

## **Response to Reviewers – Paper Submission (AURO-D-17-00250)**

### **Title: Disambiguation of Human Intent Through Control Space Selection**

We sincerely thank all the reviewers for taking the time to provide us with useful feedback and insight. We will be responding to all the major concerns raised by each reviewer separately and will discuss the relevant changes that have been incorporated in the revised version of the document. All minor comments (e.g. grammar typos) have been incorporated into the text. Please also find attached to this letter a version of the article with all changes marked in blue.

The summary of our major changes are as follows. (1) We have made significant changes to the text that aim to improve the clarity and rigor of our writing, and to motivate the problem more clearly. (2) We have clarified the intuition behind our approach and choice of disambiguation metric. (3) We have presented new results, that compare our dynamic field theory based intent inference approach to a Bayesian inference approach used widely in shared autonomy settings. (4) We have included previously omitted survey results from our subject study, highlighting users' perception of our disambiguation system. We believe these changes to have significantly improved the quality of the paper. We again thank the reviewers, and look forward to their feedback.

#### **Response to Guest Editor:**

##### *(1) Improve clarity and rigor*

Throughout the text, we have made significant changes with the aim of addressing clarity, rigor and motivation. (To help spot these changes, all modifications to the text are marked in the version of the article attached to this letter.) - We have also provided more explanation as to the intuition behind our approach, thereby addressing some of the concerns raised by Reviewers #1 and #3 (Section 3.1).

##### *(2) Baseline comparison*

In this version of the article, we have implemented a Bayesian inference approach often used in the shared autonomy literature, and provide a qualitative comparison to our dynamic field theory based intent inference approach. We provide intuition on under what conditions the approaches perform similarly and differently, and provide data on an illustrative example (Section 4.2).

Regarding information gain, we agree that an information theoretic notion of entropy could also be used to solve the disambiguation problem---and towards this end, we have added text to this revision on the topic of information theory. In this article we introduce the idea of control subspace selection for the purpose of intent disambiguation. As a first exploration of this idea, we took the approach of investigating what features of shape of the probability distribution over goals can be most beneficial for intent disambiguation purposes. We note that a change in entropy of a distribution is typically accompanied by a change in the contour/shape of the distribution. Therefore, by designing a disambiguation heuristic such as the one presented in this paper, we get to have a closer look at which low-level features of the distribution contribute the most to the intent disambiguation. Using information theoretic ideas would be the next step in our work.

##### *(3) Intuition and clarity*

We have added and revised text with the aim of providing more intuition for our choice of metrics and approach (Section 3.1). (Again, this text is marked in blue in the attached document.)

| (4) *Discussion addressing Reviewer #2's concerns regarding the experimental validation:*

We have presented clarifications for Reviewer #2's concerns regarding the experimental validations, especially with respect to baseline comparison of our intent inference approach to Bayesian schemes (Section 4.2). We have also included survey results that support some of our claims regarding how the disambiguation system helped in easier task execution (Section 7.4).

**Reviewer #1**

a. *Further analysis is required*

In the current version of the paper, we have supplemented our analysis by including previously omitted user survey results, which indicate that the users generally found task execution to be easier when controlling the robot in the disambiguating control modes that the algorithm selected (Section 7.4). In the revised version, we also supplement our analysis of the efficacy of the intent inference algorithm by presenting some baseline comparisons to a standard Bayesian approach found used commonly in shared-control domain (Section 4.2).

b. *Detailed analysis for intent inference*

In this revised version, we have included a baseline comparison of our dynamic field theory based intent inference approach to a standard Bayesian inference scheme (Dragan et al., 2013) widely used in the shared autonomy literature (Section 4.2). This comparison includes both a qualitative discussion, as well as an illustrative example in which there are three discrete goals and the user teleoperates and moves the robot to each one of the objects sequentially. In general, we found performance between the two approaches to be similar, except in scenarios where the Bayesian approach's delta function collapsed (as in the figure shown in the paper). We were unable to uncover any scenarios in which our DFT approach performed worse than the Bayesian approach.

c. *User perception of disambiguation system*

| We have included the results of a user survey from our subject study, highlighting users' perception of our disambiguation system. Subjects were asked to fill out a questionnaire after each task in which they evaluated the system and reported how much they liked to operate the robot in the control modes selected by the algorithm (section 7.4).

d. *The paper states that "a higher value [of the mode of the probability distribution] implies that the robot has a good idea," however this is not always true.*

| We agree with the reviewer, and believe that there in fact has been a misinterpretation due to a lack of clarity on our part, which we hope has been resolved in the current revision of the text. Our claim is that higher probability indicates a higher confidence in the robot's prediction (and not that a higher distribution mode will result in better disambiguation). It is precisely because of the reasons mentioned by the reviewer that we consider other features (the remaining 3 components/features) of the probability distribution as well, to determine which control mode has higher *disambiguation* capabilities. In the current version of the text we have clarified the intuition behind the choice of the

different features that inform the disambiguation metric (Section 3.5).

(In short, a single feature by itself is unlikely to disambiguate the goals. But by considering multiple features in a combined fashion, it adds to the disambiguation power.)

*e. In Fig. 3..., One way to improve it might be to visualize only one goal and illustrate the change of confidence for one goal.*

We thank the reviewer for this suggestion. After careful consideration however, in the present version of the document we have decided to retain the three goals, because disambiguation is more relevant when there are multiple goals (or else there is nothing to disambiguate between). The shaded bars indicate how the probabilities vary for robot motion along each dimension. For this illustration we use a simple directedness-based heuristic to determine the probabilities.

*f. Fig. 4 is not clear; why is best control dimension  $x$  in the right column? I suggest simplifying it using only two goals. Also, how are the C1 and C2 specified? It appears that some information was omitted from the RSS version of this work.*

Our reason for choosing four goals is to illustrate the robustness of the disambiguation algorithm in identifying disambiguating control dimensions effectively in a scene with higher number of goals. Intent disambiguation typically becomes harder as the number of potential goals in the scene increase. With two goals, the disambiguation problem in many cases becomes trivial. As mentioned in the figure caption, the right column shows those parts of the workspace in which the best disambiguating control dimension is  $Z$ . Due to space constraints we have referred our readers to our original RSS paper for detailed specifications of what the two confidence functions (C1 and C2) are. However, the figure caption does mention that C1 and C2 correspond to an instantaneous proximity-based and directedness-based heuristic confidence function.

*g. I suggest that the authors extend their related work section with recent studies on shared autonomy (see recent survey by Javdani et al., "Shared Autonomy via Hindsight Optimization for Teleoperation and Teaming").*

We have taken this into account and revised our related work section accordingly.

## **Reviewer #2**

*a. Why not Information Theoretic alternatives?*

We agree with the reviewer about the utility of information theoretic concepts in relation to the problem posed in our article, and have expanded the discussion information theoretic concepts for the purposes of active learning and information gathering actions in the related work section.

In this article we introduce the idea of control subspace selection for the purpose of intent disambiguation. As a first exploration of this idea, our motivation is to investigate what aspects/features of the probability distribution over goals (more precisely, the shape of the distribution) inform intent disambiguation the most. A change in entropy is typically accompanied by a change in the shape of the probability distribution over goals. We take a bottom-up approach wherein we investigate how the shape of the probability distribution evolves as the user operates the robot and moves it in space. We

hand-engineer four different features (components) that characterize different aspects of the shape of the distribution based on empirical insights we had during our algorithm design phase. The combination of these individual components into a single disambiguation metric is also a design decision, but carefully done in such a way that higher values of the disambiguation metric would imply greater disambiguation capability. Using information theoretic ideas would be the next step in our work.

*b. Erroneous claims regarding the metric?*

We respectfully disagree with the statement that this is erroneous. By virtue of design, a higher value of the metric indeed corresponds to better intent disambiguation and therefore the intent inference mechanism will be able to infer the human's intent unambiguously and accurately. We have made this aspect clear in the revised text to avoid any potential misunderstanding in interpretation (Section 3.5 – 5).

*c. Why not Bayesian?*

We agree that a clear motivation for the development of our new intent inference approach was missing from the article, and thank the reviewer for pointing this out. The proposed dynamic field theory based system for intent inference is an alternative to Bayesian and other heuristic approaches. We have added text to the beginning of Section 4.1 that motivates our development of this alternative, as well as a comparison in Section 4.2 to a Bayesian approach from the shared control literature.

In addition to the new text in Section 4.1, we note that while it is true that if a process is truly finite-order Markovian then the state is a 'sufficient statistic', for human-robot interaction in the context of assistive robotics this assumption is not always correct. Furthermore, in a Bayesian scheme, if at any time-step the likelihood is peaked (a delta function), it can result in the collapse of the posterior to a single value, thereby eliminating any memory trace.

By framing the evolution of the probability distribution as a dynamical system, our method is similar to approaches in which recurrent neural nets are used for intent inference purposes (except in our case we hand-engineer the features that drive the dynamical system).

*d. Benchmarking of proposed work.*

In the revised version we have provided baseline comparison of our intent inference mechanism and a Bayesian approach that is widely used in shared autonomy setting (proposed by Dragan et al. (2013)). We have provided a qualitative comparison through an illustrative example, and discussion of the mathematical underpinings of each approach (Section 4.2).

In essence, our approach, relies on the features of the raw input to determine how the probability distribution should evolve in time and does not assume that the human behaves optimally. This is crucial in the setting of assistive robotic manipulation, in which subjects have inherent motor limitations that make optimal behavior an unrealistic assumption.

*e. Experimental Validation*

In the present version of the work, we have included previously omitted user survey results from our subject study (Section 7.4). The survey results indicate that the subjects did find task execution to be easier when operating the robot in disambiguating control modes.

### **Reviewer #3**

*a. There are a few run-on sentences throughout the work. Please make sure all sentences are clear and concise. Similarly, there are a few very long paragraphs that could be broken up to best present one idea at a time. There are a few widows and orphans (hanging words or phrases). For aesthetic purposes, please watch for unnecessary white space.*

Thank you for the suggestions. We have taken them into consideration and have revised our work accordingly.

#### *b. Mathematical Notation*

We have clarified our math notation to avoid ambiguity.

*c. There are a lot of equations that were selected to fit the task / goals of this work, but it might be nice to add a little intuition about how you selected these parameters (and maybe what didn't work) so readers can get useful insight from your work.*

We have included text throughout the article with the aim of providing more intuition. Please see in particular Sections 3.5, and also Section 8 for a discussion of what did not work well.

