Identification of control modes that maximizes goal disambiguation using Fisher Information

Context: The subject performs reaching tasks using an assistive robotic manipulator. The set of discrete goals is denoted by $\mathcal{G}$ with $n = |\mathcal{G}|$. Our aim is to identify those control dimensions/modes that will help the robot to disambiguate between the different goals and thereby help the robot perform better intent inference.
Intent inference: In order for the assistive robot to provide the right kind of assistance, it needs to have a good idea of what the user's underlying intentions are; or in other words, the robot needs to know which one of the n discrete goals is the user's intended goal. Furthermore, we assume that there is one and only one intended goal and is denoted by g^*.
Intent estimation: Intent inference essentially boils down to estimating $P(g|\boldsymbol{u_h}, \boldsymbol{x_r})$. The problem can be formally described in terms of POMDPs, in which a prior over goals and likelihood of user actions can be assumed. Bayesian inference techniques can be used to estimate the posterior distribution over goals to estimate $P(g|\boldsymbol{u_h}, \boldsymbol{x_r})$.

$$P(g|\boldsymbol{u_h}, \boldsymbol{x_r}) = \eta P(\boldsymbol{u_h}|g, \boldsymbol{x_r})P(g)$$

where η is a normalization factor.

Alternately, we could rely on heuristic methods based on *confidence functions* to estimate intent. Different types of confidence functions can be used. One choice of confidence function is a "directedness" based confidence function which measures the directedness of the user's control command towards a particular goal location. The higher the confidence associated with a goal, the more the system is confident that the goal is indeed the user's intended goal. Confidence functions can be interpreted as a "proxy" for $P(g|\boldsymbol{u_h}, \boldsymbol{x_r})$.

Another interpretation of a confidence function is that it is like a "virtual sensor" whose measurements contain information regarding the user's intended goal.
Control dimensions/modes: Due to the dimensionality mismatch between the robotic device and the control device used to control these devices, the entire control space is partitioned into smaller subsets called *control modes*.

In the most restricted cases (head arrays and sip n' puffs) the control modes correspond to the control dimensions themselves (1D interfaces). In such cases, the modes can be characterized by the control commands that can be generated along the corresponding control dimension.
Maximizing goal disambiguation: Given a particular form of intent estimation technique (Bayesian or confidence functions), certain control commands (or equivalently control dimensions) issued *by* the human will be *more* useful *for* the robot in performing goal disambiguation and therefore inferring the human's intended goal. Our aim is to develop a metric that will estimate the "goodness" of being in a control dimension (and as a result of issuing control commands along those control dimensions).

In our RSS 2017 work, we formulate a disambiguation metric that characterizes the disambiguation capability of each control dimension/mode. A mode switch assistance scheme is developed utilizing this metric in which the control mode with maximum disambiguation capability is chosen *for* the human upon assistance request. The hypothesis is that subsequent operation of the robot in the optimal control mode with "help" the robot to perform better intent inference and will result in appropriate kinds of assistance behavior which will in turn improve task related performance.

Fisher Information for characterizing control modes: The disambiguation metric developed in our previous work was completely ad hoc and heuristic. We hope to rely on more theoretically sound measures to characterize the disambiguation capability of a control mode. To this end, we rely on the Fisher Information metric.

Fisher information quantifies the ability of a random variable (in this case the confidence measure) to estimate an unknown parameter (which is the position of the intended goal). This idea is directly related to the approach taken by Silverman et al. and Miller et al., in which Fisher Information is used within the context of optimal control for information acquisition. In their work, the expected value of the Fisher Information with respect to the probability density function of the unknown parameter is computed (the positions of the intended goal) and is known as the *Expected Information Density* (EID). In our work we use EID to characterize the "information content" contained in a control mode in estimating the unknown parameter.

Scenario: We will restrict to a 2D reaching task for the sake of illustration.

The robot is a 2D robot that can perform reaching tasks. The control interface has 2 modes: corresponding to the x and y dimensions. The user can initiate motion along any dimension (one at a time). There are n discrete goals in the scene.

The set of goals are characterized by their position in space and is denoted by $\{\boldsymbol{x_{g_1}}, \boldsymbol{x_{g_2}}, \dots, \boldsymbol{x_{g_n}}\}$. Let $\boldsymbol{x_r}$ denote the end-effector position with respect to the world frame and $\boldsymbol{u_h}$ denote the user control command. Let c_g denote the confidence measure associated with a goal g. We can interpret this confidence measure as a sensor reading.

Aim: Given all the goal locations, the prior distribution over goals and the robot position what we are interested in the *expected information density* of a control dimension. That is, how much information can be gained by moving in a particular control dimension regarding the location of the intended goal ($\boldsymbol{x_{g^*}}$).

Mathematical Formalism: The unknown parameter that we are trying to estimate is the location of the intended goal denoted by $\boldsymbol{x_{g^*}}$. The "measurement" model for the confidence "sensor" that captures the confidence of the intended goal, g^*, is given by

$$c_{g^*} = \Upsilon(\boldsymbol{x_{g^*}}, \boldsymbol{x_r}, \boldsymbol{u_h}) + \delta, \quad \delta \sim \mathcal{N}(0, \sigma^2)$$

We assume that our measurement model is stochastic. The noise is modeled as a zero-mean Gaussian distribution with a predefined variance. (Note that since this sensor is purely a mathematical construct and therefore the variance is artificially introduced into the picture by the system designer). Assuming a "directedness" based confidence formulation we have

$$\Upsilon(\boldsymbol{x_{g^*}}, \boldsymbol{x_r}, \boldsymbol{u_h}) = \frac{1 + cos(\theta)}{2} \quad \in [0, 1]$$

where

$$cos(\theta) = \frac{\boldsymbol{u_h} \cdot (\boldsymbol{x_{g^*}} - \boldsymbol{x_r})}{\|\boldsymbol{u_h}\| \cdot \|\boldsymbol{x_{g^*}} - \boldsymbol{x_r}\|}$$

The unknown parameter is the location of the intended goal g^* and this could be any one of the n goals from the set $\mathcal{G}$.

