Identification of control modes that maximizes goal disambiguation using Fisher Information

Context: The subject performs reaching tasks using an assistive robotic manipulator. The set of discrete goals is denoted by $\mathcal{G}$ with $n_g = |\mathcal{G}|$. Our aim is to identify those control dimensions/modes that will help the robot to disambiguate between the different goals and thereby help the robot perform better intent inference.
Intent inference: In order for the assistive robot to provide the right kind of assistance, it needs to have a good idea of what the user's underlying intentions are; or in other words, the robot needs to know which one of the n_g discrete goals is the user's intended goal. Furthermore, we assume that there is one and only one intended goal and is denoted by g^*.
Intent estimation: Intent inference essentially boils down to estimating $P(g|\boldsymbol{u_h}, \boldsymbol{x_r})$. The problem can be formally described in terms of POMDPs, in which a prior over goals and likelihood of user actions can be assumed. Bayesian inference techniques can be used to estimate the posterior distribution over goals to estimate $P(g|\boldsymbol{u_h}, \boldsymbol{x_r})$.

$$P(g|\boldsymbol{u_h}, \boldsymbol{x_r}) = \eta P(\boldsymbol{u_h}|g, \boldsymbol{x_r})P(g)$$

where η is a normalization factor.

Alternately, we could rely on heuristic methods based on *confidence functions* to estimate intent. Different types of confidence functions can be used. One choice of confidence function is a "directedness" based confidence function which measures the directedness of the user's control command towards a particular goal location. The higher the confidence associated with a goal, the more the system is confident that the goal is indeed the user's intended goal. Confidence functions can be interpreted as a "proxy" for $P(g|\boldsymbol{u_h}, \boldsymbol{x_r})$.

Another interpretation of a confidence function is that it is like a "virtual sensor" whose measurements contain information regarding the user's intended goal.
Control dimensions/modes: Due to the dimensionality mismatch between the robotic device and the control device used to control these devices, the entire control space is partitioned into smaller subsets called *control modes*.

In the most restricted cases (head arrays and sip n' puffs) the control modes correspond to the control dimensions themselves (1D interfaces). In such cases, the modes can be characterized by the control commands that can be generated along the corresponding control dimension.
Maximizing goal disambiguation: Given a particular form of intent estimation technique (Bayesian or confidence functions), certain control commands (or equivalently control dimensions) issued *by* the human will be *more* useful *for* the robot in performing goal disambiguation and therefore inferring the human's intended goal. Our aim is to develop a metric that will estimate the "goodness" of being in a control dimension (and as a result of issuing control commands along those control dimensions).

In our RSS 2017 work, we formulate a disambiguation metric that characterizes the disambiguation capability of each control dimension/mode. A mode switch assistance scheme is developed utilizing this metric in which the control mode with maximum disambiguation capability is chosen *for* the human upon assistance request. The hypothesis is that subsequent operation of the robot in the optimal control mode with "help" the robot to perform better intent inference and will result in appropriate kinds of assistance behavior which will in turn improve task related performance.

Fisher Information for characterizing control modes: The disambiguation metric developed in our previous work was completely ad hoc and heuristic. We hope to rely on more theoretically sound measures to characterize the disambiguation capability of a control mode. To this end, we rely on the Fisher Information metric.

Fisher information quantifies the ability of a random variable (in this case the confidence measure) to estimate an unknown parameter (which is the position of the intended goal). This idea is directly related to the approach taken by Silverman et al. and Miller et al., in which Fisher Information is used within the context of optimal control for information acquisition. In their work, the expected value of the Fisher Information with respect to the probability density function of the unknown parameter is computed (the positions of the intended goal) and is known as the *Expected Information Density* (EID). In our work we use EID to characterize the "information content" contained in a control mode in estimating the unknown parameter.

