CBCS SCHEME

15CS/IS834

Eight Semester B.E. Degree Examination, November 2020

System Modeling & Simulation

Module 1

1.a. List any four circumstances, when the simulation is the appropriate tool and when it is not? (08 Marks)

Ans:

WHEN SIMULATION IS APPROPRIATE TOOL:

1. Simulation enables the study of, and experimentation with, the internal interactions of a complex system or of a subsystem within a complex system.

2. Informational, organizational, and environmental changes can be simulated, and the effect of these alterations on the model's behavior can be observed.

3. The knowledge gained during the designing of a simulation model could be of great value toward suggesting improvement in the system under investigation.

4. Changing simulation inputs and observing the resulting outputs can produce valuable insight into which variables are the most important and into how variables interact.

5. Simulation can be used as a pedagogical device to reinforce analytic solution methodologies.

6. Simulation can be used to experiment with new designs or policies before implementation, so as to prepare for what might happen.

7. Simulation can be used to verify analytic solutions.

8. Simulating different capabilities for a machine can help determine the requirements on it.

9. Simulation models designed for training make learning possible without the cost and disruption of on-the-job instruction.

WHEN SIMULATION IS NOT APPROPRIATE:

1. Simulation should not be used when the problem can be solved by common sense.
2. Simulation should not be used if the problem can be solved analytically.
3. Simulation should not be used if it is easier to perform direct experiment.
4. Fourth rule says not to use simulation if the costs exceed the savings.
5. Simulation should not be performed if the resources or time are not available.
6. If there is not enough time or if the personnel are not available, simulation is not appropriate.
7. If system behavior is too complex or can't be defined, simulation is not appropriate.

1.b. Consider the grocery store with one check out counter. Prepare the simulation table for 8 customers and find out average waiting time of customer in queue, idle time of server and average service time. The inter arrival time and service time are given in minutes. (08 Marks)

Inter Arrival time (IAT)	3,2,6,4,4,5,8
Service time (ST)	3,5,5,8,4,6,2,3

Ans:

Customer	Inter Arrival Time (Minutes)	Arrival Time	Service Time (Minutes)	Time Service Begins	Waiting Time in Queue (Minutes)	Time Service Ends	Time Customer spends in system (Minutes)	Idle Time of Server (Minutes)
1	--	0	3	0	0	3	3	0
2	3	3	5	3	0	8	5	0
3	2	5	5	8	3	13	8	0
4	6	11	8	13	2	21	10	0
5	4	15	4	21	6	25	10	0
6	4	19	6	25	6	31	12	0
7	5	24	2	31	7	33	9	0
8	8	32	3	33	1	36	4	0
			36		25			

Average waiting time of customer in queue:

$$\text{Average waiting time} = \frac{\text{Total time customers wait in queue}}{\text{Total number of customers}}$$

$$= \frac{25}{8}$$

$$= 3.125 \text{ minutes}$$

The proportion of Idle time of the server is 0 minutes.

Average Service Time:

$$\text{Average service time} = \frac{\text{Total service time}}{\text{Total number of customers}}$$

$$= \frac{36}{8}$$

$$= 4.5 \text{ minutes}$$

2.a. Define: i) System ii) Event iii) FEL (Future Event List) (03 Marks)

<u>Ans</u>:

i) **System:** A system is defined as a group of objects that are joined in regular fashion to accomplish a task.

Example: Production System

ii) **Event:** An instantaneous occurrence that changes the state of a system. (such as an arrival of a new customer).

Example: Arrival, Departure

iii) **FEL (Future Event List):** A list of event notices for future events, ordered by time of occurrence; also known as the future event list (FEL).

2.b. Explain different types of world views. (06 Marks)

Ans:

event-scheduleing world view:

- We view the simulation as a sequence of events scheduled according to their event time.
- The simulation is proceeded by a sequence of snap-shots of the system. Each snap-shot is triggered by a event from the event list.
- Only one sanp-shot (the current one) is kept in computer memory. A new snap-shot can be derived only from the previous snap-shot, newly generated random variable values, and the event logic.
- Past snap-shots should be ignored when advancing the clock.
- The current snap-shot must contain all information necessary to continue the simulation.

process-interaction world view:

- In process-interaction world view, the simulation is considered as a collection of interactions among processes.
- It is similar to the object-oriented programming paradigm. Processes interact with each other by messages.
- See Figure 3.4 for an example. From the view point that two processes interact with each other.
- Often specialized simulation package can support this view. These simulation packages take care of the time advancing issues for the programmers.
- Programming in general purpose high level language is difficult to use this process-interaction world view because it will be too complicated for programmers to specify all the details.

activity-scanning world view:

- With activity-scanning approach, a modeler concentrates on the activities of a model and those conditions that allow an activity to begin.
- At each clock advance, the conditions for each activity are checked and if the conditions are true, the corresponding activity begins.

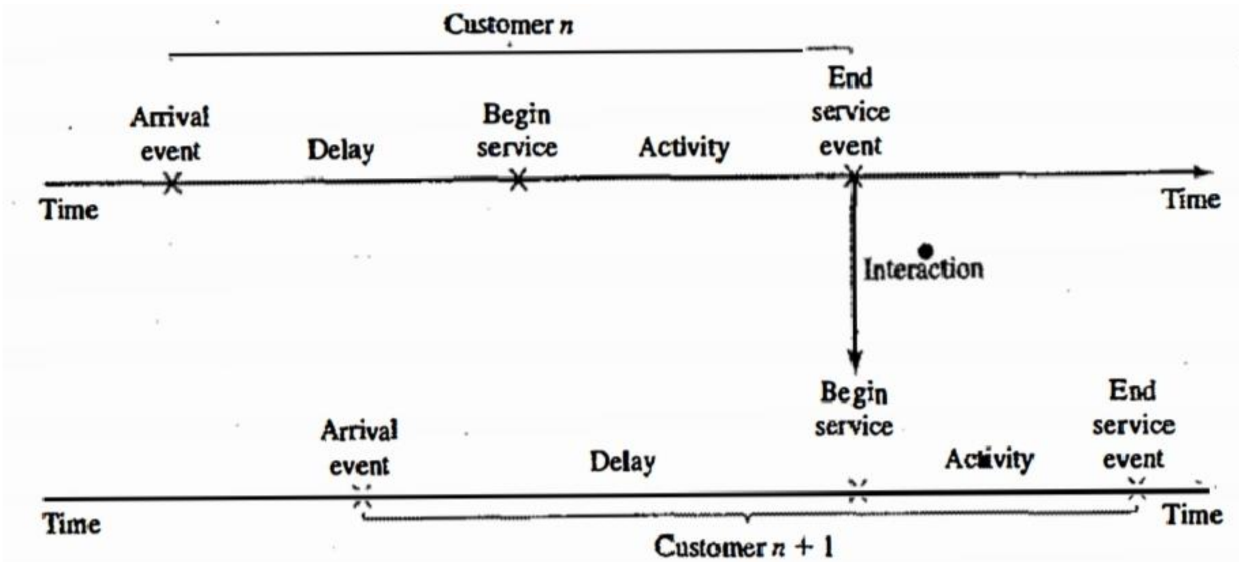


Figure 3.4 Two interacting customer processes in a single-server queue.

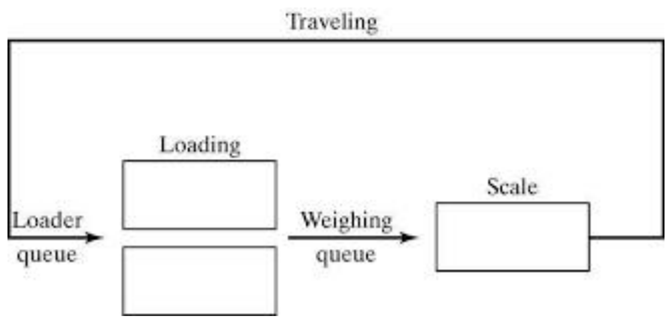
2.c. Six dump trucks are used to haul coal from the entrance of a small mine to the rail road. Each truck is loaded by one of two loaders. After loading, truck immediately moves to the scale to be weighed. Loader and Scale have First-Come-First-Serve (FCFS) queue. The travel time from loader to scale is negligible. After being weighed, a truck begins a travel time, afterwards unload the coal and returns to the loader queue. It is assumed that Five trucks are at the loader and one is at the scale at time=0.Carryout simulation process till the completion of two weighing from the scale. The activities of loading, weighing and travel time are given in the following table:

Loading time	10	5	5	10	15	10	19
Weighing time	12	12	12	16	12	16	
Travel time	60	100	40	40	80		

Calculate: i) The busy time of both the loaders and scale.

ii) Average loader and scale utilization. (07 Marks)

Ans:



The model has the following components:

System state:

LQ(t) : Number of trucks in loader queue

L(t) : Number of trucks (0,1 or 2) being loaded

WQ(t): Number of trucks in weigh queue

W(t) : Number of trucks (0 or 1) being weighed, all at simulation time t.