Since c_{g^*} is distributed as a Gaussian random variable (by definition), the Fisher information (the amount of information a "measurement" has regarding $\boldsymbol{x_{g^*}}$) is given by

$$\mathcal{I}_{i,j}(\boldsymbol{x_{g^*}}, \boldsymbol{u_h}, \boldsymbol{x_r}) = \frac{1}{\sigma^2} \frac{\partial^2 \Upsilon(\boldsymbol{x_{g^*}}, \boldsymbol{x_r}, \boldsymbol{u_h})}{\partial x^i_{g^*} x^j_{g^*}}$$

Since $\boldsymbol{x_{g^*}}$ is a 2D parameter, the Fisher Information is a 2 by 2 matrix and not a scalar. Since the intended goal could be any one of the goals from $\mathcal{G}$ we take expectation over $\boldsymbol{x_{g^*}}$. Therefore,

$$\Phi_{i,j}(\boldsymbol{u_h}, \boldsymbol{x_r}) = \frac{1}{\sigma^2} \int_{x^i_{g^*}} \int_{x^j_{g^*}} \frac{\partial^2 \Upsilon(\boldsymbol{x_{g^*}}, \boldsymbol{x_r}, \boldsymbol{u_h})}{\partial x^i_{g^*} x^j_{g^*}} p(x^i_{g^*}, x^j_{g^*}) dx^j_{g^*} dx^i_{g^*}$$

The priors over the goals are only defined at discrete points in the domain, since the goal locations themselves are discrete. Therefore,

$$p(\boldsymbol{x_{g^*}}) = \begin{cases} p_i & for\ \boldsymbol{x_{g^*}} = \boldsymbol{x_{g_i}} \\ 0 & otherwise \end{cases}$$

such that $\sum_{i=1}^{n} p_i = 1$.
Marginalizing over $\boldsymbol{x_r}$

$$\Phi_{i,j}(\boldsymbol{u_h}) = \int_{\boldsymbol{x_r}} \left[\frac{1}{\sigma^2} \int_{x^i_{g^*}} \int_{x^j_{g^*}} \frac{\partial^2 \Upsilon(\boldsymbol{x_{g^*}}, \boldsymbol{x_r}, \boldsymbol{u_h})}{\partial x^i_{g^*} x^j_{g^*}} p(x^i_{g^*}, x^j_{g^*}) dx^j_{g^*} dx^i_{g^*} \right] p(\boldsymbol{x_r}) d\boldsymbol{x_r}$$

(The integral with respect to $\boldsymbol{x_r}$ is also a double integral, but is written vectorially due to space constraints.)

We assume perfect knowledge of where the robot is. Therefore the probability density function for $\boldsymbol{x_r}$ is given by

$$p(\boldsymbol{x_r}) = \begin{cases} 1 & \boldsymbol{x_r} = \boldsymbol{x_{true}} \\ 0 & otherwise \end{cases}$$

Therefore $\Phi_{i,j}(\boldsymbol{u_h})$ reduces to

$$\Phi_{i,j}(\boldsymbol{u_h}) = \frac{1}{\sigma^2} \int_{x^i_{g^*}} \int_{x^j_{g^*}} \frac{\partial^2 \Upsilon(\boldsymbol{x_{g^*}}, \boldsymbol{x_r}, \boldsymbol{u_h})}{\partial x^i_{g^*} x^j_{g^*}} p(x^i_{g^*}, x^j_{g^*}) dx^j_{g^*} dx^i_{g^*} \bigg|_{\boldsymbol{x_r} = \boldsymbol{x_{true}}}$$

The EID for $\boldsymbol{u_h}$ is then computed as

$$EID(\boldsymbol{u_h}) = \det(\bar{\Phi}(\boldsymbol{u_h}))$$

The EID for control command along x dimension can be computed by summing up $EID([1,0])$ and $EID([-1,0])$ and similarly for y dimension can be computed by the summation of $EID([0,1])$ and $EID([0,-1])$.

Simulation Results

Simulations were done in MATLAB to understand what kind of information $\Phi(\boldsymbol{u_h})$ encoded. The number of goals were varied from 2 to 4. EID was calculated for x and y dimensions.

The EID was consistently higher for that control dimension which would result in an increase of confidences for **ALL** goals. The system does not know which is the user's intended goal. It assigns a probability for each one of the n goals to be the intended goal (the prior). The EID is computed by taking the expectation over all possible intended goals. Higher EID will be for that control command (control dimension) which would result in a higher confidence regardless of which one of the n goals the user chose as the intended goal.

However, this is not exactly what we are looking for as this is not goal disambiguation. In order for the system to be able to disambiguate it should be able to identify dimensions along with confidence related to the true intended goal rises and all other confidences get suppressed.

Possible solutions

Instead of using the confidence in the intended goal as the "virtual sensor" we could possibly use the negative of entropy of the confidence distribution as the sensor readings. By doing so, the control dimension which will maximize this quantity will have higher EID. I am still trying to work out the details for this now.

References

1. Silverman, Y., L. M. Miller, M. A. MacIver, and T. D. Murphey, "Optimal Planning for Information Acquisition", IROS 2013, 2013.
2. Miller, L. M., and T. D. Murphey, "Optimal Planning for Target Localization and Coverage Using Range Sensing", IEEE Int. Conf. on Automation Science and Engineering (CASE): 2015, pp. 501-508, 2015.