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Managing demand uncertainty in supply chain planning

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Abstract

In this work, we provide an overview of our previously published works on incorporating demand uncertainty in midterm planning of multisite supply chains. A stochastic programming based approach is described to model the planning process as it reacts to demand realizations unfolding over time. In the proposed bilevel-framework, the *manufacturing decisions* are modeled as 'here-and-now' decisions, which are made before demand realization. Subsequently, the *logistics decisions* are postponed in a 'wait-and-see' mode to optimize in the face of uncertainty. In addition, the trade-off between customer satisfaction level and production costs is also captured in the model. The proposed model provides an effective tool for evaluating and actively managing the exposure of an enterprises assets (such as inventory levels and profit margins) to market uncertainties. The key features of the proposed framework are highlighted through a supply chain planning case study.
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Keywords: Supply chain management; Production planning; Demand uncertainty; Stochastic optimization; Mathematical programming

1. Introduction

The increasing competitive pressures in the global marketplace coupled with the rapid advances in information technology have brought supply chain planning into the forefront of the business practices of most manufacturing and service organizations. Unwillingness to commit to large irreversible investments for expanding the existing manufacturing asset base while still generating shareholder value has forced most chemical companies to re-evaluate and improve the way in which they run and manage their existing facilities. Given the complexity of large chemical operations and the often conflicting objectives of the various business divisions, such as marketing, distribution, planning, manufacturing and purchasing, it is imperative to develop a unified and rigorous framework for capturing the various synergies and trade-offs involved. Effective integration of these various functionalities is the primary objective of supply chain planning.

Supply chain planning is concerned with the coordination and integration of key business activities undertaken by an enterprise, from the procurement of raw materials to the distribution of the final products to the customer. The decision making process in these highly complex and interacting networks can be decomposed according to the time horizons considered (Gupta & Maranas, 1999). This results in the following temporal classification of the decisions/models: *strategic*, *tactical* and *operational*. Strategic or long-term planning models aim to identify the optimal timing, location and extent of additional investments in processing networks over a relatively long time horizon ranging from 5 to 10 years (Sahinidis, Grossmann, Fornari, & Chathrathi, 1989; Sahinidis & Grossmann, 1991; Norton & Grossmann, 1994). These decisions affect the long-term performance of the system from a design and planning perspective. Short-term operational scheduling models (Shah, Pantelides, & Sargent, 1993; Xueya & Sargent, 1996; Karimi & McDonald, 1997) constitute the other extreme of the spectrum of planning models. These models are characterized by very short timeframes, such as 1–2 weeks, over which they address the exact sequencing of the manufacturing tasks while accounting for the various resource and timing constraints. The third class, com-

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Nomenclature

Sets

$\{i\}$	set of products
$\{f\}$	set of product families
$\{j\}$	set of processing units
$\{s\}$	set of production sites
$\{t\}$	set of time periods

Parameters

FC_{fjs}	fixed production cost for family f on unit j at site s
v_{ijs}	variable production cost for product i on unit j at site s
p_{is}	price of raw material i at site s
$t_{ss'}$	transportation cost from site s to site s'
t_{sc}	transportation cost from site s to site c
h_{ist}	inventory holding cost for product i at site s in period t
ζ_{is}	safety stock violation penalty for product i at site s
μ_{ic}	revenue per unit of product i sold to customer c
R_{ijst}	rate of production of product i on unit j at site s in period t
$\beta_{i'is}$	yield adjusted amount of product i consumed to produce product i' at site s
λ_{if}	0–1 parameter indicating whether product i belongs to family f
H_{jst}	production capacity of unit j at site s in period t
MRL_{fjs}	minimum run length for family f on unit j at site s
d_{ict}	demand for product i at customer c in period t
I_{ist}^{L}	safety stock for product i at site s in period t