Event notices:

(ALQ,t,DT_i) : Dump truck arrives at loader queue (ALQ) at time t.

(EL,t,DT_i) : Dump truck DT_i ends loading (EL) at time t.

(EW,t,DT_i) : Dump truck DT_i ends weighing (EW) at time t.

Entities:

The six dump trucks (DT_1, -----,DT_6)

Lists:

Loader queue : All trucks waiting to begin loading ordered on a first-come, first served basis.

Weigh queue : All trucks waiting to be weighed, ordered on a first-come, first served basis.

Clock t	System State				Lists		Future Event List	Cumulative statistics	
	LQ(t)	L(t)	WQ(t)	W(t)	Loader queue	Weigh queue		B_L	B_S
0	3	2	0	1	DT_4 DT_5 DT_6		$(EL,5,DT_3)$ $(EL,10,DT_2)$ $(EW,12,DT_1)$	0	0
5	2	2	1	1	DT_5 DT_6	DT_3	$(EL,10,DT_2)$ $(EL,5+5,DT_4)$ $(EW,12,DT_1)$	10	5
10	1	2	2	1	DT_6	DT_3 DT_2	$(EL,10,DT_4)$ $(EW,12,DT_1)$ $(EL,10+10,DT_5)$	20	10
10	0	2	3	1		DT_3 DT_2 DT_4	$(EW,12,DT_1)$ $(EL,20,DT_5)$ $(EL,10+15,DT_6)$	20	10
12	0	2	2	1		DT_2 DT_4	$(EL,20,DT_5)$ $(EW,12+12,DT_3)$ $(EL,25,DT_6)$ $(ALQ,12+60,DT_1)$	24	12
20	0	1	3	1		DT_2 DT_4 DT_5	$(EW,24,DT_3)$ $(EL,25,DT_6)$ $(ALQ,72,DT_1)$	40	20
24	0	1	2	1		DT_4 DT_5	$(EL,25,DT_6)$ $(EW,24+12,DT_2)$ $(ALQ,72,DT_1)$ $(ALQ,24+100,DT_3)$	44	24

i) Total busy time of both loaders = $B_L =$ 44 minutes

Total busy time of scale = $B_S =$ 24 minutes

ii) Average loader utilization = $\dfrac{B_L / \text{Number of loaders}}{\text{Total time}}$

$$= \frac{44/2}{24} = 0.9166$$

$$\text{Average scale utilization} = \frac{B_S}{\text{Total time}}$$

$$= \frac{24}{24} = 1.00$$

Module 2

3.a. Explain i) Exponential Distribution ii) Binomial Distribution (06 Marks)

Ans:

Exponential distribution: A random variable X is said to be exponentially distributed with parameter $\lambda > 0$ if its pdf is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

The exponential distribution has mean and variance given by

$$E(X) = \frac{1}{\lambda} \quad \text{and} \quad V(X) = \frac{1}{\lambda^2}$$

cdf of exponential distribution is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

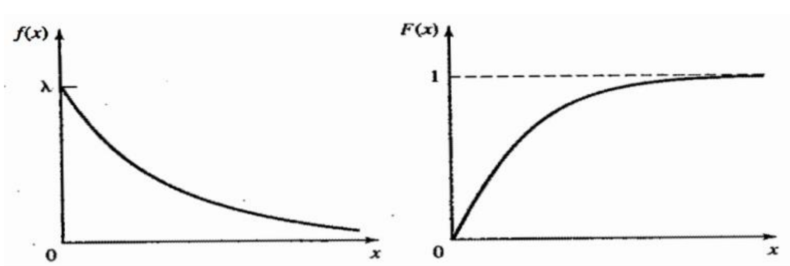


Fig : Exponential density function and cumulative distribution function

<u>Binomial distribution</u>: The random variable X that denotes the number of successes in n Bernoulli trials has a binomlal distribution given by p(x), where

$$p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x = 0,1,2,\text{---},n \\ 0, & \text{Otherwise} \end{cases} \qquad (5.12)$$

Where,

$q=1-p$

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

An easy approach to calculating the mean and variance of the binomial distribution is to consider X as a sum of n independent Bernoulli random variables, each with mean p and variance p(1 - p) = pq. Then,

$X=X_1+ X_2+\text{---}+X_n$

and the mean, E(X), is given by

$E(X) = p + p + ... + p = np$

and the variance V(X) is given by

$V(X) = pq + pq + \cdot\cdot\cdot + pq = npq$

3.b. With example explain the properties of poisson process. (06 Marks)

<u>Ans</u>:

<u>Properties of Poisson Process</u>:

1. Random Splitting
2. Pooled Process

1. Random Splitting:
   - Consider a Poisson process $\{N(t), t \geq 0\}$ having rate λ, as represented by the left side of Figure 5.25.
   - Suppose that, each time an event occurs, it is classified as either a type I or a type II event. Suppose further that each event is classified as a type I event with probability p and type II event with probability 1-p, independently of all other events.
   - Let $N_1(t)$ and $N_2(t)$ be random variables that denote, respectively, the number of type I and type II events occurring in [0, t].
   - Note that $N(t) = N_1(t) + N_2(t)$.
   - It can be shown that $N_1(t)$ and $N_2(t)$ are both Poisson processes having rates p and (1-p), as shown in Figure 5.25. Furthermore, it can be shown that the two processes are independent.

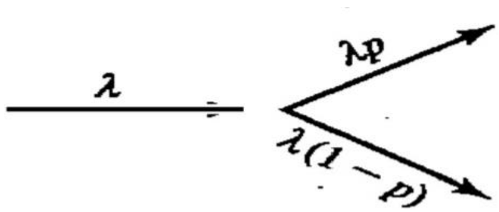


Figure 5.25 Random splitting

2. Pooled Process:

- Now, consider the opposite situation from random splitting, namely the pooling of two arrival streams.
- The process of interest is illustrated in Figure 5.26.
- It can be shown that, if $N_i(t)$ are random variables representing independent Poisson processes with rates λ_i for i= 1 and 2, then $N(t) = N_1(t) + N_2(t)$ is a Poisson process with rate $\lambda_1+\lambda_2$.

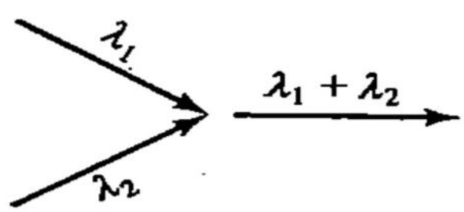


Figure 5.26 Pooled process

3.c. The time to failure of a battery is weibull-distributed with location parameter=0, α=½ years and β= ¼.what fraction of batteries are expected to last longer than the mean life. (04 Marks)

Ans:

Given,

$v = 0$

$\alpha = \frac{1}{2}$ years

$\beta = \frac{1}{4}$

Mean of weibull distribution is given by

$E(X) = v+\alpha\Gamma(1/\beta+1)$

$= 0+\frac{1}{2}\Gamma(4+1)$

$= \frac{1}{2}\Gamma(5)$

$= \frac{1}{2}\{(5-1)!\}$

$= \frac{1}{2}(4!)$

$E(X) = 12$ years

Fraction of batteries expected to last longer than the mean life = $P(X>12)$

$P(X>12) = 1-P(X\leq 12)$

$= 1-F(12)$

$= 1-\left\{1-\exp\left[-\left(\frac{12-0}{0.5}\right)^{1/4}\right]\right\}$

$= 1-\left\{1-\exp\left[-24\right]^{1/4}\right\}$

$= 1-\left\{1-\exp\left[-2.213\right]\right\}$

$= 1-\left\{1-0.10937\right\}$

= 1-{0.89063}

P(X>12) = 0.10937

4.a. Explain the characteristics of Queueing System. List different queueing notations. (10 Marks)

Ans:

Characteristics of Queuing System:

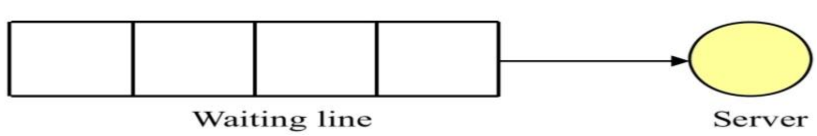
1. Calling population:

- The population of potential customers is refered to as the *calling population*.
- The calling population can be finite or infinite.
- The key difference between “finite” and “infinite” population model is how the arrival rate is defined.
- ➢ In an infinite population model, the arrival rate is not affected by the number of customers in the system. Usually the system is viewed as an *open* system, customers come from outside the system and leave the system after finishing the work.
- ➢ In a finite population model, the arrival rate at a server is affected by the population in the system. Usually the system is viewed as a *closed* system, customers (with a fixed population) don't leave the system, they merely move around system from one server to another, from one queue to another.

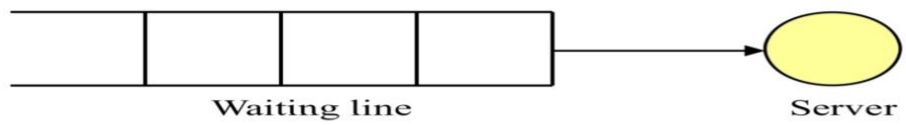
2. System capacity:

A limit on the number of customers that may be in the waiting line or system.

