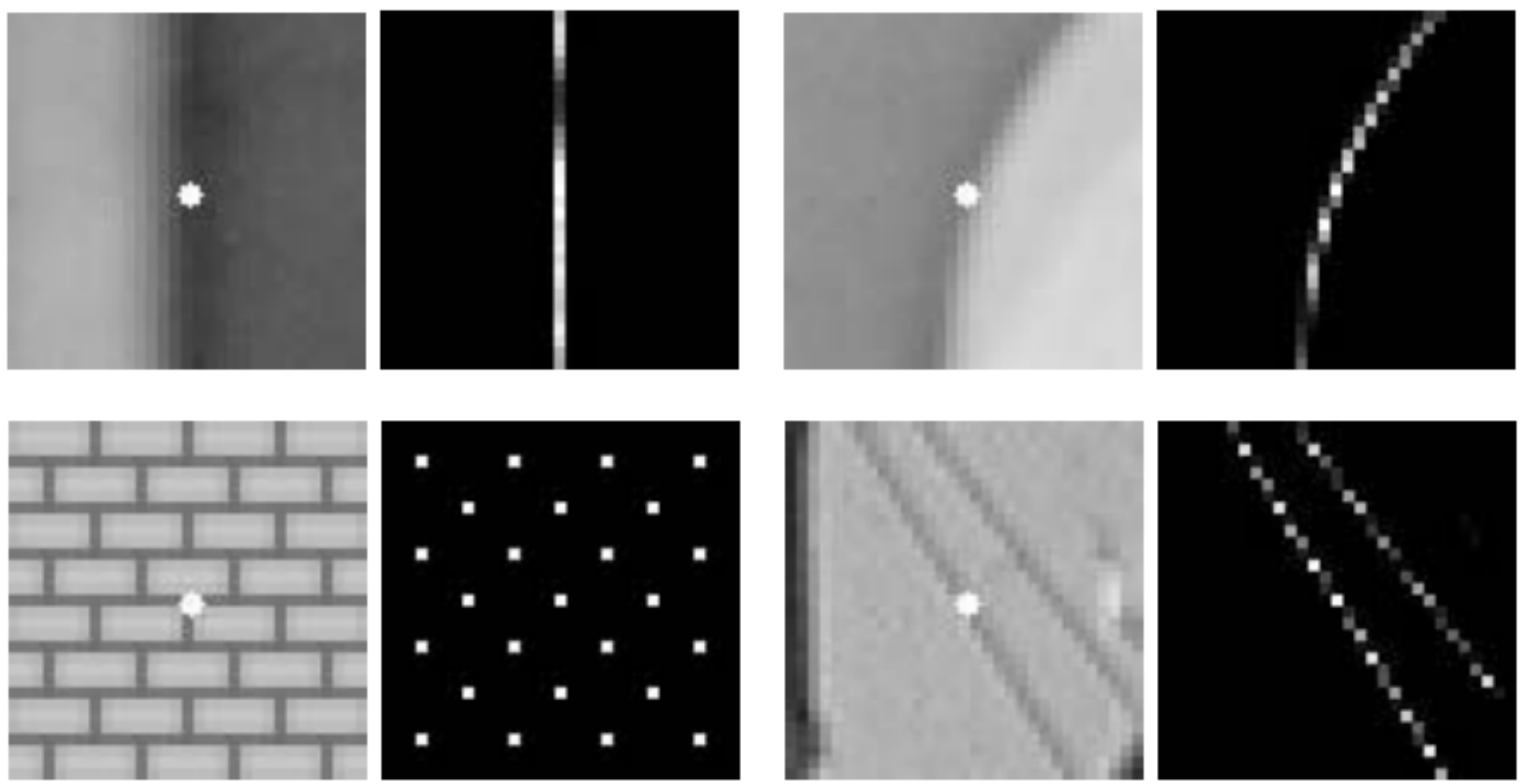


image source: Buades et al cvpr 2005

Denoising results

image source: Buades et al cvpr 2005

Method noise

