ME721 Design Engineering Lab

Second Order Systems: Vibration Measurements, Data Acquisition and Analysis

In this experiment, you will learn to analyse a typical second order system, such as a mass-spring system or a combination of several such systems. A single mass-spring system (“M-K system”) exhibits behaviour typical of a second order system, i.e. a system governed by an ordinary linear differential equation of the second order, where the highest order of the derivative is two. The ‘free’ vibration of any system is defined as its motion in response to initial conditions, and in the absence of sustained/continuous forcing. For the ‘free’ response of a single degree-of-freedom (‘1-DoF’) M-K system, the ODE governing its motion is

$$m\ddot{x} + kx = 0$$

where *m* and *k* are the mass and stiffness, and *x* is the displacement w.r.t equilibrium. Note that the RHS of the above ODE is zero for the ‘free’ response. The system oscillates at its natural frequency when it is displaced from its equilibrium, and this quantity is $\omega = \sqrt{\frac{k}{m}}$ for an M-K system.

In this experiment, you will start with experimentally identifying the natural frequency of a 1-DoF system and correlate it with its properties (here, the mass and stiffness). Later, you will be acquiring vibration data with the help of data acquisition system and analyze it to get useful information and verify the natural frequency obtained manually and after the post-processing of signal acquired through sensor.

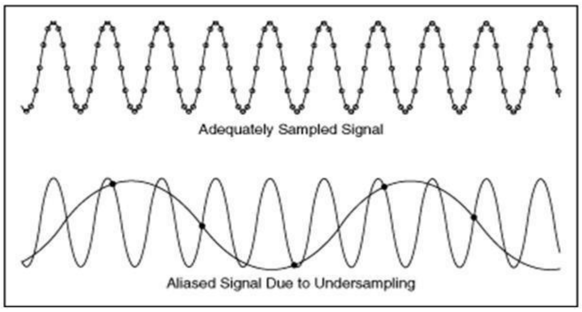
The basics of data acquisition and post-processing has to be studied first.

Vibration Data Analysis

Vibration data is acquired using measurement devices (sensors, data acquisition systems etc.). In order to properly acquire and analyse vibration data, certain aspects concerning the measurement process need to be kept in mind. This lab will introduce you to some of the issues taken into consideration in the analysis of vibration data. You will see how signal processing is employed to condition the data and extract required information from it.

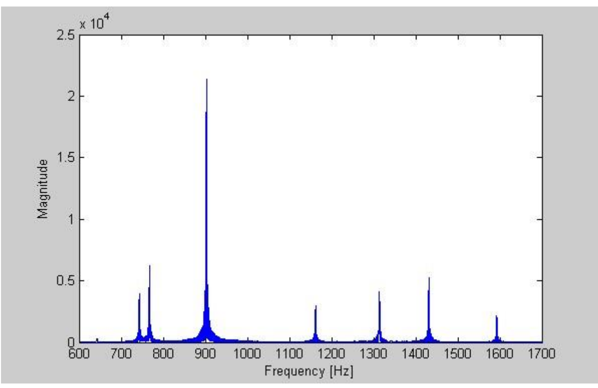
Sampling frequency. : The sampling rate or sampling frequency defines the number of samples per unit of time (usually seconds) taken from a continuous signal to make a discrete signal. Any digital measurement system acquires data in a discrete manner i.e. it does not continuously make measurements but only at particular time instants. Each such measurement made at a particular time instant is called a sample. These samples are used to reconstruct the signal we are trying to measure. The more such samples acquired, the more accurately will the reconstructed signal resemble the actual signal. As the sampling frequency is increased, it starts to approximate a continuous measurement closer. Sampling rates of several MHz is easily achievable today. However, there are certain overheads with high sample rates, such as cost, data management etc. Thus it is desirable to find out an optimum sampling rate which will accurately acquire the signal but will not prove too costly.

If the sampling rate is too small, then the signal won't be measured correctly and a phenomenon known as aliasing occurs. In order to avoid this, frequency of sampling must be suitably chosen. The Nyquist–Shannon sampling theorem states that perfect reconstruction of a signal is possible when the sampling frequency is greater than twice the maximum frequency of the signal being sampled. In practice, since in most cases it is not known in advance what the maximum frequency in a measurement will be, a suitable initial sampling rate is chosen and a measurement is made. Subsequently the sampling rate is increased and the measurement is repeated. This process is continued till subsequent measurements don't show much difference.



