



# Dynamic analysis of planar mechanical system with clearance joints using a new nonlinear contact force model

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**Assignment 2 : ME 748 Computer Aided Simulation of Machines | Prof. Anirban Guha**

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7. Conclusion

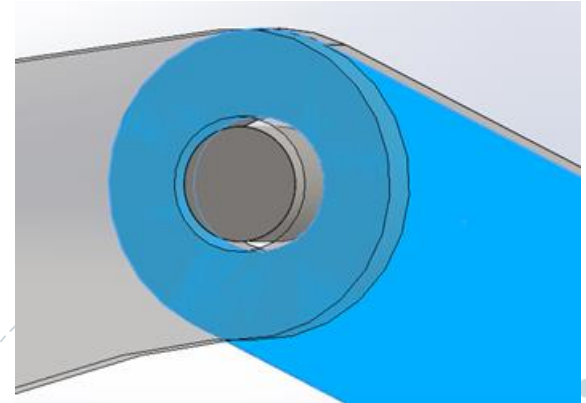
# 1. Introduction

**Mechanical systems consist of Kinematic joints with inevitable clearances.**

**With the demand of relative motions, clearance joints have many effects on mechanical system such as vibration, wear and noise**

**Reduces system reliability, kinematic accuracy and stability, even the life of mechanism**

**Hence modelling of an accurate contact force model for dynamic simulation of mechanical system with clearance joints is critical**





## 2. Need for this model

Various contact force model proposed but all these models are purely elastic and do not consider the energy dissipation during contact process.

This drawback was solved using different models based on Hertz's contact force models.

But these modified models works well for larger coefficient of restitution  
(  $C_r \geq 1$  )



### 3. What's considered and not considered

- A nonlinear contact law with nonlinear contact stiffness
- Frictional effect measured by modified Coulomb friction
- Flexibility of links
- Lubrication action in the clearance joint
- Plastic deformation in the clearance joint

## 4. Hertz Model

- Hertz contact law is a purely elastic contact force
- It does not account for the energy dissipation during the impact process

$$F = K\delta^n,$$

$K$  - Generalized stiffness parameter,  
 $n$  - Nonlinear power exponent (1.5 for metallic contact)  
 $\Delta$  - Relative deformation depth.

$$K = \frac{4}{3\pi(\sigma_1 + \sigma_2)} \sqrt{\frac{R_B R_J}{R_B - R_J}},$$

$R_B$  and  $R_J$  - Radius of bearing and journal  
 $\sigma_i$  - Material parameter

$$\sigma_i = \frac{1 - \nu_i^2}{\pi E_i}, (i = 1, 2),$$

$\nu_i$  - Poisson's ratio  
 $E_i$  - Young's modulus associated with journal and bearing

## 4. Various development of contact law

### MODEL

### CONTACT FORCE MODEL

Hunt and  
Crossley

Pure elastic Hertz's law, combined with a nonlinear viscoelastic element

Lankarani  
and Nikravesh

Based on Hunt and Crossley work,  
Frequently used contact force model

Bai et al.

A new hybrid contact force model, based on the Lankarani-Nikravesh model and the improved Winkler elastic foundation model,

Wang and Liu

Based on the contact force model of Liu et al. [7] and contact force model of Lankarani and Nikravesh

\*\*\* These modified models works well for larger coefficient of restitution (  $C_r \geq 1$  )

## 5. New nonlinear contact force model

$$F_N' = \frac{\pi E^* L \delta^n}{2} \left( \frac{1}{2(c + \delta)} \right)^{1/2} = K_i \delta^n.$$

$K_i$  - Improved Generalized nonlinear stiffness coefficient,

$$D_i = \frac{3(1 - c_r^2) e^{2(1 - c_r)} \delta^n}{4 \delta^{(-)}} K_i.$$

Improved damping coefficient

Improved nonlinear contact force model

$$\begin{aligned} F_{Ni} &= K_i \delta^n + D_i \delta' \\ &= \frac{\pi E^* L \delta^n}{2} \left( \frac{1}{2(c + \delta)} \right)^{1/2} \left( 1 + \frac{3(1 - c_r^2) e^{2(1 - c_r)} \delta'}{4 \delta^{(-)}} \right). \end{aligned}$$

