Theoretical Assignment № 2

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Problem 1

The pseudo code is as follows.

Listing 1: Pseudo code – Search in an Infinite Array.

```
1  BOOL infinite_search{
2  isfound = FALSE;
3  i = 1;
4  s = s;
5  while (isfound == 0)
6  {
7      if (A[i] == s)
8          isfound = TRUE;
9      else if (A[i] < s)
10         i = i * 2;
11     else if (A[i] > s OR A[i] == empty)
12         modif_bin_search (i);
13 }
14 return isfound;
15 }
```

Listing 2: modif_bin_search.

```
1  void modif_bin_search (int i)
2  {
3      L = i / 2;
4      R = i;
5      mid = (L + R) / 2
6      while (mid != R)
7      {
8          if (A[mid] == s)
9              return;
10         else if (A[mid] < s)
11             L = mid + 1;
12         else if (A[mid] > s OR A[mid] == empty)
13             R = mid -1;
14     }
15 }
```

Proof of correctness

TODO fill here.

Time Complexity

The function modif_bin_search takes log(i) steps to give us a solution (Similar to a binary search). Now the original function in the worst case takes log(n) steps. Thus in the worst possible case our algorithm shall consume 2 * log (n). Hence, the order of the given algorithm is log (n).

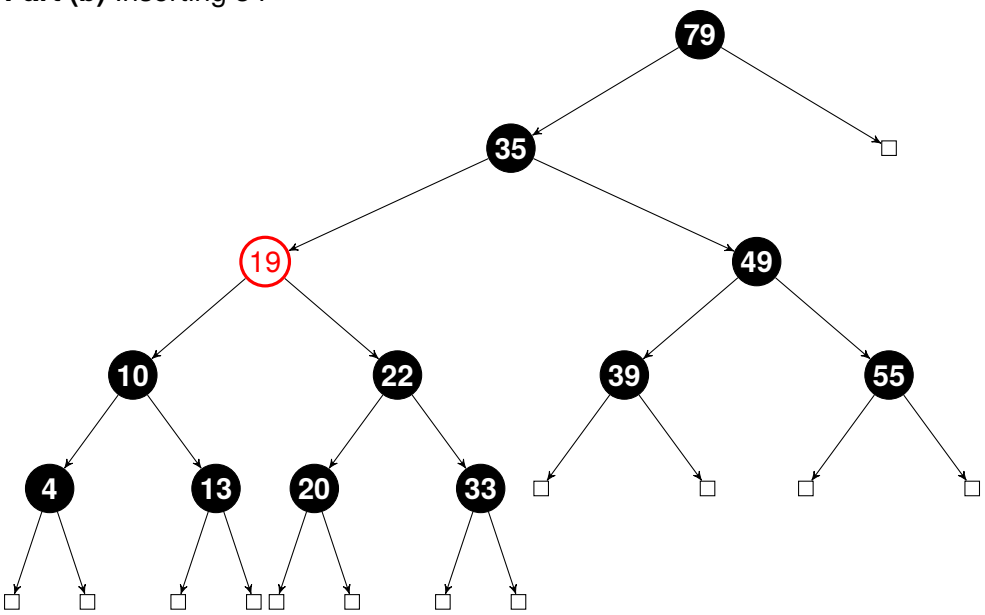
This can also be proposed by saying that the number of elements that this algorithm skips keeps on doubling every loop and so the order should be log (n).