*(d) Can you compare your methods to some baseline techniques for intent / goal inference? The results show how well your approach works, but doesn't give compare to baselines.*

We have provided a baseline comparison to a standard heuristic Bayesian approach (Dragan et al. (2013)). A qualitative comparison through an illustrative example, and discussion of the mathematical underpinnings of each approach, can now be found in Section 4.2.

*(e) Please discuss how this would be useful in other applications (e.g. how would this extend to cases where there are less discrete goals) so readers can gain insight and use it in their work.*

We thank the reviewer for the suggestion, and have included the discussion in Section 8.

# Disambiguation of Human Intent Through Control Space Selection

Deepak E. Gopinath\* · Brenna D. Argall

Received: date / Accepted: date

**Abstract** Assistive shared-control have the potential to transform the lives of millions of people afflicted with severe motor impairments as a result of spinal cord or brain injuries. The effectiveness and usefulness of shared-control robots is closely related to their ability to infer the user's needs and intentions and is often a limiting factor for providing appropriate assistance quickly, confidently and accurately. The contributions of this paper are three-fold: first, we propose a goal disambiguation algorithm which enhances the intent inference and assistive capabilities of a shared-control assistive robotic arm. Second, we introduce a novel intent inference algorithm that works in conjunction with the disambiguation scheme, inspired by *dynamic field theory* in which the time evolution of the probability distribution over goals is specified as a dynamical system. Third, we present a pilot human subject study to evaluate the

efficacy of the disambiguation system. This study was performed with eight subjects. Our results suggest that (a) the disambiguation system has a greater utility value as the control interface becomes more limited and the task becomes more complex, (b) subjects demonstrated a diverse range of disambiguation request behavior with a greater concentration in the earlier parts of the trial and (c) there are no differences in the onset of robot assistance between different mode switching paradigms across tasks or across interfaces.

**Keywords** Shared Autonomy · Intent Inference · Intent Disambiguation · Assistive Robotics

## 1 Introduction

Assistive and rehabilitation machines—such as robotic arms and smart wheelchairs—have the potential to transform the lives of millions of people with severe motor impairments (LaPlante et al 1992). These machines can promote independence, boost self-esteem and help to extend the mobility and manipulation capabilities of such individuals, and revolutionize the way motor-impaired people interact with society (Scherer 1996; Huete et al 2012). With rapid technological strides in the domain of assistive robotics, the machines have become more capable and complex, and with this complexity the control of these machines has become a greater challenge.

The control of an assistive machine is typically enacted through a control interface. Moreover, the greater the motor impairment of the user, the more limited are the interfaces available for them to use. These interfaces (for example, sip-and-puffs and switch-based head arrays) are low-dimensional, discrete interfaces that can operate only in subsets of the entire control space (Simpson et al 2008; Nuttin et al 2002). The dimensionality

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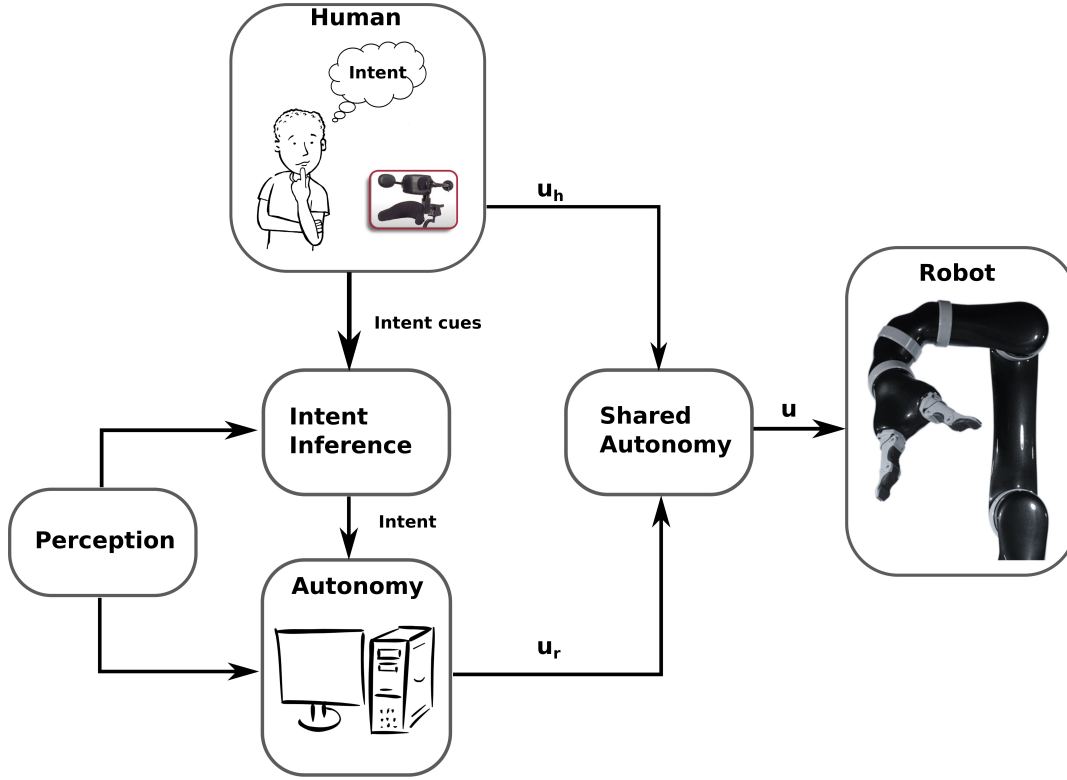
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**Fig. 1** Illustration of the core components of a shared control architecture.  $u_h$  denotes the human control command,  $u_r$  denotes the autonomy control command and  $u$  is the final control command issued to the robot. The intent inference module can extract intent cues from the human in multiple ways, for example via the human control actions or biometric measures that indicate the cognitive and physical load of the user during task execution. In our work we intentionally restrict this to be similar to operating the machine without autonomy and so intent information is  $u_h$  exclusively.

mismatch between the control interfaces and the controllable degrees-of-freedom (DoF) of the assistive robot necessitates the partitioning of the entire control space into smaller subsets called *control modes*. Moreover, the more limited and lower-dimensional the control interface, the greater the number of control modes. In order to achieve full control of the machine, the user must switch between the control modes, which is referred to as *mode switching* or *modal control*. Mode switching adds to the cognitive and physical burden of task execution and has a detrimental effect on the performance (Eftring and Boschian 1999).

The introduction of *shared autonomy* to these assistive machines seeks to alleviate some of these issues. In a shared control system the task responsibility is shared between the user and the robot, with the aim of reducing human effort in achieving a goal. Shared autonomy systems arbitrate between the human control commands and the autonomous control commands using different strategies depending on the task context, user preference and robotic platform. Figure 1 depicts the

key components of a typical shared control architecture and how they interact with each other.

Any assistive robotic system needs to have a good idea of the user’s needs and intentions. Therefore, *intent inference* is a necessary and crucial component to ensure appropriate assistance. In assistive robotic manipulation specifically, often the first step of a manipulation task is to reach for and grasp discrete objects in the environment. Intent inference therefore can be framed as a problem of estimating a probability distribution of intent likelihood over all possible goals (objects) in the environment. This inference is usually informed by various cues from the human and the environment, such as the human control actions, biometric measures and task-relevant features such as robot and goal locations. With a greater number of sensor modalities available, it is likely that the intent inference becomes more accurate.

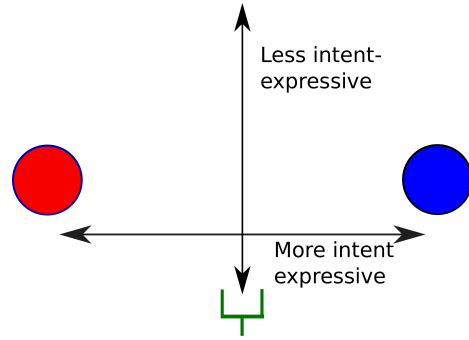
However, in the assistive domain, user acceptance and adoption is of paramount importance. Adding more sensors to track biometric data and object locations can become expensive and cumbersome (e.g. if the sensor

must be worn by the user). For reasons of user adoption and cost, we intentionally design our assistance add-ons to be as invisible and close to the manual system as possible. As a result the intent inference is exclusively informed by the human’s control commands issued to the assistive machine. The low-dimensionality, sparsity and noise in these control commands make the inference task even harder, prompting the need for robust intent inference formalisms.

Our key insight is that certain control commands issued by the human are *more intent expressive* than others, and may help the autonomy in inference accuracy. This is the notion of *inverse legibility* (Gopinath and Argall 2017) in which human-generated actions *help the robot* to infer the human’s intent unambiguously. Consider the hypothetical reaching experiment illustrated in Figure 2. Since the spatial locations of the goals are maximally spread along the horizontal axis, any human control command issued along the horizontal dimension conveys a lot of information about the intended goal to the robot. In other words, motion along  $x$  is more *intent expressive* and will help the robot to draw accurate inference more quickly and confidently.

In this work, we investigate how the selection of a subset of the operational control dimensions/modes improves the intent inference and disambiguation capabilities of the robot. As our primary contribution we develop a control mode selection paradigm which selects the control mode in which a user-initiated motion will *maximally disambiguate* human intent by eliciting *more intent expressive* control commands from the user. Furthermore, the disambiguation power of the algorithm is closely linked to, and is dependent on, the success and accuracy of the underlying intent inference mechanism (Gopinath and Argall 2017). Therefore, as our secondary contribution, we also develop a novel intent inference scheme which utilizes ideas from *dynamic field theory* that casts the time-evolution of the probability distribution over goals as a dynamical system. By doing so, the distribution evolves in continuous time and information from past states can be easily incorporated using a time-scale parameter and thereby improves the performance of the disambiguation algorithm.

In Section 2 we present an overview of relevant research in the areas of shared autonomy in assistive robotics, types of shared autonomy assistance paradigms, intent inference and synergies in human-robot interaction. Section 3 presents the mathematical formalism developed for intent inference and disambiguation. Section 5 focuses on the implementation details of the shared control system. The study design and experimental methods are discussed in Section 6 followed by results



**Fig. 2** Illustration of goal disambiguation along various control dimensions. Any motion of the end effector (green) along the  $y$ -axis will not help the system to disambiguate the two goals (A and B). However, motion along the  $x$ -axis provides cues as to which goal.

in Section 7. Discussion and conclusions are presented in Sections 8 and 9 respectively.