Variables

Y_{fjst}	binary variable indicating whether product family f is manufactured on unit j at site s in period t
P_{ijst}	production amount of product i on unit j at site s in period t
RL_{ijst}	run length of product i on unit j at site s in period t
FRL_{fjst}	run length of product f on unit j at site s in period t
C_{ist}	consumption of product i at site s in period t
$W_{iss't}$	intersite shipment of product i from site s to site s' in period t
A_{ist}	amount of product i available for supply at site s in period t
I_{ist}	inventory of product i at site s in period t
S_{isct}	supply of product i from site s to customer c in period t
I_{ict}^{-}	shortage of product i at customer c in period t
I_{ist}^{Δ}	deviation below safety stock of product i at site s in period t

prising of midterm tactical models (Gupta & Maranas, 1999; Dimitriadis, Shah, & Pantelides, 1997; McDonald & Karimi, 1997), is intermediate in nature. These models address planning horizons of 1–2 years and incorporate some features from both the strategic and operational models. For instance, they account for the carryover of inventory over time and various key resource limitations much like the short-term scheduling models. Similarly, in spirit with the strategic planning models and unlike the operational models, they account for the presence of multiple production sites in the supply chain. The midterm planning models derive their value from this overlap and integration of modelling features and thus form the focus of our work.

In today's ever changing markets, maintaining an efficient and flexible supply chain is critical for every enterprise, especially given the prevailing volatilities in the business environment with constantly shifting and increasing customer expectations. Various sources of uncertainty can be identified in these systems. Based on the timeframe over which these uncertainties affect the system, they can be categorized into short-term or long-term uncertainties (Subrahmanyam, Pekny, & Reklaitis, 1994). Short-term uncertainties may include day-to-day processing variations, cancelled/rushed orders, equipment failure, etc. Long-term uncertainty refers to raw material/final product unit price fluctuations, seasonal demand variations and production rate changes occurring over longer time frames. Underestimating uncertainty and its impact can lead to planning decisions that neither safeguard a company against the threats nor take advantage of the opportunities that higher levels of uncertainty provide. For instance, one of the key sources of uncertainty in any production-distribution system is the product demand. Failure to account for significant demand fluctuations could either lead to unsatisfied customer demand translating to loss of

market share or excessively high inventory holding costs (Petkov & Maranas, 1997), both highly undesirable scenarios in the current market settings where the profit margins are extremely tight. The former scenario corresponds to a failure in recognizing an opportunity to capture additional market share while the later translates to a failure in effectively managing the downside risk exposure of the company. Deterministic planning models, which do not recognize the uncertainty in the future demand forecasts, can thus be expected to result in inferior planning decisions as compared to models that explicitly account for the uncertainty.

The rest of the paper is organized as follows. In the next section, the deterministic midterm supply chain model which has been the basis of our work is described. Subsequently, the need for incorporating uncertainty into the planning decisions is motivated and the various issues involved in achieving this are discussed. Next, the resulting mathematical framework is presented followed by a description of the benefits and challenges associated with the proposed framework for describing uncertainty. Application of the developed framework to a supply chain planning case study is then presented. Finally, the work is summarized and concluding remarks are provided.

2. Deterministic midterm production planning model

The deterministic planning model originally proposed by McDonald and Karimi (1997) is adopted as the representative formulation for our work. This model is aimed at determining the optimal sourcing and allocation of an enterprise's limited resources to its manufacturing assets so as to satisfy the market demands in the most cost-effective way. The supply chain network considered in the model consists of multiple production sites, potentially located globally, manufacturing multiple products. The demand for these products exists at a set of customer locations. The planning horizon, in keeping with the midterm nature of the model, ranges from around 1 to 2 years. Each production site is characterized by one or more single stage semi-continuous processing units having limited capacity. The various products, which are grouped into product families, compete for the limited capacity of these processing units. The decision making process at the tactical level can be decomposed into two distinct phases: the *manufacturing phase* and the *logistics phase* (Gupta & Maranas, 1999; McDonald & Karimi, 1997; Gupta & Maranas, 2000). The manufacturing phase focuses on the efficient allocation of the production capacity at the various production sites with an aim to determining the optimal operating policies. Subsequently, in the logistics phase, the post-production activities such as demand satisfaction and inventory

management are considered for effectively meeting the customer demand. This classification of supply chain activities translates into the following model formulation.

