

THEORETICAL ASSIGNMENT № 2

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Problem 1

The pseudo code is as follows.

Listing 1: Pseudo code – Search in an Infinite Array.

```
1  BOOL infinite_search{
2  isfound = FALSE;
3  i = 1;
4  s = s;
5  while (isfound == 0)
6  {
7      if (A[i] == s)
8          isfound = TRUE;
9      else if (A[i] < s)
10         i = i * 2;
11     else if (A[i] > s OR A[i] == empty)
12     {
13         isfound = modif_bin_search (i);
14         break;
15     }
16 }
17 return isfound;
18 }
```

Listing 2: modif_bin_search.

```
1  BOOL modif_bin_search (int i)
2  {
3      L = i / 2;
4      R = i;
5      mid = (L + R) / 2
6      while (L < R)
7      {
8          if (A[mid] == s)
9              return TRUE;
10         else if (A[mid] < s)
11             L = mid + 1;
12         else if (A[mid] > s OR A[mid] == empty)
```

```

13         R = mid - 1;
14     }
15     return FALSE;
16 }

```

Proof of correctness

For the `infinite_search()` function, in order to prove correctness we can take the assertion that if the loop breaks after i iterations then the given key must lie between i and $i/2$.

The above assertion is true for all cases irrespective the key values being found or not. Now, this function will, if the key is found, get to modified binary search. Now, for the modified binary search, we have a normal binary search only difference being, even if the value at R not be there, we are basically assuming it to be some high value. Note that this value shall always be higher than the key as that is the only case when the function is called.

Now this means that after i th iteration if the loop exits then the key value was lying between $k = 2^i$ and $k/2$. Now the modified binary search works the same way as a normal Binary search only except it ensures that the absence of value at k will not affect the end result.

Hence, this algorithm will only return correct results.

Time Complexity

The function `modif_bin_search` takes $\log(i)$ steps to give us a solution (Similar to a binary search). Now the original function in the worst case takes $\log(n)$ steps. Thus in the worst possible case our algorithm shall consume $2 * \log(n)$. Hence, the order of the given algorithm is $\log(n)$.

This can also be proposed by saying that the number of elements that this algorithm skips keeps on doubling every loop and so the order should be $\log(n)$.

Notice that the base of $\log(n)$ is decided based on our choice of the multiplier in every step.

Problem 2

Yes, it is possible to give the maximum size of the banner. Algorithm of the same is as following

-

Listing 3: Pseudo Code.

```

1 left = 0; right = n;
2 max_rectangle_area (height[n],width[n],left,right)
3 {
4     if (left==right) return {height(left),width(left)};
5     m = right - left + 1;
6     return merge( max_rectangle_area (height[n],width[n],left,right - m), ←
                    max_rectangle_area (height[n],width[n],right - m + 1,right));
7 }
8 merge({h1,d1},{h2,d2})
9 {

```

```

10     if(h1 >= h2) temp = {h2,(d1+d2)};
11     else temp = {h1*(d1+d2)};
12     return max_area({h1,d1},{h2,d2},temp);
13 }
14 max_area(area1,area2,area3)
15 {
16     if(area1[1]*area1[2] >= area2[1]*area2[2]){
17         if(area1[1]*area1[2] >= area3[1]*area3[2]) return area1;}
18     else if(area2[1]*area2[2] >= area3[1]*area3[2]) return area2;
19     else return area3;
20 }

```

Given algorithm is similar to merge sort.

Proof Of Correctness -

Base Case is $n = 1$. For base case max area is equal to the area of building which is correct.

Assume the algo is correct for all $n < k$ the for $n = k$,

buildings will be divided in 2 sets with $n = k/2$. As $(k/2 < k)$ so the algo will give correct output for k also provided that merge function is correct.

As merge function gives the max possible area of two halves so the function is correct.