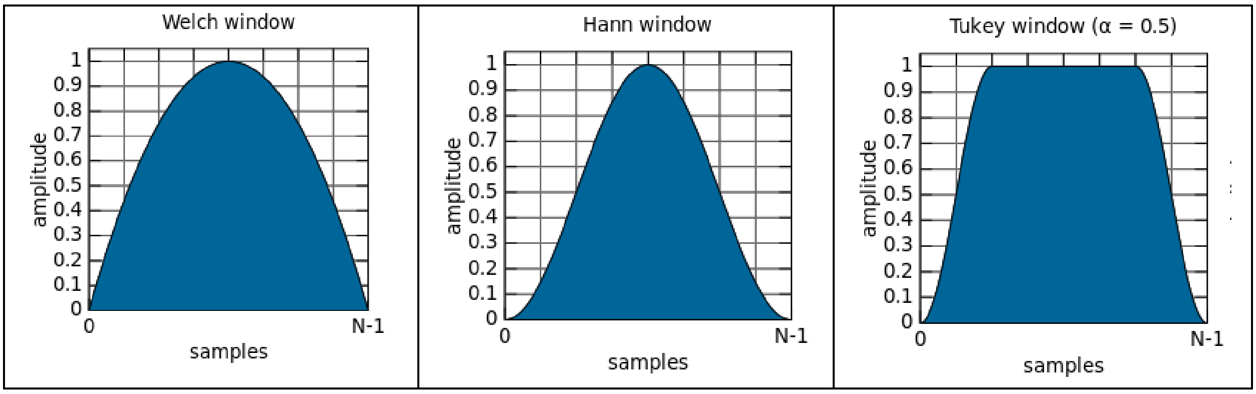
Signal Processing: Often the data acquired in a measurement needs to be processed before analysis can be done. There are several reasons for this. One reason is that, along with the signal that we are interested in measuring, unwanted signals, known as noise are also captured by the measurement device. This could be due to a problem in the system being measured itself, but often it is a result of extraneous signals that get captured from the surroundings, the most common being the 50Hz line frequency noise, which is almost always present in any environment. Thus before analysing the data we would like to remove this noise. This process is known as filtering. Different kinds of filters exist classified based on their application. Based on the frequencies they filter out and allow to pass, they are classified as low-pass, high-pass or band pass.

Windowing: Another kind of operation is windowing. A measurement is started at a certain time instant arbitrarily chosen and similarly the measurement is stopped at some arbitrary instant, once we believe we have enough data or samples to analyse the system. In order to find out the frequency content of the signal from the data obtained, an operation known as Fast Fourier Transform (FFT) is performed which gives the various frequencies contained



in a signal and their relative amplitudes. This data is typically plotted with frequency on the X-axis and magnitude on the Y-axis. The FFT computation presumes that the input data repeats over and over i.e. it concatenates the acquired data to form an endless data set. Since the measurement was started and ended at arbitrary time instants, it is possible that the initial and final values of the data set are not the same; the discontinuity causes aberrations in the spectrum computed by the FFT. In order to mitigate this effect, a "Windowing" operation is done which smoothens the ends of the data to eliminate these aberrations. A window is shaped so that it is exactly zero at the beginning and end of the data block and has some special shape in between. This function is then multiplied with the time data block forcing the signal to be continuous.

However, since a windowing function alters the data, it affects the frequency spectrum. Thus there are many windowing functions available (such as Hanning, Barlett, Tukey), each with their own advantages and disadvantages and are suitable for specific kinds of signals (such as random, sinusoidal)