## 2 Related Work

This section provides an overview of related research in the domains of shared autonomy in assistive robotics, robot assistance for modal control, intent inference in human-robot interaction and information acquisition in robotics.

Shared-autonomy in assistive systems aims to reduce the user’s cognitive and physical burden during task execution without having the user relinquish complete control (Philips et al 2007; Demeester et al 2008; Gopinath et al 2017; Muelling et al 2017). The most common strategies to share control between the user and the assistive system include (a) hierarchical paradigms in which the higher level goals are entrusted with the user and the autonomy generates low-level control (Tsui et al 2011; Kim et al 2010, 2012), (b) control allocation in distinct partitions of the control space (Driessen et al 2005) and (c) blending user controls and robot autonomy commands (Downey et al 2016; Storms and Tilbury 2014; Muelling et al 2017).

In order to offset the drop in task performance due to shifting focus (task switching) from the task at hand to switching between different control modes different mode switch assistance paradigms have been proposed. For example, a simple time-optimal mode switching scheme has shown to improve task performance (Herlant et al 2016; Pilarski et al 2012).

Shared control systems often require a good estimate of the human’s intent—for example, their intended reaching target in a manipulation task or a target location in the environment in a navigation task (Liu et al 2016). Intent can either be explicitly communicated by the



user (Choi et al 2008) or can be inferred from their control signals or sensor data using various algorithms.

Intent recognition and inference are actively studied by cognitive scientists and roboticists and can be broadly categorized into two main classes: model-based approaches and heuristic approaches. In model-based approaches, intent inference is typically cast within a Bayesian framework, and the posterior distribution over goals (belief) at any time is determined by the iterative application of Bayes theorem. Evidence in this context can be derived from a combination of factors such as task-relevant features in the environment, trajectory information, human control actions or biometric data from the user. Within the context of shared autonomy based teleoperation a Bayesian scheme for user intent prediction is used (Dragan and Srinivasa 2012b; Javdani et al 2017; Admoni and Srinivasa 2016) in which the user is modeled within the Markov Decision Process framework and is typically assumed to be noisily optimizing some cost function for their intended goal. For example, in low-dimensional spaces, this cost function can be learned from expert demonstrations using Inverse Reinforcement Learning such as MaxEnt IRL (Ziebart et al 2008). However for high-dimensional spaces such as that of robotic manipulation, learning cost functions that generalize well over the entire space requires large number of samples. In such cases, heuristic cost functions such as sum of squared velocities along a trajectory have been found to be useful for goal prediction (Dragan and Srinivasa 2013). Furthermore, the inference is made tractable via a second-order approximation of the cost function with respect to an optimal trajectory under a quadratic cost function assumption. On the other hand simple heuristic approaches can also be used to find direct mappings from instantaneous cues and the underlying human intention. Heuristic approaches can incorporate domain-specific knowledge easily and are computationally less expensive. For example, the use of instantaneous confidence functions for estimating intended reaching target in robotic manipulation (Dragan and Srinivasa 2012a; Gopinath et al 2017). However, heuristic methods in general are not sophisticated enough to incorporate histories of past states and actions, making them less robust to external noise.

Eliciting more legible and information-rich control commands from the user to improve intent estimation can be thought of as an information acquisition process. Intent information acquisition can be an *active* process in which the robot takes actions that will probe the human’s intent (Sadigh et al 2016). Designing optimal control laws that maximize information gain can be accomplished by having the associated reward structure reflect some measure of information gain (Atanasov et al

2014). Autonomous robots designed for exploration and data acquisition tasks can benefit from exploring more information-rich regions in the environment. If the spatial distribution of information density is known *a priori*, information maximization can be accomplished by maximizing the ergodicity of the robot’s trajectory with respect to the underlying information density map (Miller et al 2016; Miller and Murphey 2013). **In this paper, we take the novel step of casting intent disambiguation as a problem of active inference. As a first exploratory step, we do not yet rely on information theoretic approaches, but instead adopt a heuristic approach in order to investigate how various features of the shape of the probability distribution over goals play a role in disambiguation.**

From the robot’s perspective, the core idea behind our disambiguation system is that of “*Help Me, Help You*”—that is, if the user can help the robot with more information-rich actions, then the robot in turn can provide accurate and appropriate task assistance more quickly and confidently. By having the human assist the robot, our work leverages the underlying synergies that are inherent in human-robot cooperation. A framework for “*people helping robots helping people*” in which the robot relies on semantic information and judgments provided by the human to improve its own capabilities has been developed in (Sorokin et al 2010). In order to overcome the various types of communication bottlenecks that can hamper performance, different types of communication interfaces have been developed that account for the restricted capabilities of the robot (Goodfellow et al 2010). Lastly, more intent-expressive human actions is related to the idea of legibility in robot actions. In human-robot interaction, the legibility and predictability of robot motion *to* the human has been investigated (Dragan et al 2013) and various techniques to generate legible robot motion have been proposed (Holladay et al 2014). Our work relies on the idea of *inverse legibility* (Gopinath and Argall 2017) in which the assistance scheme is intended to bring out more legible intent-expressive control commands *from* the human.

### 3 Mathematical Formalism for Intent Disambiguation

This section describes our intent disambiguation algorithm that computes the control mode that can maximally disambiguate between the goals and the intent inference mechanism that works in conjunction with the disambiguation algorithm. Section 3.1 motivates the utility of active inference accomplished through a disambiguation metric. Section 3.2-3.4 describe the formulation and computation of the disambiguation metric.

### 3.1 Motivation

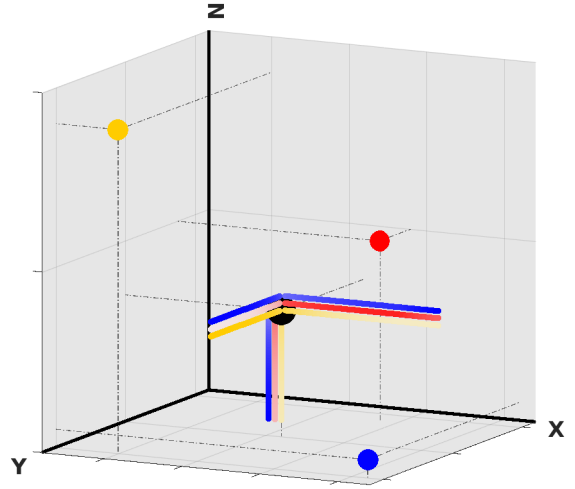
The need for intent (goal) disambiguation arises in situations in which there are multiple potential goals and the autonomy has to infer the user’s goal unambiguously to provide accurate and appropriate assistance. This becomes even harder in scenarios in which the user input is low-dimensional and sparse—for example, due to limited control interfaces or the user’s inherent motor or cognitive impairments.

Given an intent inference scheme that is dependent on robot pose or movement (Kelley et al 2008; Wang et al 2013), as the user controls the robot and moves it in space, the probability distribution over goals will evolve in time. This time-evolution of the probability distribution is also accompanied by a change in the shape/contour of the probability distribution. When the user is only able to control the robot within a subset of the operational space (as dictated by the current control mode), this impacts the evolution of the probability distribution. By investigating how the shape of the distribution evolves as a result of user operation of the robot in different control modes, we aim to differentiate and characterize when the weight of the distribution narrows and accumulates on a single goal, which correlates directly with the disambiguation capability of a control mode. A narrow probability distribution indicates that the system has more confidence in its prediction of the intended goal. With this characterization of the shape of the distribution, we investigate the low-level features that contribute the most to disambiguation.

Figure 3 shows simulations which motivate the development of a disambiguation metric. For motions along different control dimensions, the confidences associated with each goal evolve differently. Moreover, motions in some control dimensions result in sharper rise in some goal confidences compared to others. This indicates the existence of control dimensions that can better disambiguate between the goals.

### 3.2 Notation

Let  $\mathcal{G}$  denote the set of all candidate goals with  $n_g = |\mathcal{G}|$  and let  $g^i$  refer to the  $i^{th}$  goal with  $i \in [1, 2, \dots, n_g]$ . A *goal* represents the human’s underlying intent. Specifically, in assistive robotic manipulation, since the robotic device is primarily used for reaching toward and grasping of discrete objects in the environment, intent inference is the estimation of the probability distribution over all possible discrete goals (objects) in the environment. At any time  $t$ , the robot actively maintains a probability distribution over goals denoted by  $\mathbf{p}(t)$  such that  $\mathbf{p}(t) = [p^1(t), p^2(t), \dots, p^{n_g}(t)]^T$  where  $p^i(t)$  denotes the



**Fig. 3** Illustration of how goal confidences change upon motion along control dimensions. The three colored lines along each dimension represent the confidences associated with a goal of corresponding color. Brighter and dimmer colors correspond to high and low confidences respectively. One can see that motion along certain control dimensions results in sharper rise in confidences for some goals compared to others, suggesting that the goal disambiguation capabilities are different for different control dimensions.

probability associated with goal  $g^i$ . The probability  $p^i(t)$  represent the robot’s *confidence* that goal  $g^i$  is the human’s intended goal.

Let  $\mathcal{K}$  be the set of all controllable dimensions of the robot and  $k^i$  represent the  $i^{th}$  control dimension where  $i \in [1, 2, \dots, n_k]$ . The cardinality of  $\mathcal{K}$  is denoted as  $n_k$  and typically depends on the robotic platform used. For example, for a smart wheelchair  $n_k = 2$ , since the controllable degrees-of-freedom are velocity and heading. Likewise, for a six DoF robotic arm  $n_k = 6$ , since the controllable degrees-of-freedom correspond to the translation and rotation of the end-effector in the task-space.

In this work, we assume a kinematic model for the robot and the kinematic state (the robot pose) at any time  $t$  is denoted as  $\mathbf{x}_r(t) \in \mathbb{R}^3 \times \mathbb{S}^3$  and consists of a position component and orientation component.<sup>1</sup> The pose for goal  $g \in \mathcal{G}$  is denoted as  $\mathbf{x}_g \in \mathbb{R}^3 \times \mathbb{S}^3$ . The control command issued by the human via the control interface is denoted as  $\mathbf{u}_h$  and is mapped to the Cartesian velocity of the robot’s end-effector. For a six DoF robotic arm,  $\mathbf{u}_h \in \mathbb{R}^6$ . We also assume the existence of an autonomous robot control policy that generates a robot control command denoted as  $\mathbf{u}_r \in \mathbb{R}^6$ . The control command issued to the robot, which is a synthesis of  $\mathbf{u}_h$  and  $\mathbf{u}_r$  (described further in Section 5) is denoted

<sup>1</sup> $\mathbb{S}^3$  is the space of all unit quaternions.

by  $\mathbf{u} \in \mathbb{R}^6$ . We also denote the control command that corresponds to a unit velocity vector along the positive and negative directions of control dimension  $k$  as  $\mathbf{e}^k$  and  $-\mathbf{e}^k$  respectively.

