Introduction To Problem Solving

7:05 am

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To Everyone

FAQs:
1. Notes will be uploaded after the class
2. Once session ends, assignments will unlock
3. No deadline for assignments
4. During doubt session ⇒ attendance won't be counted
5. Pseudo code {Language independant}.

Q. Count the no. of factors. [divisor]

24 → 4 → perfectly divisible

$4 \times \frac{6}{} = 24$

%

N = 10

[1 - 10]

$1 \times \frac{10}{} = 10$

[1, 2, 5, 10]

N = 12

↳ [1, 2, 3, 4, 6, 12]

Pseudo code:

```
int cntFactors (N) {        → input
    cnt = 0
    for(i = 1; i <= N; i++) {
        if(N % i == 0)
            cnt++
    }
    return cnt              → output
}
```

iterations

i = 1, 2, 3 ... (N+1)

N iterations

Assumption: 10^8 iter $\Rightarrow$ 1 sec

$N = 10^9$

10^8 iter $\Rightarrow$ 1 sec

1 iter $= 1/10^8$ sec

10^9 iter $= 10^9/10^8 =$ 10 sec

$N = 10^{18}$

1 iter $= 1/10^8$ sec

10^{18} iter $= 10^{18}/10^8 =$ 10^{10} secs

$\downarrow$

$\sim$ 317 years

you - child - grand child - 4th gen - 5th/6th

Optimize

if i is a factor of N

$i \times j = N$

$j \times i = N$

i & j are factor

$j = N/i$

$N = 24$ [1-24]

i	N/i	
1	24	+2
2	12	+2
3	8	+2
4	6	+2
6	4	8
8	3	
12	2	
24	1	

$N = 27$

i	N/i	
1	27	+2
3	9	+2
9	3	4
27	1	

$N = 12$

i	N/i	
1	12	2
2	6	2
3	4	2
4	3	

left <= right

$i <= N/i$

$i \times i <= N$

$i^2 <= N$

$i <= \sqrt{N}$

$\sqrt{25} = 5$

25/2

N = 100

i	N/i	
1	100	+2
2	50	+2
4	25	+2
5	20	+2
10	10	+1
20	5	

N = 25

i	N/i
1	25
5	5
25	1

Pseudo code

```
int cntfactors(N) {
    cnt = 0
    for(i=1; i<=√N ; i++) {
        if(N%i == 0) {
            if(i != N/i)
                cnt = cnt+2
            else
                cnt++
        }
    }
    return cnt
}
```

floor(sqrt(N))

1. sqrt(N) i<=sqrt(N)
2. S = sqrt(N) i<=S ✓
3. i*i <= N ✓
4. i <= N/i

avoid

iter N=100 10 iter

input: N → √N iterations

$N = 10^{18}$

first → 10^{18} iter ⇒ 317 years

2nd → $\sqrt{10^{18}} = 10^9$ iter = 10 secs

diff

N = 23

i	N/i	
1	23	+2
2		
3		
4		

4x4 = 16

5x5 = 25

$N = 10^{24}$ $\sqrt{10^{24}}$

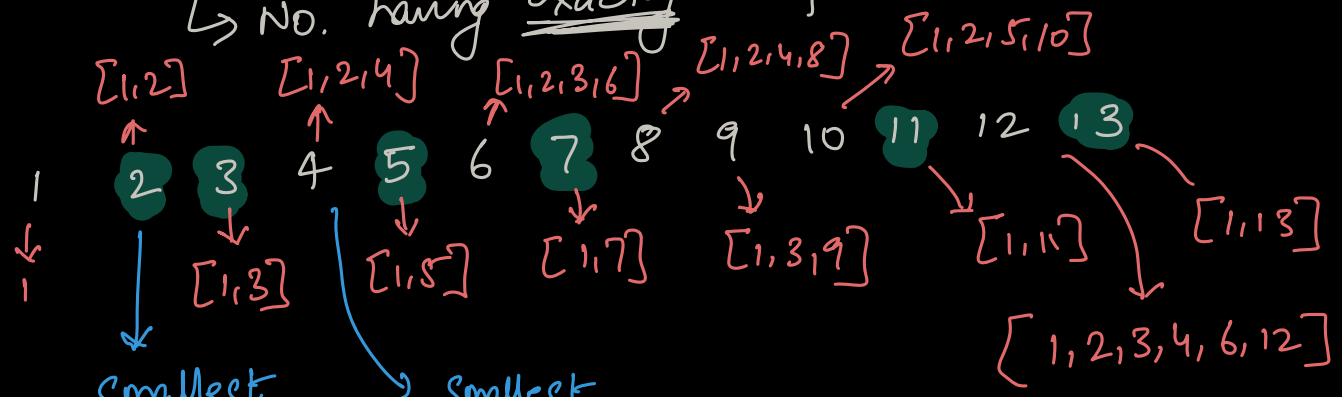
$10^{24/2} = 10^{12}$

10^8 iter = 1

10^{12} iter = 10^4 secs

Prime numbers

↳ No. having exactly 2 factors



Smallest prime

Smallest composite

#factors > 2

33

[1 3 11 33]

Q. Given N. Check if N is prime.

(N > 0)

```
bool isPrime(N) {
    if(cntFactors(N) == 2)
        return true;
    else
        return false;
}
```

H.W.