Time complexity -

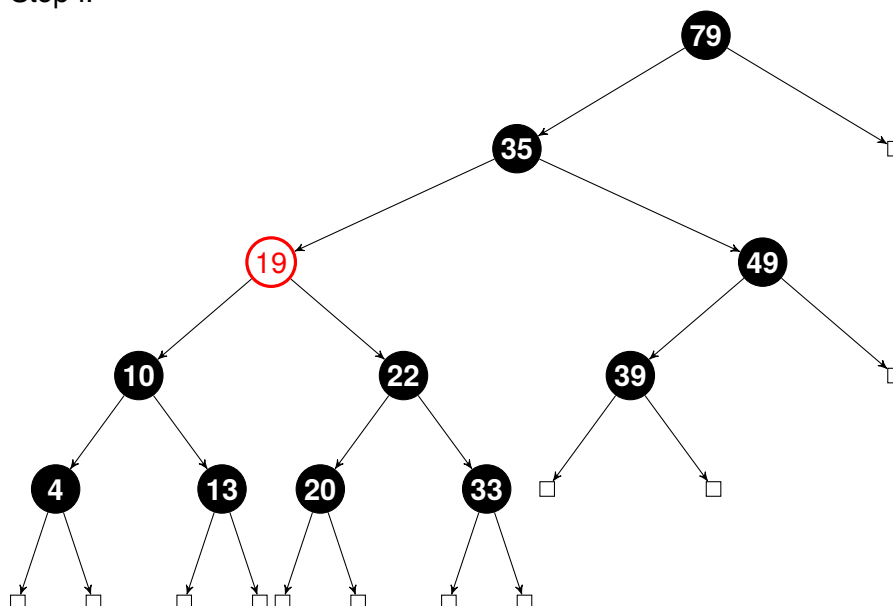
$T(n) = 2*T(n/2) + N$ and $T(1) = 1$.