The limitations of the control interfaces necessitate the control space  $\mathcal{K}$  to be partitioned into control modes. Let  $\mathcal{M}$  denote the set of all control modes with  $n_m = |\mathcal{M}|$ . Additionally, let  $m^i$  refer to the  $i^{\text{th}}$  control mode where  $i \in [1, 2, \dots, n_m]$ . Each control mode  $m^i$  is a subset of  $\mathcal{K}$  such that  $\bigcup_{i=1}^{n_m} m^i$  spans all of the controllable dimensions. A dimension  $k \in \mathcal{K}$  can be an element of multiple control modes.

### 3.3 Disambiguation Metric

The disambiguation metric that we develop in this paper is a heuristic measure that characterizes the intent disambiguation capabilities of a control dimension  $k \in \mathcal{K}$  and is denoted by  $D_k \in \mathbb{R}$ . We explicitly define disambiguation metrics for both positive negative motions along  $k$  as  $D_k^+$  and  $D_k^-$  respectively. We also define a disambiguation metric  $D_m \in \mathbb{R}$  for each control mode  $m \in \mathcal{M}$ .

The disambiguation metric  $D_m$  is a measure of how useful the user control commands would be for the robot to perform more accurate intent inference if the user were to operate the robot in control mode  $m$ . Both  $D_k$  and  $D_m$  will be formally defined in Section 3.5. The computation of  $D_k$  depends on four features (denoted as  $\Gamma_k, \Omega_k, \Lambda_k$  and  $\Upsilon_k$ ), that capture different aspects of the shape of a projection of the probability distribution over intent. These projections and computations are described in detail in Section 3.4 and Section 3.5, and as a pseudocode in Algorithm 1. The disambiguation formalism developed in this section is agnostic to the particular form of intent inference. However, the algorithm assumes that  $\mathbf{p}(t)$  can be forward projected in time by iteratively applying the intent inference algorithm.

### 3.4 Forward Projection of $\mathbf{p}(t)$

The first step towards the computation of  $D_k$  is the forward projection of the probability distribution  $\mathbf{p}(t)$  from the current time  $t_a$  to  $t_b$  and  $t_c$  ( $t_a < t_b < t_c$ ), Algorithm 1, lines 3-13. We consider two projection times ( $t_b$  and  $t_c$ ) in order to compute short-term and long-term behavior and evolution of the probability distribution. Application of control command  $\mathbf{e}^k$  results in probability distributions  $\mathbf{p}_k^+(t_b)$ ,  $\mathbf{p}_k^+(t_c)$  and  $-\mathbf{e}^k$  results in  $\mathbf{p}_k^-(t_b)$  and  $\mathbf{p}_k^-(t_c)$ , where the subscript  $k$  denotes the fact that the projection is the result of the application of

---

#### Algorithm 1 Intent Disambiguation

---

**Require:**  $\mathbf{p}(t_a), \mathbf{x}_r(t_a), \Delta t, t_a < t_b < t_c, \Theta$

```

1: for  $k = 0 \dots n_k$  do
2:   Initialize  $D_k = 0, t = t_a$ 
3:   while  $t \leq t_c$  do
4:      $\mathbf{p}_k(t + \Delta t) \leftarrow \text{UpdateIntent}(\mathbf{p}_k(t), \mathbf{u}_h; \Theta)$ 
5:      $\mathbf{x}_r(t + \Delta t) \leftarrow \text{SimulateKinematics}(\mathbf{x}_r(t), \mathbf{u}_h)$ 
6:     if  $t = t_b$  then
7:       Compute  $\Gamma_k, \Omega_k, \Lambda_k$ 
8:     end if
9:     if  $t = t_c$  then
10:      Compute  $\Upsilon_k$ 
11:    end if
12:     $t \leftarrow t + \Delta t$ 
13:  end while
14:  Compute  $D_k$ 
15: end for
```

---

a control command only along control dimension  $k$ . Note that Algorithm 1 is run twice to compute the projected probability distributions for  $\mathbf{e}^k$  and  $-\mathbf{e}^k$ . All parameters and features which affect the computation of  $\mathbf{p}(t)$  are denoted as  $\Theta$ .

### 3.5 Features of $D_k$

Informed by our initial empirical investigations, we hand-design four features that encode different aspects of the shape of the probability distribution as it evolves under user control in a specific control dimension  $k$ . For each control dimension  $k$ , each of the four features is computed for projections along both positive and negative directions independently. The four features are computed in lines 7 and 10 in Algorithm 1.

1) *Maximum probability:* The maximum of the projected probability distribution  $\mathbf{p}_k(t_b)$  is a good measure of the robot's *overall certainty* in accurate predicting human intent (The maximum of this discrete probability distribution is the mode of the distribution). We define the distribution maximum as  $\Gamma_k$ .

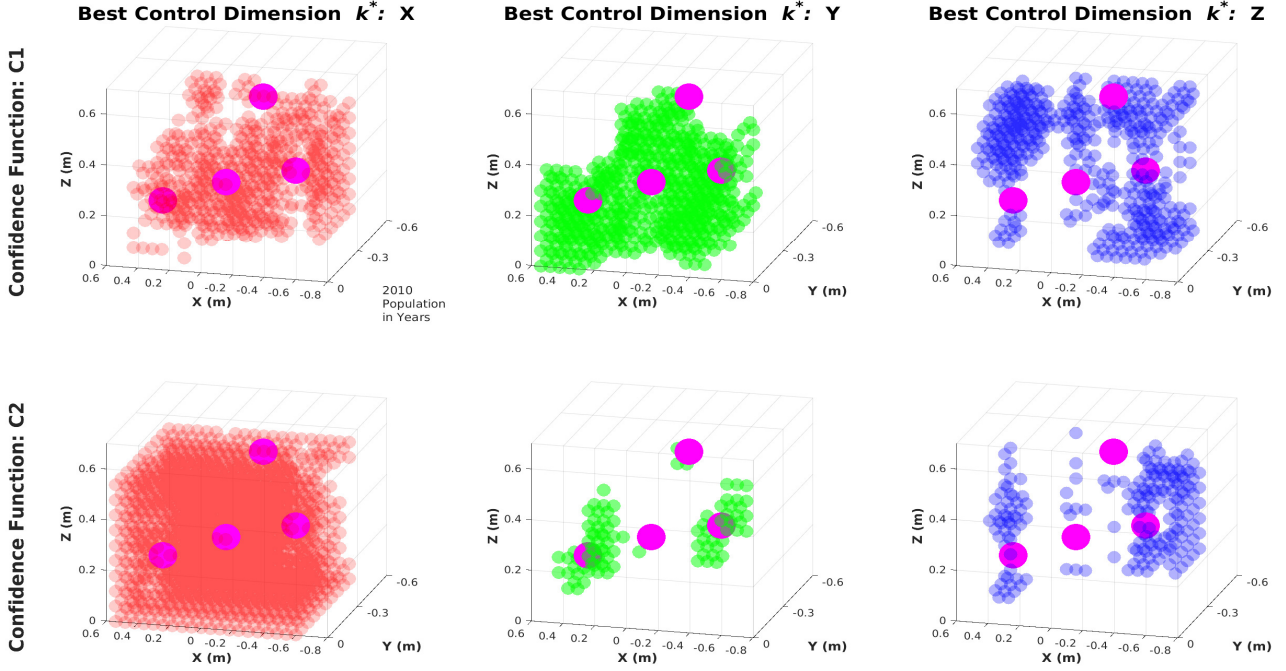
$$\Gamma_k = \max_{1 \leq i \leq n_g} p_k^i(t_b) \quad (1)$$

A higher value implies that the robot has a greater confidence in its prediction of the human's intended goal.

2) *Difference between largest probabilities:* Disambiguation accuracy benefits from greater differences between the first and second most probable goals. This difference is denoted as  $\Omega_k$ .

$$\Omega_k = \max(\mathbf{p}_k(t_b)) - \max(\mathbf{p}_k(t_b) \setminus \max(\mathbf{p}_k(t_b))) \quad (2)$$

$\Omega_k$  becomes important in situations in which the distribution can have multiple modes and a measure of



**Fig. 4** Control dimensions best able to disambiguate intent. Left column:  $k^*$  is X. Middle Column:  $k^*$  is Y. Right Column:  $k^*$  is Z. Magenta spheres indicate the goal locations (intent). In this example, the goals are spread maximally along the  $x$  and  $z$  dimensions, and so inference happens more quickly if the human control commands are along  $x$  or  $z$ . We see that  $x$  and  $z$  are chosen more often as the most disambiguating dimensions when using intent inference function C2 (bottom row). Function C2 considers the instantaneous directedness of the human’s control command towards that goal, while inference function C1 (top row) encodes only proximity to a given goal. Function C2 is considered to encode more information about the human’s intent than C1, with the result of stronger inference power—which is inherently linked to the disambiguation power of our algorithm. Further details in (Gopinath and Argall 2017).

overall certainty ( $\Gamma_k$ ) alone is not sufficient for successful disambiguation.

3) *Pairwise separation of probabilities*: If the difference between the largest probabilities fails to disambiguate, then the separation,  $\Lambda_k$ , in the remaining goal probabilities will further aid in intent disambiguation. The quantity  $\Lambda_k$  is computed as the *sum of the pairwise distances* between the  $n_g$  probabilities.

$$\Lambda_k = \sum_{i=1}^{n_g} \sum_{j=i}^{n_g} |p_k^i(t_b) - p_k^j(t_b)| \quad (3)$$

4) *Gradients*:  $\Gamma_k$ ,  $\Omega_k$  and  $\Lambda_k$  are local measures that encode shape characteristics of the short-term temporal projections of the probability distribution over goals. However, the quantity  $p_k(t)$  can undergo significant changes upon long-term continuation of motion along control dimension  $k$ . The spatial gradient of  $p_k(t)$  encodes this propensity for change and is approximated by

$$\frac{\partial p_k(t)}{\partial x_k} \simeq \frac{p_k(t_c) - p_k(t_b)}{x_k(t_c) - x_k(t_b)}$$

where  $x_k$  is the component of robot’s projected displacement along control dimension  $k$ . The greater the difference between individual spatial gradients, the greater will the probabilities deviate from each other, thereby helping in disambiguation. In order to quantify the “spread” of gradients we define a quantity  $\Upsilon_k$

$$\Upsilon_k = \sum_{i=1}^{n_g} \sum_{j=i}^{n_g} \left| \frac{\partial p_k^i(t)}{\partial x_k} - \frac{\partial p_k^j(t)}{\partial x_k} \right| \quad (4)$$

where  $|\cdot|$  denotes the absolute value.

