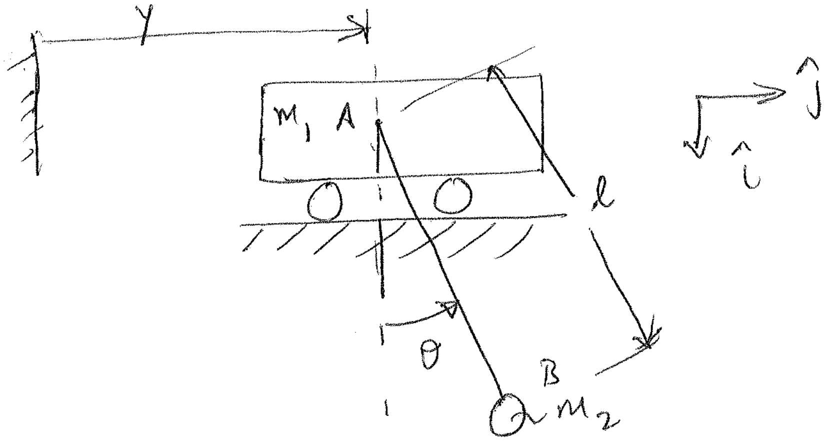
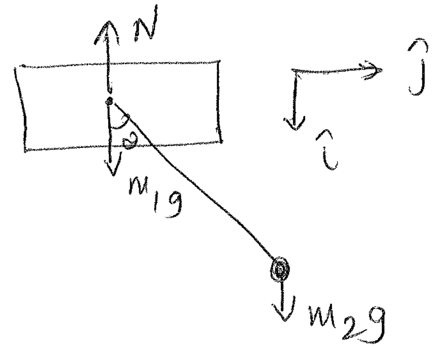
04-21-09

~~IV~~ Pendulum on cart

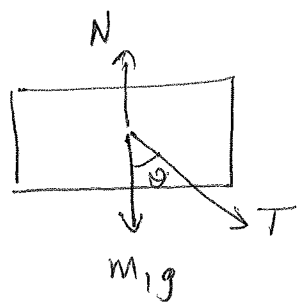


Find equations of motion

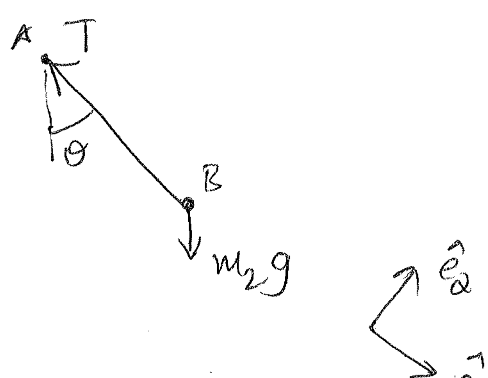
FBD: A



B



C



Unknowns: $N, T, \ddot{\theta}, \ddot{y}$

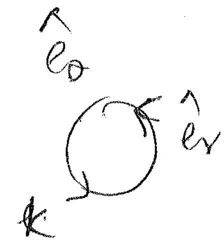
Need $\ddot{y}, \ddot{\theta}$.

(A) $\{LMB \text{ for } (A)\} \cdot \hat{j} \Rightarrow \{\sum \vec{F} = \sum \dot{\vec{L}}\} \cdot \hat{j}$

$$\{(N - m_1 g - m_2 g)\hat{i}\} \cdot \hat{j} = \{m_1 \vec{a}_A + m_2 \vec{a}_B\} \cdot \hat{j}$$

$$\vec{a}_A = \ddot{y}\hat{j}$$

$$\vec{a}_B = \vec{a}_A - \dot{\theta}^2 \vec{r}_{B/A} + \ddot{\theta}\hat{k} \times \vec{r}_{B/A}$$



Now $\vec{r}_{B/A} = l\hat{e}_r$

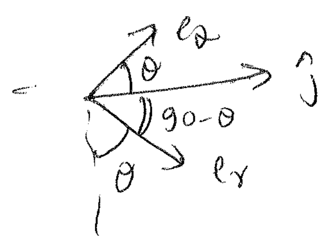
$$\vec{a}_B = \ddot{y}\hat{j} - \dot{\theta}^2 l \hat{e}_r + \ddot{\theta}\hat{k} \times l\hat{e}_r$$

$$\vec{a}_B = \ddot{y}\hat{j} - \dot{\theta}^2 l \hat{e}_r + \ddot{\theta} l \hat{e}_\theta$$

Thus

$$\{\{N - m_1 g - m_2 g\}\hat{i}\} \cdot \hat{j} = \{m_1 \ddot{y}\hat{j} + m_2(\ddot{y}\hat{j} - \dot{\theta}^2 l\hat{e}_r + \ddot{\theta} l\hat{e}_\theta)\} \cdot \hat{j}$$

$$0 = m_1\ddot{y} + m_2\ddot{y} - \dot{\theta}^2 l(\hat{e}_r \cdot \hat{j}) + \ddot{\theta} l(\hat{e}_\theta \cdot \hat{j})$$