$\implies T(n) = O(n \log n)$

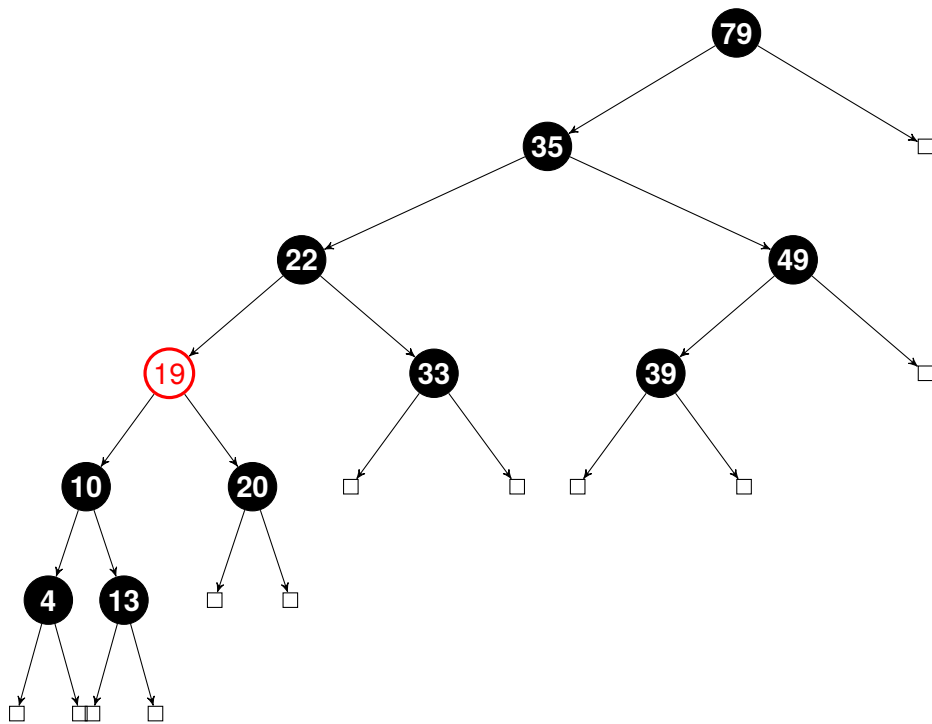
Problem 3

Part (a) Deleting 55

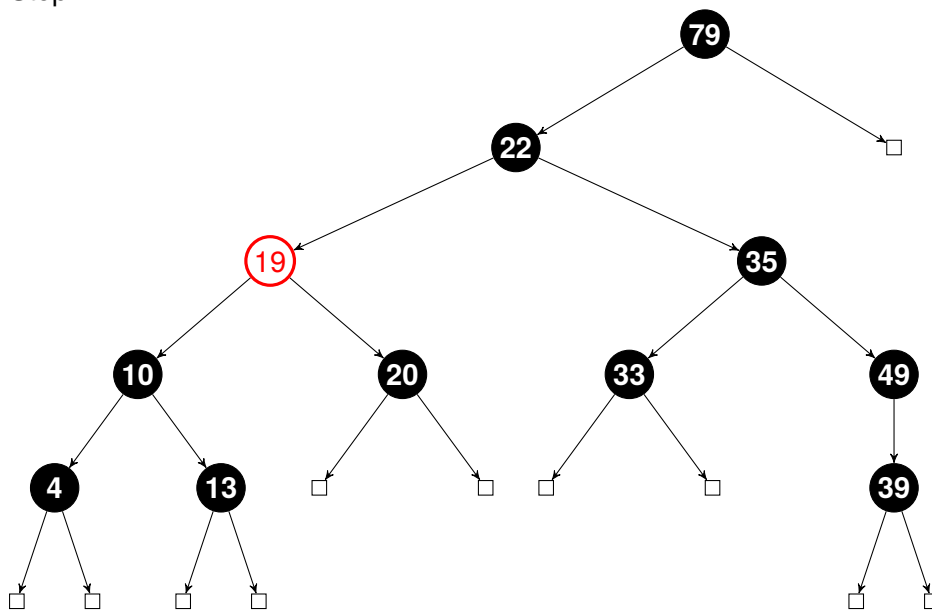
Step I.



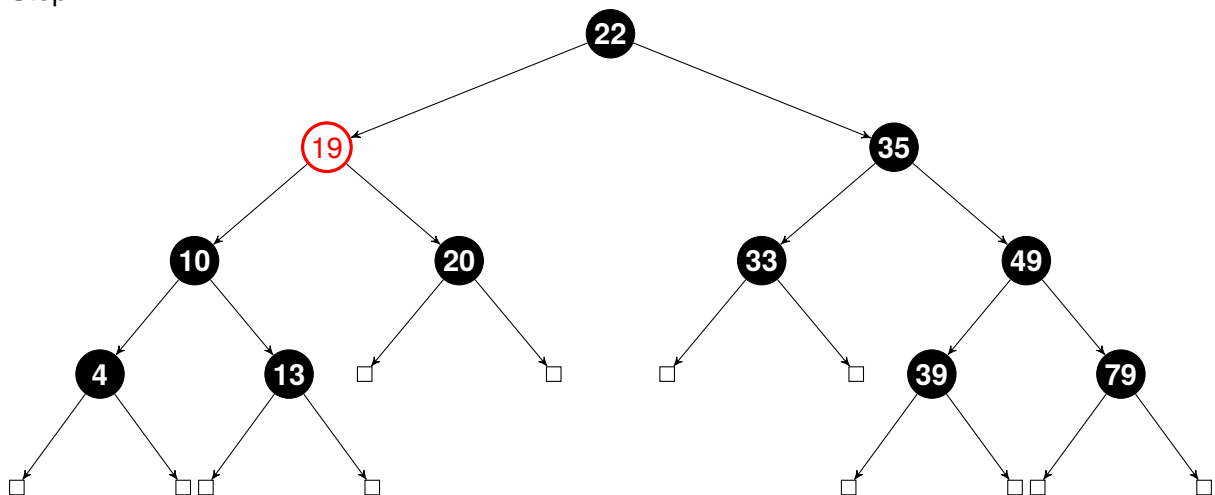
Step II.