(MP)

$$\min \sum_{f,j,s,t} FC_{fjs} Y_{fjst} + \sum_{i,j,s,t} v_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist}$$
$$+ \sum_{i,s,s',t} t_{ss'} W_{iss't} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,t} h_{ist} I_{ist}$$
$$+ \sum_{i,s,t} \zeta_{is} I_{ist}^{\Delta} + \sum_{i,c,t} \mu_{ic} I_{ict}^{-} \tag{1}$$

subject to

$$P_{ijst} = R_{ijst} RL_{ijst} \tag{2}$$

$$C_{ist} = \sum_{i'} \beta_{i'is} \sum_{j} P_{i'jst} \tag{3}$$

$$C_{ist} = \sum_{s'} W_{iss't} \tag{4}$$

$$FRL_{fjst} = \sum_{\lambda_{if}=1} RL_{ijst} \tag{5}$$

$$\sum_{f} FRL_{fjst} \leq H_{jst} \tag{6}$$

$$MRL_{fjs} Y_{fjst} \leq FRL_{fjst} \leq H_{jst} Y_{fjst} \tag{7}$$

$$A_{ist} = I_{is(t-1)} + \sum_{j} P_{ijst} - \sum_{s'} W_{iss't} \tag{8}$$

$$I_{ist} = A_{ist} - \sum_{c} S_{isct} \tag{9}$$

$$\sum_{s,t' \leq t} S_{isct'} \leq \sum_{t' \leq t} d_{ict'} \tag{10}$$

$$I_{ic(t-1)}^{-} + d_{ict} - \sum_{c} S_{isct} \leq I_{ict}^{-} \leq \sum_{t' \leq t} d_{ict'} \tag{11}$$

$$I_{ist}^{L} - I_{ist} \leq I_{ist}^{\Delta} \leq I_{ist}^{L} \tag{12}$$

$P_{ijst}, \ RL_{ijst}, \ FRL_{fjst}, \ C_{ist}, \ W_{iss't}, \ A_{ist}, \ I_{ist}, \ I_{ict}^{-}, \ S_{isct}, \ I_{ist}^{\Delta}$
$\geq 0, \qquad Y_{fjst} \in \{0, \ 1\}$

The objective function of the deterministic midterm-planning model (MP), Eq. (1), captures the combined costs incurred in the manufacturing and logistics phases. The manufacturing phase costs include fixed and variable production charges, cost of raw material purchase and transportation charges incurred for the intersite shipment of intermediate products. The subsequent logistics phase costs are comprised of the transportation charges incurred for shipping the final product to the customer, inventory holding charges, safety stock violation penalties and penalties for lost sales. The decisions made in the manufacturing phase establish the location and timing of production runs, length of campaigns, production amounts and consumption of raw materials. Specifically, $P_{ijst}, RL_{ijst}, FRL_{fjst}, A_{ist}, C_{ist}, W_{iss't}$ and Y_{fjst} constitute the *manufacturing variables*, and uniquely define the production levels and resource utiliza-

tions in the supply chain. These manufacturing variables are constrained by the *manufacturing constraints* given by Eqs. (2)–(8). The production amount of a particular product is defined in terms of the rate of production and the campaign run length by Eq. (2). The input-output relationships between raw materials and final products, accounting for the bill-of-materials, are given through Eq. (3). Redundancy in the intersite shipment of intermediate products is eliminated by Eq. (4), which forces the products shipped to a particular site in a particular period to be consumed in the same period. The allocation of products to product families is achieved through Eq. (5). Grouping of products into product families is typically done to account for the relatively small transition times and costs between similar products. Eq. (6) models the capacity restrictions while Eq. (7) provides upper and lower bounds for the family run lengths. The amount available for supply in the logistics phase following the manufacturing phase is defined through Eq. (8). The decisions made in the logistics phase, termed the *logistics variables*, are S_{isct}, I_{ist}, I_{ist}^{Δ} and I_{ist}^{-}. The corresponding *logistics constraints* are given by Eqs. (9)–(12). The linking between the manufacturing and logistics phases is captured by Eq. (9). The inventory level, which is determined by the amount available for supply and the actual supplies to the various customers, is defined by Eq. (9). No over-stocking is permitted at the customer (Eq. (10)) and the customer shortages are carried over time (Eq. (11)). Eq. (12) models the violation of the safety stock levels. Establishing of safety stock targets for the inventory level can be viewed as an aggregate deterministic attempt to buffer against unpredicted contingencies such as demand variations and production rate fluctuations.