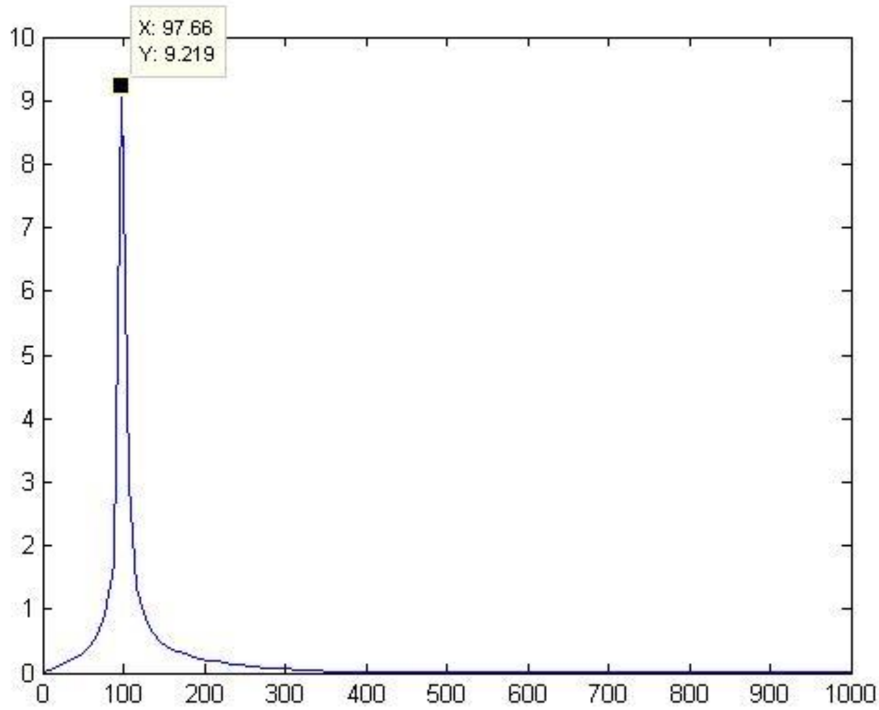
In order to perform the windowing operation, first choose a suitable windowing function. Apply this window function to the sampled data.

In order to do the FFT analysis, use the command fftin MATLAB. fft(x)will return the frequency content of the data stored in vector x. Read the following MATLAB Help page for details on how to use the fft function. http://www.mathworks.in/help/matlab/ref/fft.html

Details about FFT and Windowing

FFT stands for "Fast Fourier Transform" which is an algorithm to compute discrete Fourier Transform and its inverse. Many times analysing the signal in the frequency domain provides more information than time domain. Fourier analysis converts time domain signal to frequency domain signal and vice versa.

Generally, any periodic function f(x) can be written as infinite sum of sine and cosine functions with the help of Fourier series (as you would have studied in MA 10x in first year). FFT is the plot between frequencies (on X-axis) and amplitudes (on Y-axis) of those sine and cosine functions. For example, FFT of the function $f(x) = 10 * \sin(100x)$ is as shown below.

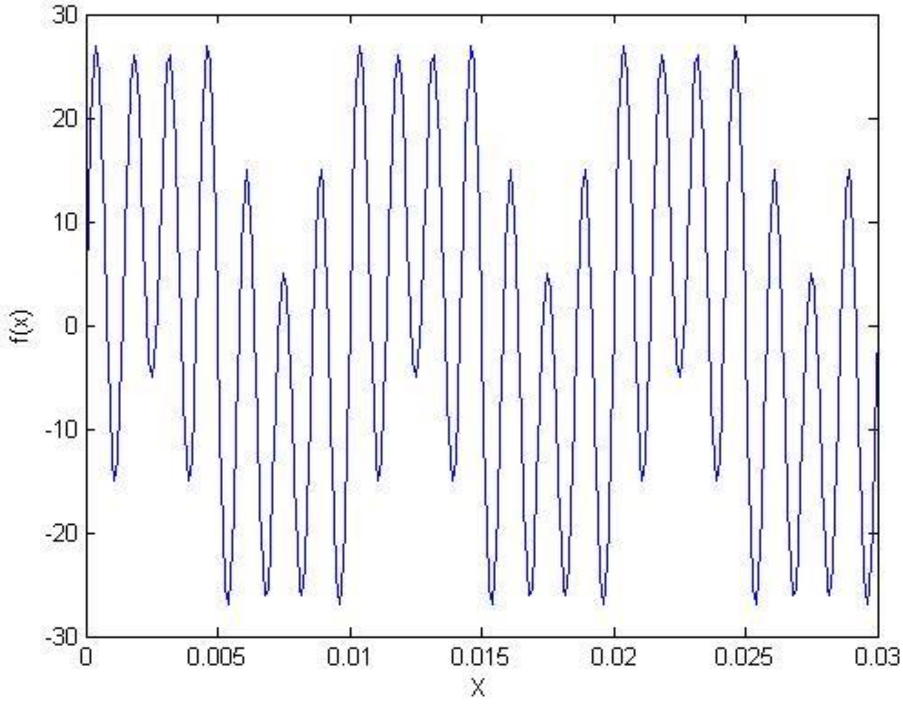


Here, X-axis is frequency and Y-axis is amplitude. So at 100Hz, we get one sharp peak whose amplitude is approximately 10.