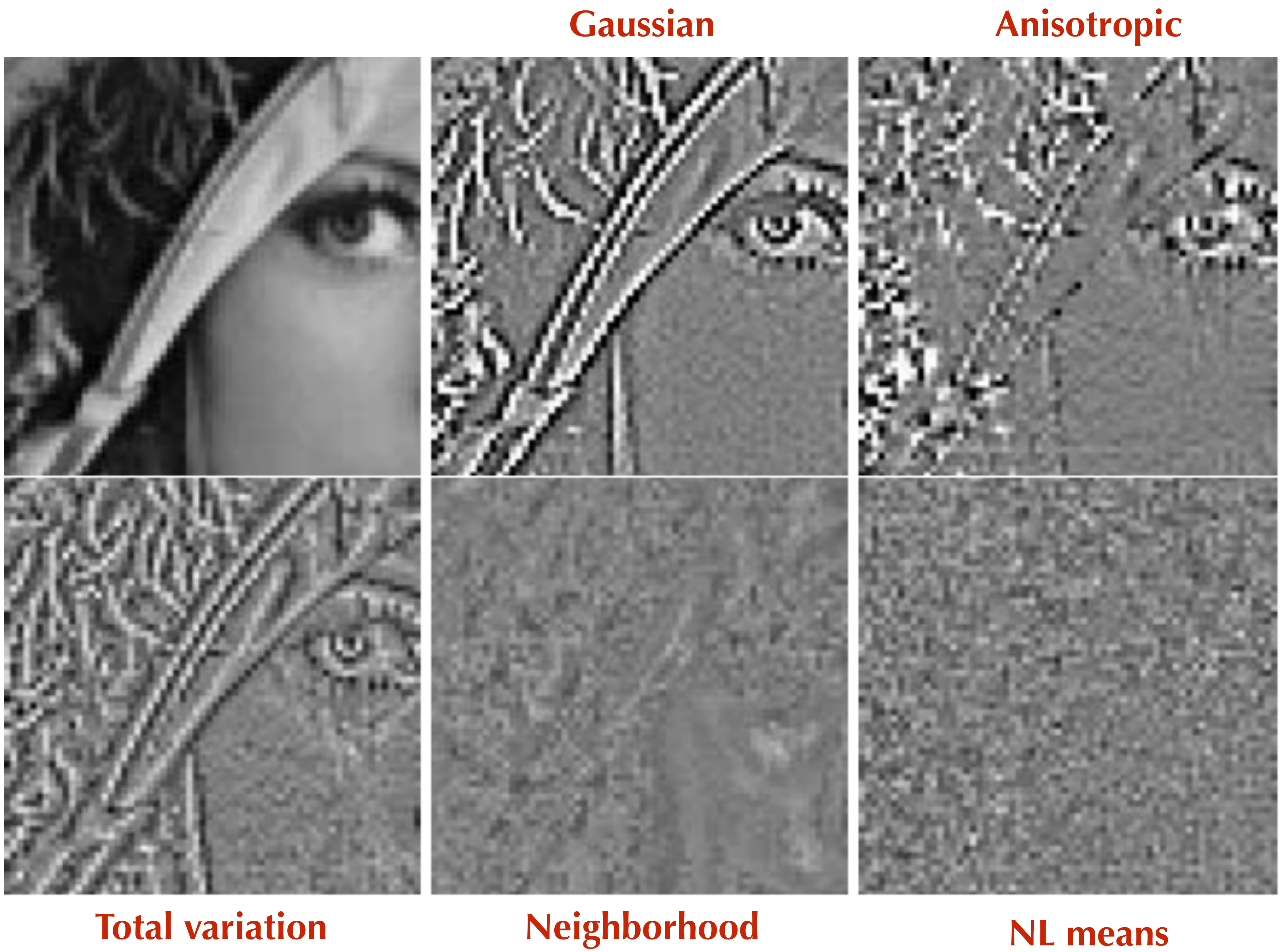


image source: Buades et al cvpr 2005

Lecture outline

- Image denoising
  - Bilateral filtering
  - Non-local means
  - The diffusion perspective