Modified Coulomb friction force model to depict the friction effect in clearance joint

$c_f$  - friction coefficient

$F_N$  - Normal force

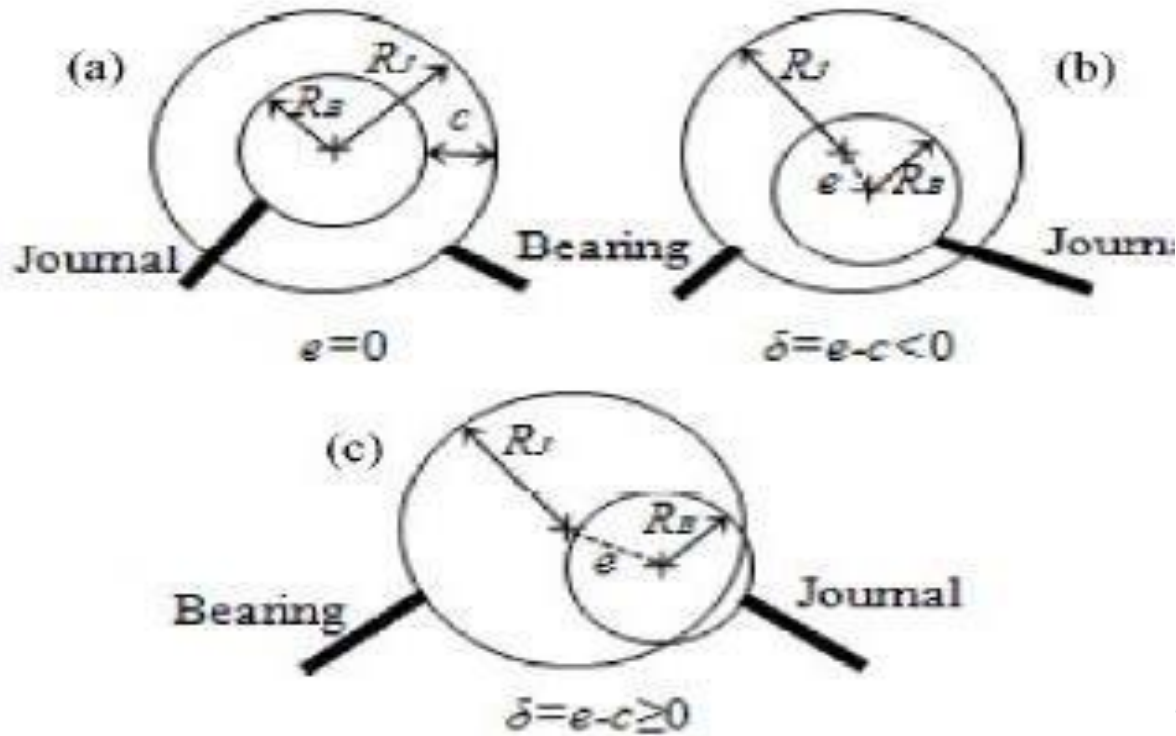
$V_T$  - Relative tangential velocity

$c_d$  - Dynamic correction coefficient

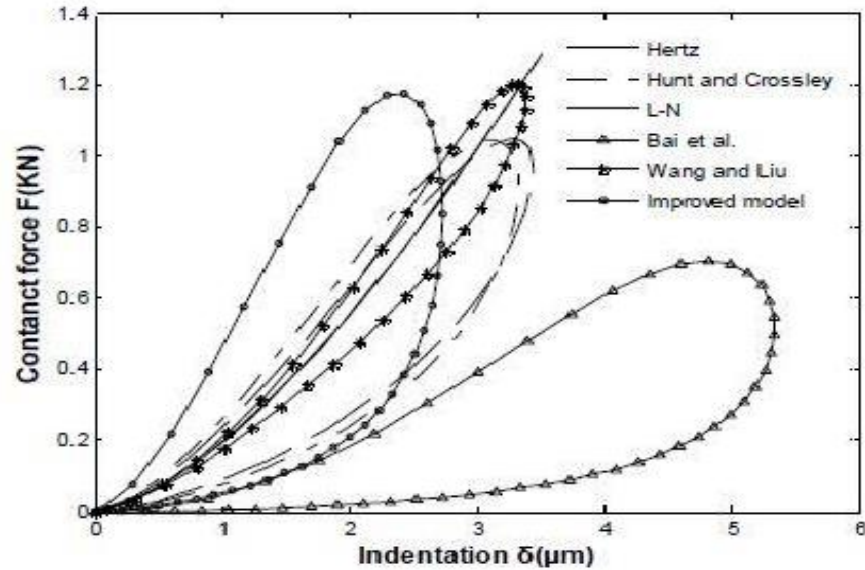
$$F_T = -c_f c_d F_N \operatorname{sgn}(V_T),$$