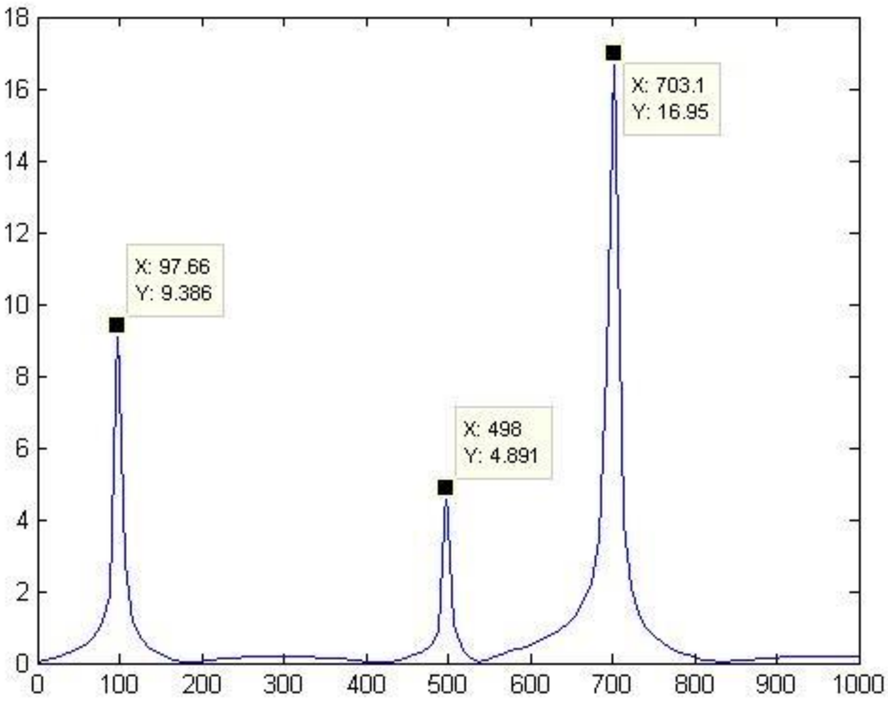
Similarly, if function is the sum of different sines e.g.

$$f(x) = 10\sin(100x) + 5\sin(500x) + 20\sin(700x)$$

F(x) when plotted vs x, looks like as shown in the figure below.



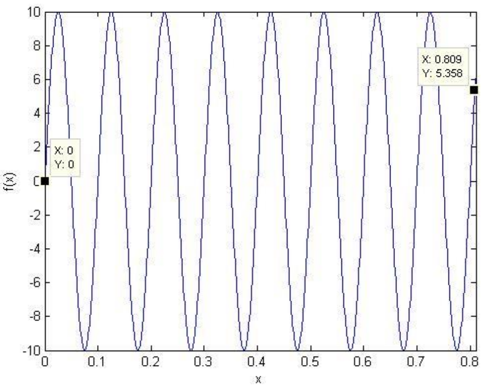
FFT of this function will have 3 peaks at frequency equal to 100, 500 and 700 Hz with corresponding amplitudes.



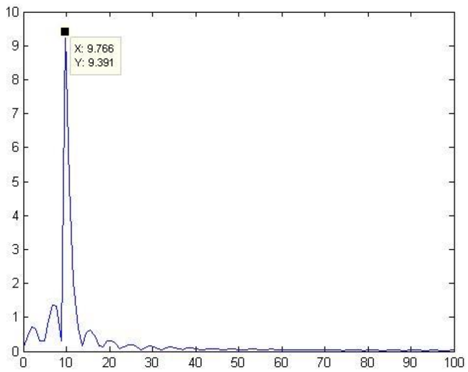
Since any periodic signal can be written in the infinite sum of sine and cosine functions, when plotted on FFT, it can have large number of frequency components.

