TRIBHUWAN UNIVERSITY

INSTITUTE OF ENGINEERING

KATHMANDU ENGINEERING COLLEGE

KALIMATI, KATHMANDU

LAB-REPORT ON

Computer Graphics

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LAB-4 : MID-POINT CIRCLE ALGORITHM

Objectives:

- To draw a circle using mid-point circle algorithm.

Theory:

Circle:
A circle is the set of all points that lie at an equal distance from a fixed point. The equal distance is called radius, and the fixed point is called center. A circle is a symmetric figure. A circle has 4-way symmetry i.e. symmetry in quadrant and 8-way symmetry i.e. symmetry in octant.

In 8-way symmetry, if we know the octant, then we can plot the whole circle by finding the corresponding coordinate just by reflecting with respect to axis and 45º.

The equation of circle in Cartesian form is:

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

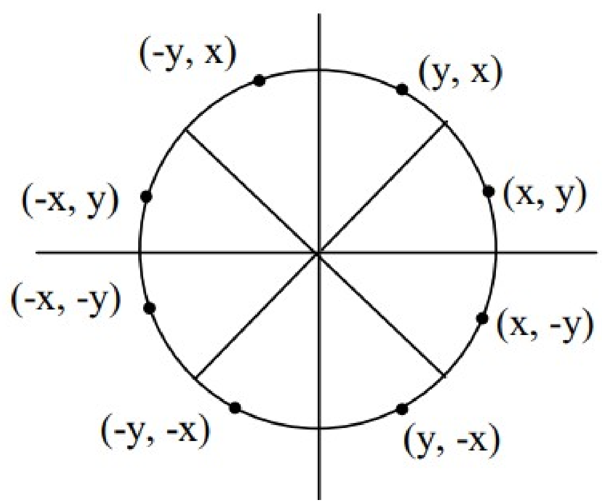

Figure: Circle with symmetry points

Derivation:

The equation of circle is $x^2 + y^2 = r^2$
With center (0, 0) and radius r, let's define a circle function as circle (x, y) = $x^2 + y^2 = r^2$.

The distance of pixel to adjacent pixel is unit.

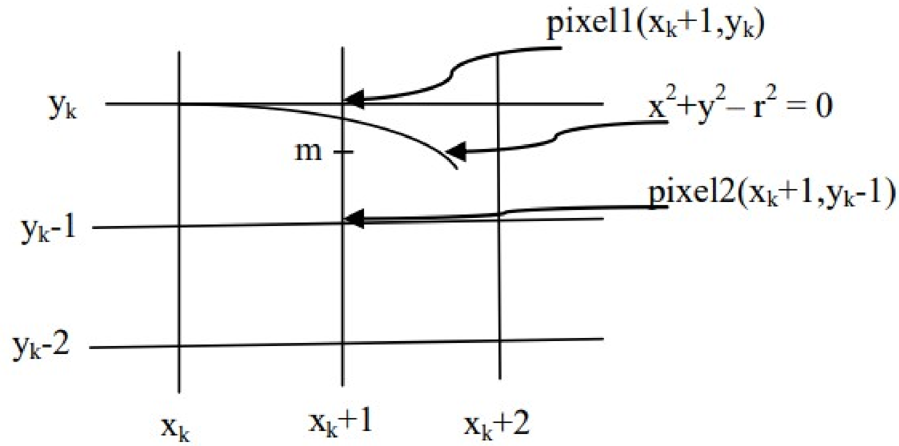

Figure: Mid-point circle pixels

In figure, length between pixel 1 and pixel 2 is $(y_k - y_{k+1}) = 1$ and half the length = 1/2.

$$Circle\ (x,y)\begin{cases} < 0 \rightarrow \text{if (x, y) is inside the circle boundary} \\ = 0 \rightarrow \text{if (x, y) is on the circle boundary} \\ > 0 \rightarrow \text{if (x, y) is outside the circle boundary} \end{cases}$$

Assume(x_k, y_k) is plotted, then next point close to the circle is (x_{k+1},y_k) or (x_{k+1},y_{k-1})

Decision parameter is the circle function evaluated at the midpoint between these two points.

