

7:07 am

Decimal No. System:  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$\begin{array}{r} 210 \\ 342 \end{array} \rightarrow 300 + 40 + 2 \rightarrow 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$

$$\begin{array}{r} 3210 \\ 2563 \end{array} \rightarrow 2000 + 500 + 60 + 3 \rightarrow 2 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 3 \times 10^0$$

Binary No System:  $\{0, 1\}$

$$\begin{array}{r} 110 \\ 210 \end{array} \rightarrow 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \rightarrow 4 + 2 + 0 = 6$$

$$\begin{array}{r} 1011 \\ 3210 \end{array} \rightarrow 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \rightarrow 8 + 0 + 2 + 1 = 11$$

0	10	20
1	11	21
2	12	22
⋮	⋮	⋮
9	19	29

$$\begin{array}{l} 0 \rightarrow 0 \quad 10 \rightarrow 2 \quad 100 \rightarrow 4 \quad 110 \rightarrow 6 \\ 1 \rightarrow 1 \quad 11 \rightarrow 3 \quad 101 \rightarrow 5 \quad 111 \rightarrow 7 \end{array}$$

Binary to decimal:

$$\begin{array}{r} 43210 \\ 10110 \end{array}$$

$$\begin{array}{l} \rightarrow 0 \times 2^0 = 0 \\ \rightarrow 1 \times 2^1 = 2 \\ \rightarrow 1 \times 2^2 = 4 \\ \rightarrow 0 \times 2^3 = 0 \\ \rightarrow 1 \times 2^4 = 16 \\ \hline 22 \end{array}$$

$$(10110)_2 \rightarrow (22)_{10}$$

Quiz:

6	5	4	3	2	1	0
1	0	1	1	0	1	0

						$\hookrightarrow 0 \times 2^0 = 0$
						$\hookrightarrow 1 \times 2^1 = 2$
						$\hookrightarrow 0 \times 2^2 = 0$
						$\hookrightarrow 1 \times 2^3 = 8$
						$\hookrightarrow 1 \times 2^4 = 16$
						$\hookrightarrow 0 \times 2^5 = 0$
						$\hookrightarrow 1 \times 2^6 = 64$
						<u>90</u>

lengthy  
process  
but  
correct.

6	5	4	3	2	1	0
1	0	1	1	0	1	0

$\downarrow$		$\downarrow$	$\downarrow$	$\downarrow$		
$2^6$		$2^4$	$2^3$	$2^1$		

  
$$= 64 + 16 + 8 + 2 = \underline{90}$$

$$(1011010)_2 \rightarrow (90)_{10}$$

$(102010)_2$  X

$\hookrightarrow$  invalid

## Decimal to Binary

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
0		

$$\begin{array}{r}
 4 \ 3 \ 2 \ 1 \ 0 \\
 1 \ 0 \ 1 \ 0 \ 0 \\
 \downarrow \quad \downarrow \\
 2^4 \quad 2^2 = 16 + 4 = 20
 \end{array}$$

$$(20)_{10} \rightarrow (10100)_2$$

Quiz 2:

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
0		

$$\begin{array}{r}
 5 \ 4 \ 3 \ 2 \ 1 \ 0 \\
 \Rightarrow 1 \ 0 \ 1 \ 1 \ 0 \ 1 \\
 \downarrow \quad \downarrow \downarrow \downarrow \\
 2^5 \quad 2^3 \quad 2^2 \quad 2^0 \\
 = 32 + 8 + 4 + 1 \\
 = 45
 \end{array}$$

$$(45)_{10} \rightarrow (101101)_2$$

Base 10                      Base 2

## Addition of No.

$$\begin{array}{r}
 1 \quad 1 \\
 3 \quad 6 \quad 8 \\
 (+) \quad 4 \quad 5 \quad 3 \\
 \hline
 8 \quad 2 \quad 1
 \end{array}$$

$$9 + 9 = 18$$

↳ max carry value

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10 \quad (2)_{10} \rightarrow (10)_2$$

$$\begin{array}{r}
 5 \rightarrow \begin{array}{c} 1 \ 1 \\ 1 \ 0 \ 1 \end{array} \\
 3 \rightarrow (+) \begin{array}{c} 0 \ 1 \ 1 \end{array} \\
 \hline
 8 \quad \begin{array}{c} 1 \ 0 \ 0 \ 0 \\ \hline \end{array} \\
 \uparrow \quad \downarrow \\
 2^3 \quad 2^3
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{c} 1 \ 1 \ 1 \\ 1 \ 0 \ 1 \end{array} \rightarrow 5 \\
 (+) \begin{array}{c} 1 \ 1 \ 1 \end{array} \rightarrow 7 \\
 \hline
 \begin{array}{c} 1 \ 1 \ 0 \ 0 \\ \hline \end{array} \rightarrow 12 \\
 \downarrow \downarrow \\
 2^3 \ 2^2 = \underline{8+4}
 \end{array}$$