Windowing: -

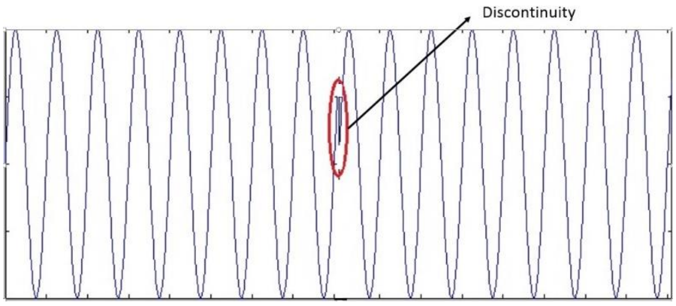
A fundamental assumption in Fourier analysis is that the signal is **periodic** from $t = -\infty\ to + \infty$. Since any measured signal has to be finite, the FFT computation presumes that the measured data repeats over and over till $t = \infty$, i.e. it concatenates the acquired data to form an endless data set. Measurement of the signal can start and stop at any arbitrary time instant, and need not be such that complete cycles of the signals (at different frequencies) within it should be present within the time range measured. So, it is possible that the initial and final values of the data set are not the same. When the FFT algorithm concatenates such a measured signal, the 'recreated' periodic signal deviates from the actual signal. This difference between the actual and recreated signals causes aberrations in the spectrum computed by the FFT as shown below; this is because the Fourier series of the recreated signal is computed, and not for the actual one.



Actual Signal

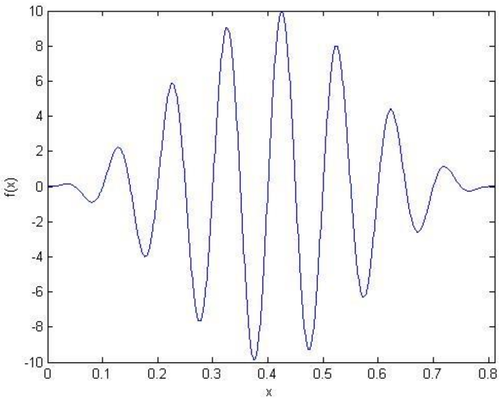


FFT of Recreated Signal

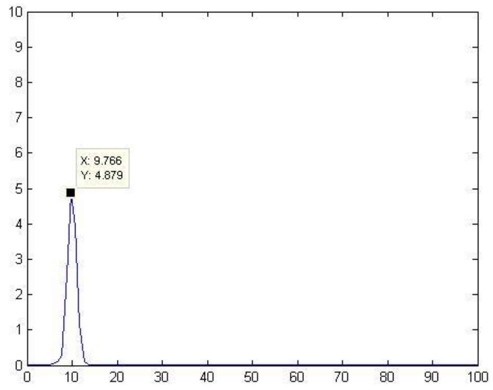


Recreated Concatenated Signal

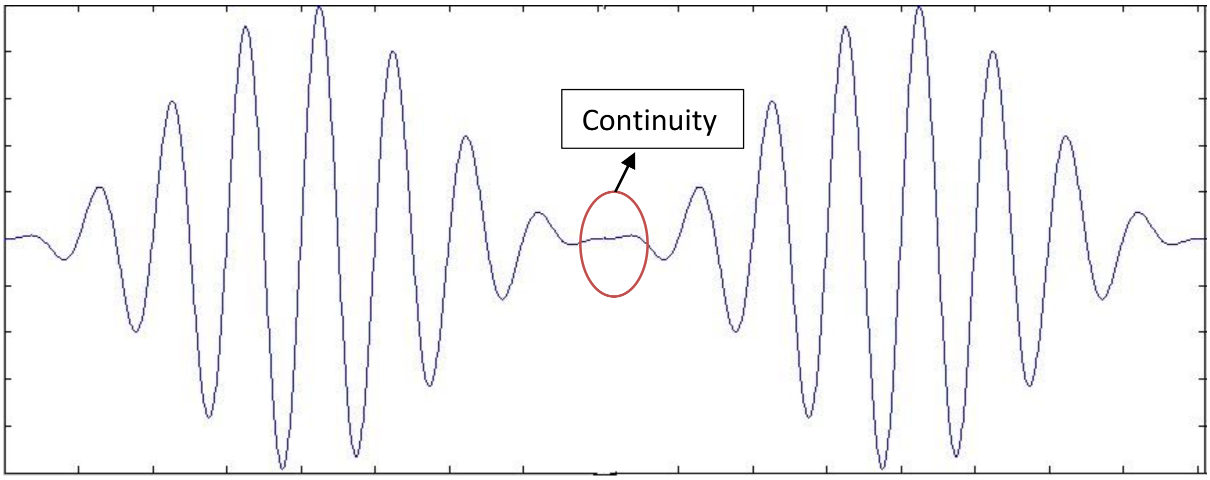
In order to mitigate this effect, a "Windowing" operation is done which smoothens the ends of the measured data set, such that the aberrations due to concatenation are reduced/eliminated. However, such a windowing function alters the data, and thereby affects the frequency spectrum. Signal is multiplied with Hanning function for demonstration. Modified signal and its FFT looks as shown below. Note that after windowing, the windowed signal on concatenation shows much lower aberrations as compared with the non-windowed ('raw') signal.