Formulation MP takes a deterministic view of supply chain planning by considering all model parameters, such as cost coefficients, production rates, demand, etc. to be known with complete certainty. This assumption of complete and deterministic information, though desirable from a model complexity point of view, is highly optimistic. This is especially so given the highly dynamic nature of most supply chains which are characterized by numerous sources of technical and commercial uncertainty. The former typically arises in the manufacturing phase of the supply chain where issues such as rates of production and bill-of-material relationships are subject to considerable variation. Commercial uncertainty or market risk manifests into the planning decisions through the exposure of the supply chain to commodity prices fluctuations and seasonal demand variations. Within this spectrum of sources of uncertainty, probably the most important and extensively studied one is demand uncertainty (Gupta & Maranas, 2000; Petkov & Maranas, 1998; Ierapetritou & Pistikopoulos, 1996; Liu & Sahinidis,

1998). The emphasis on incorporating demand uncertainty into the planning decisions is well placed given the fact that effectively meeting customer demand is what primarily drives most supply chain planning initiatives. In view of this, development of a framework for incorporating demand uncertainty in the midterm planning model is described next.

3. Decision making under uncertainty

The need to account for uncertainty in the planning decisions can essentially be traced back to the core functionality of planning models, which is to allocate resources for the future based on current information and future projections. The foremost consideration in incorporating uncertainties into the planning decisions is the determination of the appropriate representation of the uncertain parameters. Two distinct methodologies for representing uncertainty can be identified. These are the *scenario-based* approach and the *distribution-based* approach. In the former approach, the uncertainty is described by a set of discrete scenarios capturing how the uncertainty might play out in the future. Each scenario is associated with a probability level representing the decision maker's expectation of the occurrence of a particular scenario. For instance, suppose a company is waiting for the result of a pending legislation in Congress that would give it access to new markets in Asia. Clearly, the resolution of this source of uncertainty results in two discrete scenarios with their respective probabilities. However, the applicability of the scenario-based approach is limited by the fact that it requires forecasting all possible outcomes of the uncertain parameter. In cases where a natural set of discrete scenarios cannot be identified and only a continuous range of potential futures can be predicted, the distribution-based approach is used. By assigning a probability distribution to the continuous range of potential outcomes, the need to forecast exact scenarios is obviated. The distribution-based approach is adopted in this work by modelling the demand as normally distributed with a specified mean and standard deviation. The normality assumption is widely invoked in literature (Wellons & Reklaitis, 1989; Nahmias, 1989) as it captures the essential features of demand uncertainty and is convenient to use.

An enterprise can adopt two strategic 'postures' when faced with demand uncertainty. It can either position itself as a *shaper* or as an *adapter* to combat uncertainty. In the former strategy, the company aims to restructure the demand distribution so that the associated downside risk is limited while the upside potential is retained. This is typically achieved through contracting agreements with the customer. For example, the company may offer a supply contract with a minimum/maximum quantity

commitment to its customer in return for a price discount (Anupindi & Bassok, 1999). By contrast, in the adapter strategy, the enterprise does not attempt to influence the uncertainty level in the market. It controls the risk exposure of its assets, such as inventory levels and profit margins, by constantly adapting its operations to unfolding demand realizations. An adapter view to the planning process is taken in this work.

One of the most popular frameworks for planning under uncertainty is *two-stage stochastic programming* (Birge & Louveaux, 1997; Dantzig, 1955). In this approach, the decisions and constraints of the system are classified into two sets. The *first-stage* variables, also known as *design* variables, are determined prior to the resolution of the underlying uncertainty. Contingent on these 'here-and-now decisions and the realizations of the uncertain parameter, the *second-stage* or *control* variables are determined to optimise in the face of uncertainty. These 'wait-and-see' recourse decisions model how the decision maker adapts to the unfolding uncertain events. The presence of uncertainty is reflected by the fact that both the second-stage decisions as well as the second stage costs are probabilistic in nature. The objective is, therefore, to minimize the sum of the first-stage costs, which are deterministic, and the *expected* value of the second-stage costs. The classification of the decisions of the midterm planning model into manufacturing and logistics naturally fits into the two-stage stochastic programming framework as described next.

4. Midterm production planning with uncertain demand

The midterm production-planning model under demand uncertainty is formulated as the following two-stage stochastic program (2SMP) (Gupta & Maranas, 2000; Gupta, Maranas, & McDonald, 2000).