Diffusion process

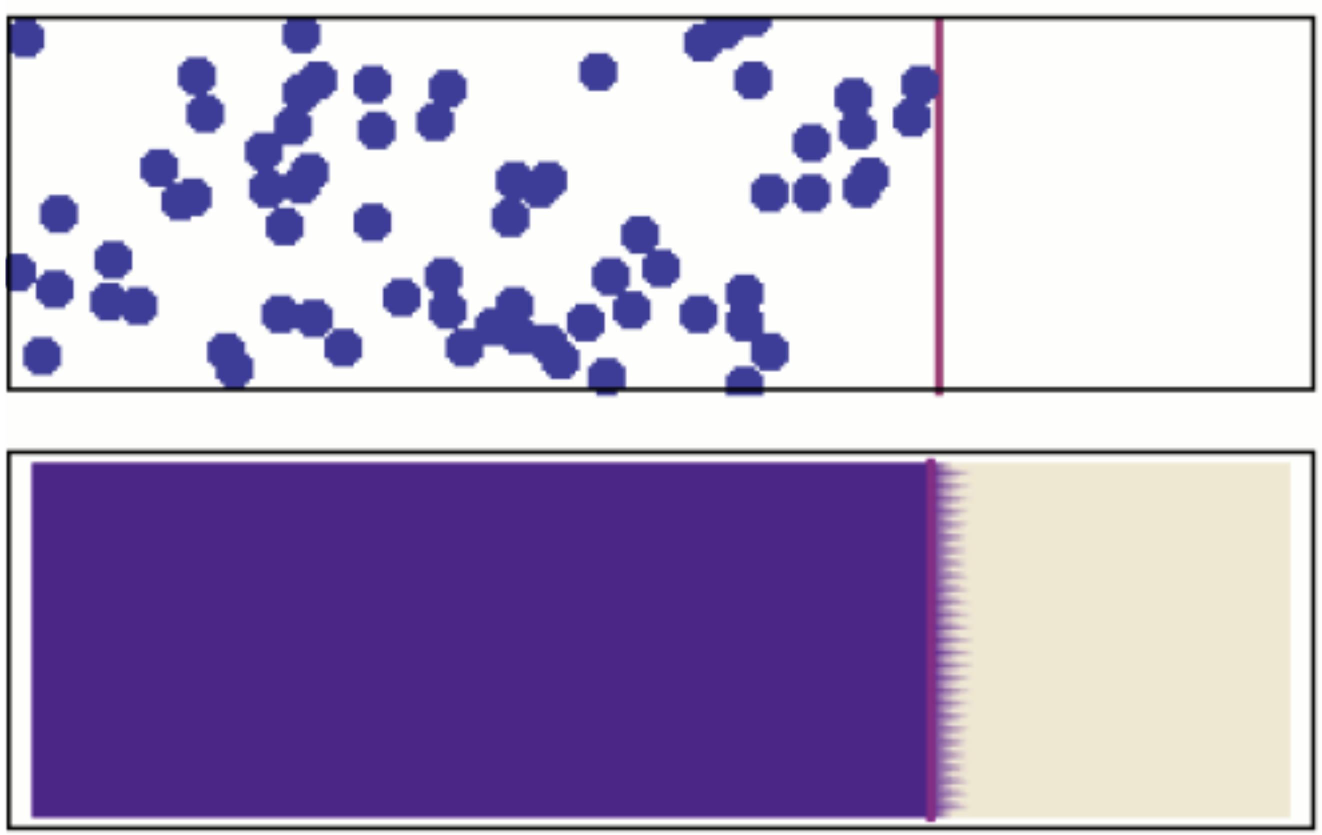


image source: Wikipedia

Diffusion process

- Consider $u(x, y, t)$ diffusing with time
- Concentration in space $\nabla u = [\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}]$
- **Fick's law:**

$$\frac{\partial u}{\partial t} = \operatorname{div}(D\nabla u)$$

- *D* is the diffusion coefficient (scalar or vector)

Diffusion process

- D can be thought of as a diagonal matrix as follows:

$$D = \begin{bmatrix} d_x & 0 \\ 0 & d_y \end{bmatrix}$$

- $d_x = d_y \longrightarrow$ **isotropic**, else **anisotropic**

- d_x and d_y independent of $u \longrightarrow$ **linear**

- d_x , d_y dependent on $u \longrightarrow$ **non-linear**

Diffusion process

- For $d_x = d_y = 1$ we get the **heat equation**

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y}$$

- It turns our that the solution of the above equation is

$$G_{\sqrt{2}t}(x, y) * u(x, y, t = 0)$$

- This is nothing but Gaussian blurring.

- Gaussian blurring => isotropic diffusion.