Signal after windowing



Modified FFT



Modified Periodic Signal after Windowing

For further reading, you may refer the following:

http://www.physik.uni-wuerzburg.de/~praktiku/Anleitung/Fremde/ANO14.pdf

http://www.tmworld.com/electronics-news/4383713/Windowing-Functions-Improve-FFT-Results- Part-I

Accelerometer MMA7361: -

Accelerometer is the device that measures the acceleration of the object to which it is connected in terms of voltage. Accelerometer given to you is the analogue accelerometer MMA7361 which works on the supply voltage given by the computer through labjack. It has a micro-cantilever beam which vibrates according to the excitation and other electronic circuitry converts this motion into voltage. It is 3 axes accelerometer i.e. it can sense the accelerations of the moving object in 3 directions X, Y and Z. For this experiment since we need to collect data for only vertical motion of the mass, only Z-axis of the accelerometer is connected to the labjack. This sensor will give the output in terms of the voltage which can be seen and saved on the computer. We need to calibrate the accelerometer if we want the exact amplitude of the acceleration.

Calibration: -

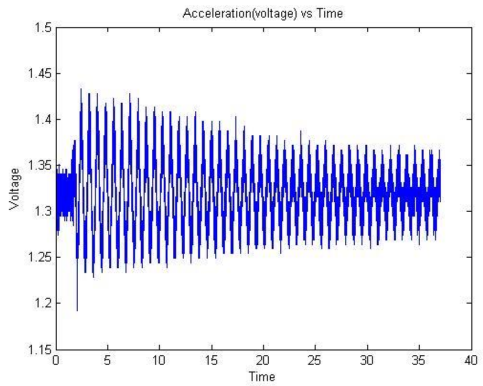
Calibration is the process of getting the relation between the units of measured value and actual engineering units of the measurement. E.g. in this case we will have to find the relation between voltage given by the accelerometer and actual acceleration of the object in terms of 'g' or 'm/s^2'. This can be done by measuring known acceleration acting on the accelerometer i.e. when accelerometer is stationary and its face on which marking is done is upwards, downward 'g' acceleration will be acting on the micro-cantilever present in the accelerometer. Note the voltage given by it at this moment. Now, turn the accelerometer by 90^0. Now zero acceleration will be experienced by the beam. Note the corresponding voltage. And finally turn the accelerometer 90^0 again i.e. upside down to very initial position. Now '-g' acceleration will be experienced by the beam. Note the corresponding voltage. So by assuming linear variation between voltage and acceleration, we can get the calibration equation.

Filtering: -Filtering is the class of signal processing by which unwanted component or feature of the signal is removed from the actual signal. Here we use the filtering to minimize the noise in the acquired signal. There are different kinds of filters for different purpose. E.g.

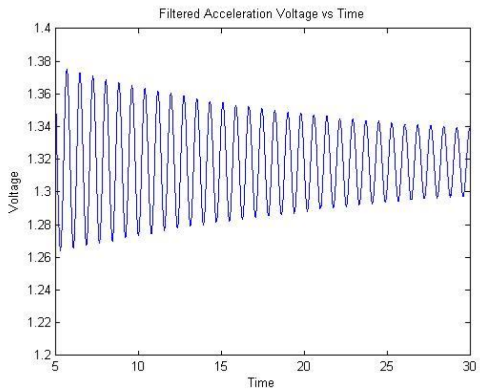
Low pass filter: - Low frequencies are passed and high frequencies are attenuated. E.g. 100Hz low pass filter will allow all frequencies below 100Hz to be present and all above this will be eliminated.

High Pass Filter: - High frequencies are passed and low frequencies are attenuated. E.g. 100Hz high pass filter will allow all frequencies above 100Hz to be present and all below this will be eliminated.

Band Pass Filter: - Only frequencies in the defined frequency band are passed. E.g. 50-100Hz band pass filter will allow frequencies only between 50 and 100 Hz to be present and all frequencies below 50Hz and above 100Hz will be eliminated.