Quiz:

$$\begin{array}{r}
 \begin{array}{c} 1 \ 1 \\ 1 \ 0 \ 1 \ 1 \ 0 \end{array} \\
 \begin{array}{c} 0 \ 0 \ 1 \ 1 \ 1 \end{array} \\
 \hline
 \begin{array}{c} 1 \ 1 \ 1 \ 0 \ 1 \end{array} \\
 \hline
 \end{array}$$

Bitwise operations: { AND, OR, XOR, NOT, Left Shift, Right Shift }

$\&$     $|$     $\wedge$     $!/\sim$     $\ll$     $\gg$

$0 \rightarrow 1$   
 $1 \rightarrow 0$

A	B	A & B	A   B	A ^ B	
0	0	0	0	0	(0+0)
0	1	0	1	1	(0+1)
1	0	0	1	1	(1+0)
1	1	1	1	0	(1+1 = 10)

same same  
puppy shame

Add w/o  
carry = XOR

## Bitwise operations on numbers

$$5 \& 6$$

$$\begin{array}{r} 5 \quad 101 \\ 6 \quad 110 \\ \hline 100 \rightarrow \boxed{4} \end{array}$$

$$5 \& 6 = 4$$

Try:

$$\begin{array}{r} 010100 \\ \& 101101 \\ \hline 000100 = 4 \end{array}$$

$$\begin{array}{r} \swarrow \quad 010100 \rightarrow 20 \\ (OR) \quad 1 \quad 101101 \rightarrow 45 \\ \hline \quad \quad 111101 \\ \quad \quad \swarrow \downarrow \downarrow \downarrow \downarrow \\ 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^0 \end{array}$$

$$\begin{aligned} &= 32 + 16 + 8 + 4 + 1 \\ &= 61 \end{aligned}$$

$$(20 | 45) = 61$$

$$\begin{array}{r} \swarrow \quad 010100 \rightarrow 20 \\ (XOR) \quad 1 \quad 101101 \rightarrow 45 \\ \hline \quad \quad 111001 \\ \quad \quad \swarrow \downarrow \downarrow \downarrow \downarrow \\ 2^5 \quad 2^4 \quad 2^3 \quad 2^0 \\ = 32 + 16 + 8 + 1 = \boxed{57} \end{array}$$

# Properties

1)  $A \& 1$

$$\begin{array}{r} A = 10 \\ \& 1 \\ \hline 0000 \end{array}$$

$$\begin{array}{r} A = 9 \\ \& 1 \\ \hline 0001 \end{array}$$

$$\begin{array}{r} \phantom{A = } \\ \phantom{\& } \\ \hline 0001 \end{array}$$

$A \& 1 \rightarrow 0$  : if last bit is 0 (even no)  
 $\rightarrow 1$  : if last bit is 1 (odd no)  
 $\hookrightarrow 0^{\text{th}}$  bit

What no. have last bit set/unset?

$$\begin{array}{r} \phantom{A = } \\ \phantom{\& } \\ \hline 0001 \end{array}$$

$2^5 \downarrow$   
 $2^3 \downarrow$   
 $2^2 \downarrow$   
 $2^0 = 1$   
 $\leftarrow \text{even} + \leftarrow \text{odd} = \text{odd}$

$$\begin{array}{r} \phantom{A = } \\ \phantom{\& } \\ \hline 0000 \end{array}$$

$0 \times 2^0$   
 $\leftarrow \text{even} + \leftarrow \text{even} = \text{even}$