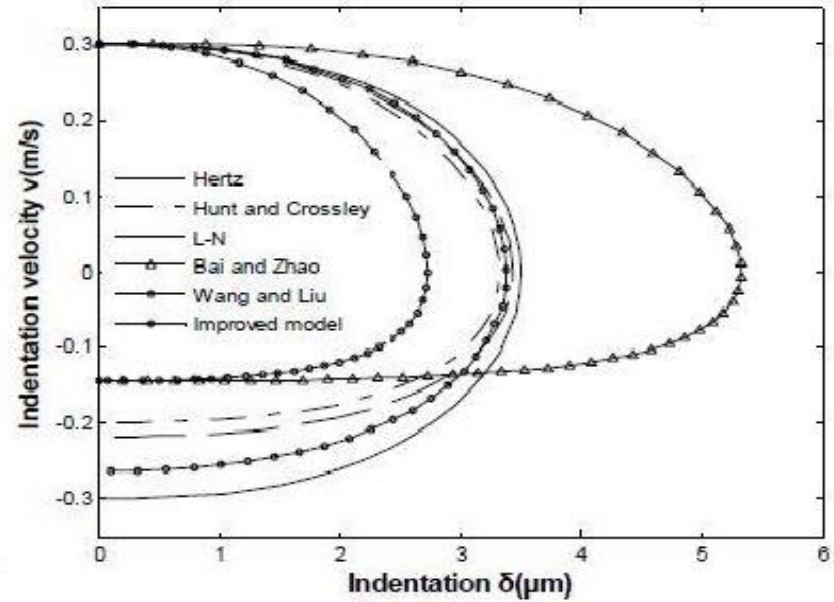
## 5. New nonlinear contact force model



## 6. Comparison of new model with different other model and experimental values



(a)



(b)

Fig. (a) Force-indentation relation; (b) velocity indentation relation for different contact force models For journal and bearing