$\hat{j} \cdot \hat{e}_r = \sin\theta$

$\hat{j} \cdot \hat{e}_\theta = \cos\theta$

Thus

$$m_1\ddot{y} + m_2\ddot{y} - m_2\dot{\theta}^2 l \sin\theta + m_2\ddot{\theta}\, l\cos\theta = 0$$

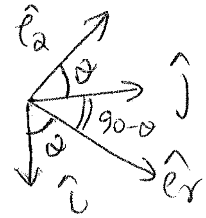
$$\boxed{(m_1+m_2)\ddot{y} + m_2 l\cos\theta\,\ddot{\theta} = m_2\dot{\theta}^2 l\sin\theta} \quad \text{— (I)}$$

AMB$_{/A}$ for (C)

$$\cancel{\vec{M}_{/A} = \vec{M}_{/A}} \qquad \vec{M}_{/A} = \dot{\vec{H}}_{/A}$$

$$\vec{r}_{B/A} \times m_2 g\,\hat{i} = \vec{r}_{B/A} \times m_2\,\vec{a}_B$$

$$l\hat{e}_r \times m_2 g\,\hat{i} = l\hat{e}_r \times m_2\{\ddot{y}\,\hat{j} - \dot{\theta}^2 l\,\hat{e}_r + \ddot{\theta}\, l\,\hat{e}_\theta\}$$



$$\hat{i} = \hat{e}_r\cos\theta - \hat{e}_\theta\sin\theta$$

$$\hat{j} = \hat{e}_r\sin\theta + \hat{e}_\theta\cos\theta$$

$$\Rightarrow \quad m_2 g l\,(\underbrace{\hat{e}_r\times\hat{i}}_{-\sin\theta\,\hat{k}}) = m_2 l\ddot{y}\,(\underbrace{\hat{e}_r\times\hat{j}}_{\cos\theta\,\hat{k}}) - l^2 m_2\dot{\theta}^2(0) + m_2 l^2\ddot{\theta}\,\hat{k}$$

$$\Rightarrow \quad -\cancel{m_2} g\cancel{l}\sin\theta\,\hat{k} = \cancel{m_2}\cancel{l}\cos\theta\,\ddot{y}\,\hat{k} + \cancel{m_2}\, l^{\cancel{2}}\ddot{\theta}\,\hat{k}$$

$$\{\ \}\cdot\hat{k} \quad \boxed{l\ddot{\theta} + \ddot{y}\cos\theta = -g\sin\theta} \quad \text{— (II)}$$

Combine equations I & II

$$\begin{bmatrix} (m_1 + m_2) & m_2 l \cos\theta \\ \cos\theta & l \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} m_2 l \dot{\theta}^2 \sin\theta \\ -g \sin\theta \end{bmatrix}$$

$$[M]\,[X] = [RHS]$$

How to code this in RHS of matlab

sample rhs file

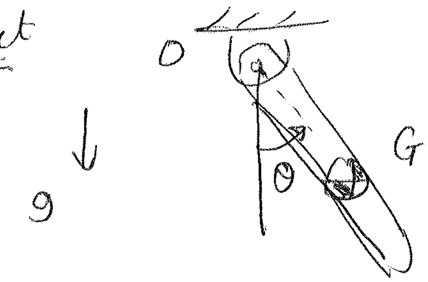
```
=> zdot = cartpendulum (t, z)

   m1 = 1; m2 = 1; g = 1; l = 1;

   y = z(1);  ydot = z(2);
   theta = z(3);  thetadot = z(4);

   M = [(m1+m2), m2 l cos(theta); cos(theta) l];

   RHS = [m2 l thetadot^2 sin(theta); -g sin(theta)];

   X = M \ RHS; % equivalent to X = M^-1 RHS

   ydddot = X(1);
   thetaddot = X(2);

   zdot = [ydot, ydddot, thetadot, thetaddot]';
```

— animation

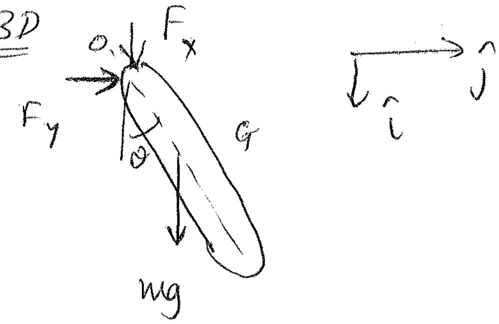
Simple pendulum Revisited — Brute force

— The way commercial packages solve ~~Rgn~~ this problem

~~Appr~~ Recollect



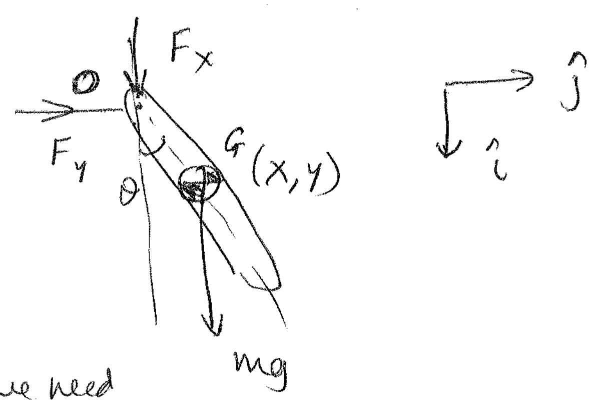
FBD



$AMB|_O$ gave $\ddot{\theta} = \dfrac{-gl}{\left(I_G + \frac{ml^2}{4}\right)} \sin\theta$