$P_k = f_{circle}\ (x_{k+1},\ y_k - 1/2\ )$

$P_k = (x_k + 1)^2 + (y_k - 1/2\ )^2 - r^2$ ………..(1)

if $p_k < 0$, then midpoint is inside the circle and y_k is closer to circle boundary. Else, midpoint is outside the circle. And y_{k-1} is closer to the circle boundary.

Successive decision parameters are obtained using incremental calculations i.e. next decision parameter is obtained as:

$x_{k+1}+1 = x_k+1+1$ and $y_{k+1} - 1/2$

$P_{k+1} = f_{circle}\ (x_{k+1}+1,\ y_{k+1} - 1/2\ )$

$P_{k+1} = (x_k+1+1)^2 + (y_{k+1} - 1/2)^2 - r^2$ ……..(2)

Subtracting equation (1) from (2)

$P_{k+1} - p_k = (x_k+1+1)^2 + (\ y_{k+1} - 1/2\ )^2 - r^2 - (\ x_k+1)^2 - (\ y_k - 1/2\ )^2 + r^2$

$= (x_k+1)^2 + 2(x_k+1)+1+ y_k^2 +1 - 2y_{k+1}\times 1/2 + 1/4 -(x_k+1)^2 - y_k^2 +2y_k \times 1/2 - 1/4$

$= 2(x_k+1)+(y_{k+1}^2 - y_k^2) -(y_{k+1} - y_k)+1$

Where y_{k+1} is either y_k or y_{k-1} depending on sign of p_k. If $p_k<0$, then next pixel is at (x_{k+1}, y_k)

$P_{k+1} = p_k+2x_{k+1}+1$

If $p_k\geq 0$, then next pixel is at (x_{k+1}, y_{k-1})

$P_{k+1} = p_k+ 2x_{k+1} + [(y_k-1)^2 - y_k^2] -(y_k - 1 - y_k)+1$

$= p_k+ 2x_{k+1}+(y_k^2 -2y_k+1 -y_k^2)+1+1$

$= p_k +2x_{k+1} -2y_k+1+1+1$

$= p_k+2x_{k+1} -2(y_k -1)+1$

$= p_k+2x_{k+1} -2y_{k+1}+1$

Initial decision parameter

The initial decision parameter is obtained by evaluating the circle function at starting point $(x_0, y_0) = (0,r)$.

$P_0 = f_{circle}((x_0+1), (y_0 - 1/2))$

$= f_{circle}((0+1), (r - 1/2))$

$= 1/2 +(r - 1/2)^2 -r^2$

$= 1+r^2 -2r \times 1/2 + 1/4 -r^2$

$P_0 = 5/4 - r$

If the radius r is specified as an integer, we can simply round to $P_0=1-r$

Algorithm:

Step 1: Start
Step 2: Declare variables $x_c,y_c,r,x_0,y_0,p_0, p_k,p_{k+1}$.
Step 3: Read values of x_c , y_c, r.
Step 4: Initialize the x_0 and y_0 i.e., set the co-ordinates for the first point on the circumference of the circle centered at origin as

$x_0 = 0$

$y_0 = r$

Step 5: Calculate initial value of decision parameter $p_0 = 5/4 - r$

Step 6: At each x_k position, starting from $k = 0$

If $p_k < 0$

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k$

$p_{k+1} = p_k + 2x_{k+1} + 1$

else

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k - 1$

$p_{k+1} = p_k + 2x_{k+1} - 2y_{k+1} + 1$

Step 7: Determine the symmetry in the other seven octants.

Step 8: Move each calculated pixel position (x,y) onto the circular path centered on (x_c,y_c)

Step 9: Plot the co-ordinates values

$x = x + x_c$

$y = y + y_c$

Step 10: Repeat steps 6 to 9 until $x \geq y$.

Step 11: Stop