- Limited capacity: e.g., an automatic car wash only has room for 10 cars to wait in line to enter the mechanism.
- If system is full no customers are accepted anymore



- Unlimited capacity, e.g., concert ticket sales with no limit on the number of people allowed to wait to purchase tickets.



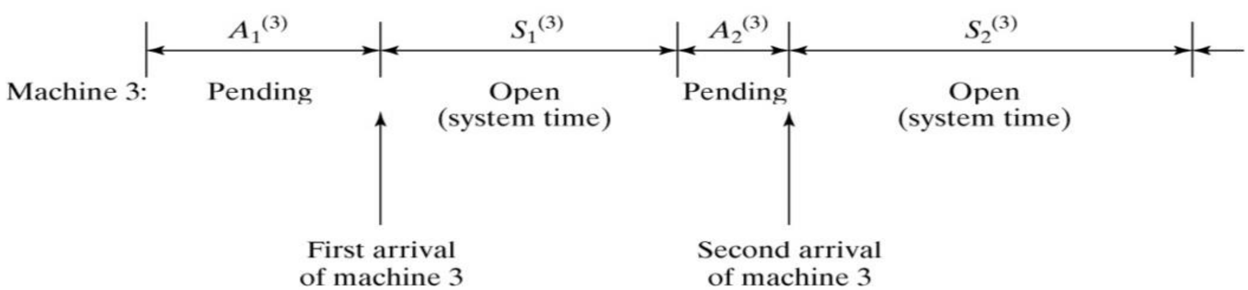
3. Arrival Process:

infinite-population models:

- The arrival process for infinite-population models is usually characterized in terms of interarrival times of successive customers.
- Arrivals may occur at scheduled times or at random times.

For finite-population models:

- Customer is pending when the customer is outside the queueing system, e.g., machine-repair problem: a machine is “pending” when it is operating, it becomes “not pending” the instant it demands service from the repairman.
- Runtime of a customer is the length of time from departure from the queueing system until that customer’s next arrival to the queue, e.g., machine-repair problem, machines are customers and a runtime is time to failure (TTF).
- Let $A_1^{(i)}$, $A_2^{(i)}$, … be the successive runtimes of customer i, and $S_1^{(i)}$,$S_2^{(i)}$ be the corresponding successive system times:



4. Queue Behavior and Queue Discipline:

Queue behavior: the actions of customers while in a queue waiting for service to begin, for example:

- Balk: leave when they see that the line is too long
- Renege: leave after being in the line when its moving too slowly
- Jockey: move from one line to a shorter line

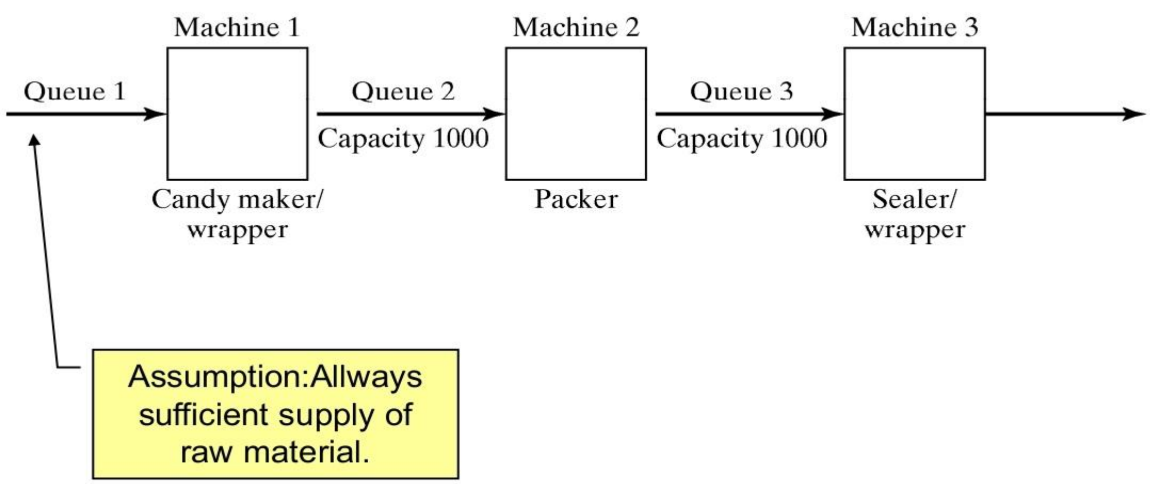
Queue discipline: the logical ordering of customers in a queue that determines which customer is chosen for service when a server becomes free, for example:

- First-in-first-out (FIFO)
- Last-in-first-out (LIFO)
- Service in random order (SIRO)
- Shortest processing time first (SPTF)
- Service according to priority (PR)

5. Service Times and Service Mechanism:

- Service times of successive arrivals are denoted by S_1, S_2, S_3. They May be constant or of random duration.
- $\{S_1, S_2, S_3, \ldots\}$ is usually characterized as a sequence of independent and identically distributed (IID) random variables.
- A queueing system consists of a number of service centers and interconnecting queues. Each service center consists of some number of servers, c, working in parallel; that is, upon getting to the head of the line, a customer takes the first available server.
- Parallel service mechanisms are either single server ($c = 1$), multiple server ($1 < c < \infty$), or unlimited servers ($c = \infty$).
- A self-service facility is usually characterized as having an unlimited number of servers.

Example of Queueing System:



Candy production line

- Three machines separated by buffers
- Buffers have capacity of 1000 candies

Queueing Notations:

- A notation system for parallel server queues: A/B/c/N/K
  →A represents the interarrival-time distribution
  →B represents the service-time distribution
  →c represents the number of parallel servers
  →N represents the system capacity
  →K represents the size of the calling population
  →N, K are usually dropped, if they are infinity
- Common symbols for A and B

  →M Markov,exponential distribution

  →D Constant, deterministic

  →Ek Erlang distribution of order k

  →H Hyperexponential distribution

  →G General, arbitrary

• P_n	steady-state probability of having n customers in system
• $P_n(t)$	probability of n customers in system at time t
• λ	arrival rate
• λ_e	effective arrival rate
• μ	service rate of one server
• ρ	server utilization
• A_n	interarrival time between customers n-1 and n
• S_n	service time of the n-th arriving customer
• W_n	total time spent in system by the n-th customer
• W_n^Q	total time spent in the waiting line by customer n
• $L(t)$	the number of customers in system at time t
• $L_Q(t)$	the number of customers in queue at time t
• L	long-run time-average number of customers in system
• L_Q	long-run time-average number of customers in queue
• W	long-run average time spent in system per customer
• w_Q	long-run average time spent in queue per customer

Table 6.2 Queueing Notation for Parallel Server Systems

4.b. What is network of queue? Mention the general assumption for a stable system with infinite calling population. (06 Marks)

Ans:

Network of Queue:

Networks of queues are systems in which a number of queues are connected by what's known as customer routing. When a customer is serviced at one node it can join another node and queue for service, or leave the network.

Assumption for a stable system with infinite calling population:

1. Provided that no customers are created or destroyed in the queue, then the departure rate out of a queue is the same as the arrival rate into the queue; over the long run.
2. If customers arrive to queue i at rate λ_i , and a fraction $0 \leq p_{ij} \leq 1$ of them are routed to queue j upon departure, then the arrival rate from queue i to queue j is $\lambda_i p_{ij}$ over the long run.
3. The overall arrival rate into queue j, λ_j, is the sum of the arrival rate from all sources. If customers arrive from outside the network at rate a_j, then

$$\lambda_j = a_j + \sum_{\text{all } i} \lambda_i p_{ij}$$

4. If queue j has $c_j < \infty$ parallel servers, each working at rate μ_j ,then the long-run utilization of each server is

$$\rho_j = \frac{\lambda_j}{c_j \mu_j}$$

and $\rho_j < 1$ is required for the queue to be stable

5. If, for each queue j, arrivals from outside the network form a Poisson process with rate a_j, and if there are c_j identical servicers delivering exponentially distributed service times with mean $1/\mu_j$ (where c_j may be ∞), then, in steady state, queue j behaves like an M/M/c_j queue with arrival rate $\lambda_j = a_j + \sum_{\text{all } i} \lambda_i p_{ij}$

Module 3

5.a. What are the problems that occur while generating pseudo-random numbers? Also list the important considerations during generation of random numbers.

(08 Marks)

Ans:

Problems that occur while generating pseudo-random numbers:

1. The generated numbers might not be uniformly distributed.

2. The generated numbers might be discrete-valued instead of continuous-valued.

3. The mean of the generated numbers might be too high or too low.

4. The variance of the generated numbers might be too high or too low.

5. There might be dependence. The following are examples:

   (a) autocorrelation between numbers;

   (b) numbers successively higher or lower than adjacent numbers;

   (c) several numbers above the mean followed by several numbers below the mean.