$$(A \& 1) == 0 \rightarrow \text{even}$$

else  $\rightarrow \text{odd}$

2)  $A \& 0 = 0$

$$\begin{array}{r} A = 101 \\ \& 000 \\ \hline 000 \end{array}$$

3)  $A \& A = A$

$$\begin{array}{r} A = 101 \\ \& 101 \\ \hline 101 = A \end{array}$$

$$(4) A \vee 0 = A$$

$$A = 101$$

$$0 = \frac{000}{101}$$

$$(5) A \wedge A = A$$

$$A = 101$$

$$A = \frac{101}{101}$$

$$(6) A \vee 1 = 1$$

$$A = 101$$

$$1 = \frac{000}{101}$$

$$(7) A \wedge 0 = 0$$

$$A = 101$$

$$0 = \frac{101}{000}$$

### (8) Commutative Property

$$a \& b = b \& a$$

$$a | b = b | a$$

$$a \wedge b = b \wedge a$$

$$\begin{array}{c} a \& b \& c \\ \hline x \end{array} = \begin{array}{c} c \& a \& b \\ \hline x \end{array}$$

$c \& b \& a$  ✓  
 $a \& c \& b$  ✓  
 $b \& a \& c$  ✓  
 $b \& c \& a$  ✓

### (9) Associative Property:

$$\left. \begin{array}{l} (a \& b) \& c = a \& (b \& c) \\ (a | b) | c = a | (b | c) \\ (a \wedge b) \wedge c = a \wedge (b \wedge c) \end{array} \right\}$$

Q:  $a^1 b^1 a^1 d^1 b = ??$

$$\frac{a^1 a^1 b^1 b^1 d}{\downarrow \quad \downarrow \quad \downarrow \quad \downarrow} = \frac{0^1 0^1 d}{0^1 0^1 d} = d$$

Q:  $\frac{1^1 3^1 5^1 3^1 2^1 1^1 5}{2 \quad 7 \quad 4 \quad 6 \quad 7 \quad 2} = \frac{1^1 1^1 3^1 3^1 5^1 5^1 2}{\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow} = \frac{0^1 0^1 0^1 0^1 2}{0^1 0^1 0^1 0^1 2} = 2$

$$1 \rightarrow 001$$

$$2 \rightarrow 010$$

$$3 \rightarrow 011$$

$$4 \rightarrow 100$$

$$5 \rightarrow 101$$

$$6 \rightarrow 110$$

$$7 \rightarrow 111$$

Qn: Given an int  $a[]$ , containing +ve values. All elements appear twice, except for 1 element, which appears only once. Find the element appearing once.

$$A = [6 \ 9 \ 6 \ 10 \ 9]$$

$\hookrightarrow \text{ans} = 10$

$$= \frac{6^1 9^1 6^1 10^1 9}{\underline{6^1 6^1} \underline{9^1 9^1} 10} = 10$$

$$A = [2 \ 3 \ 5 \ 6 \ 3 \ 2 \ 6]$$

$\hookrightarrow \text{ans} = 5$

```

ans = 0
for (i = 0; i < N; i++) {
    ans = ans ^ a[i]
}
return ans

```

TC:  $O(N)$   
SC:  $O(1)$

$$\begin{aligned} & \frac{2^1 3^1 5^1 6^1 3^1 2^1 6}{\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow} \\ &= \frac{2^1 2^1 3^1 3^1 5^1 6^1 6^1}{0 \quad 0 \quad 0 \quad 0 \quad 5} \\ &= 5 \end{aligned}$$