Casting the problem in the space of u_h

We restrict ourselves to the 2D scenario for ease of illustration. For convenience, we can assume that the control interface is 1D and can only generate unit magnitude control commands. A single point in the 2D space of u_h corresponds to a specific control command issued by the human. All points on the unit circle in this space is the set of all points whose magnitude is 1. The intersection of the unit circle and the x-y axes denote set of control commands available in control mode 1 and 2 respectively. For the purposes of disambiguation, magnitude is irrelevant and therefore can be fixed at unity.

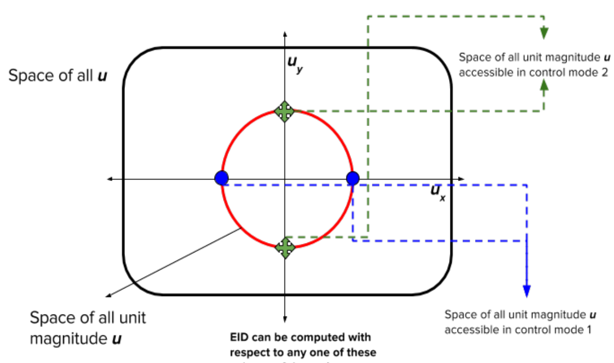

Figure 1: Illustration of subspaces of control command space.

Imagine that the virtual sensor that measures the "entropy of confidence distributions" is moving in the space of u_h. In some parts of the space, the sensor readings will peak and in others they will not. This is akin to the electric field sensor getting close to the true spatial location of the goal in Miller et al.

Math Formalism

The (negative of) entropy of the confidence distribution is the measurement model and can be written as,

$$e = \Upsilon(\boldsymbol{u_h}; \boldsymbol{x_r}, \boldsymbol{x_{g_1}}, \ldots, \boldsymbol{x_{g_n}}) + \delta, \quad \delta \sim \mathcal{N}(0, \sigma^2)$$

Let $\boldsymbol{x_r}, \boldsymbol{x_{g_1}}, \ldots, \boldsymbol{x_{g_n}}$ be denoted by $\boldsymbol{\Theta}$. The above equation then simplifies to

$$e = \Upsilon(\boldsymbol{u_h}; \boldsymbol{\Theta}) + \delta, \quad \delta \sim \mathcal{N}(0, \sigma^2)$$

where Υ is the entropy (negative of) of the confidence distribution and is a function of robot position and user control command. The robot and goal positions are treated as fixed parameters. Using a directedness based confidence function, individual confidences are given as

$$c_{g^i} = \frac{1 + cos(\theta)}{2}$$

where

$$cos(\theta) = \frac{\boldsymbol{u_h} \cdot (\boldsymbol{x_{g^i}} - \boldsymbol{x_r})}{\|\boldsymbol{u_h}\| \cdot \|\boldsymbol{x_{g^i}} - \boldsymbol{x_r}\|}$$

and θ is the angle between the control command vector and the vector joining the current robot position and the goal g^i. Normalizing the confidences across the goals will result in a "proxy" probability distributions over goal confidences. The probabilities are computed as

$$P(g^i) = \eta c_{g^i}, \quad \eta = \Sigma_{i=1}^{n_g} c_{g^i}$$

η is the normalization constant.

Entropy of this distribution over goals is defined as

$$H(g) = -\Sigma_{i=1}^{n_g} \, P(g^i) \, log[P(g^i)]$$

and

$$\Upsilon(\boldsymbol{u_h}; \boldsymbol{\Theta}) = -H(g) = -(-\Sigma_{i=1}^{n_g} \, P(g^i) \, log[P(g^i)])$$

It can be seen that $H(g)$ is a function of $\boldsymbol{u_h}$, given the robot position and goal positions (treated as fixed parameters). Given a set of goals and the form of the confidence function, entropy is a scalar defined for each robot position and user control command.