Important considerations during generation of random numbers:

1. The routine should be fast. Individual computations are inexpensive, but simulation could require many millions of random numbers. The total cost can be managed by selecting a computationally efficient method of random-number generation.

2. The routine should be portable to different computers and, ideally, to different programming languages. This is desirable so that the simulation program will produce the same results wherever it is executed.

3. The routine should have a sufficiently long cycle. The cycle length, or period, represents the length of the random number sequence before previous numbers begin to repeat themselves in an earlier order. Thus, if 10,000 events are to be generated, the period should be many times that long. A special case of cycling is degenerating. A routine degenerates when the same random numbers appear repeatedly. Such an occurrence is certainly unacceptable. This can happen rapidly with some methods.

4. The random numbers should be replicable. Given the starting point (or conditions) it should be possible to generate the same set of random numbers. completely independent of the system that is being simulated. This is helpful for debugging purposes and is a means of facilitating comparisons between systems. For the same reasons, it should be possible to easily specify different starting points, widely separated, within the sequence.

5. Most important, the generated random numbers should closely approximate the ideal statistical properties of uniformity and independence.

5.b. Consider the sequence of random numbers 0.12,0.01,0.23,0.28,0.89,0.31,0.64, 0.28,0.83,0.93,0.99,0.15,0.33,0.35,0.91,0.41,0.60,0.27,0.75,0.88,0.68,0.49,0.05, 0.43,0.95,0.58,0.19,0.36,0.69,0.87. Test whether 3rd,8th,13th and so on numbers in the above sequence are auto-correlated. At significance level =0.05, Normal critical table value is given as 1.96. (08 Marks)

Ans:

Here,

$i = 3$ (beginning with the third number)

$m = 5$ (every five numbers)

$N = 30$ (30 numbers in the sequence)

$i+(M+1)m \leq N$

$3+(M+1)5 \leq 30$

$3+5M+5 \leq 30$

$5M+8 \leq 30$

$5M \leq 22$

$M \leq 4.4$

$M = 4$ (largest integer such that $3+(M + 1)5 \leq 30$)

$$\hat{\rho}_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km}R_{i+(k+1)m}\right] - 0.25$$

$$\hat{\rho}_{35} = \frac{1}{4+1}[R_3R_8+R_8R_{13}+R_{13}R_{18}+R_{18}R_{23}+R_{23}R_{28}]-0.25$$

$$= \frac{1}{4+1}[(0.23)(0.28)+(0.28)(0.33)+(0.33)(0.27)+(0.27)(0.05)+(0.05)(0.36)]-0.25$$

$$= -0.1945$$

$$\sigma_{\hat{\rho}_{im}} = \frac{\sqrt{13M+7}}{12(M+1)}$$

$$\sigma_{\hat{\rho}_{35}} = \frac{\sqrt{13(4)+7}}{12(4+1)} = 0.1280$$

The test statistics is given by:

$$Z_o = \frac{\hat{\rho}_{im}}{\sigma_{\hat{\rho}_{im}}} = \frac{-0.1945}{0.1280} = -1.516$$

From the standard normal distribution table, the critical value is:

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$

Since $-Z_{0.025} \leq Z_o \leq Z_{0.025}$ That is $-1.96 \leq -1.516 \leq 1.96$

Therefore, the hypothesis of independence cannot be rejected on the basis of this test.

6.a. Explain inverse transform technique for, (i) Exponential distribution (ii) Triangular distribution. (08 Marks)

Ans:

(i) Exponential Distribution:

The exponential distribution has the probability density function (pdf)

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x<0 \end{cases}$$

and the cumulative distribution function (cdf)

$$F(x)= \begin{cases} 0 & x<0 \\ 1-e^{-\lambda x} & x \geq 0 \end{cases}$$

The parameter λ can be interpreted as the mean number of occurrences per time unit.

$$E(Xi) = \frac{1}{\lambda}$$

step-by-step procedure for the inverse-transform technique, by the exponential distribution, consists of the following steps:

Step 1: Compute the cdf of the desired random variable X.

For the exponential distribution, the cdf is F(x) = $1-e^{-\lambda x}$ $x \geq 0$

Step 2: Set F(X) = R on the range of X.

For the exponential distribution, it becomes $1-e^{-\lambda x}$ =R on the range x≥0.

X is a random variable ,so $1-e^{-\lambda x}$ is also a random variable, here called R.

Step 3: Solve the equation F(X) = R for X in terms of R.

$$1-e^{-\lambda x} = R$$

$$e^{-\lambda x} = 1-R$$

$$-\lambda X = \ln(1-R)$$

$$X = -\frac{1}{\lambda}\ln(1-R) \qquad \text{------} \quad (8.1)$$

Equation (8.1) is called a random-variate generator for the exponential distribution. In general, Equation (8.1) is written as $X = F^{-1}(R)$.

Step 4: Generate (as needed) uniform random numbers $R_1, R_2, R_3, \ldots$ and compute the desired random variates by $X_i = F^{-1}(R_i)$

For the exponential case, $F^{-1}(R) = (-1/\lambda)\ln(1 - R)$ by Equation (8.1), so

$$X_i = -\frac{1}{\lambda}\ln(1-R_i) \qquad \text{------- (8.2)}$$

for i= 1, 2, 3, ... One simplification that is usually employed in Equation (8.2) is to replace $1 - R_i$ by R_i to yield

$$X_i = -\frac{1}{\lambda}\ln(R_i) \qquad \text{------- (8.3)}$$

This alternative is justified by the fact that both R_i and $1 - R_i$ are uniformly distributed on [0, 1].

(ii) Triangular Distribution:

Consider a random variable X that has pdf

$$f(x) = \begin{cases} x\,, & 0 \le x \le 1 \\ 2-x\,, & 1 < x \le 2 \\ 0\,, & \text{Otherwise} \end{cases}$$

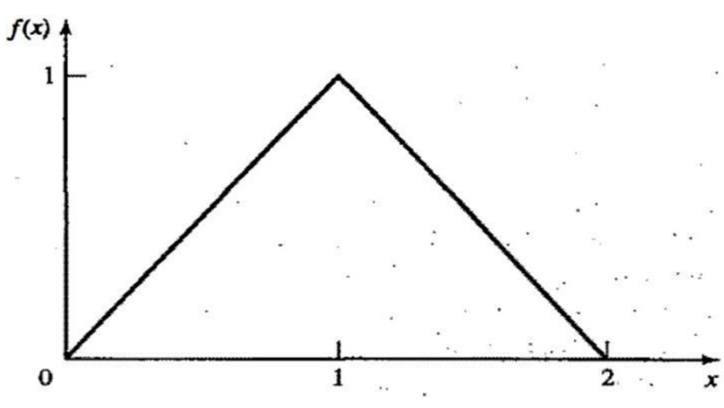


Figure 8.3 Density function for a particular triangular distribution.

As shown in Figure 8.3. This distribution is called a triangular distribution with endpoints (0, 2) and mode at 1. Its cdf is given by

$$F(x)=\begin{cases} 0, & x \le 0 \\ \frac{x^2}{2}, & 0 < x \le 1 \\ 1-\frac{(2-x)^2}{2}, & 1 < x \le 2 \\ 1, & x > 2 \end{cases}$$

For $0 \le X \le 1$,

$$R=\frac{X^2}{2} \qquad (I)$$

For $1 \le X \le 2$,

$$R= 1-\frac{(2-X)^2}{2} \qquad (II)$$

By equation (I), $0 \le X \le 1$ implies that $0 \le R \le \frac{1}{2}$, in which case $X=\sqrt{2R}$. By equation (II), $1 \le X \le 2$ implies that $\frac{1}{2} \le R \le 1$, in which case $X= 2-\sqrt{2(1-R)}$.Thus X is generated by

$$X = \begin{cases} \sqrt{2R}, & 0 \le R \le \frac{1}{2} \\ 2-\sqrt{2(1-R)}, & \frac{1}{2} < R \le 1 \end{cases}$$

6.b. What is Acceptance Rejection technique? Generate three poisson variates with mean α=0.2. Take the random number as: 0.4357, 0.4146, 0.8353, 0.9952, 0.8004, 0.7945. (08 Marks)

Ans:

Acceptance Rejection Technique:

Suppose that an analyst needed to devise a method for generating random variates, X, uniformly distributed between 1/4 and 1. One way to proceed would be to follow these steps:

Step 1. Generate a random number R.

Step 2a. If R l/4, accept X= R, then go to Step 3.

Step 2b. If R < ll4, reject R, and return to Step 1.

Step 3. If another uniform random variate on [ll4, 1] is needed, repeat the procedure beginning at Step 1. If not, stop.

'Each time Step 1 is executed, a new random number R must be generated. Step 2a is an "acceptance" and Step 2b is a "rejection" in this acceptance-rejection technique.