## 6. Comparison of new model with different other model and experimental values

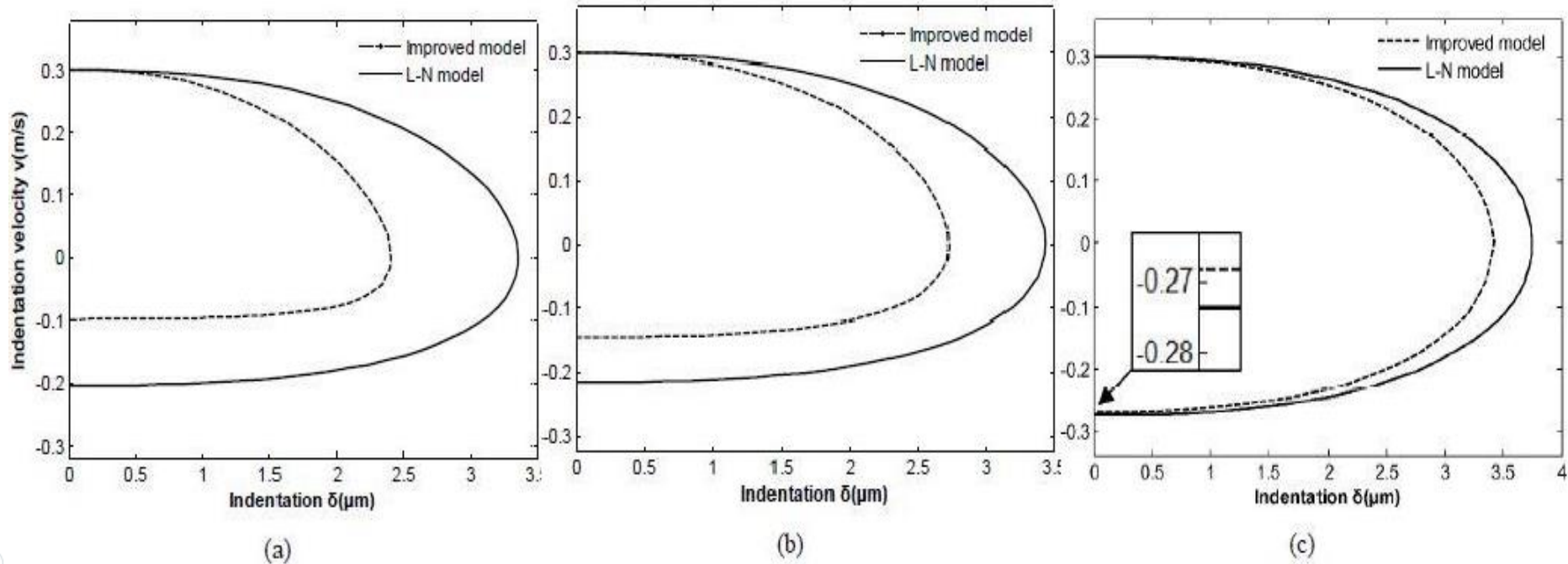


Fig. Velocity-indentation curves with improved model and Lankarani and Nikravesh models:  
(a)  $C_r = 0.3$ ; (b)  $C_r = 0.5$ ; (c)  $C_r = 0.9$ .

## 6. Comparison of new model with different other model and experimental values

### *Application Case : Slider crank mechanism*

Table 4. Mass and inertia properties of slider-crank mechanism.

Body	Length [m]	Mass [kg]	Moment of inertial [kgm <sup>2</sup> ]
Crank	0.05	17.900	0.460327
Connecting rod	0.30	1.130	0.015300
Slider	-	1.013	0.000772

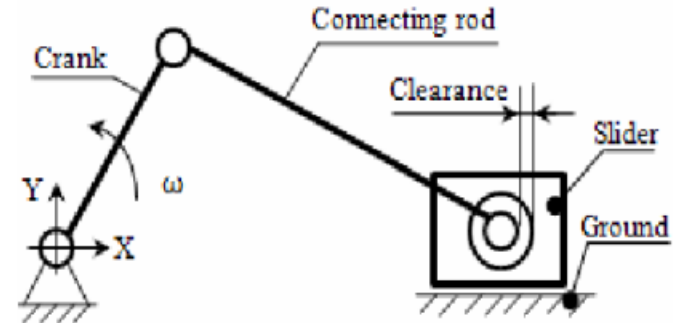


Fig. 6. Slider-crank mechanism with a revolute clearance.

## 6. Comparison of new model with different other model and experimental values

Table 1. Simulation results with fixed  $c_r$ ,  $\delta'(-)$  and varied  $c$ .

Name	c	0.01	0.1	0.5	1
$M_{Imp}$	$\delta^{(+)}(m/s)$	0.1448	0.1448	0.1448	0.1448
	$c_r'$	0.4827	0.4827	0.4827	0.4827
	Error	3.46%	3.46%	3.46%	3.46%
$M_{L-N}$	$\delta^{(+)}(m/s)$	0.2176	0.2176	0.2176	0.2176
	$c_r'$	0.7253	0.7253	0.7253	0.7253
	error	45.06%	45.06%	45.06%	45.06%