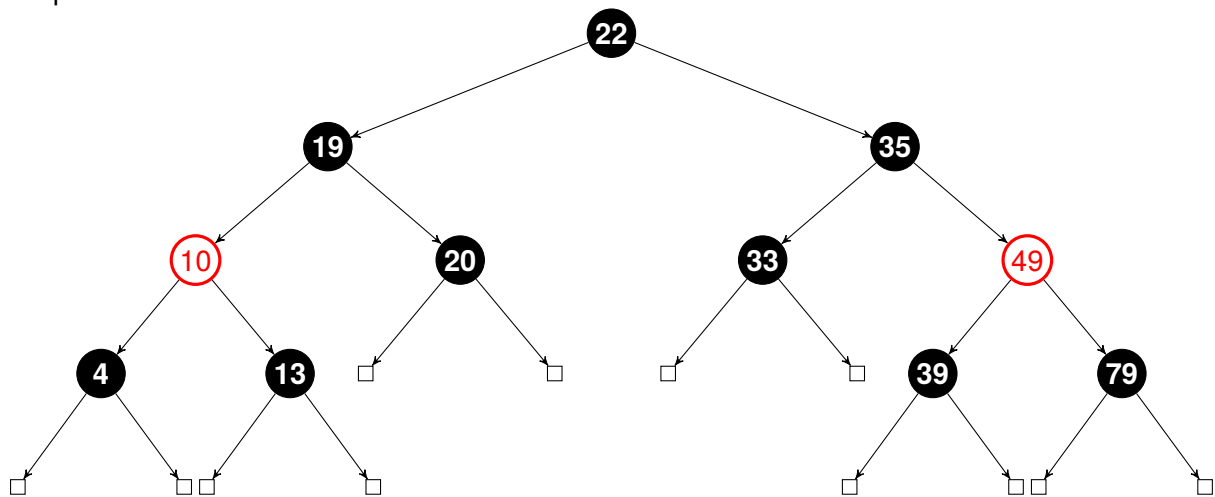
Step III.



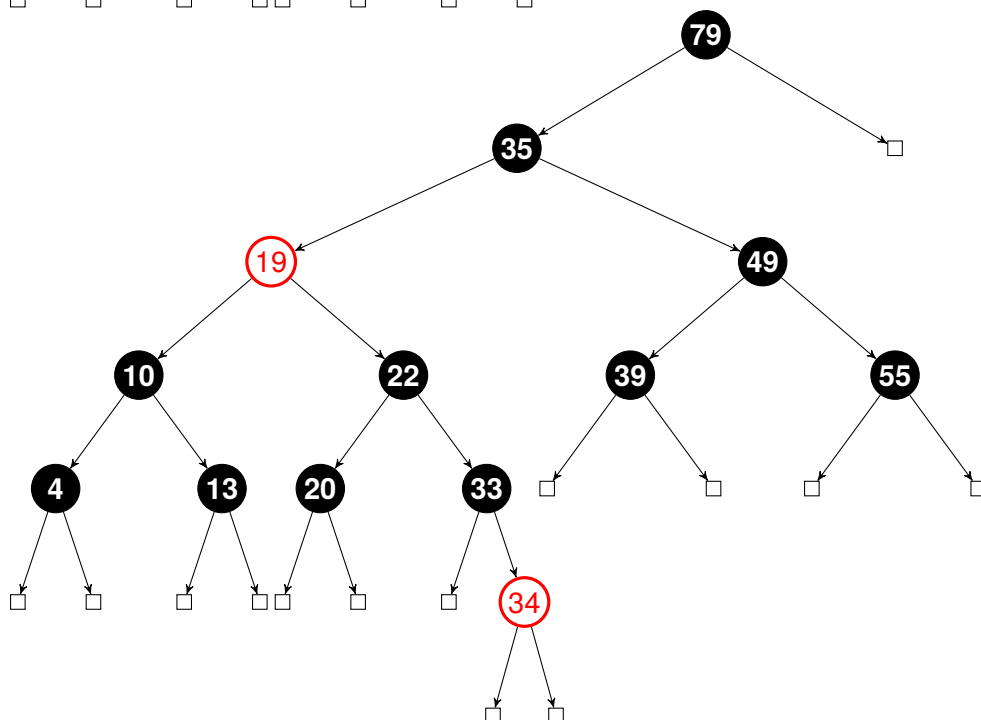
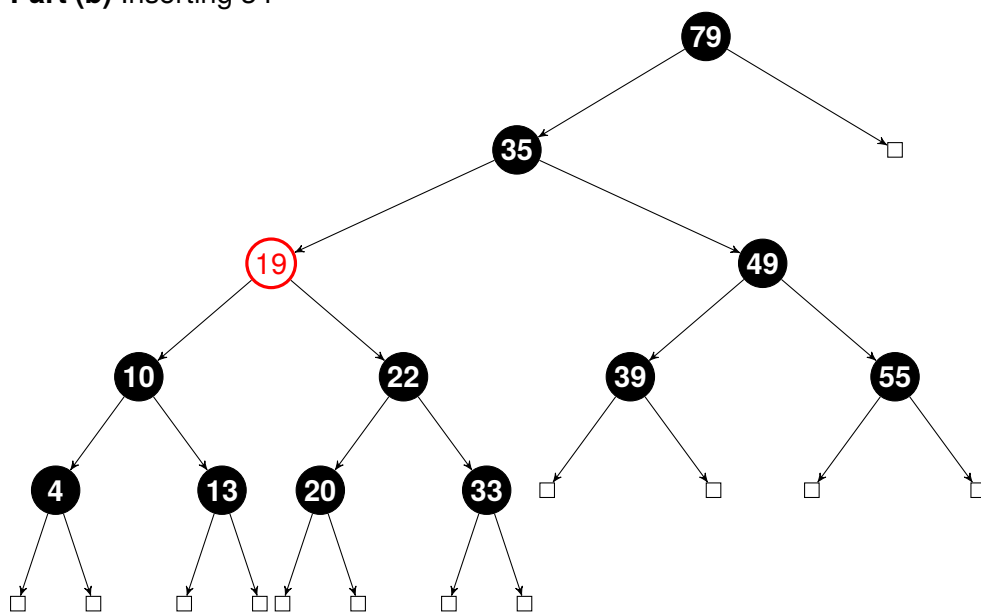
Step IV.