5) *Putting it all together*: The individual features  $\Gamma_k$ ,  $\Omega_k$ ,  $\Lambda_k$  and  $\Upsilon_k$  are then combined to compute  $D_k$  in such a way that, by design, higher values of  $D_k$  imply greater disambiguation capability for the control dimension  $k$ . More specifically,  $D_k$  is denoted as

$$D_k = \underbrace{w \cdot (\Gamma_k \cdot \Omega_k \cdot \Lambda_k)}_{\text{short-term}} + \underbrace{(1 - w) \cdot \Upsilon_k}_{\text{long-term}} \quad (5)$$

where  $w$  is a task-specific weight that balances the contributions of the short-term and long-term components (in our implementation,  $w$  was set at 0.5 empirically).



Equation 5 actually is computed twice, once in each of the positive ( $e^k$ ) and negative directions ( $-e^k$ ) along  $k$ , and the results ( $D_k^+$  and  $D_k^-$ ) are then summed.

The computation of  $D_k$  is performed for each control dimension  $k \in \mathcal{K}$ . The disambiguation metric  $D_m$  for control mode  $m$  then is calculated as

$$D_m = \sum_k D_k$$

where  $k \in m$  iterates through the set of control dimensions on which  $m$  is able to operate. The control mode with highest disambiguation capability  $m^*$  is given by

$$m^* = \operatorname{argmax}_m D_m$$

while  $k^* = \operatorname{argmax}_k D_k$  gives the control dimension with highest disambiguation capability  $k^*$ . Disambiguation mode  $m^*$  is the mode that the algorithm chooses for the human to better estimate their intent.

## 4 Intent Inference

This section describes the intent inference scheme used in this paper. Our preliminary work (Gopinath and Argall 2017) revealed that the power of our disambiguation algorithm proposed in Section 3.3 is intimately linked with the inference power of different choices of intent inference mechanisms. In this work, we propose a novel intent inference scheme inspired by *dynamic field theory*. We detail this approach in Section 4.1 and provide in Section 4.2 a qualitative baseline comparison to an inference formulation from shared-control literature.

### 4.1 Intent inference based on Dynamic Field Theory

Our novel intent inference approach is inspired by *dynamic field theory*, in which the time evolution of the probability distribution  $p(t)$  is specified as a dynamical system with constraints. Unlike Bayesian approaches used in shared-control teleoperation systems, our approach does not assume continuous user input (Dragan and Srinivasa 2013; Pellegrinelli et al 2016); this is important especially in the domain of assistive robotics in which user input is typically intermittent due to inherent motor or cognitive limitations. By casting the time-evolution of the distribution as a dynamical system, information from the past states can be easily incorporated (for example, via a low-pass filter that smooths the evolution). Furthermore, Bayesian approaches require a model for how the user operates the robot (the likelihood function) and typically this model assumes that the user optimizes a cost function according to the

maximum entropy principle (Javdani et al 2017; Ziebart et al 2008). Our intent inference approach does not require any such assumptions and allows for suboptimal human performance. The critical piece in the dynamical system is a ‘forcing function’ that drives the evolution of the distribution to different states, the design of which is informed by what features of the human input capture their underlying intent most effectively.

Section 4.1.1 provides a primer on the basic principles and features of *dynamic field theory* and its application in the fields of neuroscience and cognitive robotics. Section 4.1.2 describes our novel formulation that makes use of dynamic field theory for the purposes of intent inference.

#### 4.1.1 Dynamic Field Theory

In Dynamic Field Theory (DFT) (Schöner and Spencer 2015), variables of interest are treated as dynamical state variables. To represent the information about these variables requires two dimensions: one which specifies the value the variables can attain (the domain) and the other which encodes the *activation level* or the amount of information about a particular value. These *activation fields* are analogous to probability distributions defined over a random variable.

Following Amari’s formulation (Amari 1977) dynamics of an activation field  $\phi(x, t)$  are given by

$$\tau \dot{\phi}(x, t) = -\phi(x, t) + h + S(x, t) + \int dx' b(x - x') \sigma(\phi(x', t))$$

where  $x$  denotes the variable of interest,  $t$  is time,  $\tau$  is the time-scale parameter,  $h$  is the constant resting level, and  $S(x, t)$  is the external input,  $b(x - x')$  is the interaction kernel and  $\sigma(\phi)$  is a sigmoidal nonlinear threshold function. The interaction kernel mediates how activations at all other field sites  $x'$  drive the activation level at  $x$ . Two types of interactions are possible: excitatory (when interaction is positive) which drives up the activation, and inhibitory (when the interaction is negative) which drives the activation down.

Dynamic neural fields possess some unique characteristics that make them ideal candidates for modeling higher-level cognition. First, a peak in the activation field can be *sustained* even in the absence of external input due to the recurrent interaction terms. Second, information from the past can be *preserved* over much larger time scales quite easily by tuning the time-scale parameter thereby endowing the fields with memory. Third, the activation fields are *robust* to disturbance

and noise in the external output (Schöner 2008). As a result, DFT principles have found widespread application in the area of cognitive robotics (Erlhagen and Bicho 2006), specifically in the contexts of efficient human-robot interaction (Erlhagen and Bicho 2014), robotic scene representation (Zibner et al 2011), obstacle avoidance and target reaching behaviors in both humans and robots (Schöner et al 1995), and for object learning and recognition (Faubel and Schöner 2008).

#### 4.1.2 Dynamic Neural Fields for Intent Inference

Recurrent interaction between the state variables, robustness to noise and inherent memory make dynamic neural fields an ideal candidate for intent inference.

Our insight is to use the framework of dynamic neural fields to specify the time evolution of the probability distribution  $\mathbf{p}(t)$ , in which we treat the individual goal probabilities  $p^i(t)$  as constrained dynamical state variables such that  $p^i(t) \in [0, 1]$  and  $\sum_1^{n_g} p^i(t) = 1$ . The dynamical system can be generically written as

$$\dot{\mathbf{p}}(t) = F(\mathbf{p}(t), \mathbf{u}_h; \Theta)$$

where  $F$  represents a nonlinear vector field,  $\mathbf{u}_h$  is the human control input and  $\Theta$  represents all other task-relevant features and parameters that affect the time-evolution of the probability distribution. The full specification of the neural field is given by

$$\frac{\partial \mathbf{p}(t)}{\partial t} = \frac{1}{\tau} \left[ -\mathbb{I}_{n_g \times n_g} \cdot \mathbf{p}(t) + \underbrace{\frac{1}{n_g} \cdot \mathbb{1}_{n_g}}_{\text{rest state}} \right] + \underbrace{\lambda_{n_g \times n_g} \cdot \sigma(\xi(\mathbf{u}_h; \Theta))}_{\text{excitatory + inhibitory}} \quad (6)$$

where time-scale parameter  $\tau$  determines the memory capacity of the system,  $\mathbb{I}_{n_g \times n_g}$  is the identity matrix,  $\mathbb{1}_{n_g}$  is a vector containing all ones of dimension  $n_g$ ,  $\lambda$  is the control matrix that controls the excitatory and inhibitory aspects,  $\xi$  is a function that encodes the nonlinearity through which human control commands and task features affect the time evolution, and  $\sigma$  is a biased sigmoidal nonlinearity given by  $\sigma(\xi) = \frac{1}{1+e^{-\xi}} - 0.5$ .

The design of  $\xi$  is informed by what features of the human control input and environment capture the human's underlying intent most effectively. For example, motion cues reveal a great deal about the underlying intention and have been successfully used in different intent inference schemes (Barrett et al 2005). We rely on the *directedness* of the robot motion towards a goal, the *agreement* between the human commands

and robot autonomy and the *proximity* to a goal. With  $\Theta = \{\mathbf{x}_r, \mathbf{x}_{g^i}, \mathbf{u}_{r,g^i}\}$ , one dimension  $i$  of  $\xi$  is defined as

$$\xi^i(\mathbf{u}_h; \mathbf{x}_r, \mathbf{x}_{g^i}, \mathbf{u}_{r,g^i}) = \underbrace{\frac{1+\eta}{2}}_{\text{directedness}} + \underbrace{\mathbf{u}_h^{\text{rot}} \cdot \mathbf{u}_{r,g^i}^{\text{rot}}}_{\text{agreement}} + \underbrace{\max\left(0, 1 - \frac{\|\mathbf{x}_{g^i} - \mathbf{x}_r\|}{R}\right)}_{\text{proximity}}$$

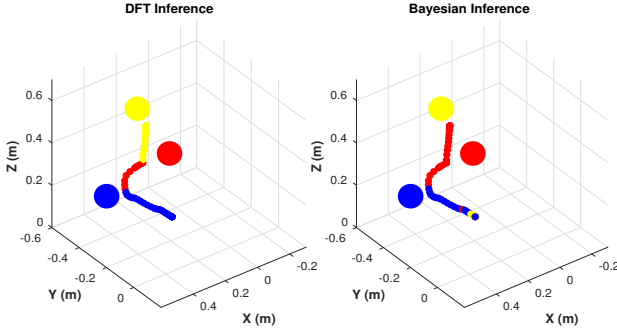
where  $\eta = \frac{\mathbf{u}_h^{\text{trans}} \cdot (\mathbf{x}_{g^i} - \mathbf{x}_r)^{\text{trans}}}{\|\mathbf{u}_h^{\text{trans}}\| \|(\mathbf{x}_{g^i} - \mathbf{x}_r)^{\text{trans}}\|}$ ,  $\mathbf{u}_{r,g^i}$  is the robot autonomy command for reaching goal  $g^i$ , *trans* and *rot* refer to the translational and rotational components of a command  $\mathbf{u}$  or position  $\mathbf{x}$ ,  $R$  is the radius of the sphere beyond which the proximity component is always zero, and  $\|\cdot\|$  is the Euclidean norm. The *directedness* component captures the likelihood of a user taking the shortest straight line path towards a goal  $g$  (equivalent to a user optimizing a straight line distance based cost function), whereas the *agreement* measures the alignment of the human's input to the autonomy command in the end-effector's rotation space and serves as an indicator of how the human cooperates with autonomy.