Fisher Information Metric

$$\mathcal{I}_{i,j}(\boldsymbol{u_h}; \boldsymbol{\Theta}) = \frac{1}{\sigma^2} \frac{\partial^2 \Upsilon(\boldsymbol{u_h}; \boldsymbol{\Theta})}{\partial u_h^i u_h^j}$$

In order to compute the expected Fisher information density over different subspaces of the control command space we can compute the expectation of the above quantity over any subspace of interest $\mathcal{U}$. Each subspace (set of points) corresponds to a control mode. In the 2D case, with 1D control modes each mode contains two points; one corresponding to positive and negative unit magnitude control commands along the dimension of interest.

$$\Phi^{\mathcal{U}}_{i,j}(\boldsymbol{\Theta}) = \frac{1}{\sigma^2}\int_{\boldsymbol{u_h}\in\mathcal{U}} \frac{\partial^2 \Upsilon(\boldsymbol{u_h};\boldsymbol{\Theta})}{\partial u_h^i u_h^j} p(\boldsymbol{u_h}) d\boldsymbol{u_h}$$

Note that the above integral is a multidimensional integral, but has been written using a "vector" notation for convenience sake.

Illustration of computation with 1D interface

We will assume that only unit magnitude control commands are possible with the interface.

Control mode 1 ($\mathcal{U}_1$) corresponds to the set $[\{1,0,0\},\{-1,0,0\}]$, control mode 2 ($\mathcal{U}_2$) corresponds to the set $[\{0,1,0\},\{0,-1,0\}]$ and control mode 3 ($\mathcal{U}_3$) corresponds to the set $[\{0,0,1\},\{0,0,-1\}]$. Let $p(\boldsymbol{u_h}) = 0.5$ for all $\boldsymbol{u_h}$.

Computation of $\Phi^{\mathcal{U}_1}_{i,j}$:

$$\Phi^{\mathcal{U}_1}_{i,j}(\boldsymbol{\Theta}) = \frac{1}{\sigma^2}\Big[\left.\frac{\partial^2 \Upsilon(\boldsymbol{u_h};\boldsymbol{\Theta})}{\partial u_h^i u_h^j}\right|_{\boldsymbol{u_h}=\{1,0,0\}} p(\{1,0,0\}) + \left.\frac{\partial^2 \Upsilon(\boldsymbol{u_h};\boldsymbol{\Theta})}{\partial u_h^i u_h^j}\right|_{\boldsymbol{u_h}=\{-1,0,0\}} p(\{-1,0,0\})\Big]$$

Computation of $\Phi^{\mathcal{U}_2}_{i,j}$:

$$\Phi^{\mathcal{U}_2}_{i,j}(\boldsymbol{\Theta}) = \frac{1}{\sigma^2}\Big[\left.\frac{\partial^2 \Upsilon(\boldsymbol{u_h};\boldsymbol{\Theta})}{\partial u_h^i u_h^j}\right|_{\boldsymbol{u_h}=\{0,1,0\}} p(\{0,1,0\}) + \left.\frac{\partial^2 \Upsilon(\boldsymbol{u_h};\boldsymbol{\Theta})}{\partial u_h^i u_h^j}\right|_{\boldsymbol{u_h}=\{0,-1,0\}} p(\{0,-1,0\})\Big]$$

$$\Phi^{\mathcal{U}_3}_{i,j}(\boldsymbol{\Theta}) = \frac{1}{\sigma^2}\Big[\left.\frac{\partial^2 \Upsilon(\boldsymbol{u_h};\boldsymbol{\Theta})}{\partial u_h^i u_h^j}\right|_{\boldsymbol{u_h}=\{0,0,1\}} p(\{0,0,1\}) + \left.\frac{\partial^2 \Upsilon(\boldsymbol{u_h};\boldsymbol{\Theta})}{\partial u_h^i u_h^j}\right|_{\boldsymbol{u_h}=\{0,0,-1\}} p(\{0,0,-1\})\Big]$$

The expected information density for each control mode can then be computed as

$$EID(\mathcal{U}_n) = det(\bar{\Phi}^{\mathcal{U}_n}(\boldsymbol{\Theta}))$$

where $n \in [1,2,3]$ and $\bar{\Phi}$ refers to the 3 by 3 Fisher Information Matrix.