Solution to Problem:

$e^{-\alpha}= e^{-0.2}$=0.8187

step1: Set n=0, P=1

step2: $R_{n+1}=R_{0+1}=R_1$= 0.4357

$P=P*R_{n+1}=P*R_1$= 0.4357

step3:Since $P< e^{-\alpha}$ that is, 0.4357<0.8187

Accept N=0

step1: Set n=0, P=1

step2: $R_{n+1}=R_{0+1}=R_1$= 0.4146

$P=P*R_{n+1}=P*R_1$=0.4146

step3:Since $P< e^{-\alpha}$ that is, 0.4146<0.8187

Accept N=0

step1: Set n=0, P=1

step2: $R_{n+1}=R_{0+1}=R_1= 0.8353$

$P=P*R_{n+1}=P*R_1= 0.8353$

step3:Since $p \geq e^{-\alpha}$ that is, $0.8353 \geq 0.8187$

reject n=0, go to step 2 with n=1

step2: $R_{n+1}=R_{1+1}=R_2= 0.9952$

$P=P*R_{n+1}=P*R_2=0.8313$

step3:Since $P \geq e^{-\alpha}$ that is, $0.8183 \geq 0.8187$

reject n=1, go to step 2 with n=2

step2: $R_{n+1}=R_{2+1}=R_3= 0.8004$

$P=P*R_{n+1}=P*R_3= 0.6654$

step3:Since $P < e^{-\alpha}$ that is, $0.6654<0.8187$

Accept N=2

n	R_{n+1}	P	Accept/reject	Result
0	0.4357	0.4357	$P<e^{-\alpha}$(Accept)	N=0
0	0.4146	0.4146	$P<e^{-\alpha}$(Accept)	N=0
0	0.8353	0.8353	$P\geq e^{-\alpha}$(reject)	
1	0.9952	0.8313	$P\geq e^{-\alpha}$(reject)	
2	0.8004	0.6654	$P<e^{-\alpha}$(Accept)	N=2

Three poisson variates are,

N=0 , N=0 , and N=2

Module 4

7.a. Explain the steps involved in development of a useful model of input data.

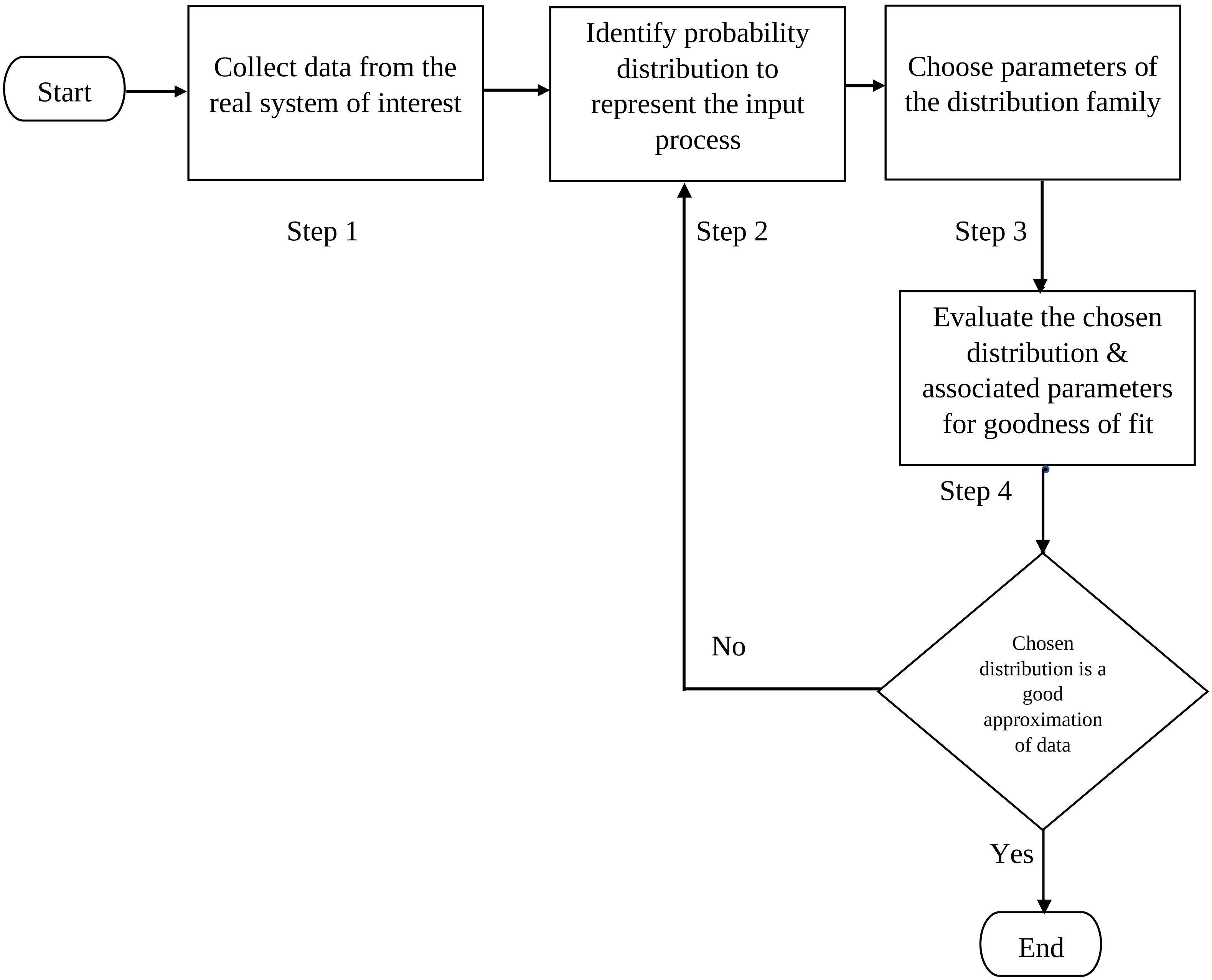
(08 Marks)

Ans:

Steps involved in development of a useful model of input data:

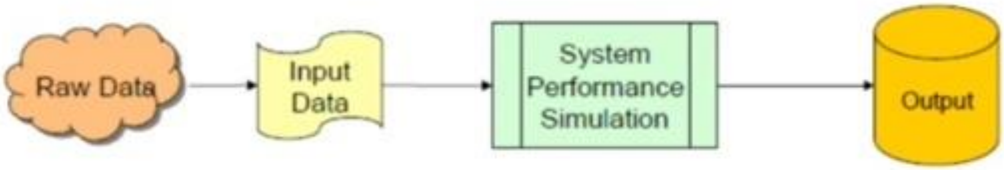
1. Data Collection
2. Identifying the Distribution with Data
3. Parameter Estimation
4. Goodness of fit tests

Flow Chart:



1. Data Collection:

- Data collection is One of the biggest task in solving a real problem.
- GIGO: Garbage-In-Garbage-Out



- Even when model structure is valid simulation results can be misleading, if the input data is
  →Inaccurately collected
  →Inappropriately analyzed
  →Not representative of the environment

2. Identifying the Distribution with Data:

Histograms

A frequency distribution or histogram is useful in identifying the shape of a distribution. A histogram is constructed as follows:

1. Divide the range of the data into intervals. (Intervals are usually of equal width; however, unequal widths may be used if the heights of the frequencies are adjusted.)
2. Label the horizontal axis to conform to the intervals selected.
3. Find the frequency of occurrences within each interval.
4. Label the vertical axis so that the total occurrences can be plotted for each interval.
5. Plot the frequencies on the vertical axis.

Selecting the Family of Distributions

A family of distribution is selected based on:

→Context of the input variable

→Shape of the histogram

Frequently encountered distributions:

i. Easier to analyze: Exponential, Normal and poisson
ii. Difficult to analyze: Beta, Gamma and Weibull

Quantile-Quantile Plots

A quantile-quantile (q - q) plot is a useful tool for evaluating distribution fit.

If X is a random variable with cdf F, then the q-quantile of X is that value such that $F(\gamma)=P(X\leq \gamma)=q$, for $0 < q < 1$. When F has an inverse, we write $\gamma= F^{-1}(q)$.