Given the initial probability distribution at time  $t_a$  Equation 6 can be solved numerically from  $t \in [t_a, t_b]$  using a simple Euler algorithm with a fixed time-step  $\Delta t$ . At every time-step the constraints on  $p^i(t)$  are enforced thereby ensuring that  $\mathbf{p}(t)$  is a valid probability distribution. The most confident goal  $g^*$  is computed as  $g^* = \operatorname{argmax}_i p^i(t)$ .

#### 4.2 Baseline Comparison

In this section we provide a qualitative baseline comparison of our proposed intent inference method against a Bayesian inference scheme that is used in shared autonomy in teleoperation (Dragan and Srinivasa 2012a; Admoni and Srinivasa 2016; Javdani et al 2017). Under the Bayesian approach, the user is typically modeled as a Markov Decision Process and is assumed to noisily optimize a cost function according to the maximum entropy principle (Ziebart et al 2008).

A few fundamental differences between the Bayesian scheme and our dynamical field theory based approach are as follows. First, the DFT approach does not assume continuous input from the user and in the absence of user input the in-built 'forgetting' mechanism causes the distribution to gradually settle down to a uniform distribution. Secondly, unlike the Bayesian approach the DFT approach does not rely on an explicit user model and therefore can handle suboptimal user input seamlessly. Third, the Bayesian approach is made tractable



**Fig. 5** Goal inference using our dynamic field theory (DFT) approach (left) and the Bayesian approach Dragan and Srinivasa (2013) (right). Goals shown as circles. The motion trajectory is colored according to the goal prediction made at that location during the execution. The human operator first reaches towards the blue goal, then the red and finally the yellow.

using a Laplacian approximation of the cost function around an optimal trajectory; however for trajectories that deviate a great deal from the optimal trajectory the second-order approximation is less accurate.

Figure 5 shows an illustrative comparison of goal inference using our dynamic field theory based approach and the Bayesian approach using a distance based cost function. In this example, the user’s intended goal changes during the execution: first reaching towards the blue goal, then switching to the red and finally settling on the yellow goal. We can see that the two inference approaches are comparable for the first two segments of the trajectory. For the final segment, the Bayesian inference method is unable to produce the correct inference due to the posterior distribution collapsing to a delta function. However, the dynamic neural field based approach is successful in producing the correct inference at all times. The posterior collapse to a delta function in Bayesian inference is primarily due to numerical errors and can be avoided by adding some amount of noise to the prior at every time-step, however then this becomes yet another heuristic for practical success.

In general, both inference approaches consistently produce similar performances for a wide variety of goal configurations. For goal poses that include orientation information, the Bayesian approach will work only if the cost function also depends on the ‘distance’ traversed in the orientation space. The inference accuracy is highly dependent on the choice of the user model (the likelihood function) for Bayesian schemes, and on  $\xi$  for the DFT based approach.

## 5 Shared Control

The shared-control paradigm implemented on our robot is a blending-based approach in which the final control command issued to the robot is a linear composition of the human control command and an autonomous robot policy. The autonomous control policy generates control command  $\mathbf{u}_r \leftarrow f_a(\mathbf{x}_r)$  where  $f_a(\cdot) \in \mathcal{F}_a$ , and  $\mathcal{F}_a$  is the set of all control behaviors corresponding to different tasks. This set could be derived using a variety of techniques such as *Learning from Demonstrations* (Argall et al 2009; Schaal 1997), motion planners (Hsu et al 2002; Ratliff et al 2009) or navigation functions (Rimon and Koditschek 1992; Tanner et al 2003). Specifically, let  $\mathbf{u}_{r,g}$  be the autonomy command associated with goal  $g$ . The final control command  $\mathbf{u}$  issued to the robot then is given by

$$\mathbf{u} = \alpha \cdot \mathbf{u}_{r,g^*} + (1 - \alpha) \cdot \mathbf{u}_h$$

where  $g^*$  is the most confident goal. Similar to  $\mathbf{u}_h$ , the autonomy command  $\mathbf{u}_{r,g^*}$  is mapped to the 6D Cartesian velocity of the end-effector,  $\mathbf{u}_{r,g^*} \in \mathbb{R}^6$ . The robot state evolves according to the kinematic equation given by,

$$\mathbf{x}_r(t + \Delta t) = f_{kin}(\mathbf{x}_r(t), \mathbf{u}(t))$$

where,  $f_{kin}(\cdot)$ , denotes the nonlinear kinematics of the robot and  $\Delta t$  is the Euler time-step. The blending factor  $\alpha$  is a piecewise linear function of the probability  $p(g^*)$  associated with  $g^*$  and is given by

$$\alpha = \begin{cases} 0 & p(g^*) \leq \rho_1 \\ \frac{\rho_3(p(g^*) - \rho_1)}{\rho_2 - \rho_1} & \text{if } \rho_1 < p(g^*) \leq \rho_2 \\ \rho_3 & p(g^*) > \rho_2 \end{cases}$$

with  $\rho_i \in [0, 1] \forall i \in [1, 2, 3]$  and  $\rho_2 > \rho_1$ . In our implementation, we empirically set  $\rho_1 = \frac{1.2}{n_g}$ ,  $\rho_2 = \frac{1.4}{n_g}$  and  $\rho_3 = 0.7$ .

The robot control command  $\mathbf{u}_{r,g}$  is generated using a simple potential field which is defined in all parts of the state space (Khatib 1986). Every goal  $g$  is associated with a potential field  $\gamma_g$  which treats  $g$  as an attractor and all the other goals in the scene as repellers. For potential field  $\gamma_g$ , the attractor velocity is given by

$$\dot{\mathbf{x}}_r^{attract} = \mathbf{x}_g - \mathbf{x}_r$$

where  $\mathbf{x}_g$  is the location of goal  $g$ .<sup>2</sup> The repeller velocity is given by

$$\dot{\mathbf{x}}_r^{repel} = \sum_{i \in \mathcal{G} \setminus g} \frac{\mathbf{x}_r - \mathbf{x}_{g^i}}{\mu(\|\mathbf{x}_r - \mathbf{x}_{g^i}\|^2)}$$

<sup>2</sup>In position space, the ‘-’ operator computes the difference between the goal position and current robot position in  $\mathbb{R}^3$ . In orientation space, the ‘-’ operator computes the *quaternion difference* between the goal orientation and the current robot orientation.

where  $\dot{\mathbf{x}}_r$  indicates the velocity of the robot in the world frame,  $\mu$  controls the magnitude of the repeller velocity and  $\|\cdot\|$  is the Euclidean norm. The autonomy command is computed as a summation of these attractor and repeller velocities.

$$\mathbf{u}_{r,g} = \dot{\mathbf{x}}_r^{attract} + \dot{\mathbf{x}}_r^{repel}$$

$\gamma_g$  operates in the full six dimensional Cartesian space, and treats position and orientation as independent potential fields.

## 6 Study Methods

In this section, we describe the study methods used to evaluate the efficacy of the disambiguation system.

### 6.1 Participants

For this study eight subjects were recruited (mean age:  $31 \pm 11$ , 3 males and 5 females). All participants gave their informed, signed consent to participate in the experiment, which was approved by Northwestern University's Institutional Review Board.

### 6.2 Hardware

The experiments were performed using the MICO 6-DoF robotic arm (Kinova Robotics, Canada), specifically designed for assistive purposes. The software system was implemented using the Robot Operating System (ROS) and data analysis was performed in MATLAB.

The subjects teleoperated the robot using two different control interfaces: a 2-axis joystick and a switch-based head array. The control signals captured from the interfaces were mapped to the Cartesian velocities of the end-effector (Figure 7).

The joystick generated continuous control signals and allowed for control of a maximum of two dimensions at a time. The 6-D control space was partitioned into four

control modes that could be accessed using the buttons on the joystick interface.

The switch-based head array consisted of three switches embedded in the headrest and generated 1-D discrete signals. The switch at the back of the headrest was used to cycle between the different control modes, and the switches to the left and right controlled the motion of the robot's end effector in the positive and negative directions along the dimension corresponding to the selected control mode. All switches were operated via head movements. An external button was provided to request the mode switch assistance. For both control interfaces the gripper had a dedicated control mode.

#### 6.2.1 Switching Paradigms

Two kinds of mode switching paradigms were evaluated in the study. Note that the blending assistance was always active for both paradigms. Under the blending paradigm, the amount of assistance was directly proportional to the probability of the most confident goal  $g^*$ , and thus to the strength of the intent inference. Therefore, if intent inference improved as a result of goal disambiguation, more assistance would be provided by the robot. All trials started in a randomized initial control mode and robot position.


*Manual:* During task execution the user performed all mode switches.

*Disambiguation:* The user additionally could request a disambiguation mode switch at any time during task execution. Upon disambiguation request, the algorithm identified and switched the current control mode to the best disambiguating mode  $m^*$ . The user was required to request disambiguation at least once during the task execution.

### 6.3 Study Protocol

A within-subjects study was conducted using a fractional factorial design in which the manipulated variables were the tasks, control interfaces and the switching paradigm conditions. Each subject underwent an initial training period that lasted approximately thirty minutes after which the subject performed both tasks using both interfaces under the *Manual* and *Disambiguation* paradigms. The trials were balanced and the control interfaces and the paradigms were randomized and counterbalanced across all subjects to avoid ordering effects. Three trials were collected for the *Manual* paradigm and five trials for the *Disambiguation* paradigm.

