Ans (4.) (a) Conditional Probability Tables (CPTs) for the Naïve Bayes Model are —

P(T)

T	P(T)
b	$\frac{35+22+35+12+8+4+7+8}{200} = 131/200 = 0.655$
c	$\frac{16+1+2+43+3+4}{200} = 69/200 = 0.345$

Here in the Ist row, numerator is the sum of all tweets for T=b i.e. 131, denominator is total sum of all tweets i.e. 200.

P(B|T) :- Probability of B given T.

B	T	P(B|T)
t	b	$\frac{8+4+7+8}{35+22+35+12+8+4+7+8} = \frac{27}{131} = 0.206$
t	c	$\frac{43+3+4}{16+1+2+43+3+4} = \frac{50}{69} = 0.725$
f	b	$\frac{35+22+35+12}{35+22+35+12+8+4+7+8} = \frac{104}{131} = 0.794$
f	c	$\frac{16+1+2}{16+1+2+43+3+4} = \frac{19}{69} = 0.275$

Here, in the first row, numerator is the sum of all tweets corresponding to B=t & T=b i.e. 27, denominator is the sum of all tweets corresponding to T=b i.e. 131. Similarly, all other values are written from table.

P(C|T) :- Probability of C given T

C	T	P(C|T)
t	b	$\frac{35+12+7+8}{35+22+35+12+8+4+7+8} = \frac{62}{131} = 0.473$
t	c	$\frac{2+4}{16+1+2+43+3+4} = \frac{6}{69} = 0.087$
f	b	$\frac{35+22+8+4}{35+22+35+12+8+4+7+8} = \frac{69}{131} = 0.527$
f	c	$\frac{16+1+43+3}{16+1+2+43+3+4} = \frac{63}{69} = 0.913$

Here, in the first row, we have taken numerator as the sum of all tweets corresponding to C = t & T = b i.e. 62, Denominator as the sum of all tweets corresponding to T = b i.e. 131 & similarly other values are written from table & then calculated.

P(S|T) :- Probability of S given T

S	T	P(S|T)
t	b	$\frac{22+12+4+8}{35+22+35+12+8+4+7+8} = \frac{46}{131} = 0.351$
t	c	$\frac{1+3}{16+1+2+43+3+4} = \frac{4}{69} = 0.058$
f	b	$\frac{35+35+8+7}{35+22+35+12+8+4+7+8} = \frac{85}{131} = 0.649$
f	c	$\frac{16+2+43+4}{16+1+2+43+3+4} = \frac{65}{69} = 0.942$

Here, in the first row, we have taken numerator as the sum of all tweets corresponding to S = t & T = b i.e. 46, Denominator as the sum of all tweets corresponding to T = b i.e. 131 & similarly other values are written from table & then calculated.

Note * All calculations are rounded off to 3-digits after decimal place.

(b) In Naïve Bayes Model

$$P(T=b \mid B=t, C=t, S=f) = \frac{P(T=b, B=t, C=t, S=f)}{P(B=t, C=t, S=f)}$$

Using Naïve Bayes assumption:

$$P(T=b, B=t, C=t, S=f) = P(T=b) \cdot P(B=t \mid T=b) \cdot P(C=t \mid T=b) \cdot P(S=f \mid T=b)$$

$$= 0.655 \times 0.206 \times 0.473 \times 0.649$$ {values taken from CPT in (a)}

$$= 0.041$$

And $P(B=t, C=t, S=f)$

$$= P(T=b, B=t, C=t, S=f) + P(T=c, B=t, C=t, S=f)$$

$$= 0.041 + P(T=c) \cdot P(B=t \mid T=c) \cdot P(C=t \mid T=c) \cdot P(S=f \mid T=c)$$

$$= 0.041 + (0.345 \times 0.725 \times 0.087 \times 0.942)$$

$$= 0.041 + 0.020$$

$$= 0.061$$

So $$P(T=b \mid B=t, C=t, S=f) = \frac{P(T=b, B=t, C=t, S=f)}{P(B=t, C=t, S=f)}$$

$$= \frac{0.041}{0.061} = 0.672$$

This conditional probability, $P(T=b \mid B=t, C=t, S=f)$, represents probability of $T=b$, when the event $B=t$, $C=t$ & $S=f$ has already happened & they are conditionally independent. In other words, probability of Business Type being B2B, when we know that the words 'Bank', 'Consulting' appear & the word 'Services' doesn't appear in the Company name, as per Naive Bayes model.

