Lab Session 2

MA- 423 : Matrix Computations July - November, 2019 S. Bora

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1. Write a function program `[L,U,p] = gepp(A)` to find a unit lower triangular matrix `L`, an upper triangular matrix `U` and a column vector `p` satisfying `A(p,:) = LU` via Gaussian Elimination with Partial Pivoting (GEPP) as explained in the theory class.

2. Write a function program `x = geppsolve(A,b)` to solve a system $Ax = b$ via GEPP. Your program should call the program `[L,U,p] = gepp(A)` and the programs written in previously for solving upper and lower triangular systems. Compare your answers with that of the MATLAB command $A \setminus b$.

3. Given $A \in \mathbb{R}^{n\times n}$, if P be a permutation matrix such that $PA = LU$ is an LU factorization of PA, then the determinant of A satisfies $\det(A) = \sigma\Pi_{i=1}^{n} u_{ii}$ where $u_{ii}, i = 1, 2, \ldots, n$ are the diagonal entries of U and $\sigma = 1$ or $\sigma = -1$ depending upon whether P is a product of an even or odd number of transpositions respectively.

   Use this fact to write a function program `d = mydet(A)` to compute the determinant of A. Verify that your program costs $\frac{2}{3}n^3$ flops for $A \in \mathbb{R}^{n\times n}$ as against at least $n!$ flops via the conventional formula for the $\det(A)$.

4. Write a function program `R` $=$ `mychol(A)` that *recursively* finds the Cholesky factor R of a positive definite matrix A using the outer product formulation of the Cholesky Decomposition of A. Your program should use $n^3/3 + O(n^2)$ flops if A is of size n and exit with an error message if A is not positive definite.