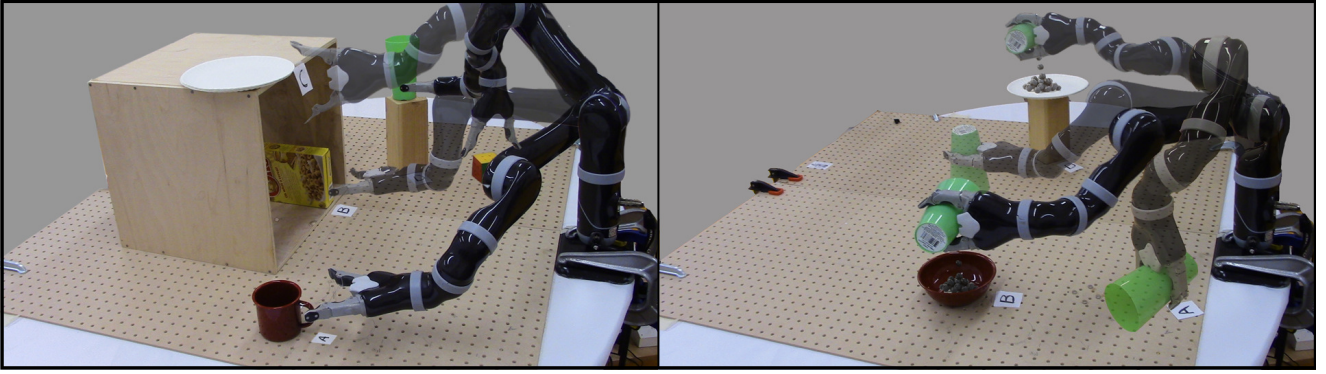
**Training:** The training period consisted of three phases and two different task configurations. The subjects used both interfaces to perform the training tasks.



Control Mappings		
Mode	Head Array	Joystick
1	$v_x$	$v_x, v_y$
2	$v_y$	$v_x, v_z$
3	$v_z$	$\omega_z, \omega_y$
4	$\omega_z$	$\omega_x$
5	$\omega_y$	—
6	$\omega_x$	—

**Fig. 7** A 2-axis joystick (left) and switch-based head array (center) and their operational paradigms (right).  $v$  and  $\omega$  indicate the translational and rotational velocities of the end-effector, respectively.





**Fig. 6** Study tasks performed by subjects. *Left*: Single-step reaching task. *Right*: Multi-step Pouring task.

**Phase One:** The subjects were asked to perform a simple reaching motion towards a single goal in the scene. This phase was intended for the subjects to get familiarized with the control interface mappings and teleoperation of the robotic arm.

**Phase Two:** In the second phase of training, the blending-based shared autonomy was introduced. The subjects experienced how the autonomy could provide assistance during the task execution. The subjects were informed that the robot autonomy would be present for all trials of the experiment.

**Phase Three:** For the third phase of the training, multiple objects were introduced in the scene. Subjects were explicitly informed by the experimenter that upon a disambiguation request the robot would select a control mode that would help the robot figure out which goal the subject was going for and thereby enable it to assist the user more effectively. Subjects were able to explore this request feature during a reaching task, and were instructed to move as much as s/he can in the control mode chosen by the robot and observe the effects of the mode switch request and subsequent change in robot assistance.

**Testing:** Two different task types were evaluated during testing. Both control interfaces were employed, for all tasks.

**Single-step:** The task is to reach one of five objects on the table, each with a target orientation (Figure 6, Left).

**Multi-step:** Each trial began with a full cup held by the robot gripper. The task required first that the contents of the cup be poured into one of two containers and then that the cup be placed at one of the two specified locations and with a specific orientation (Figure 6, Right).

**Metrics:** The objective metrics used for evaluation include the following. *Number of mode switches* refers to the number of times a user switched between various control modes during task execution. *Number of disambiguation requests* refers to the number of times user

pressed the button requesting disambiguating assistance. *Number of button presses* is the sum of *Number of mode switches* and *Number of disambiguation requests*, and is also an indirect measure of user effort. *Onset of robot assistance* refers to the earliest time at which blending assistance to became active. We also characterize the temporal distribution of disambiguation requests using a skewness measure.

At the end of each testing phase, subjective data was gathered via a brief questionnaire. Users were given statements about the usefulness and capability of the assistance system to rate on a 7-point Likert Scale according to their agreement. The questions primarily concerned whether the disambiguating control modes helped to make task execution easier (the utility value of the assistance system), and user preference about and perception of the disambiguation system and the different control interfaces.

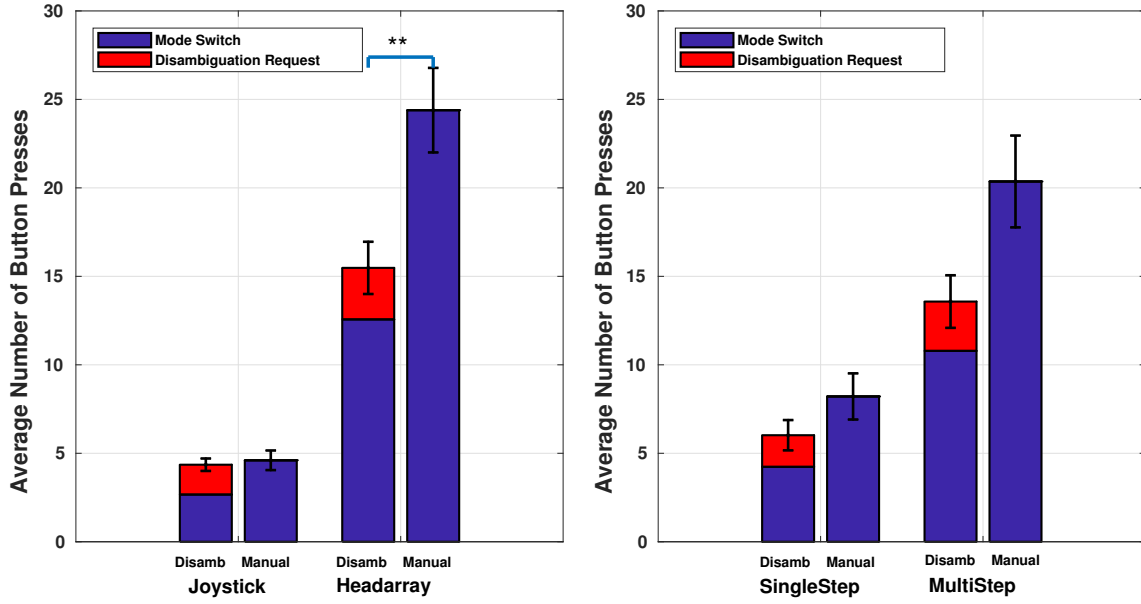
## 7 Results

Here we report the results of our subject study. Statistical significance was determined by the Wilcoxon Rank-Sum test in where (\*\*\*) indicates  $p < 0.001$ , (\*\*)  $p < 0.01$  and (\*)  $p < 0.05$ .

### 7.1 Impact of Disambiguation on Task Performance

Figure 8 presents the number of button presses under each mode switching paradigm, for both interfaces and tasks.

A statistically significant decrease in the number of button presses was observed between the *Manual* and *Disambiguation* paradigms when using the headarray (Figure 8, Left). Due to the low-dimensionality of headarray and cyclical nature of mode switching, the number of button presses required for task completion is inherently high. That the disambiguation paradigm



**Fig. 8** Comparison of average number of button presses between *Disambiguation* and *Manual* Paradigms. *Left*: Grouped by control interfaces. *Right*: Grouped by tasks.

was helpful in reducing the number of button presses likely is due to higher robot assistance in the disambiguating control mode and therefore reduced the need for subsequent user-initiated mode switches.

For joystick, statistically significant differences were observed for the number of manual mode switches between the two paradigms ( $p < 0.05$ , not pictured in Figure 8). However, the gain due to the reduction of user-initiated mode switches was offset by the button presses that were required for disambiguation requests.

A general trend (although not statistically significant) of a decrease in the number of button presses was also observed for the more complex multi-step task (Figure 8, Right).

These results suggest that disambiguation is more useful as the control interface becomes more limited and the task becomes more complex. *Intuitively, intent prediction becomes harder for the robot when the control interface is low-dimensional and does not reveal a great deal about the user’s underlying intent. By having the users operate the robot in the disambiguating mode, the robot is able to elicit more intent-expressive control commands from the human which in turn helps in accurate goal inference and subsequently appropriate robot assistance.*

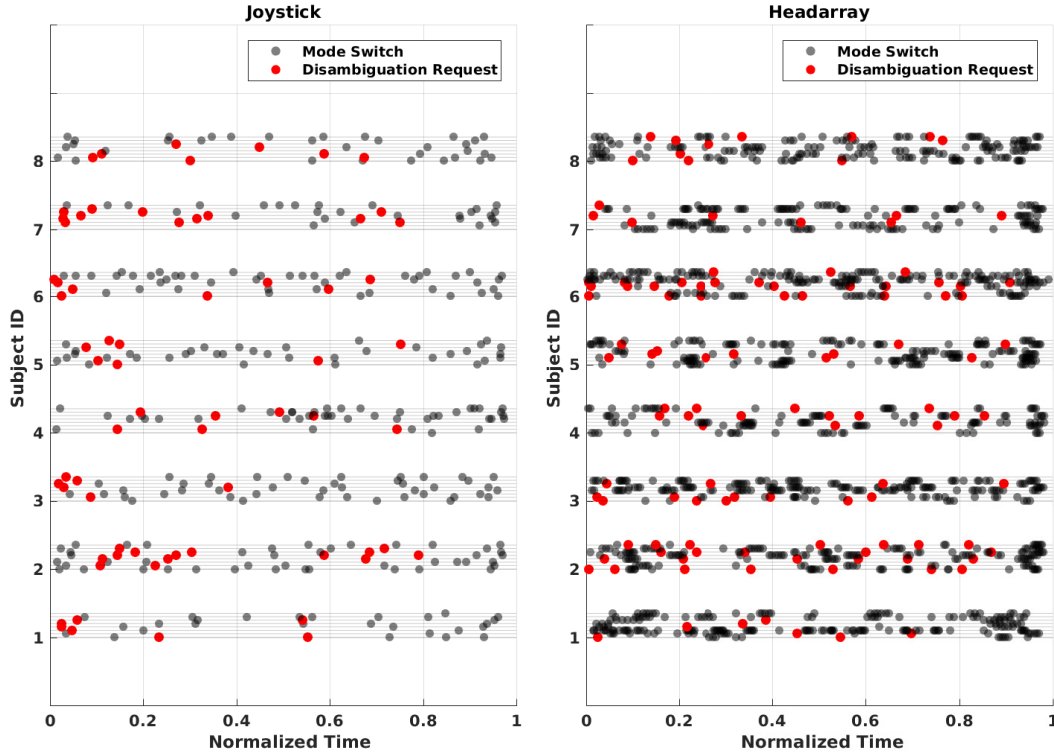
**Table 1** Characterization of the temporal distribution of disambiguation requests. The values in the table denote the deviation of the temporal distribution from a uniform distribution. Smaller values mean more concentrated and earlier disambiguation requests.

