EE604A – Assignment 1

Deepak Gangwar

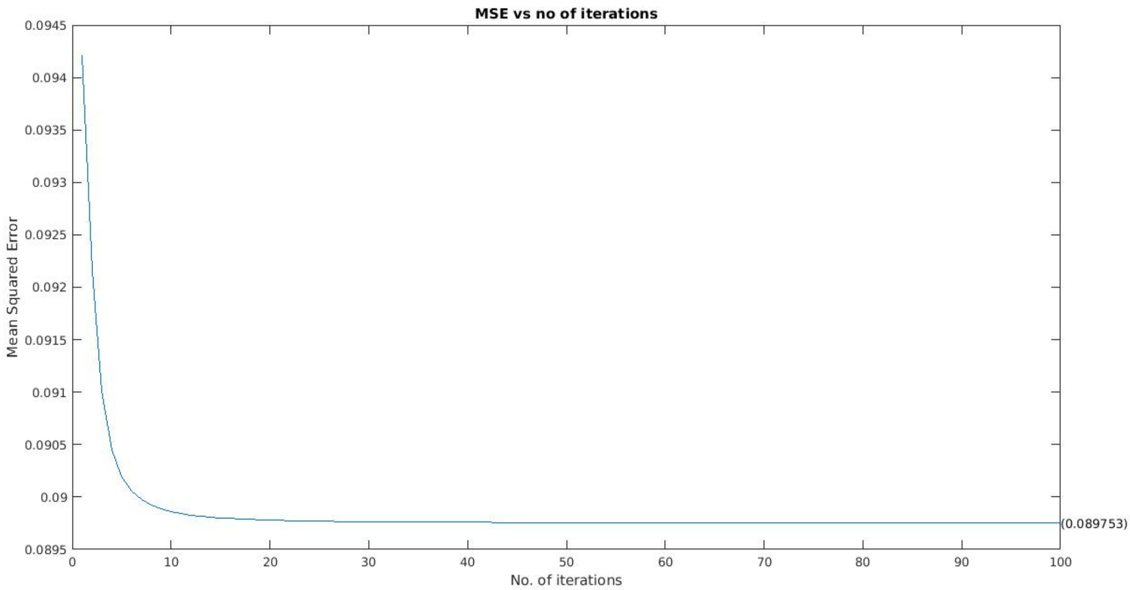
Roll No. 14208 06/Sep/2017

1

Transition Levels: -1.8271, -0.5958, 0.3626, 1.2933, 2.6195

Representation Levels: -1.0835, -0.1081, 0.8332, 1.7533

Figure 1: Sample Output - 1



Initialization affects number of iterations required to converge.

For uniform distribution, Llyod-Max quantizer is basically a uniform linear quantizer. **Proof:**

$$\text{Let, } p_m(m) = \left\{ \begin{array}{c} 1/(m_{L+1} - m_1)\, \forall\, x \in [m_1, m_{L+1}] \\ 0 \quad \text{otherwise} \end{array} \right\}$$

$$\text{Now, } a_j = \left(\int_{m_j}^{m_{j+1}} m p_m(m) dm \right) / \left(\int_{m_j}^{m_{j+1}} p_m(m) dm \right)$$

$$where, \quad (p_m = constant \text{ for given limits })$$

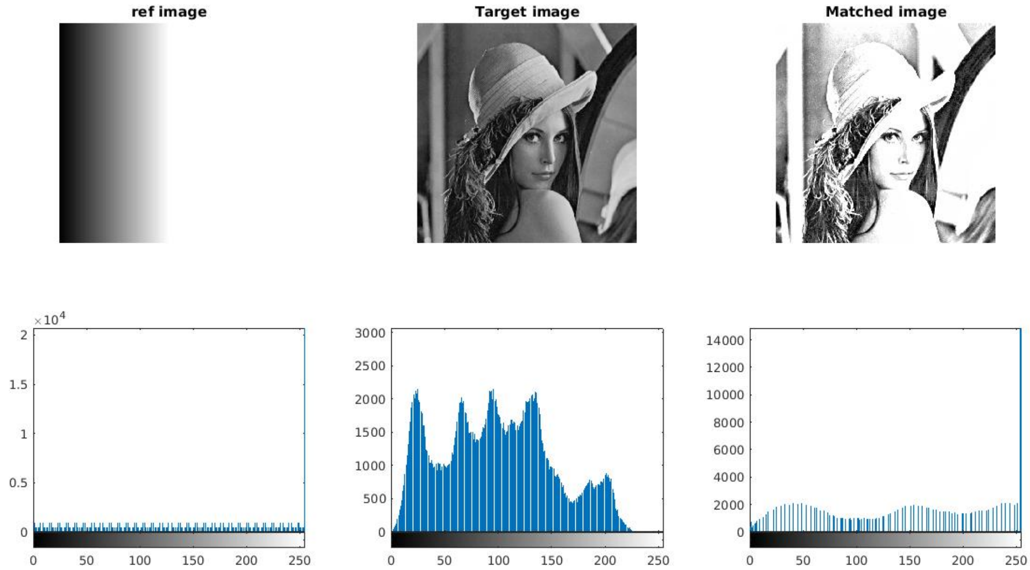
$$\implies \text{Representation Level } \boxed{a_j = \frac{m_j + m_{j+1}}{2}}$$