Original Signal



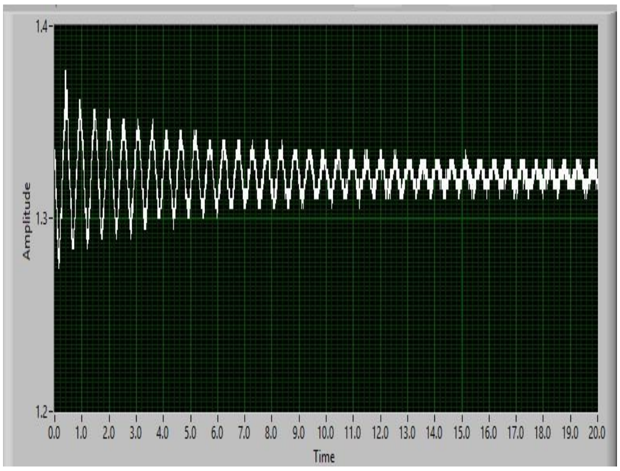
Filtered Signal

The hardware for this experiment comprises the following:

- A mounting bracket and hooks
- Two springs, of different spring stiffnesses *k1* and *k2*
- Two masses, of mass *m1* and *m2*
- Sensor – Accelerometer (MMA7361)
- Data acquisition block – LabJack
- Mountboard

Setup: -

Setup



Captured data

<u>Tasks To Be Done: -</u>

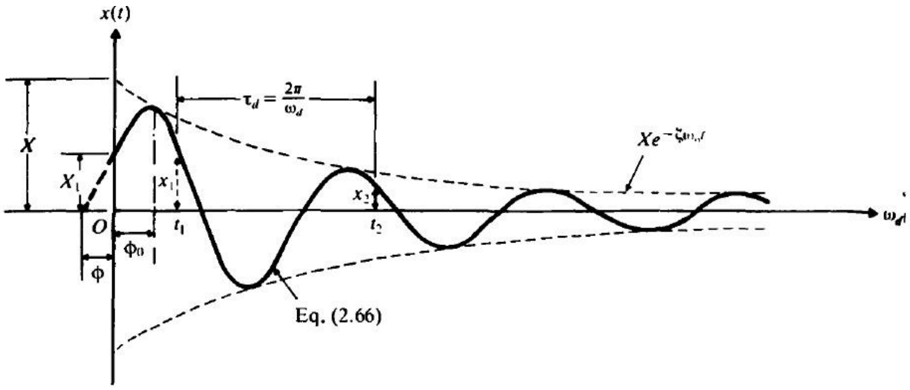
- **Free response of a 1-DoF system:** Suspend one of the springs from the bracket, and then suspend any one of the masses from the spring. Displace the mass by a short distance vertically and release. As the mass oscillates, use the stopwatch to note the time taken for a large number (>10) of full oscillations. Repeat the experiment several (>3) times. (A large number of counted oscillations and repetitions will reduce the measurement error.) Calculate the natural frequency of this 1-DoF system. Repeat the above process for the second spring.
- **Obtain the spring stiffnesses of each of the two springs:** For either of the two springs, use the known values of the masses used in Step 1 and the experimentally-determined value of the natural frequency to calculate the stiffness of that spring.
- **Data Acquisition:** Now with the help of TAs connect the accelerometer to the labjack and computer. Now, give small initial displacement to the mass and release. Mass will oscillate whose acceleration data will be captured by the accelerometer. Fix the sampling rate to 500Hz for this trial. Save the data into computer.
- Using the calibration done earlier, plot the absolute acceleration vs time with the help of MATLAB.