To solve specify $\theta_0, \dot{\theta}_0$ & integrate using ODE 45

To day



Rigid body ~~has~~ we need $\ddot{x}, \ddot{y}, \ddot{\theta}, F_x, F_y$? — need 5 equations

LMB $(F_x + mg)\,\hat{i} + F_y\,\hat{j} = m\ddot{x}\,\hat{i} + m\ddot{y}\,\hat{j}$

$\{\ \}\cdot\hat{i}$ $F_x + mg = m\ddot{x}$

$$\boxed{m\ddot{x} - F_x = mg} \quad —(1)$$

$\{\ \}\cdot\hat{j} \qquad F_y = m\ddot{y}$

$$\boxed{m\ddot{y} - F_y = 0} \quad \text{— (2)}$$

AMB/G $\qquad \vec{r}_{O/G} \times \vec{F} = I_G\ddot{\theta}\,\hat{k}$

$$\left\{-\tfrac{l}{2}\sin\theta\,\hat{j} - \tfrac{l}{2}\cos\theta\,\hat{i}\right\} \times \left\{F_x\hat{i} + F_y\hat{j}\right\} = I_G\ddot{\theta}\,\hat{k}$$

$$-\tfrac{l}{2}\sin\theta\, F_x(-\hat{k}) - \tfrac{l}{2}\cos\theta\, F_y\hat{k} = I_G\ddot{\alpha}\,\hat{k}$$

$\{\ \}\cdot\hat{k} \qquad \tfrac{l}{2}\sin\theta\, F_x - \tfrac{l}{2}\cos\theta\, F_y = I_G\ddot{\theta}$

$$\boxed{I_G\ddot{\theta} - \tfrac{l}{2}\sin\theta\, F_x + \tfrac{l}{2}\cos\theta\, F_y = 0} \quad \text{— (3)}$$

Hinge constraints:

$$x = \tfrac{l}{2}\cos\theta$$

$$\dot{x} = -\tfrac{l}{2}\sin\theta\,\dot{\theta}$$

$$\ddot{x} = -\tfrac{l}{2}\sin\theta\,\ddot{\theta} - \tfrac{l}{2}\cos\theta\,\dot{\theta}^2$$

$$\cancel{-\tfrac{l}{2}\cos\theta\,\dot{\theta}^2 - \ddot{x} = \tfrac{l}{2}\sin}$$

$$\boxed{-\tfrac{l}{2}\sin\theta\,\ddot{\theta} - \ddot{x} = \tfrac{l}{2}\cos\theta\,\dot{\theta}^2} \quad \text{— (4)}$$

$$y = \frac{l}{2}\sin\theta$$

$$\dot{y} = \frac{l}{2}\cos\theta\,\dot{\theta}$$

$$\ddot{y} = \frac{l}{2}\cos\theta\,\ddot{\alpha} - \frac{l}{2}\sin\theta\,\dot{\alpha}^2$$

$$\boxed{\frac{l}{2}\cos\theta\,\ddot{\theta} - \ddot{y} = \frac{l}{2}\sin\theta\,\dot{\theta}^2} \quad \text{— } 5$$

Put 1-5 in matrix form

$$\begin{bmatrix} m & 0 & 0 & -1 & 0 \\ 0 & m & 0 & 0 & -1 \\ 0 & 0 & I_G & -\frac{l}{2}\sin\theta & +\frac{l}{2}\cos\theta \\ -1 & 0 & -\frac{l}{2}\sin\theta & 0 & 0 \\ 0 & -1 & \frac{l}{2}\cos\theta & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ F_x \\ F_y \end{bmatrix} = \begin{bmatrix} mg \\ 0 \\ 0 \\ \frac{l}{2}\cos\theta\,\dot{\theta}^2 \\ \frac{l}{2}\sin\theta\,\dot{\theta}^2 \end{bmatrix}$$

IC's $\cancel{x_0 \text{ and } y_0}$ θ_0 $\dot{\theta}_0$

$$x_0 = \frac{l}{2}\cos\theta_0 \; ; \; \dot{x}_0 = -\frac{l}{2}\sin\theta_0\,\dot{\theta}_0$$

$$y_0 = \frac{l}{2}\sin\theta_0 \; ; \; \dot{y}_0 = +\frac{l}{2}\cos\theta_0\,\dot{\theta}_0$$

consistency.

ANIMATION