(c) Let's assume $T = z$, where $z \in \{b, c\}$

So we need to calculate probabilities for both & see which is more.

So. $\underset{z \in \{b,c\}}{\arg\max}\ P(T=z \mid B=t, C=t, S=f)$

$$= \underset{z}{\arg\max} \frac{P(T=z, B=t, C=t, S=f)}{P(B=t, C=t, S=f)}$$

Since the denominator part $P(B=t, C=t, S=f)$ doesn't depend on the value of z. So we can just maximize the numerator.

So it becomes =>

$$= \underset{z}{\arg\max}\ P(T=z) \cdot P(B=t \mid T=z) \cdot P(C=t \mid T=z) \cdot P(S=f \mid T=z)$$

For $z = b$

$\Rightarrow P(T=b) \cdot P(B=t \mid T=b) \cdot P(C=t \mid T=b) \cdot P(S=f \mid T=b)$

$\Rightarrow 0.655 \times 0.206 \times 0.473 \times 0.649 = 0.041$

{values taken from already calculated values in part (b)}.

For $z = c$

$\Rightarrow P(T=c) \cdot P(B=t \mid T=c) \cdot P(C=t \mid T=c) \cdot P(S=f \mid T=c)$

$\Rightarrow 0.345 \times 0.725 \times 0.087 \times 0.942$ {values taken from already calculated values in part (b).}

$\Rightarrow 0.020$

Since $0.041 > 0.020$

Hence $\boxed{T = b}$ is more likely. Hence the Business Type is most likely to be B2B business for the partial configuration $(B=t, C=t, S=f)$.

(d) $P(T=b \mid B=t, C=t, S=f)$ as per Joint Distribution model is $\Rightarrow$

$$= \frac{P(B=t, C=t, S=f, T=b)}{P(B=t, C=t, S=f)}$$

$$= \frac{\frac{7}{200}}{\frac{7+4}{200}}$$

$$= \frac{\frac{7}{200}}{\frac{11}{200}} = \frac{7}{11} = 0.636$$

(e) Probability Tables of independent variables, as per the Fully Independent Model are -

B	P(B)
t	$\frac{8+43+4+3+7+4+8}{200} = \frac{77}{200} = 0.385$
f	$\frac{35+16+22+1+35+2+12}{200} = \frac{123}{200} = 0.615$

For the Ist row, numerator is sum of all tweets corresponding to B = t i.e. 77 & denominator is sum of all tweets i.e. 200. In the similar way, all other tables are made $\rightarrow$

C	P(C)
t	$\frac{35+2+12+7+4+8}{200} = \frac{68}{200} = 0.340$
f	$\frac{35+16+22+1+8+43+4+3}{200} = \frac{132}{200} = 0.660$

S	P(S)
t	$\frac{22+1+12+4+3+8}{200} = \frac{50}{200} = 0.250$
f	$\frac{35+16+35+2+8+43+7+4}{200} = \frac{150}{200} = 0.750$

T	P(T)
b	$\frac{35+22+35+12+8+4+7+8}{200} = \frac{131}{200} = 0.655$
c	$\frac{16+1+2+43+3+4}{200} = \frac{69}{200} = 0.345$

$$\therefore P(T=b \mid B=t, C=t, S=f) = \frac{P(B=t, C=t, S=f, T=b)}{P(B=t, C=t, S=f)}$$

Since its a fully independent model.

So it becomes $\Rightarrow \dfrac{P(B=t) \cdot P(C=t) \cdot P(S=f) \cdot P(T=b)}{P(B=t) \cdot P(C=t) \cdot P(S=f)}$

$\Rightarrow P(T=b)$

$= 0.655$ {from Probability Table}