Try to optimize

N = 1

1

i = 1 i <= 1

5 min break

8:22 am

Q. Find sum of all no. from 1-100. Gauss

5050

$$S = 1 + 2 + 3 + 4 + \dots + 100$$
$$S = 100 + 99 + 98 + 97 + \dots + 1$$
$$2S = \underbrace{101 + 101 + 101 + 101 + \dots + 101}_{100 \text{ times}}$$

$$S = \frac{101 \times 100}{2} = 5050$$

$$S_n = 1 + 2 + 3 + \dots + n$$
$$S_n = n + (n-1) + (n-2) + \dots + 1$$
$$2S_n = \underbrace{(n+1) + (n+1) + (n+1) + \dots + (n+1)}_{n \text{ times}}$$

$$2S_n = n \times (n+1)$$

$$S_n = \frac{n(n+1)}{2}$$

$n+1$, $n \times (n+1)$, $\frac{n \times (n+1)}{2}$ — 3 ops

(*) Basics of Log

"log b, base a"

$\log_a b = c$

To what pow should I raise a to get b

$$a^c = b$$

$$\log_a (a^x) = x$$

$$a^{(x)} = a^{(k)}$$

$\log_2 64 = c = 6 \qquad 2^{6} = 64$

$\log_3 27 = 3 \qquad 3^{3} = 27$

$\log_2 32 = 5 \qquad 2^{5} = 32$

$\log_2 10 = 3 \ldots \qquad 2^{3} = 8 \quad 2^{4} = 16$

$\log_2 40 = 5 \ldots \qquad 2^{5} = 32 \quad 2^{6} = 64$

$\log_b b^{K} = K \qquad b^{K} = b^{K}$

$\log_2 N = K \iff 2^{K} = N$

$\log_a a = 1 \qquad a^{1} = a$

$\log_a (m \times n) = \log_a m + \log_a n$

$\log_a (m/n) = \log_a m - \log_a n$

Q. Given N. How many times divide N by 2 to get 1.

int (N>0)

N = 45

45 /2
22 /2
11 /2
5 /2
2 /2
1

= 5

H.W.

Q. Perfect sq nums.

↳ Product of 2 same integers

4 → 2×2

16 → 4×4

25 → 5×5

6 → X

Given N, check if its a perfect sq. num or not.

sqrt(N) X

N = 100

1 ——— 100

1 1×1 ≠ 100

2 2×2 ≠ 100

3

⋮

10 10×10 = 100 ⇒ STOP

```
int sqrt(N) {
    for(i=1; i<=N; i++) {
        if(i*i == N)
            return i;
    }
}
```

Amazon mcq

N = Perfect sq

N = 16

i = 1 2 3 4

1 4 9 16

$\sqrt{16} = 4$

N = 100

1 2 3 4 ... 10

[1 ... $\sqrt{N}$] iterations

Q. Given N. Find floor(sqrt(N)) (N>0)

$5^2 = 25$ $6^2 = 36$

28 → 5

$4^2 = 16$

27 → 5

24 → 4

15 → 3

```
int sqrt(N) {
    ans = 0
    for(i=1; i<=N; i++) {
        if(i*i <= N)
            ans = i
        else
            break
    }
    return ans
}
```

N = 6

i = 1 2 3 4 5 6

ans = ✗ 2 ✗

ans = 2

$\sqrt{N}$ iter

i*i <= N

i <= $\sqrt{N}$

N = 12

i = 1 2 3 4 ... 12

i = 1 $1 \times 1 <= 12$ pot. cand ans = 1 ✓

i = 2 $2 \times 2 <= 12$ pot. cand ans = 2 ✓

i = 3 $3 \times 3 <= 12$ pot. cand ans = 3 ✓

i = 4 4×4 X

iter : $\sqrt{N}$

Sqrt (N) → 1st → $\sqrt{N}$ iter

→ 2nd → $\log_2 N$ iter (Binary Search)

Advanced module

$\log_2 N < \sqrt{N}$

$\log_2 100 = 6.\text{--} < 10$ N = 100

$\log_2 2^{100} = 100$ $\log_b b^x = x$

$N = 2^{100}$

$\sqrt{2^{100}} = 2^{50}$

$2^{10} = 1024$