Now let $\{x_i, i = 1, 2, \ldots, n\}$ be a sample of data from X. Order the observations from the smallest to the largest, and denote these as $\{y_j, j=1, 2, \ldots, n\}$, where $y_1\leq y_2\leq \ldots \leq y_n$. Let j denote the ranking or order number. Therefore, j = 1 for the smallest and j = n for the largest. The q - q plot is based on the fact that y_j is an estimate of the $(j - 1/2)/n$ quantile of X. In other words,

$$y_j \text{ is approximately } F^{-1}\left(\frac{j-\frac{1}{2}}{n}\right)$$

3. Parameter Estimation:

Preliminary statistics: Sample mean and sample variance

i. If observations in a sample size of n are $X_1,X_2,X_3,\ldots,X_n$ (continuous or discrete), the sample mean and variance are:

$$\bar{X}= \frac{\sum_{i=1}^{n} X_i}{n} \qquad S^2 = \frac{\sum_{i=1}^{n} X_i^2-n\bar{X}^2}{n-1}$$

ii. If the data are discrete and have been grouped in a frequency distribution:

$$\bar{X} = \frac{\sum_{j=1}^{k} f_j X_j}{n} \qquad S^2 = \frac{\sum_{j=1}^{k} f_j X_j^2-n\bar{X}^2}{n-1}$$

$k \rightarrow$ number of distinct values of X

$f_j \rightarrow$ Observed frequency of value X_j

iii. When raw data are unavailable (data are grouped in to class intervals), the approximate sample mean and variance are:

$$\bar{X} = \frac{\sum_{j=1}^{c} f_j m_j}{n} \qquad S^2 = \frac{\sum_{j=1}^{c} f_j m_j^2-n\bar{X}^2}{n-1}$$

f_j → observed frequency in the jth class interval
m_j →midpoint of the jth interval
c →number of class intervals

Suggested Estimators:

Distribution	Parameter	Estimator
Poisson	α	$\hat{\alpha} = \overline{X}$
Exponential	λ	$\hat{\lambda} = \frac{1}{\overline{X}}$
Gamma	β, θ	$\hat{\theta} = \frac{1}{\overline{X}}$
Normal	μ, σ^2	$\hat{\mu} = \overline{X}, \hat{\sigma}^2 = S^2$
Lognormal	μ, σ^2	$\hat{\mu} = \overline{X}, \hat{\sigma}^2 = S^2$

Table: Suggested Estimators for Distributions Often Used in Simulation

4. Goodness-of-Fit Tests:

Conduct hypothesis testing on input data distribution using

→Kolmogorov-Smirnov test

→Chi-square test

No single correct distribution in a real application exists

→If very little data are available, it is unlikely to reject any candidate distributions

→If a lot of data are available, it is likely to reject all candidate distributions

7.b. Apply chi-square goodness of fit test to poisson assumption with mean α =3.64, Data size=100 and observed frequency Oi = 12,10,19,17,10,8,7,5,5,3,3,1 and $\chi^2_{0.05,5}$=11.1 (08 Marks)

Ans:

Step 1:

H_0 : The random variable is poisson distributed.

H_1 : The random variable is not poisson distributed.

Step 2: Compute $\hat{\alpha}$ (Estimator)

Given $\hat{\alpha}$ =3.64

Step 3: Compute theoretical hypothesized probability using pmf of poisson distribution.

$$p(x) = \begin{cases} \dfrac{e^{-\alpha}\alpha^{x}}{x!}, & x=0, 1,2,\ldots \\ 0, & \text{Otherwise} \end{cases}$$

$$p(0) = \frac{e^{-3.64}(3.64)^{0}}{0!} = 0.026$$

$$p(1) = \frac{e^{-3.64}(3.64)^{1}}{1!} = 0.096$$

$$p(2) = \frac{e^{-3.64}(3.64)^{2}}{2!} = 0.174$$

$$p(3) = \frac{e^{-3.64}(3.64)^{3}}{3!} = 0.211$$

$$p(4) = \frac{e^{-3.64}(3.64)^{4}}{4!} = 0.192$$

$$p(5) = \frac{e^{-3.64}(3.64)^{5}}{5!} = 0.140$$

$$p(6) = \frac{e^{-3.64}(3.64)^6}{6!} = 0.085$$

$$p(7) = \frac{e^{-3.64}(3.64)^7}{7!} = 0.044$$

$$p(8) = \frac{e^{-3.64}(3.64)^8}{8!} = 0.020$$

$$p(9) = \frac{e^{-3.64}(3.64)^9}{9!} = 0.008$$

$$p(10) = \frac{e^{-3.64}(3.64)^{10}}{10!} = 0.003$$

$$P(\geq 11) = \frac{e^{-3.64}(3.64)^{11}}{11!} = 0.001$$

Step 4: Compute Expected value using $E_i = n \times p_i$,Where n is data size (n=100).

$E_0 = 100 \times 0.026 = 2.6$

$E_1 = 100 \times 0.096 = 9.6$

$E_2 = 100 \times 0.174 = 17.4$

$E_3 = 100 \times 0.211 = 21.1$

$E_4 = 100 \times 0.192 = 19.2$

$E_5 = 100 \times 0.140 = 14.0$

$E_6 = 100 \times 0.085 = 8.5$

$E_7 = 100 \times 0.044 = 4.4$

$E_8 = 100 \times 0.020 = 2.0$

$E_9 = 100 \times 0.008 = 0.8$

$E_{10} = 100 \times 0.003 = 0.3$

$E_{11} = 100 \times 0.001 = 0.1$

Step 5: Compute χ_0^2

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

x_i	Observed Frequency, O_i		Expected Frequency, E_i		$(O_i - E_i)^2/E_i$
0	12	22	2.6	12.2	7.87
1	10		9.6		
2	19		17.4		0.15
3	17		21.1		0.8
4	19		19.2		4.41
5	6		14.0		2.57
6	7		8.5		0.26
7	5		4.4		
8	5		2.0		
9	3	17	0.8	7.6	11.62
10	3		0.3		
> 11	1		0.1		
	100		100.0		27.68

$$E_i = n \cdot p(x) = n \cdot \frac{e^{-\alpha}\alpha^x}{x!}$$

Combined because of the assumption of min $E_i = 5$, e.g., $E_1 = 2.6 < 5$, hence combine with E_2

$\chi_0^2 = 27.68$

Step 6: Compute critical value

$$\chi^2_{\alpha,k-s-1} = \chi^2_{0.05,7-1-1} = \chi^2_{0.05,5} = 11.1$$

Where,

k-s-1 → Degree of freedom for the tabulated value of χ_0^2

k → Number of intervals after grouping

s → Number of parameters for estimation

Step 7: Check whether random numbers are uniformly distributed.

Compare χ_0^2 and $\chi^2_{0.05,5}$

∵ 27.69 > 11.1

The null hypothesis, H_0, is rejected.

⇒Random numbers are not uniformly distributed.

8.a. List and explain the different ways to obtain information about a process even if data are not available. (06 Marks)

<u>Ans</u>:

There are a number of ways to obtain information about a process even if data are not available:

Engineering data:

- Often a product or process has performance ratings provided by the manufacturer
- for example,
  1. the mean time to failure of a disk drive is 10000 hours
  2. a laser printer can produce 8 pages/minute
  3. the cutting speed of a tool is 1 cm/second; etc.
- Company rules might specify time or production standards. These values provide a starting point for input modeling by fixing a central value.

Expert option:

- Talk to the people who are experienced with the process or similar processes. Often, they can provide optimistic, pessimistic, and most-likely times.
- They might also be able to say whether the process is nearly constant or highly variable. And they might be able to define the source of variability.

Physical or conventional limitations:

- Most real processes have physical limits on performance.
- for example, computer data entry cannot be faster than a person can type. Because of company policies, there could be upper limits on how long a process may take.
- Do not ignore obvious limits or bounds that narrow the range of the input process.

The nature of the process:

- The description of the distributions can be used to justify a particular choice even when no data are available.
- When data are not available, the uniform, triangular, and beta distributions are often used as input models.

- The uniform can be a poor choice, because the upper and lower bounds are rarely just as likely as the central values in real processes.
- If, in addition to upper and lower bounds, a most-likely value can be given, then the triangular distribution can be used.

8.b. Briefly explain the types of simulation with respect to output analysis. Give examples. (06 Marks)

Ans:

Types of simulation with respect to output Analysis:

i. Terminating or transient simulation
ii. Steady state simulation

i. Terminating or Transient Simulation:

- A terminating simulation is one that runs for some duration of time T_E, where E is specified event (or set of events) that stops the simulation.

Example 1:

Communication system consists of several components plus several backup components as shown in below figure.

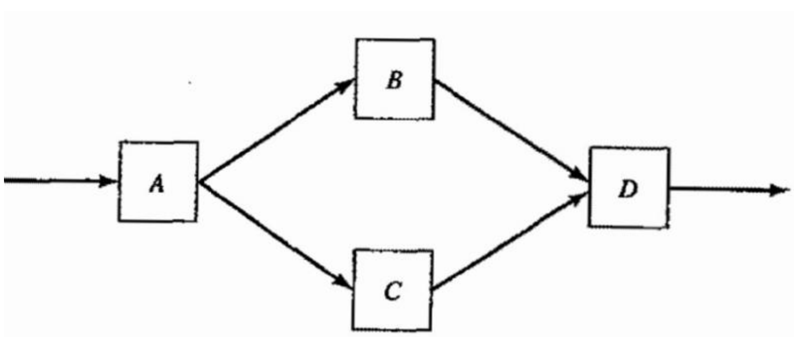


Fig: Example of Communications System

Consider system over a period of time T_E, until the system fails. The stopping event E is defined by E={A fails, or D fails, or (B and C both fail)}.Intial conditions are that all components are new at time 0.

The stopping time T_E is generally unpredictable in advance, in fact T_E is probably the output variable of interest as it represents the total time until the system breaks down. One goal of the simulation might be to estimate E(TE), the mean time of system failure.