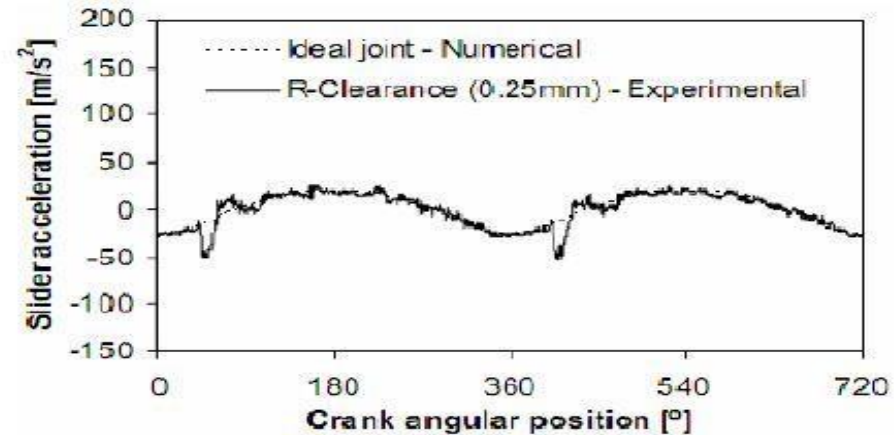
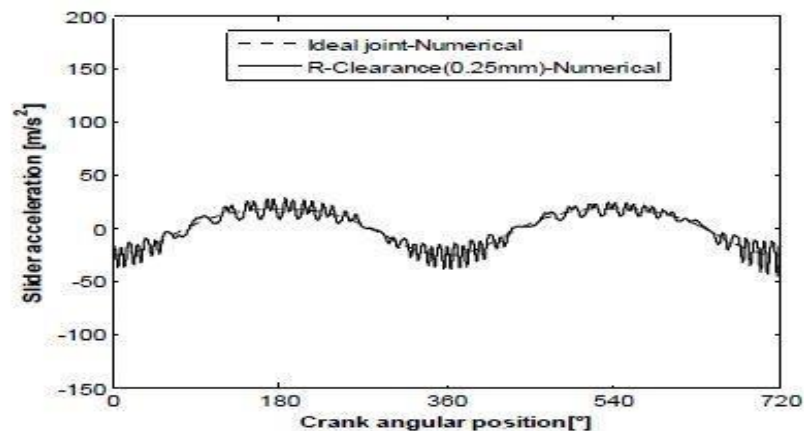
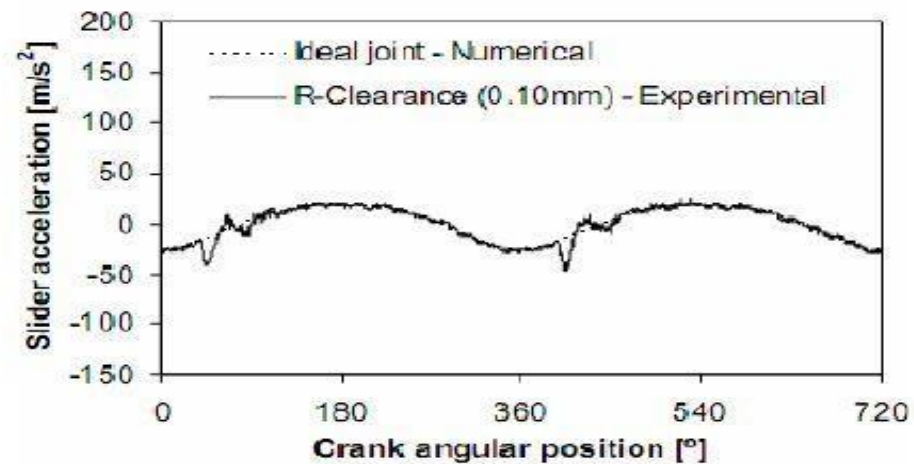
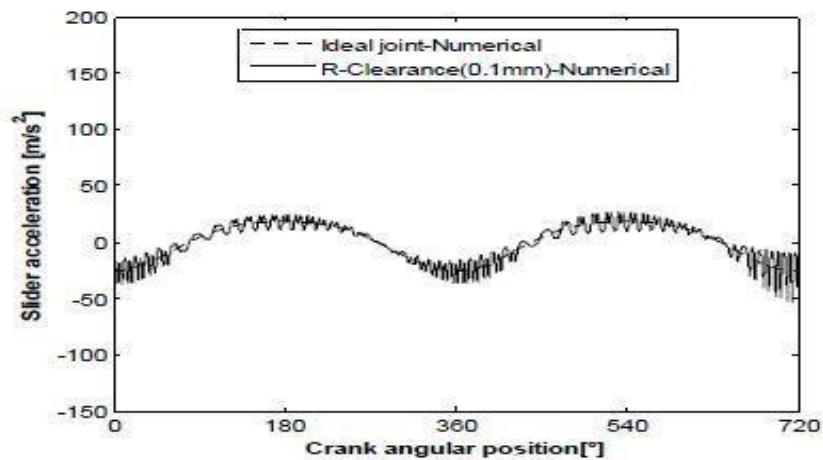
Table 2. Simulation results with fixed  $c$ ,  $\delta^{'(-)}$  and varied  $c_r$ .

