

Sorting

- Understand, what is sorting
- Few problems on sorting
- 1 Sorting algo ←
- Comparator

Sorting : Arranging data in increasing or decreasing order. based on a parameter

Eg: 2 3 7 11 13 : inc.

Eg: 15 9 7 3 2 0 : dec.

Eg: 1 2 3 7 4 9 6

factors: 1 2 2 2 3 3 4

In-built Library in Language

↳ Sort() function (in every language)

↳ How? Logic? { In adv. module }

TC : $O(N \log N)$

↳ No. of elements to sort

Q1: Elements removal

Given N elements, at every step remove an $a[i]$ element.

Cost to remove an element = sum of all elements in array present at that instance.

Find min. cost to remove all elements.

Eg: $a[] = \{ 2 \quad 1 \quad 4 \}$

remove 1 : $\frac{\text{Total Cost}}{2+1+4} = 7$

remove 4 : $2+4 = 6$

remove 2 : $\frac{2}{2} = 2$

Total Cost:

$15 \times$

remove 4 : $2+1+4 = 7$

remove 2 : $2+1 = 3$

remove 1 : $\frac{1}{1} = 1$

Total Cost: 11

Eg: $a[] = \{ 3 \quad 6 \quad 2 \quad 4 \}$

remove 6 : $\frac{\text{Cost}}{15} (3+6+2+4)$

remove 4 : $3+2+4 = 9$

remove 3 : $3+2 = 5$

remove 2 : $\frac{2}{2} = 1$

Total Cost:

31

$$a = \{ 6 \quad -3 \quad 4 \}$$

$$\text{remove } 6 : 6 - 3 + 4 = 7$$

$$\text{remove } 4 : -3 + 4 = 1$$

$$\text{remove } -3 : \underline{-3}$$

Obs: We are deleting/removing elements in desc. order.
WHY?

$$a[4] = \{ a \quad b \quad c \quad d \}$$

$$\text{remove } a : a + b + c + d$$

$$\text{remove } b : b + c + d$$

$$\text{remove } c : c + d$$

$$\text{remove } d : d$$

Reduce the cost

$$\text{Total Cost: } (a^0 + 2b^1 + 3c^2 + 4d^3)$$

↓ ↓ ↓ ↓
 1st max 2nd max 3rd max 4th max

```
int minCost(int a[], int N) {
```

Sort(arr, desc) → Sorts the array in desc. order
 ↳ check syntax in your pref. lang.

```
    int c = 0
```

```
    for(i = 0; i < n; i++) {
```

// how many times will a[i] occur = (i+1) times

```
        c = c + a[i] * (i+1)
```

```
    }
```

```
    return c
```

TC: $O(N \log N + N) = O(N \log N)$
SC: $O(1)$

Q2: Noble Integers { data is distinct }
↳ can't repeat

Given $a[N]$. Calc the no. of noble integers.

Noble integer

An element $a[i]$ is said to be noble if:

No. of elements $< a_i = a_i$ itself

Eg: {

-1	-5	3	5	-10	4
2	1	3	5	0	4

 }
less: ↳ $\boxed{ans = 3}$

Eg: {

0	1	2	3
-3	0	2	5
0	1	2	3

 }
less: ↳ $\boxed{ans = 1}$

Eg: {

0	1	2	3	4	5	6
-10	-5	1	3	4	5	10
0	1	2	3	4	5	6

 } $\boxed{ans = 3}$
less:

Brute force: for every element a_i , iterate on $a[]$ & get count of elements $< a_i$ & compare with a_i

```
ans = 0
for (i = 0; i < n; i++) {
    // a[i], need ele < a[i]
    c = 0
    for (j = 0; j < n; j++) {
        if (a[j] < a[i]) { c++; }
    }
    if (a[i] == c) { ans++; }
}
```

Taking time
∴ to find count of smaller elements, Sort

TC: $O(N^2)$
SC: $O(1)$

// Sort:

$[0 \dots i-1]$ i^{th}

↳ all elements $< a_i$

of elements $< a_i = [0 \dots i-1] \Rightarrow \underline{i}$ elements.

```
int noble (int a[], int N) {
```

```
    sort(a, a+N) // Sort in inc. order
```

```
    c = 0
```

```
    for (i = 0; i < n; i++) {
```

```
        if (a[i] == c) { c++; }
```

```
    }
```

```
    return c
```

TC: $O(N \log N + N)$
= $O(N \log N)$
SC: $O(1)$

Q3: Same Qn. with repeating data ↳ not distinct anymore

Eg: $\{ \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \begin{matrix} 1 \\ 2 \\ 1 \end{matrix} \begin{matrix} 2 \\ 2 \\ 1 \end{matrix} \begin{matrix} 3 \\ 3 \\ 3 \end{matrix} \begin{matrix} 4 \\ 3 \\ 3 \end{matrix} \begin{matrix} 5 \\ 6 \\ 5 \end{matrix} \}$

lens: Ans = 3

BF app ✓
sort idea ✗

Eg: $\{ \begin{matrix} 0 \\ -10 \\ 0 \end{matrix} \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \begin{matrix} 2 \\ 1 \\ 1 \end{matrix} \begin{matrix} 3 \\ 1 \\ 1 \end{matrix} \begin{matrix} 4 \\ 4 \\ 4 \end{matrix} \begin{matrix} 5 \\ 4 \\ 4 \end{matrix} \begin{matrix} 6 \\ 4 \\ 4 \end{matrix} \begin{matrix} 7 \\ 7 \\ 7 \end{matrix} \begin{matrix} 8 \\ 10 \\ 8 \end{matrix} \}$