$$\text{and Transition Level } \boxed{m_j = \frac{a_j + a_{j-1}}{2}}$$

$$\implies \boxed{a_{j+1} - a_j = a_j - a_{j-1} = Constant \implies Uniform\ Linear\ Quantizer}$$

2

Figure 2: Sample Output - 2



3

Bi linear interpolation:

$$f_{k1k2} = f(x_{k1}, y_{k2}) = \sum_{i=0}^{1} \sum_{j=0}^{1} a_{ij} x_{k1}^{i} y_{k2}^{j}$$

for 8 connect neighbourhood,

$$k1 \text{ and } k2 \in 1, 2, 3$$

These equations can be written as following linear algebra system

$$\implies \begin{bmatrix} f_{1,1} \\ f_{1,2} \\ \vdots \\ f_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_1 & y_2 & x_1 y_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_3 & y_3 & x_3 y_3 \end{bmatrix} \begin{bmatrix} a_{0,0} \\ a_{1,0} \\ a_{0,1} \\ a_{1,1} \end{bmatrix}$$

$$Y = AX$$

Here, the required matrix X is of dimension 4. Hence 4 equations are needed for a unique solution. In 8 connect neighbourhood, the number of equations is more than 4, which results in an over defined system with no exact solution. So a least squares fit for an approximate best fit solution. if the number of equations been less than 4, infinite solutions would be possible.

The least squares solution to this system of equations is:

$$X = (A^T A)^{-1} A^T Y$$

4

Given:

$$g(x,y) = f(x,y) + \eta(x,y)$$

variance of $\eta(x,y)$ is σ^2 and mean is zero.

$$\hat{g(x,y)} = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

To Prove:

Noise variance = $\frac{\sigma^2}{K}$

Proof:

$$\hat{g(x,y)} = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y) = \frac{1}{K}\sum_{i=1}^{K} f(x,y) + \eta_i(x,y)$$

$$\implies \hat{g(x,y)} = f(x,y) + \frac{1}{K}\sum_{i=1}^{K} \eta_i(x,y)$$

$$\implies \textit{New Noise}\ \ \delta = \frac{1}{K}\sum_{i=1}^{K} \eta_i(x,y)$$

$$\textbf{\textit{Now,}}\ \ Var(\delta) = E(\delta^2) - E(\delta)^2$$

$$\implies Var(\delta) = E(\frac{1}{K^2}[\sum_{i=1}^{K} \eta_i(x,y)]^2) - 0$$

(because mean of noise is zero)

$$\implies Var(\delta) = \frac{1}{K^2}\sum_{i=1}^{K} E([\eta_i(x,y)]^2)$$

$$\implies Var(\delta) = \frac{1}{K^2}K\sigma^2 = \frac{\sigma^2}{K}$$

$$\boxed{Var(\delta) = \frac{\sigma^2}{K}}$$ **Proved.**

5

To Prove:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}$$

where, $u = x\cos\theta + y\sin\theta$ *and* $v = -x\sin\theta + y\cos\theta$

Proof:

$$\textit{Chain Rule,}\ \ \frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} = \frac{\partial f}{\partial u}\sin\theta + \frac{\partial f}{\partial v}\cos\theta$$

$$\implies \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial u}\sin\theta + \frac{\partial f}{\partial v}\cos\theta\right) = \frac{\partial}{\partial y}\frac{\partial f}{\partial u}\sin\theta + \frac{\partial}{\partial y}\frac{\partial f}{\partial v}\cos\theta \ldots (1)$$

$$\text{Now,}\ \ \frac{\partial}{\partial u}\frac{\partial f}{\partial y} = \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\sin\theta + \frac{\partial f}{\partial v}\cos\theta\right) = \frac{\partial^2 f}{\partial u^2}\sin\theta + \frac{\partial f}{\partial u\partial v}\cos\theta$$

$$\text{similarly,}\ \ \frac{\partial}{\partial v}\frac{\partial f}{\partial y} = \frac{\partial}{\partial v}\left(\frac{\partial f}{\partial u}\sin\theta + \frac{\partial f}{\partial v}\cos\theta\right) = \frac{\partial^2 f}{\partial u\partial v}\sin\theta + \frac{\partial^2 f}{\partial v^2}\cos\theta$$