Name	$c_r$	0.3	0.5	0.9	1
$M_{Imp}$	$\delta^{(+)}(m/s)$	0.098	0.1448	0.2687	0.3
	$c_r'$	0.3267	0.4827	0.8957	1
	Error	8.889%	3.467%	0.482%	0
$M_{L-N}$	$\delta^{(+)}(m/s)$	0.2053	0.2176	0.274	0.3
	$c_r'$	0.6843	0.7253	0.9133	1
	Error	128.1%	45.06%	1.48%	0

## 6. Comparison of new model with different other model and experimental values

Table 3. Simulation results with fixed  $c$ ,  $c_r$  and varied  $\delta'(-)$ .

Name	$\delta'(-)$	$\delta'(-)$	$3\delta'(-)$	$5\delta'(-)$	$10\delta'(-)$
$M_{Imp}$	$\delta^{(+)}(m/s)$	0.1448	0.4343	0.7239	1.4478
	$c_r'$	0.4827	0.4826	0.4826	0.4826
	Error	3.46%	3.48%	3.48%	3.48%
$M_{L-N}$	$\delta^{(+)}(m/s)$	0.2176	0.6527	1.0879	2.1757
	$c_r'$	0.7253	0.7252	0.7253	0.7252
	Error	45.06%	45.04%	45.06%	45.04%

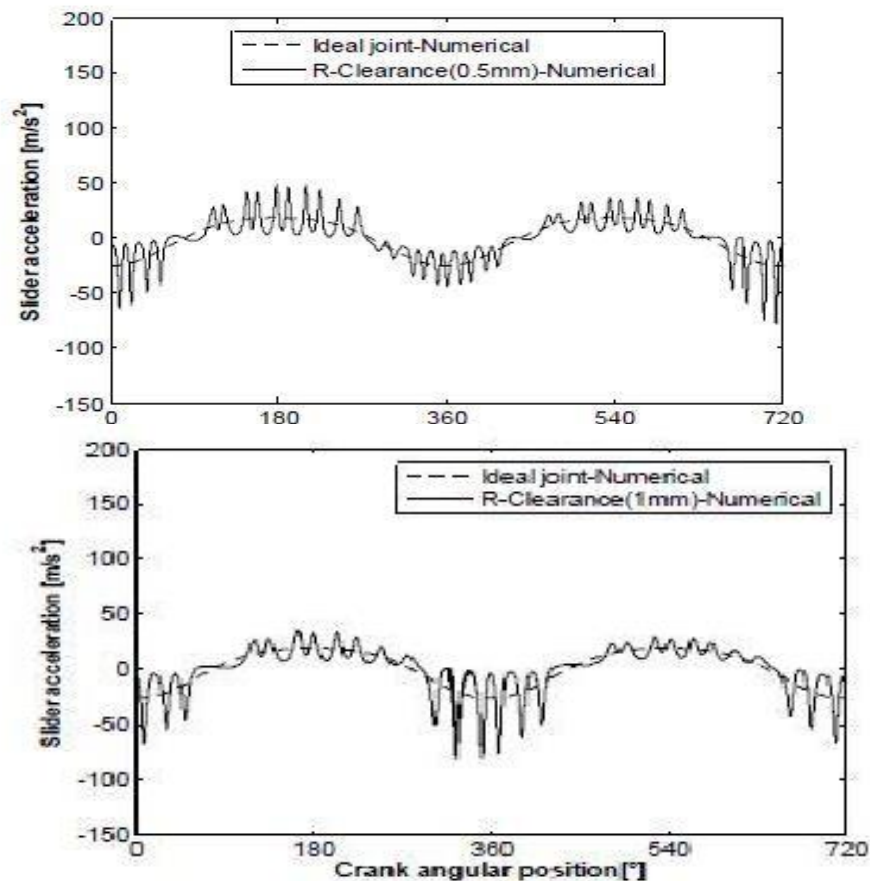


(a)