(2SMP)

In formulation (2SMP), the manufacturing variables are considered as the first-stage, here-and-now decisions while the logistics decisions are modelled as the second-stage wait-and-see decisions. Due to the appreciable lead times involved in the production process, the manufacturing decisions are made prior to the realization of the uncertain demand. The logistics decisions, which essentially aim at satisfying the customer demand in the most cost effective way while accounting for inventory management, are postponed to after the demand is realized. The objective function of model (2SMP) is composed of two terms. The first term captures the costs incurred in the manufacturing phase. The second term quantifies the costs of the logistics decisions and is obtained by applying the expectation operator to an embedded optimisation problem. This *recourse* optimisation problem determines the optimal supply policies and inventory profiles *given* the production levels and demand realisations for the various products. The role of the expectation operator is to average over all possible demand realizations the costs incurred in the logistics activities. The constraints describing the supply chain are also partitioned within this framework. Eqs. (2)–(8) provide the restrictions for the outer production setting problem while Eqs. (9)–(12) describe the logistics planning phase. The coupling between these two phases arises through Eq. (9) that models the carryover of inventory given the amount available for supply from the production phase A_{ist} and the actual supplies to the various customers S_{isct}. Model (2SMP), thus, aims at determining the optimal production settings in the supply chain that minimize the manufacturing and the expected logistics costs.

Model (2SMP) provides an effective tool for managing the risk exposure of an enterprise. The uncertainty in the product demand is translated into the uncertainty in the logistics decisions through the second stage inventory management problem. This implies that

$$\min_{\substack{P_{ijst}, C_{ist}, FRL_{fjst} \\ RL_{ijst}, W_{iss't}, A_{ist} \geq 0}} \sum_{f,j,s,t} FC_{fjs} Y_{fjst} + \sum_{i,j,s,t} \upsilon_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,s',t} t_{ss'} W_{iss't}$$

$$Y_{fjst} \in \{0,1\}$$

$$+ E_{d_{ict}} \begin{bmatrix} \min_{\substack{S_{isct}, I_{ist} \\ I_{ist}^{\Delta}, I_{ict}^{-} \geq 0}} \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,t} \zeta_{is} I_{ist}^{\Delta} + \sum_{i,c,t} \mu_{ic} I_{ict}^{-} \\ \text{s.t.} \\ \text{Eqs. (9)-(12)} \end{bmatrix}$$

subject to Eqs. (2)–(8)

inventory levels, supply policies, safety stock deficits and customer shortages are contingent on the first-stage, manufacturing decisions and the demands realized. A probability distribution can, therefore, be associated with each of these decisions. These distributions provide valuable information regarding the sensitivity of the firm's assets, such as inventory levels and profit margins, to the external demand uncertainty. For instance, by utilizing the predicted distributions for the inventory levels at the various sites in the supply chain, key strategic options such as capacity integration/expansion/disinvestments can be uncovered. Similarly, based on the variability in the supply levels and the associated shortages at the customers, quotation of guaranteed service levels can be made to strengthen the competitive advantage of the enterprise in the marketplace (Gupta et al., 2000).

The key challenge in solving two-stage stochastic formulations such as (2SMP) arises from the expectation evaluation of the inner recourse problem. For a scenario-based description of demand uncertainty this is achieved by explicitly associating a second-stage variable with each demand scenario and then solving a large-scale extensive formulation of the model. A similar methodology is also employed when the uncertainty is described with a probability distribution by explicitly/implicitly discretizing the demand distribution using techniques such as Monte Carlo sampling (Liu & Sahinidis, 1998; Diwekar & Kalagnanam, 1997) and Gaussian quadrature (Straub & Grossmann, 1990; Acevedo & Pistikopoulos, 1998). The primary advantage of such discretization methods lies in their relative insensitivity to the form of the underlying distribution of the uncertain parameter. However, these methods are characterized by an exponential increase in the problem size with the number of uncertain parameters due to the nested structure of the two-stage formulation. This translates into excessively large computational requirements thus limiting the applicability of these techniques. In view of this, an alternative methodology is based on solving the inner recourse problem analytically for the second-stage variables in terms of the first-stage variables followed by analytical integration for expectation evaluation (Gupta & Maranas, 2000; Petkov & Maranas, 1998). This obviates the need for discretizing the probability space and thus reduces the associated computational burden. In terms of model (2SMP), this corresponds to solving the inner optimisation problem for the optimal values of the logistics variables S_{isct}, I_{ist}, I_{ist}^{Δ} and I_{ist}^{-} in terms of the manufacturing variable A_{ist} and the uncertain demand d_{ict}. Consequently, the optimal second-stage costs are obtained and the expectation is evaluated analytically (Gupta & Maranas, 2000). The resulting deterministic equivalent model is then solved for the optimal production settings in the supply chain.