Example 2:

The Shady Grove Bank opens at 8:30 A.M. (time 0) with no customers present and 8 of the 11 tellers working (initial conditions) and closes at 4:30 P.M. (time T_E= 480 minutes). Here the event E is merely the fact that the bank has been open for 480 minutes. The simulation analyst is interested in modeling the interaction between customers and tellers over the entire day, including the effect of starting up and of closing down at the end of the day.

ii. Steady State Simulation:

- A steady state simulation is a simulation whose objective is to study long run or steady state behavior of a non-terminating simulation.

Example 1:

Consider the widget-manufacturing process , beginning with the second shift when the complete production process is under way. It is desired to estimate long-run production levels and production efficiencies. For the relatively long period of 13 shifts, this may be considered as a steady-state simulation. To obtain sufficiently precise estimates of production efficiency and other response variables, the analyst could decide to simulate for any length of time, T_E (even longer than 13 shifts)-that is, T_E is not determined by the nature of the problem (as it was in terminating simulations); rather, it is set by the analyst as one parameter in the design of the simulation experiment.

Example 2:

HAL Inc., a large computer-service bureau, has many customers worldwide. Thus, its large computer system with many servers, workstations, and peripherals runs continuously, 24 hours per day. To handle an increased work load, HAL is considering additional CPUs, memory, and storage devices in various configurations. Although the load on HAL's computers varies throughout the day, management wants the system to be able to accommodate sustained periods of peak load. Furthermore, the time frame in which HAL's business will change in any substantial way is unknown, so there is no fixed planning horizon. Thus, a steady-state simulation at peak-load conditions is appropriate. HAL systems staff develops a simulation model of the existing system with the current peak work load and then explores several possibilities for expanding capacity. HAL is interested in long-run average throughput and utilization of each computer. The stopping time, T_E, is determined not by the nature of the problem, but rather by the simulation analyst, either arbitrarily or with a certain statistical precision in mind.

8.c. Write a short note on point estimation. (04 Marks)

Ans:

Point Estimation for discrete time data:

Point estimator of θ based on the data $\{Y_1, ----, Y_n\}$ is defined by

$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

Where,

n→Sample size

Point estimator $\hat{\theta}$ is said to be unbiased for θ if its expected value is θ, that is if

$$E(\hat{\theta}) = \theta$$

In general, however

$$E(\hat{\theta}) \neq \theta$$

Point Estimation for continuous-time data:

The point estimator of ∅ based on the data $\{Y(t), 0 \leq t \leq T_E\}$ is defined by

$$\hat{\phi} = \frac{1}{T_E} \int_0^{T_E} Y(t)\,dt$$

TE→run length of simulation

In general,

$$E(\hat{\emptyset}) \neq \emptyset$$

And $\hat{\emptyset}$ is said to be biased for $\emptyset$.

Usually, system performance measures can be put into the common framework of θ or $\emptyset$:

- Example: The proportion of days on which sales are lost through an out-of-stock situation, let:

$$Y_i = \begin{cases} 1, & \text{if out of stock on day i} \\ 0, & \text{Otherwise} \end{cases}$$

- Example: Proportion of time that the queue length is larger than k_0

$$Y(t) = \begin{cases} 1, & \text{if } L_Q(t) > k_0 \\ 0, & \text{Otherwise} \end{cases}$$

Module 5

9.a. Explain output analysis for steady state simulation. (08 Marks)

Ans:

- Consider a single run of a simulation model to estimate a steady-state or long-run characteristics of the system.
- The single run produces observations Y_1, Y_2, ... (generally the samples of an autocorrelated time series).
- Performance measure:

$$\theta = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} Y_i, \qquad \text{for discrete measure} \qquad \text{(with probability 1)}$$

$$\phi = \lim_{T_E\to\infty} \frac{1}{T_E} \int_0^{T_E} Y(t)\,dt, \qquad \text{for continuous measure} \qquad \text{(with probability 1)}$$

Initialization Bias in Steady-State Simulations:

Methods to reduce the point-estimator bias caused by using artificial and unrealistic initial conditions:

- Intelligent initialization.
- Divide simulation into an initialization phase and data-collection phase.

<u>Intelligent initialization</u>

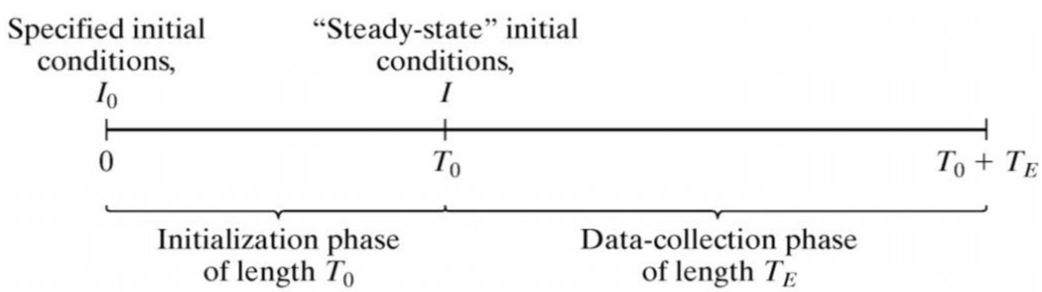
- Initialize the simulation in a state that is more representative of long run conditions.
- If the system exists, collect data on it and use these data to specify more nearly typical initial conditions.

<u>Divide each simulation into two phases:</u>

→An initialization phase, from time 0 to time T_0.
→A data-collection phase, from T0 to the stopping time T_0+T_E.

- The choice of T_0 is important:

  After T_0 , system should be more nearly representative of steady state behavior.



Replication Method for Steady-State Simulations:

- Use to estimate point-estimator variability and to construct a confidence interval.
- Approach: make R replications, initializing and deleting from each one the same way.
- Important to do a thorough job of investigating the initial-condition bias:

  →Bias is not affected by the number of replications, instead, it is affected only by deleting more data (i.e., increasing T_0) or extending the length of each run (i.e. increasing T_E).

- Basic raw output data {Yrj, r = 1, ..., R; j = 1, …, n} is derived by:

  →Individual observation from within replication r.

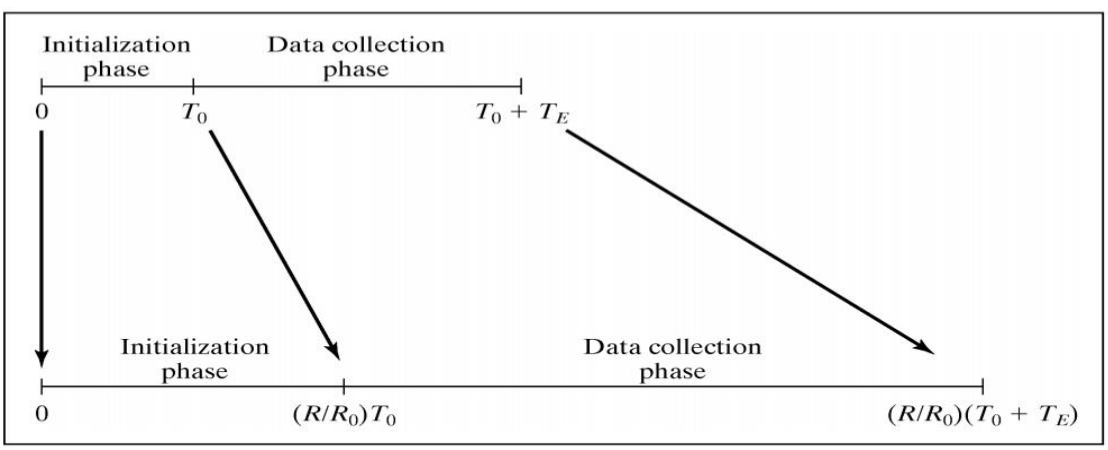
→Batch mean from within replication r of some number of discrete-time observations.

→Batch mean of a continuous-time process over time interval j.

Replication	Observations						Replication Averages
	1	$\cdots$	d	$d+1$	$\cdots$	n	
1	$Y_{1,1}$	$\cdots$	$Y_{1,d}$	$Y_{1,d+1}$	$\cdots$	$Y_{1,n}$	$\bar{Y}_{1\bullet}(n,d)$
2	$Y_{2,1}$	$\cdots$	$Y_{2,d}$	$Y_{2,d+1}$	$\cdots$	$Y_{2,n}$	$\bar{Y}_{2\bullet}(n,d)$
$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
R	$Y_{R,1}$	$\cdots$	$Y_{R,d}$	$Y_{R,d+1}$	$\cdots$	$Y_{R,n}$	$\bar{Y}_{R\bullet}(n,d)$
	$\bar{Y}_{\bullet 1}$	$\cdots$	$\bar{Y}_{\bullet d}$	$\bar{Y}_{\bullet(d+1)}$	$\cdots$	$\bar{Y}_{\bullet n}$	$Y_{\bullet\bullet}(n,d)$

Sample Size in Steady-State Simulations:

- To estimate a long-run performance measure, θ , within ±∈. with confidence 100(1- α)%.
- In a steady-state simulation, a specified precision may be achieved either by increasing the number of replications (R) or by increasing the run length (T_E). The first solution, controlling R, is carried out as given below.
- To increase R, increase total run length T_0+T_E within each replication.
- Approach:
  - ➔ Increase run length from (T_0+T_E) to (R/R_0)(T_0+T_E), and
  - ➔ delete additional amount of data, from time 0 to time (R/R_0)T_0.