Break  $\rightarrow$  8:40 am

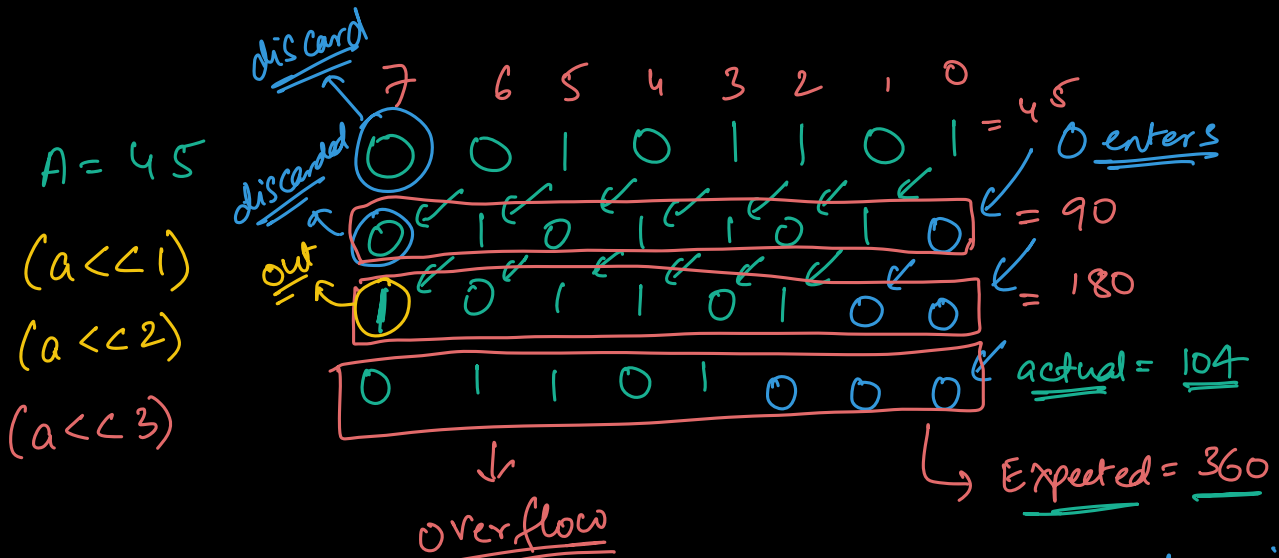


Sunday's PS session  
not present for  
some students

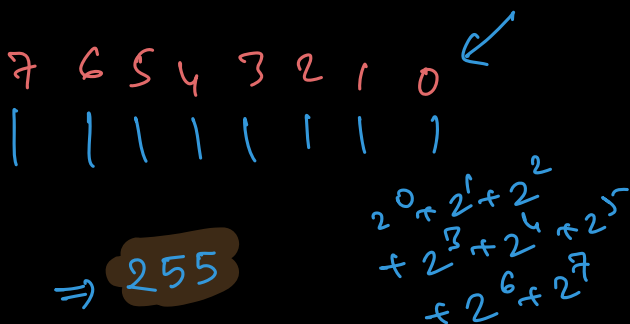
## Left Shift ( $\ll$ )