5. Planning case study

To highlight the proposed framework for managing demand uncertainty in CPI planning, it is applied to a representative supply chain network shown in Fig. 1. The supply chain consists of six production sites manufacturing a total of 30 products. Products 1 through 10 are manufactured at site 1 and 2 while products 11 through 20 are manufactured at sites 3 and 4. These 20 products are grouped into ten product families as follows: F1 = (1, 2); F2 = (3, 4); F3 = (5, 6);...; F10 = (19, 20). Thus, product families F1–F5 are associated with sites 1/2 while F6–F10 are associated with site 3/4. The demand for products 1–20 exists externally at market 1 and internally at sites 5 and 6 where they are used as intermediate products. Specifically, products 21–25 are produced at site 5 while products 26–30 are manufactured at site 6. The demand for products 21–30 exists at market 2 as shown in Fig. 1. An assembly-type product structure exists at sites 5 and 6 (shown in Fig. 2) with one unit of each intermediate product being consumed to produce one unit of the final product. All the manufacturing sites are characterized by limited capacity processing equipment, which incur a set-up charge for each production campaign. The demand is assumed to be normally distributed for each product with a coefficient of variation (standard deviation/mean) of 20%.

The solution of model 2SMP for this supply chain setting results in an average total cost of 1512 units. The resulting convex MINLP is solved using a customized implementation of the outer approximation algorithm (OA) in approximately 2000 CPU seconds on an IBM RISC 6000 machine. A total of eight iterations of the OA algorithm are required as shown in Fig. 3. The model statistics for the primal and master problem are: 1587 equations; 2351 variables (primal); 2037 equations; 2861 variables; 480 discrete variables (master). In order to benchmark the computational efficiency achieved by
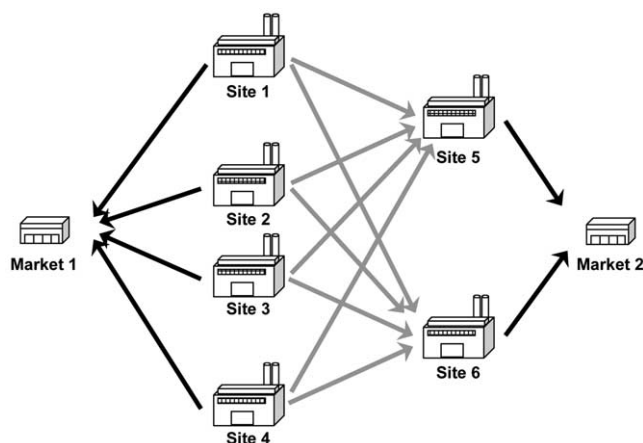

Fig. 1. Supply chain configuration for the planning case study.
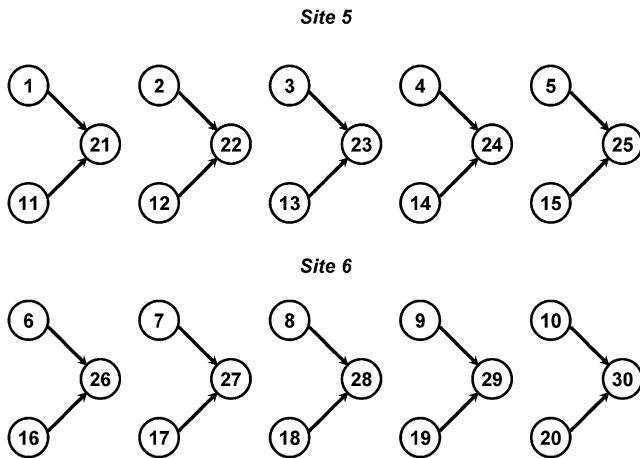
Site 5

Site 6

Fig. 2. Product structure at sites 5 and 6.

solving the inner recourse problem analytically, the planning model is also solved using a Monte Carlo sampling method. A total of 500 scenarios are generated by sampling the underlying normal distribution and the resulting MILP (136 000 equations; 156 000 variables; 120 discrete variables) is solved using the CPLEX solver accessed via GAMS. The model is found to be extremely computationally intensive and fails to converge within the specified resource limit of 10 000 CPU seconds. This highlights the significant reduction in computational complexity that can be achieved by adopting the proposed solution methodology over the naïve, brute-force approach.