	Single Step	Multi Step
Joystick	0.63	0.57
Headarray	0.35	0.22

## 7.2 Temporal Distribution of Disambiguation Requests

We also observed similar trends in the *temporal distribution* of disambiguation requests, where a higher number of disambiguation requests correlates with the more limited interface and complex task. The temporal distribution of disambiguation requests refers to *when* the subject requested assistance during the course of a trial. We use a measure of *skewness* to characterize how much the temporal distribution deviates from a uniform distribution.<sup>3</sup> Larger positive values of skewness correlate with more concentrated and earlier disambiguation requests. Earlier disambiguation requests is possibly due to the fact that the subjects wanted progress towards the goal to happen right at the start of the trial itself. Table 1 reports the skewness of the temporal distribution

<sup>3</sup>A uniform temporal distribution corresponds to a trial in which the disambiguation requests are uniformly spread out during the course of task execution. The skewness of a uniform distribution is 0.



**Fig. 9** Temporal pattern of button presses for each interface and the multi-step task on a trial-by-trial basis for all subjects. Eight trials per subject per interface/task combination.

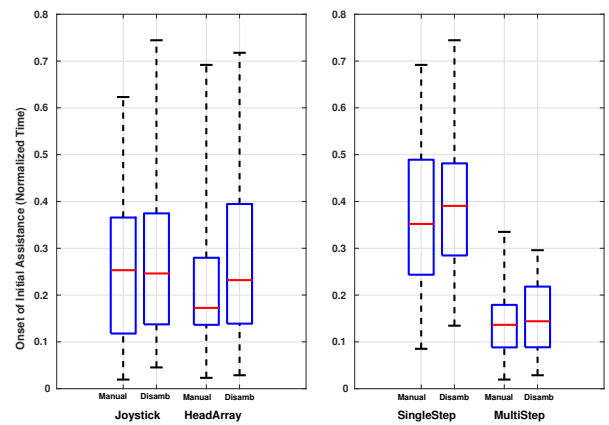
of disambiguation requests for different interface-task combinations.

Figure 9 shows the temporal pattern of disambiguation requests and mode switches for the multi-step task on a trial-by-trial basis for all subjects. From the figure it is clear that the frequency and density of button presses (disambiguation requests plus mode switches) are much higher for the more limited control interface. The subjects also demonstrated a diverse range of disambiguation request behavior, for example in regards to both when during the execution requests were made and with what frequency (e.g. Subject 8 versus Subject 2, Joystick). The variation between subjects is likely due to different factors such as the user’s comfort in operating the robot and understanding the ability of the disambiguating mode to recruit more assistance from the autonomy.

### 7.3 Onset of Robot Assistance

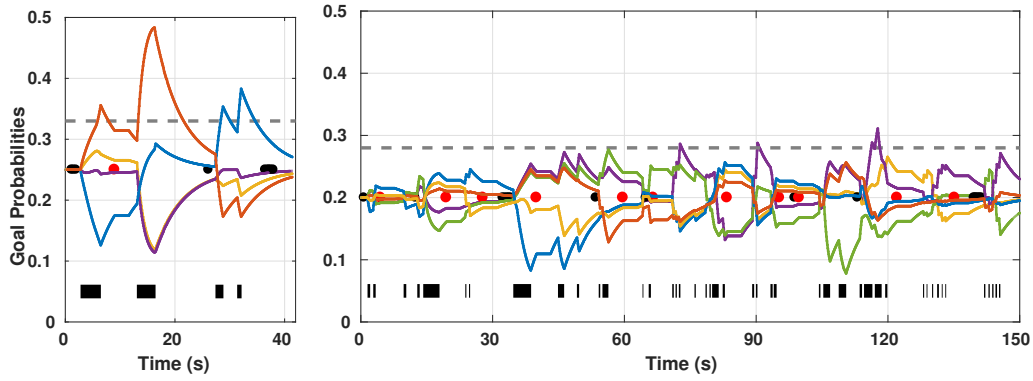
One of the motivations for developing the disambiguation system was that more accurate intent inference (as a result of intent disambiguation) should allow the autonomy to step in earlier more during the course of

task execution. However, our results did not reveal any differences between the two switching paradigms across tasks or across interfaces (Figure 10).



**Fig. 10** Onset of robot assistance normalized with respect to task completion time. *Left:* Across interfaces. *Right:* Across tasks.

One likely contributing factor is that disambiguation frequently was impaired by subjects choosing not to



**Fig. 11** Time evolution of goal probabilities. *Top*: Multi-step task. *Bottom*: Single-step task. The gray horizontal line above denotes the minimum threshold for robot assistance. The black bars at the bottom denote non-zero human control commands. The red and black dots indicate button presses that are disambiguation requests and mode switches respectively.

operate in the disambiguation mode after making a disambiguation request. The disambiguation power of a control mode depends entirely on the observation of motion within that mode, and so in such scenarios disambiguation is effectively rendered inert.

This phenomenon is illustrated in Figure 11. When control commands are issued, they are indicated by the black bars on the bottom of the plots. Disambiguation requests are shown as red dots, and mode switches as black dots. In the right plot, we see multiple instances of disambiguation requests being followed by no control commands or only very brief control commands and as a result the robot does not receive enough *intent-expressive* cues from the user and the goal probability barely crosses the threshold for providing autonomy assistance. However, in the left plot, operation in the disambiguating mode quickly elevates one goal probability above the threshold for providing autonomy assistance, and after the assistance kicks in the subject elects to stop issuing control commands.

#### 7.4 User Survey

Users rated the utility value of the system fairly high (mean =  $5.52 \pm 1.07$ ) indicating that the users thought that the control modes chosen by the system made task execution easier. Subjects on average also indicated that they liked operating the robot in the disambiguating control modes (mean  $5.00 \pm 1.15$ ). A majority of subjects (6 of 8) also preferred the disambiguation paradigm to the manual scheme, possibly due to a reduction in task effort as a result of operating the robot in the disambiguating modes. However, only 4 of 8 subjects considered the disambiguation system to be user-friendly. Likely reasons are that the user had to initiate the

disambiguation request, and that the system was not transparent about *how* their button press helped the robot figure out which goal. In the former case, an automated mode switching scheme might improve user satisfaction and friendliness; and in the latter, including transparency as an objective of in the training period.

## 8 Discussion

The disambiguation algorithm presented in our work can be utilized in any human-robot system in which there is a need to disambiguate between the different states of a discrete hidden variable can assume (for example, a discrete set of goals in robotic manipulation or a set of landmarks in navigation tasks). Our algorithm also assumes the existence of a discrete set of parameters (for example, control modes for robotic manipulation or natural language-based queries for navigation) that can help the intent inference mechanism to precisely converge to the correct inference. Although the disambiguation algorithm is task-agnostic—because it relies exclusively on the shape features of the probability distribution over the hidden variable—the disambiguation is only as good as the efficacy of the intent inference algorithm that is used for as specific task. In our experience, it becomes important that the choice of cost functions and domain-specific heuristics is appropriate for the task at hand. Furthermore, the efficacy of the disambiguation algorithm degraded when we used only a subset of the four features to inform the disambiguation metric. This only reinforces the need for a combination of different shape features for successful disambiguation.

In a *help me, help you* type of human robot system, task execution becomes seamless and more efficient when there is a sound mutual understanding of how the other



party operates. One observation from our subject study was how often participants submitted a disambiguation request, and then chose not to operate in the selected mode—effectively not letting the robot help them. This highlights the need for greater transparency in human-robot interaction so that the human has a clear picture of *how* and *why* the robot chooses to help the user in certain ways. A lack of understanding of how they might help the robot to help them might have resulted in the under-utilization of the disambiguation feature. In order to provide *intent-expressive* control commands to the robot, very likely knowledge of the assistance mechanism is critical. **Important to note is that this work was not about having the system select a control mode that the human wants to use to reach a target (which might be motivated by a wide variety of factors such as line of sight or ease of use).** In spite of this, subject responses to the user survey indicated that majority of the subjects liked operating the robot in the disambiguating modes and felt that task execution became easier as a result.

Therefore, the need for extensive and thorough training becomes apparent. The training can be made more effective in a few different ways. First, online feedback of the robot’s intent prediction at all times during training can likely help the subject gain a better understanding of the relationship between the characteristics of their control actions (sparsity, aggressiveness, persistence) and the robot’s assistive behavior. Second, the subjects could be explicitly informed of the task relevant features (directedness, proximity *et cetera*) that the robot relies on for determining the amount of assistance. Knowledge of these features might motivate the users to leverage the disambiguating mode.

The inherent time delays associated with the computation of the disambiguating mode (approximately 2-2.5s) might have been a discouraging factor and a cause for user frustration. The algorithm could be used to pre-compute a large set of most informative modes for different parts of the workspace, for different goal configurations and for different priors ahead of time, which then might be used a lookup table during task execution. Furthermore, metamodeling techniques and machine learning tools can be used to learn generalizable models that will be effective in previously unseen goal configurations.

In the present system, there is task effort associated with requesting disambiguation assistance which can discourage the users from utilizing the paradigm. Automated mode switching schemes can possibly eliminate the need for button presses for disambiguation requests. We also identify an opportunity to have adaptive assistance paradigms that explicitly take into account the characteristics of the user’s control behavior. Some users

are timid in their operation of the robot whereas some others are more aggressive and confident. Some are more comfortable operating the robot manually and do not seek assistance, whereas some others rely on assistance more frequently. Individual user characteristics could be extracted from training and be used for tuning the parameters of the intent inference engine and the shared control system to maximize robustness and efficacy of the assistive system. This would also likely improve user satisfaction and result in higher user acceptance.

## 9 Conclusion

In this paper, we have presented an algorithm for *intent disambiguation assistance* with a shared-control robotic arm using the notion of *inverse legibility*. The goal of our algorithm is to elicit more *intent-expressive* control commands from the user by placing control in those control modes that *maximally disambiguate* between the various goals in the scene. As a secondary contribution, we also present a novel intent inference mechanism inspired by *dynamic field theory* that works in conjunction with the disambiguation system. Pilot user study was conducted with eight subjects to evaluate the efficacy of the disambiguation system. Our results indicate a decrease in task effort in terms of the number of button presses when disambiguation system employed.

In our future work, as informed by our pilot study, we plan to extend the framework into an automated mode switch assistance system. A more extensive user study with motor-impaired subjects will also be conducted in the future to further evaluate the utility of the disambiguation system and explore the disambiguation request patterns of users.

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