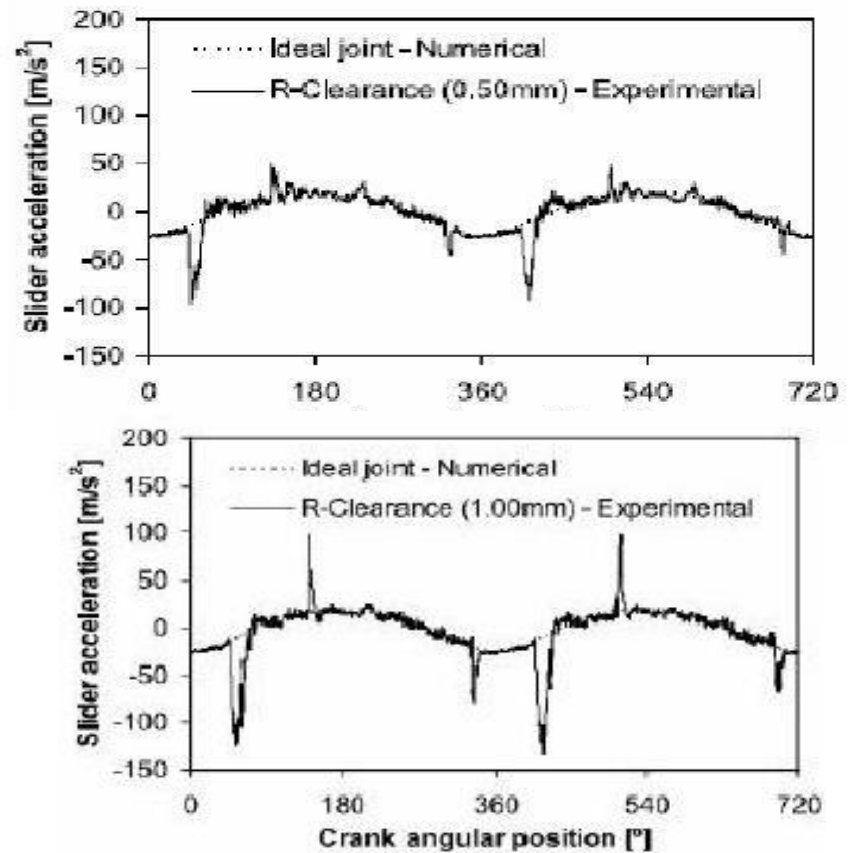
(b)

Fig. Slider accelerations with clearance size  $c = 0.1$  mm (top) and  $c = 0.25$  mm (bottom) from (a) simulation; (b) experiment





(a)



(b)

Fig. Slider accelerations with clearance size  $c = 0.5 \text{ mm}$  (top) and  $c = 1 \text{ mm}$  (bottom) mm from (a) simulation; (b) experiment



## 7. Conclusion

### Reasons for the small difference in experimental and new model values

- The clearance size is assumed to be equal and without change during simulation
- Small variations on restitution and friction coefficient during simulation
- the flexibility of links, and lubrication action and the plastic deformation in the clearance joint

Neglected in the numerical simulations which explains

### A combination of a nonlinear contact law with nonlinear contact stiffness

The dynamic response of slider acceleration using new nonlinear contact force model is similar to the experimental results published and has least error for  $C_r \leq 1$ .



# Thanks!

## Any question?

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All references from [https://inis.iaea.org/search/search.aspx?orig\\_q=RN:47084707](https://inis.iaea.org/search/search.aspx?orig_q=RN:47084707)