Next, we investigate the impact of demand uncertainty on the various supply chain decisions. To this end, the optimal decisions obtained by solving model 2SMP are compared with those obtained by solving model MP. Table 1 lists how the capacity at sites 1 through 4 is allocated amongst the various product families. As expected, differences in allocation of capacity are observed both in terms of which product families are produced at a particular site as well as how much of capacity is allocated to them. The overall capacity utilization for each site is also predicted differently by the two models with the deterministic

Fig. 3. Progress of customized OA algorithm.

model consistently underestimating it as shown in Table 1. In the light of these differences, the question that is asked is how do the manufacturing decisions obtained by neglecting variability in product demand perform when exposed to uncertainty. To answer this question, a two step procedure is applied. First, the deterministic model MP is solved. The resulting optimal first-stage manufacturing decisions are then fixed in model 2SMP and it is subsequently solved for the optimal second-stage logistic decisions. This results in a total expected cost of 1557 units implying an increase of 2.9% over the original solution of model 2SMP. These results suggest that tangible cost savings can be realized through the incorporation of demand uncertainty in the planning process.

In addition to comparing just the average cost across the two model formulations, we can also compare the entire total cost distributions as shown in Figs. 4 and 5. Given these distributions, a risk assessment analysis is performed by comparing the two distributions across three different metrics. These metrics are: (i) standard deviation of cost distributions; (ii) probabilities of exceeding a particular cost level; and (iii) worst-case costs. The standard deviations for the two distributions are 77.5 (2SMP) and 103.4 units (MP) implying that a wider spread of values is obtained by neglecting uncertainty as seen in Fig. 4. Subsequently, the cumulative probability distributions of Fig. 5 can be used to determine the probability that the cost will exceed a specified level. The results of this figure clearly indicate that for any given cost level, model MP results in a higher probability compared to model 2SMP since the curve for the later lies entirely above the curve of the former in Fig. 5. For instance, the probability that the total cost exceeds 1650 units is predicted to be as high as 21% by the MP model as compared to just 3% by the
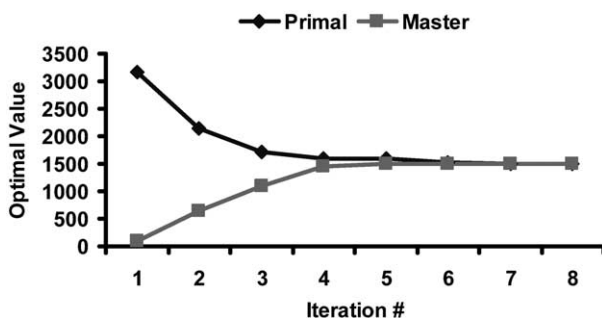
Table 1

Capacity allocation to product families forecasted by (a) model 2SMP and (b) model MP

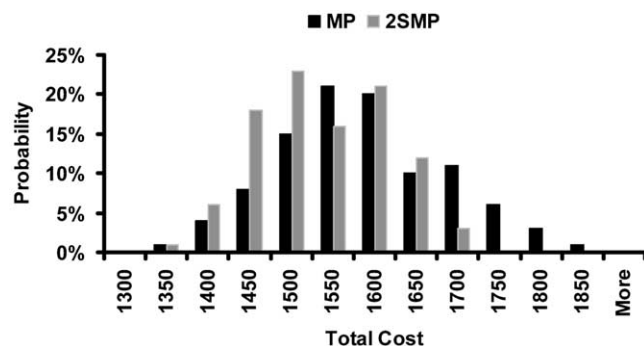| Site	Product family (% capacity utilized)	Total capacity utilization (%)
(a)		
1	F2 (42%), F3 (32%), F4 (26%)	100%
2	F1 (50%), F5 (50%)	100%
3	F7 (14%), F9 (42%), F10 (30%)	86%
4	F6 (59%), F8 (36%)	95%
(b)		
1	F1 (47%), F2 (37%)	84%
2	F3 (32%), F4 (19%), F5 (47%)	98%
3	F6 (54%), F10 (26%)	80%
4	F7 (15%), F8 (33%), F9 (29%)	77%