int  $\rightarrow$  4 bytes  $\Rightarrow$  32 bits

Assume: memory = 1 byte of storage  
= 8 bits



Why? 360 is too large to store in 8 bits

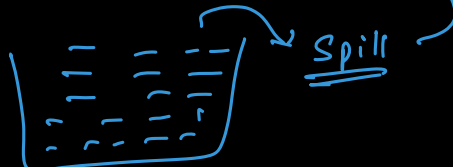


Left shift  $\Rightarrow \times 2$

$$a \ll n = a \times 2^n$$

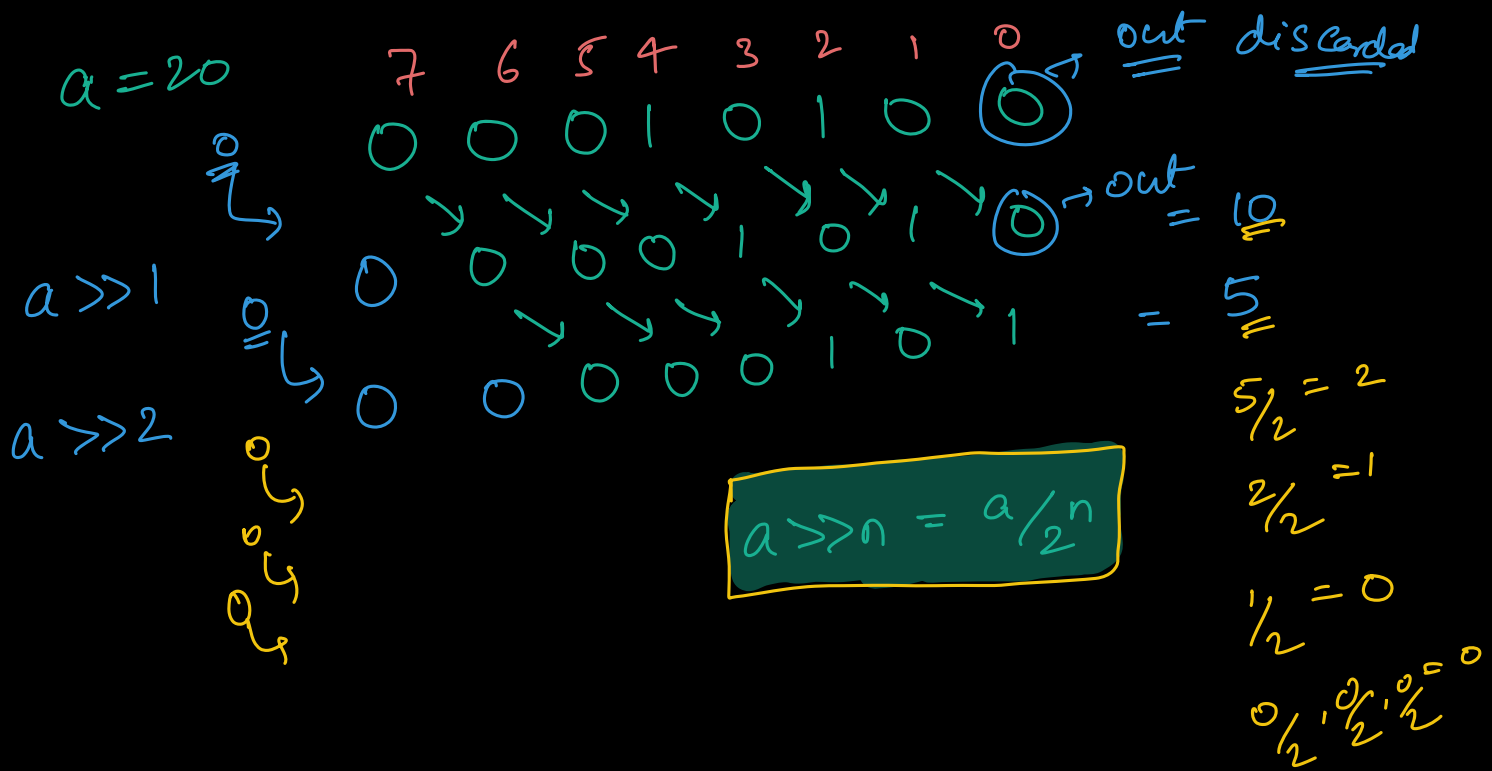
$$1 \ll n = 2^n$$

$360 > 255 \Rightarrow$  Overflow



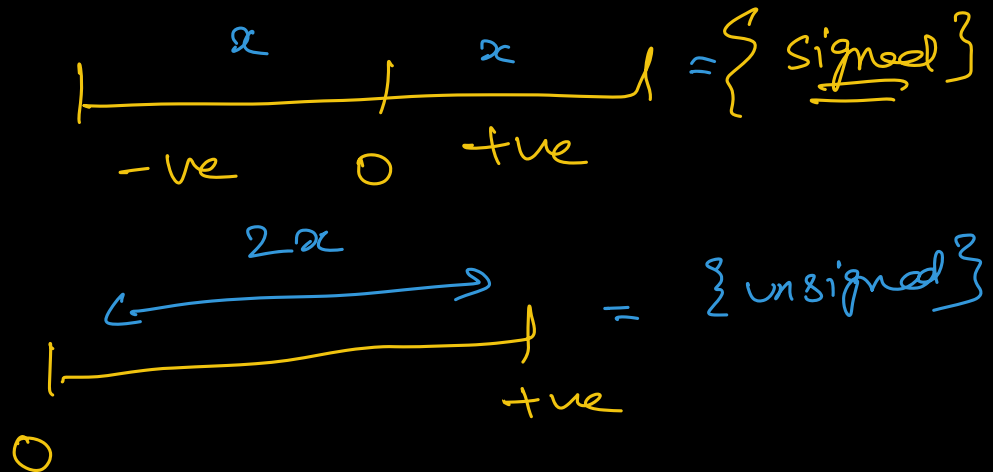
To avoid overflow, use bigger data type.

# Right Shift (>>)



Doubts: int:

$$\frac{2^{32} - 1}{-2^{31}} \quad (+ve) \quad (-ve) \quad \text{[verify]}$$



$pf_{Even} \rightarrow \text{Sum @ even idx}$   
 $pf_{Odd} \rightarrow \text{Sum @ odd idx}$

	0	1	2	3	4	5
$A =$	1	3	5	2	4	6
$pf_{Even} =$	1	1	6	6	10	10
$pf_{Odd} =$	0	3	3	5	5	11

Sum even no.

$\rightarrow pf[9] =$

2	1	3	4	5	6	7	9	8
↓	↓	↓	↓	↓	↓	↓	↓	↓
2	2	2	6	6	12	12	12	20