For example, if EID of $\mathcal{U}_1$ is greater than EID of $\mathcal{U}_2$ it implies that control mode 1 has greater disambiguation capability.

Preliminary insights and issues

This formulation seems to work the best compared to all others. There are certain numerical issues (divide by zero et cetera) that are problematic and are dealt with by making approximations to the control commands. The approximations do have an impact on the results, but they are minor I believe.

Following are some of the "point cloud" plots which depicts the best control mode for different points in the workspace. Control mode 1 (red) controls the X axis, mode 2 (green) controls the Y axis and mode 3 (blue) controls the Z axis. The big black dots refer to the goal location in the 3D space.

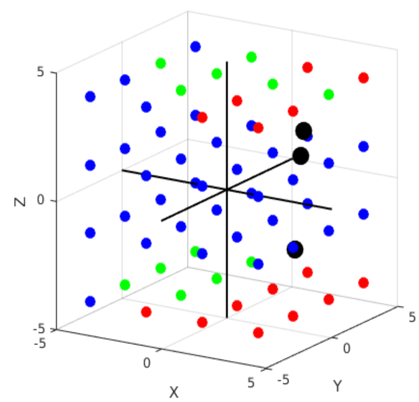

Figure 2: Disambiguating control modes - 3D

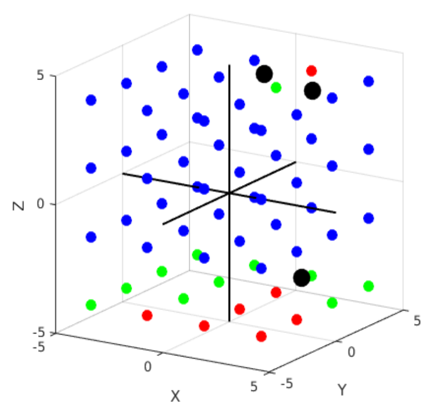

Figure 3: Disambiguating control modes - 3D

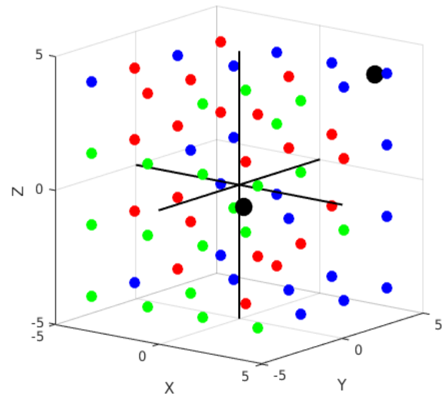

Figure 4: Disambiguating control modes - 3D

The results are intuitive and the algorithm is able to identify the mode with maximum disambiguation. For Figures 1 and 2 the goals are spread maximally along the z dimension and therefore intuitively the control dimension with maximum disambiguation is z. The algorithm was able to identify z as the best mode in much of the workspace. Note that the notion of disambiguation is not explicitly introduced in this formalism. Instead the quantity of interest is the negative entropy of the confidence distribution. The negative of entropy will be higher if the confidence distribution is 'peakier' around a particular goal and lower if the distribution is close to uniform. Maximum disambiguation occurs when the distribution is focused around a particular goal or in other words when the negative of the entropy is maximized.

Secondly, there are no simplifying assumptions made in this formalism. The disambiguation capability of a control mode is contained in the FI integral that is computed over the entire subspace that corresponds to a control mode.

An interactive GUI application has been made which lets the user move the robot around in 3D space and recompute the best disambiguating control mode repeatedly. The goal positions may also be resampled.

References

1. Silverman, Y., L. M. Miller, M. A. MacIver, and T. D. Murphey, "Optimal Planning for Information Acquisition", IROS 2013, 2013.
2. Miller, L. M., and T. D. Murphey, "Optimal Planning for Target Localization and Coverage Using Range Sensing", IEEE Int. Conf. on Automation Science and Engineering (CASE): 2015, pp. 501-508, 2015.