- Use suitable windowing function to smooth the ends of the data captured as you have done in the last lab.
- Take FFT of the windowed data to find out the dominant frequency i.e. damped natural frequency of the system.
- Compare this frequency with the calculated frequency in the first step.
- You must have observed that the oscillations get damped out after some time due to air resistance and material damping. This can also be seen on the time response of acceleration. Damping coefficient of the system can be found out from the response of the system by using logarithmic decrement of the response. Also it can be found out by using the relation between natural frequency and damped natural frequency.
- Using the MATLAB Help, use the function ***'filter'*** to get the cleaner signal. This function uses low pass moving average filter for this purpose.
- Take FFT of this filtered signal along with suitable windowing function. Note the difference in the FFTs.
- Now find out the decrement in the amplitude of the oscillation after 10 or 20 number of cycles. Find out the damping coefficient of the system with the help of logarithmic decrement. Find out the damping coefficient using the relation between natural frequency and damped natural frequency. Compare both the answers.
- Studying the effect of coulomb friction on the single DoF spring-mass system
- Now, use the piece of mountboard provided to you. Displace the mass to some distance and let it oscillate against the friction provided due to contact with mountboard. Since oscillations get damped out due to friction force, the damping is called Coulomb damping or solid friction damping.
- Capture the response of the mass with the help of data acquisition system as you have done earlier. Observe the damping characteristics. Note down the difference in both the cases i.e. nature of decay in both cases.
- Derive the equation of motion for single DoF spring-mass system for above two cases i.e. with viscous damping and with coulomb damping. Derive the expression for decrement in amplitude of vibration in both the cases. (Refer 'Mechanical Vibrations' by S. S. Rao)
- Now using the captured data, calculate the friction force experienced by the mass during oscillation.

- Now attach another spring and mass to the previous mass. Attach an accelerometer to one of the mass. Capture the vibration data by providing different initial conditions to each mass. Write down the observations after post-processing of the data.

Appendix: -

Logarithmic Decrement: -



Underdamped Solution (Ref. Mechanical Vibrations by S. S. Rao)

Here, Logarithmic Decrement δ is given by,

$$\delta = \frac{1}{m}\ln\left(\frac{x_1}{x_{m+1}}\right)$$

Where,

$$x_1 = amplitude\ of\ vibration\ of\ 1^{st}\ cycle$$

$$x_{m+1} = amplitude\ of\ vibration\ of\ m^{th}\ cycle$$

$$m = number\ of\ cylces\ uner\ consideration$$

The relation between this log decrement and damping ratio is given by,

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

Where,

$$\zeta = Damping\ Ratio$$

Hence, damping coefficient can be found using,

$$\zeta = \frac{c}{2\sqrt{km}}$$

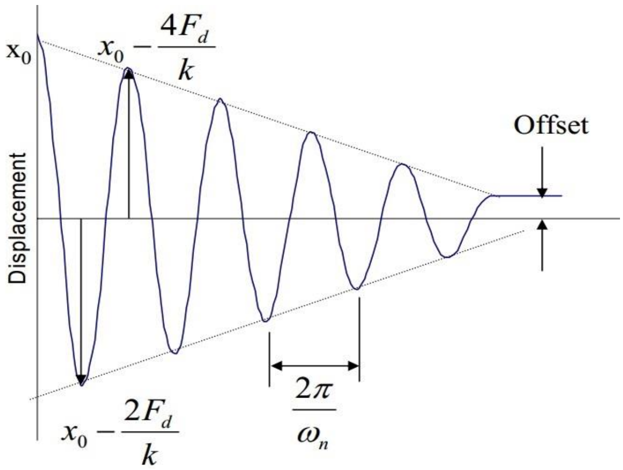
Where,

$$c = damping\ coefficient$$

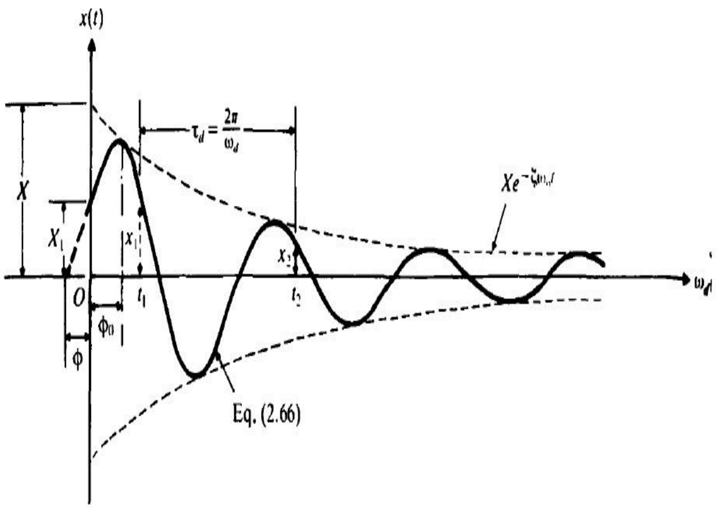
$$k = stiffness\ of\ the\ spring$$

$$m = mass$$

Appendix:



Coulomb Damping



Viscous Damping