Lab Session 8

MA-423: Matrix Computations July-November, 2019 S. Bora

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1. Write a function program $[\texttt{iter}, \texttt{lambda}] = \texttt{Powermethod}(\texttt{A}, \texttt{x}, \texttt{k})$ that performs k iterations of the Power Method with $\texttt{A} \in \mathbb{C}^{\texttt{n}\times\texttt{n}}$ and initial vector $\texttt{x} \in \mathbb{C}^{\texttt{n}}$ and returns an $n \times k$ matrix `iter` whose jth column is the jth iterate q_j and a scalar `lambda` as the dominant eigenvalue.

2. Run $[\texttt{iter}, \texttt{lambda}] = \texttt{Powermethod}(\texttt{A}, \texttt{x}, \texttt{k})$ with $x = [1\ 1\ 1]^T$ and the following matrices

$$(i)\ A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 9 & 2 \\ 0 & -1 & 2 \end{bmatrix} \qquad (ii)\ A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 9 & 2 \\ -4 & -1 & 2 \end{bmatrix} \qquad (iii)\ A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & 2 \\ -4 & -1 & 2 \end{bmatrix}$$

   In each case check if for large enough values of j, $\|\texttt{iter}(:, j+1) - v\| / \|\texttt{iter}(:, j) - v\|$ agrees with the theoretical convergence rate of $|\lambda_2|/|\lambda_1|$ where λ_1 and λ_2 are the largest and second largest eigenvalues of A in magnitude and v is an eigenvector corresponding to λ_1. Try to explain your observations. (Type `format long e` to see more digits and use the $[\texttt{V}, \texttt{D}] = \texttt{eig}(\texttt{A})$ command to find λ_1, λ_2 and v.)

3. Repeat the process in (2) for A given by (i) and initial vector $\texttt{x} = v_2 + v_3$ where v_2 and v_3 are eigenvectors corresponding to λ_2 and λ_3. Theoretically this implies that $c_1 = 0$, which violates an important necessary condition for the iterations to converge. To check if this happens in practice, run $[\texttt{iter}, \texttt{lambda}] = \texttt{Powermethod}(\texttt{A}, \texttt{x}, \texttt{k})$ for sufficiently large values of k. Can you explain your observations?

4. Write a function program $[\texttt{iter}, \texttt{lambda}] = \texttt{Shiftinv}(\texttt{A}, \texttt{x}, \texttt{s}, \texttt{k})$ that *efficiently* performs k iterations of Shift and Invert Method using $\texttt{A} \in \mathbb{C}^{\texttt{n}\times\texttt{n}}$, $\texttt{x} \in \mathbb{C}^{\texttt{n}}$ and shift s and returns an $n \times k$ matrix `iter` whose jth column is the jth iterate q_j and a scalar `lambda` as the eigenvalue of A closest to s.

5. Write a function program $[\texttt{iter}, \texttt{lambda}] = \texttt{Rayleigh}(\texttt{A}, \texttt{x}, \texttt{k})$ that *efficiently* performs k iterations of inverse iterations with Rayleigh quotient shifts using $\texttt{A} \in \mathbb{C}^{\texttt{n}\times\texttt{n}}$ and $\texttt{x} \in \mathbb{C}^{\texttt{n}}$ and returns an $n \times k$ matrix `iter` whose jth column is the jth iterate q_j and a scalar `lambda` as the eigenvalue of A to which the Rayleigh quotient shifts converge.

   Note. *Use [Q,H] = hess(A) to find an upper Hessenberg matrix H and a unitary matrix Q such that $Q^*AQ = H$ and use H in place of A in the iterations. This will reduce the flop count in each iteration from $O(n^3)$ to $O(n^2)$. However, if the program will converges, it will be to an eigenvector v corresponding to H from which an eigenvector corresponding to A will have to be found by computing Qv.*