Fig. 4. Total cost distributions.
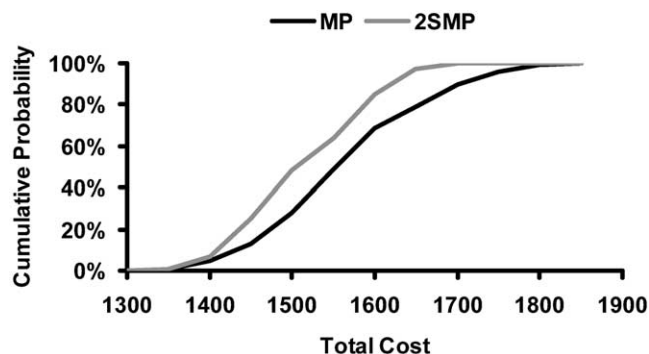

Fig. 6. Product service levels.

Fig. 5. Cumulative total cost distributions.

2SMP model. Finally, the worst-case costs obtained for the MP and 2SMP models are 1802 and 1689, respectively. Thus, the inclusion of demand uncertainty results in better risk management across the supply chain.

The proposed framework can also be used to study the performance of the supply chain from a customer service level perspective. This is especially critical in the current, highly volatile business setting given the ever-increasing expectations and constantly shifting loyalties of today's customers. In order to do that, the service level, defined as the probability of meeting the entire demand, is determined. Fig. 6 illustrates the service levels achieved for all 30 products. The results of this figure indicate that products 1–20 have relatively higher service levels as compared to products 21–30. Specifically, the average service level for products 1–20 is 79.6% (49.3%) whereas that for products 21–30 is 17.8% (18.6%) for the 2SMP (MP) model. This suggests that conditions at market 1 are relatively more favorable than at market 2.

6. Summary

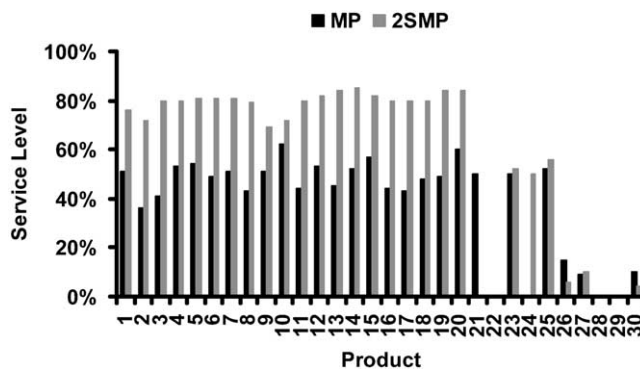In this paper, we presented an overview of our earlier work(s) addressing the problem of tactical planning of CPI supply chains under demand uncertainty. The deterministic model originally proposed by McDonald and Karimi (McDonald & Karimi, 1997) was adopted as the benchmark formulation for highlighting the various issues involved in incorporating uncertainty in the decision making process. Specifically, the supply chain networks considered were multi-product, multi-site and multi-period in nature. Other key features of the model included capacity constrained production equipment, carry-over of inventory and customer backlogs. It was shown that by appropriately partitioning the decisions variables and constraints of the deterministic model, a framework for incorporating demand uncertainty could be constructed. In particular, the supply chain decisions were classified into manufacturing and logistics decisions. The manufacturing decisions were made before the realization of the uncertain demand while the logistics decisions were postponed. The option of delaying the logistics decisions was used as recourse against the evolving uncertainty in the product demand.

Through a planning case study, the ability of the proposed framework to address key issues in managing uncertainties in CPI supply chains was highlighted. It was shown that by utilizing the presented framework, a more realistic description of the total planning costs (in terms of a probability distribution in contrast to a point estimate) could be obtained. Consequently, this information could potentially be utilized to manage the risk exposure of the company's assets. Risk management initiatives aimed at reshaping this distribution such that the downside risk is minimized while maintaining the upside potential could be undertaken based on this information. To this end, the use of derivative financial instruments, such as options, futures and swaps, in conjunction with the developed framework is currently being investigated. In addition to controlling risk in the supply chain, the proposed framework was also shown to provide valuable insights into the customer relationship aspects of the supply chain.

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