$$\text{from (1),}\ \implies \frac{\partial^2 f}{\partial y^2} = \left(\frac{\partial^2 f}{\partial u^2}\sin\theta + \frac{\partial^2 f}{\partial u\partial v}\cos\theta\right)\sin\theta + \left(\frac{\partial^2 f}{\partial u\partial v}\sin\theta + \frac{\partial^2 f}{\partial v^2}\cos\theta\right)\cos\theta \ldots (2)$$

$$\text{Similarly,}\ \ \frac{\partial^2 f}{\partial x^2} = \left(\frac{\partial^2 f}{\partial v^2}\sin\theta - \frac{\partial^2 f}{\partial u\partial v}\cos\theta\right)\sin\theta + \left(-\frac{\partial^2 f}{\partial u\partial v}\sin\theta + \frac{\partial^2 f}{\partial u^2}\cos\theta\right)\cos\theta \ldots (3)$$

$$\text{eq (2) + (3),}\ \implies \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial u^2}\left(\cos^2\theta + \sin^2\theta\right) + \frac{\partial^2 f}{\partial v^2}\left(\cos^2\theta + \sin^2\theta\right)$$

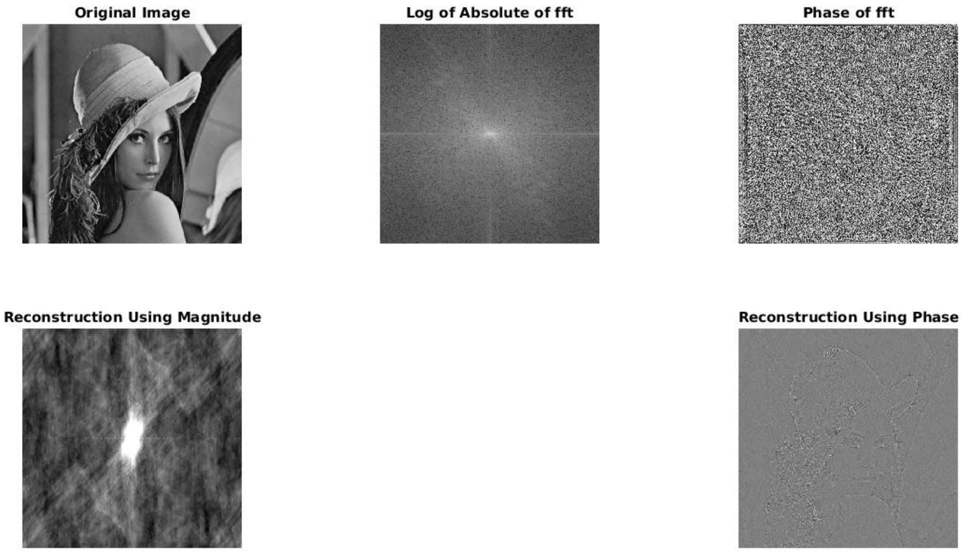
$$\implies \boxed{\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}}\ \ \textbf{Proved.}$$

$$\left(\cos^2\theta + \sin^2\theta = 1\right)$$

6

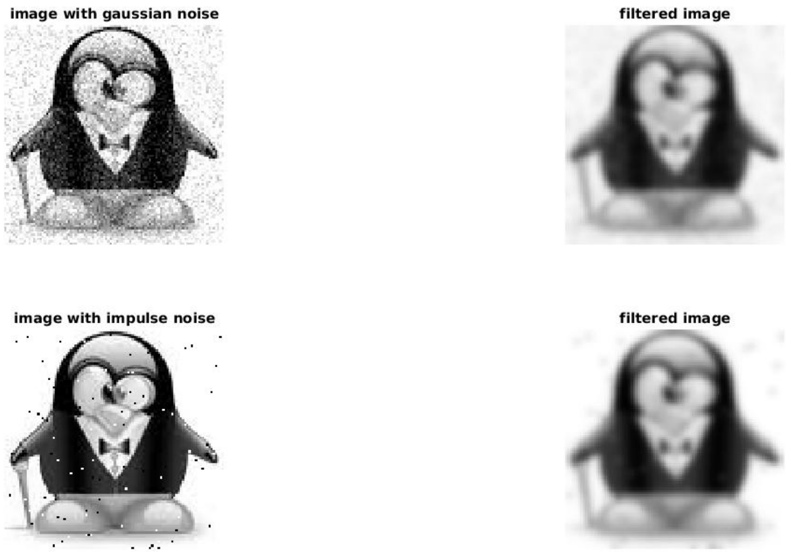
Reconstruction using magnitude only is not giving any significant location data of original image but reconstruction using phase gives the edges location of the image.

Figure 3: Sample Output - 6



7

Figure 4: Sample Output - 7



Source: stackexchange for some Tex commands.