- Advantage: any residual bias in the point estimator should be further reduced.
- However, it is necessary to have saved the state of the model at time T_0+T_E and to be able to restart the model.

Batch Means for Interval Estimation in Steady-State Simulations:

- Using a single, long replication:

  Problem: data are dependent so the usual estimator is biased.

  Solution: batch means.

- Batch means: divide the output data from 1 replication (after appropriate deletion) into a few large batches and then treat the means of these batches as if they were independent.
- Starting either with continuous-time or discrete-time data, the variance of the sample mean is estimated by:

$$\frac{S^2}{k} = \frac{1}{k}\sum_{j=1}^{k}\frac{(\bar{Y}_j - \bar{Y})^2}{k-1} = \sum_{j=1}^{k}\frac{\bar{Y}_j^2 - k\bar{Y}^2}{k(k-1)}$$

- If the batch size is sufficiently large, successive batch means will be approximately independent, and the variance estimator will be approximately unbiased.

9.b. Explain the suggestions given for use in verification process. (08 Marks)

Ans:

Many common-sense suggestions can be given for use in the verification process:

1. Have the operational model checked by someone other than its developer, preferably an expert in the simulation software being used.
2. Make a flow diagram that includes each logically possible action a system can take when an event occurs, and follow the model logic for each action for each event type.
3. Closely examine the model output for reasonableness under a variety of settings of the input parameters.

4. Have the operational model print the input parameters at the end of the simulation, to be sure that these parameter values have not been changed inadvertently.
5. Make the operational model as self-documenting as possible. Give a precise definition of every variable used and a general description of the purpose of each submodel, procedure (or major section of code), component, or other model subdivision.
6. If the operational model is animated, verify that what is seen in the animation imitates the actual system.
7. The Interactive Run Controller (IRC) or debugger is an essential component of successful simulation model building. Even the best of simulation analysts makes mistakes or commits Iogical errors when building a model. The IRC assists in finding and correcting those errors in the following ways:
   (a) The simulation can be monitored as it progresses. This can be accomplished by advancing the simulation until a desired time has elapsed, then displaying model information at that time. Another possibility is to advance the simulation until a particular condition is in effect, and then display information.
   (b) Attention can be focused on a particular entity line, of code, or procedure. For instance, every time that an entity enters a specified procedure, the simulation will pause so that information can be gathered. As another example, every time that a specified entity becomes active, the simulation will pause.
   (c) Values of selected model components can be observed When the simulation has paused, the current value or status of variables, attributes, queues, resources, counters, and so on can be observed.
   (d) The simulation can be temporarily suspended, or paused, not only to view information, but also to reassign values or redirect entities.
8. Graphical interfaces are recommended for accomplishing verification and validation. The graphical representation of the model is essentially a form of self-documentation. It simplifies the task of understanding the model.

10.a. With neat diagram, explain the iterative process of a calibrating model.

(08 Marks)

Ans:

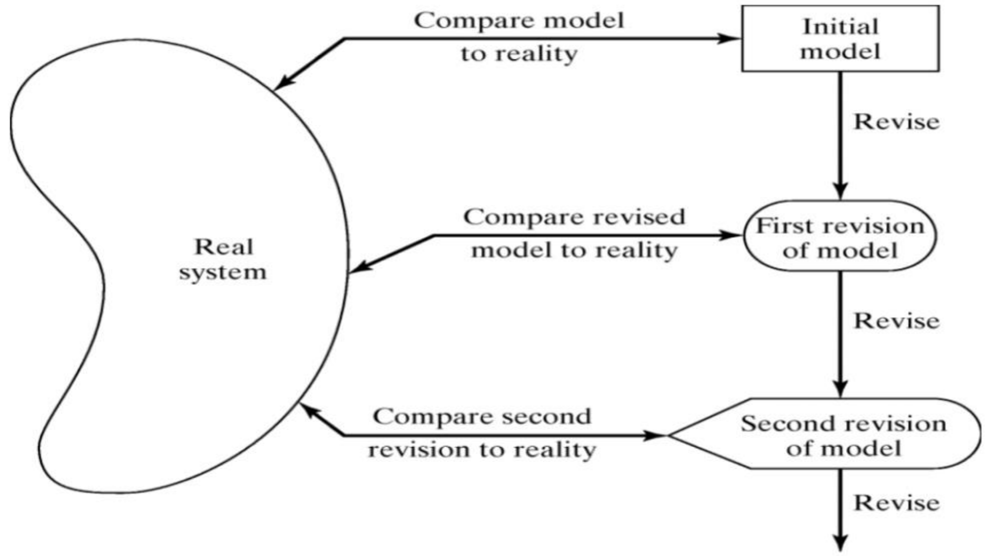


Fig: Iterative Process of Calibrating a Model

- Calibration is the iterative process of comparing the model to the real system, making adjustments to the model, comparing again and so on.
- Validation is the overall process of comparing the model and its behavior to the real system and its behavior.
- The above figure shows the relationship of the model calibration to the overall validation process.
- The comparison of the model to reality is carried out by variety of test.
- Some tests are subjective and other are objective:
    i. Subjective test usually involve people, who are knowledgeable about one or more aspects of the system, making judgments about the model and its output.
    ii. Objective tests always require data on the system's behavior plus the corresponding data produced by the model.
- Then one or more statistical tests are performed to compare some aspect of the system data set with the same aspect of the model data set.
- This iterative process of comparing model with system and then revising both the conceptual and operational models to accommodate any perceived model deficiencies is continued until the model is judged to be sufficiently accurate.

- If unacceptable discrepancies between the model and the real system are discovered in the "final" validation effort, the modeler must return to the calibration phase and modify the model until it becomes acceptable.
- each revision of the model, as pictured in Figure, involves some cost, time, and effort.
- The modeler must weigh the possible, but not guaranteed, increase in model accuracy versus the cost of increased validation effort.

10.b. Explain 3-step approach for validation process as formulated by Naylor and Finger. (08 Marks)

Ans:

As an aid in the validation process, Naylor and Finger formulated a three step approach which has been widely followed:-

1. Build a model that has high face validity.
2. Validate model assumptions.
3. Compare the model input-output transformations to corresponding input-output transformations for the real system.

Face Validity:

- The first goal of the simulation modeler is to construct a model that appears reasonable on its face to model users and others who are knowledgeable about the real system being simulated.
- The users of a model should be involved in model construction from its conceptualization to its implementation to ensure that a high degree of realism is built into the model through reasonable assumptions regarding system structure, and reliable data.
- Another advantage of user involvement is the increase in the models perceived validity or credibility without which manager will not be willing to trust simulation results as the basis for decision making.
- Sensitivity analysis can also be used to check model's face validity.
- The model user is asked if the model behaves in the expected way when one or more input variables is changed.

Validation of Model Assumptions:

- Model assumptions fall into two general classes: structural assumptions and data assumptions.
- **Structural assumptions:**
  1. Involves how system operates.
  2. Includes simplifications & abstractions of reality.

  Example: consider customer queueing and service facility in a bank. Structural assumptions are customer waiting in one line versus many lines. Customers are served according FCFS versus priority.

- **Data assumptions:**

i. based on the collection of reliable data and correct statistical analysis of the data.

Example: Inter arrival time of customers, service times of commercial accounts.

Validating Input-Output Transformation

- In this phase of validation process the model is viewed as input –output transformation.
- That is, the model accepts the values of input parameters and transforms these inputs into output measure of performance. It is this correspondence that is being validated.
- Instead of validating the model input-output transformation by predicting the future ,the modeler may use past historical data which has been served for validation purposes that is, if one set has been used to develop calibrate the model, its recommended that a separate data test be used as final validation test.

Input-Output Validation: Using Historical Input Data

- To conduct a validation test using historical input data, it is important that all input data and all the system response data, such as average delay, be collected during the same time period.

- If not taken on same time then, comparison of model responses to system responses could be misleading.
- Implementation of this technique could be difficult for a large system because of the need for simultaneous data collection of all input variables and those response variables of primary interest.

Input-Output Validation: Using a Turing Test

- when no statistical test is readily applicable, Persons knowledgeable about system behavior can be used to compare model output to system output.
- For example, suppose that five reports of system performance over five different days are prepared, and simulation outputs are used to produce five "fake" reports.
- The 10 reports should all be in exactly in the same format.
- The ten reports are randomly shuffled and given to the engineers, who are asked to decide which reports are fake and which are real.
- If engineer identifies substantial number of fake reports the model builder questions the engineer and uses the information gained to improve the model. or else the modeler will conclude that this test provides no evidence of model inadequacy.
- This type of validation test is called as TURING TEST.