Step V.



Part (b) Inserting 34



Problem 4

We propose a data structure where we augment to every node of the existing tree another value which is the pointer to the predecessor of the node. Now for this data structure we will be able to maintain the $\log(n)$ time complexity for Insert, Delete and Query operations and yet find the required k predecessor in $O(k + \log(n))$. Although for this we shall have to change the Insert and Delete operations a bit. The details for the same are as follows.

The tree

A normal RB tree where each node, in addition to its own value also stores the value of its predecessor. So the structure of each node is like

Listing 4: Node Structure.

```
1 struct Node{
2     int val;
3     struct Node *pre; // Pointer to predecessor
4     struct Node *left,*right;
5 };
```

Now the algorithms

Algorithm for k predecessor

Listing 5: k predecessor

```
1 void k_pred (int num_predecessor, int value_to_search, Node * root)
2 {
3     key ← value_to_search;
4     ptr ← search (key, root);
5     if(ptr == NULL)
6         return;
7     while (k != 0)
8     {
9         print(ptr.pre.val);
10        ptr ← ptr.pre;
11        k = k - 1;
12    }
13 }
14
15 Node * search (int key, Node * root)
16 {
17     p ← root;
18     while(p != NULL)
19     {
20         if (p.val == key)
```

```

21         return p;
22     else if (p.val > key)
23         p ← p.left;
24     else p ← p.right;
25 }
26 }

```

Proof of correctness for k predecessor

Since this is an iterative algorithm, after every loop the value that it prints is basically the immediate predecessor of the current node. Now as this loop is called exactly k times and every time the loop updates the current node and prints the immediate predecessor to it. Thus, after k iterations when the loop is finally exited it would have published k predecessors of the given number. Hence, we can conclude that this algorithm gives the correct outputs at the end of execution.

Time Complexity for the above algorithm

For each query the search effectively does Binary Search and then returns the pointer to it. Thus its time complexity will be $\log(n)$ as is known. Now, each iteration of the loop would take $3 * T(1)$ time and the loop is executed k times. Hence the total time taken is $3 * k * T(1)$. Hence, the order would be $\log(n) + k$.

Changes to Insert function

The only change we need to make in Insert is to search for the predecessor of the inserted value and augment this into the newly inserted node. Now, search for the predecessor shall be of order $\log(n)$, insert itself is of order $\log(n)$. Hence, insert is still done in $O(\log(n))$.

Changes to Delete function

In deletion we need to store the predecessor of the deleted node in a temporary variable and then after deletion travel to the successor of the deleted node and change its predecessor from the previous value (equal to the deleted node) to the value stored in the temporary variable. This too similar to the above case will be done in $O(\log(n))$.