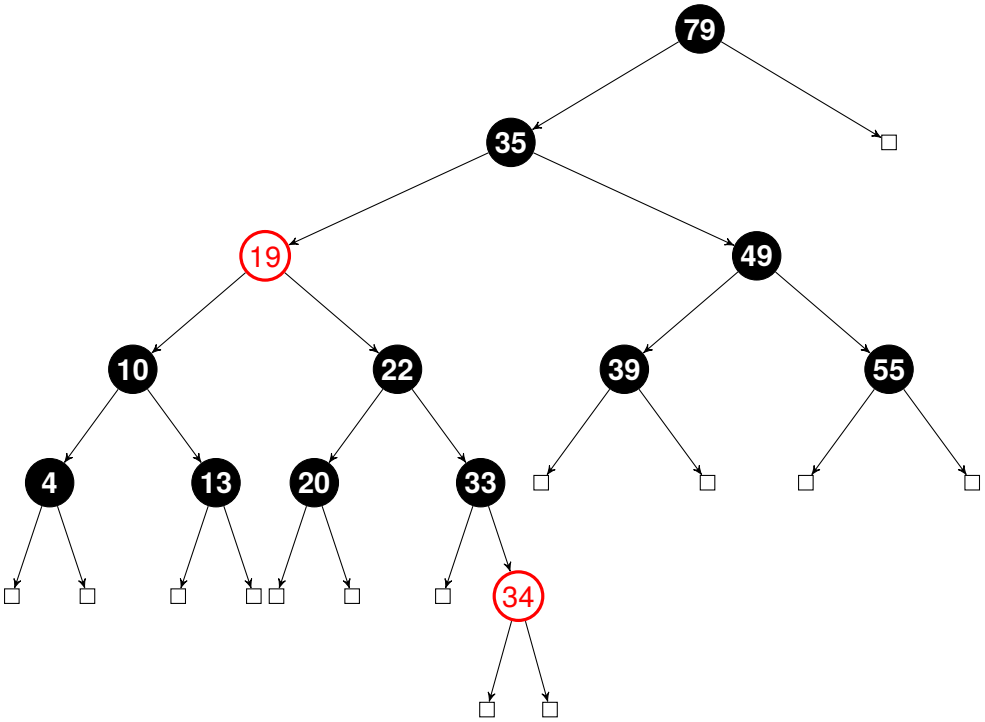
Notice that the base of log (n) is decided based on our choice of the multiplier in every step.

Problem 3

Part (a) Deleting 55

Part (b) Inserting 34





Problem 4

We propose a data structure where we augment to every node of the exiting tree another value which is the pointer to the predecessor of the node. Now for this data structure we will be able to maintain the log (n) time complexity for Insert, Delete and Query operations and yet find the required *k* predecessor in O (k + log (n)). Although for this we shall have to change the Insert and Delete operations a bit. The details for the same are as follows.

The tree

A normal RB tree where each node, in addition to its own value also stores the value of its predecessor. So the structure of each node is like

Listing 3: Node Structure.

```
1 struct Node{
2   int val;
3   struct Node *pre; // Pointer to predecessor
4   struct Node *left,*right;
5 };
```

Now the algorithms

Algorithm for *k* predecessor

Listing 4: k predecessor

```
1  void k_pred (int num_predecessor,int value_to_search, Node * root)
2  {
3      key ← value_to_search;
4      ptr ← search (key, root);
5      if(ptr == NULL)
6          return;
7      while (k != 0)
8      {
9          print(ptr.pre.val);
10         ptr ← ptr.pre;
11         k = k - 1;
12     }
13 }
14
15 Node * search (int key, Node * root)
16 {
17     p ← root;
18     while(p != NULL)
19     {
20         if (p.val == key)
21             return p;
22         else if (p.val > key)
23             p ← p.left;
24         else p ← p.right;
25     }
26 }
```

Proof of correctness for *k* predecessor

Since this is an iterative algorithm, after every loop the value that it prints is basically the immediate predecessor of the current node. Now as this loop is called exactly k times and every time the loop updates the current node and prints the immediate predecessor to it. Thus, after k iterations when the loop is finally exited it would have published k predecessors of the given number. Hence, we can conclude that this algorithm gives the correct outputs at the end of execution.

Time Complexity for the above algorithm

For each query the search effectively does Binary Search and then returns the pointer to it. Thus its time complexity will be log (n) as is know. Now, each iteration of the loop would take 3 * T(1) time and the loop is executed k times. Hence the total time taken is 3 * k * T(1). Hence,

the order would be log (n) + k.

Changes to Insert function

The only change we need to make in Insert is to search for the predecessor of the inserted value and augment this into the newly inserted node. Now, search for the predecessor shall be of order log (n), insert itself is of order log (n). Hence, insert is still done in O (log (n)).

Changes to Delete function

In deletion we need to store the predecessor of the deleted node in a temporary variable and then after deletion travel to the successor of the deleted node and change its predecessor from the previous value (equal to the deleted node) to the value stored in the temporary variable. This too similar to the above case will be done in O (log (n)).