lens: Ans = 7

Eg: $\{ \begin{matrix} 0 \\ -3 \\ 0 \end{matrix} \begin{matrix} 1 \\ 0 \\ 1 \end{matrix} \begin{matrix} 2 \\ 2 \\ 2 \end{matrix} \begin{matrix} 3 \\ 2 \\ 2 \end{matrix} \begin{matrix} 4 \\ 5 \\ 4 \end{matrix} \begin{matrix} 5 \\ 5 \\ 4 \end{matrix} \begin{matrix} 6 \\ 5 \\ 4 \end{matrix} \begin{matrix} 7 \\ 5 \\ 4 \end{matrix} \begin{matrix} 8 \\ 8 \\ 8 \end{matrix} \begin{matrix} 9 \\ 8 \\ 8 \end{matrix} \begin{matrix} 10 \\ 10 \\ 10 \end{matrix} \begin{matrix} 11 \\ 10 \\ 10 \end{matrix} \begin{matrix} 12 \\ 14 \\ 12 \end{matrix} \}$

lens: Ans = 6

Obs 1: if an element comes for very first time, no. of elements $<$ that
if ($a[i] \neq a[i-1]$) = idx itself

Obs 2: if data repeats, then count of elements $<$ that won't change
if ($a[i] == a[i-1]$)

```

int nobleRepeat(int a[], int N) {
    int less = 0 // To track the no. of elements < a[i]

    sort(arr, inc)

    if(a[0] == 0) { c++ } → Edge case

    for(i = 1; i < n; i++) {
        // for a[i], get elements < a[i]
        if(a[i] != a[i-1]) { // Edge case:
            // i=0 a[0] != a[-1]
            less = i
        }
        else {
            // Do nothing
        }
        if(less == a[i]) { c++ }
    }

    return c
}

```

TC: $O(N \log N + N) = O(N \log N)$
 SC: $O(1)$

Break : → till 8.40am

⑧ 1 Sorting algo

$a[] = \{ 3, 8, 6, 2, 4 \}$

iter 1:

	3	8	6	2	4	8
		6	8	8	4	
			2	4		

iter 2:

	3	2	4	6	8
	2	3			

iter 3:

2	3	4	6	8
---	---	---	---	---

Obs:

- 1 After 1st iter → Last 1 element is in correct pos
- 2 After 2nd iter → Last 2 elements are in correct pos
- 3 After 3rd iter → Last 3 elements are in correct pos
- ⋮
- $N-1$ After $N-1$ iter → Last $N-1$ elements are in correct pos
↳ all N are correct

→ In every iteration: $(N-1 \text{ iter})$

Iterate from first to last, Compare adjacent elements

If not in correct order
↳ we swap

Bubble Sort (int a[], int N) {

for(i=0; i < n; i++) { // N-1 times

for(j=0; j < n-1; j++) {

if(a[j] > a[j+1]) { // Edge case:

swap(a[j], a[j+1])

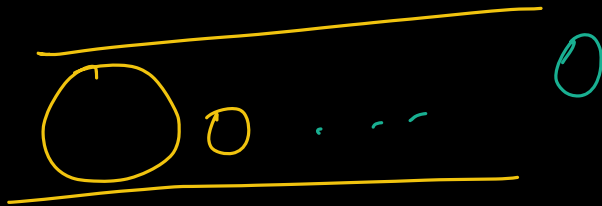
j = n-1
a[n-1] a[n]

if (condn)
has the power to
decide order of
elements.

TC: $O(N^2)$

SC: $O(1)$

Algo name: Bubble Sort



void BubbleSort (int a[], int N) {

for(i=0; i < n-1; i++) {

for(j=0; j < n-1; j++) {

if(comp(a[j], a[j+1])) {

swap(a[j], a[j+1])

bool comp (int a, int b) {

// Overriding Comparator

We need to know:

① a before b

② b before a

}

Comparator in Java / JS → check Syntax

Comparator customComparator (Integer a, Integer b) {

If you want a to come before b, return -1

If you want a & b = same, return 0

If you want b to come before a, return 1

}

arr.sort (arr, customComparator)

Comparator in Python

def compare (a, b):

If a should be before b, return -1

If a & b same, return 0

If b before a, return 1

arr.sort (key = cmp-to-key(compare))

Comparator in C++

bool compare (int a, int b) {

If a should be before b, return true

Else, return false

}

Qn: Given a $[N]$. Sort them in inc. order of their no. of factors.

If 2 elements have same factors,
element with lesser value should come first.

SC: $O(1)$

a[]: { 9, 3, 4, 8, 16, 37, 6, 13, 15 }
factors: { 3, 2, 3, 4, 5, 2, 4, 2, 4 }

Ans = { 3, 13, 37, 4, 9, 6, 8, 15, 16 }

idea: given arr[]
Sort(arr, comp)

return arr

TC: → adv module

Course: monch

PSP: 90+
↓
Referral to MSFT

```
int comp(int a, int b) {  
    int fa = factors(a) ] first class  
    int fb = factors(b)  
    // When should a come before b?  
    if (fa < fb) {  
        return -1 (a comes before b)  
    }  
    else if (fa == fb && a < b)  
        return -1  
    }  
    else { // fa > fb  
        b comes first  
        return 1  
    }  
}
```