Today's Content

→ % operator

→ modular arithmetics

→ 1 Hard problem (Google).

Range

int x = -2×10^9 to 2×10^9

long y = -9×10^{18} to 9×10^{18}

% Basics

$n \% a$ = Remainder when n is divided by a

$10 \% 4 = 2$ | $10 - ($greatest multiple of $4 <= 10$ $8) = 2$

$13 \% 5 = 3$ | $13 -$ (greatest multiple of $5 <= 13$) $= 3$
10

Dividend = Quotient × Divisor + Remainder

∴ Remainder = Dividend − $\underbrace{\text{Quotient} \times \text{Divisor}}_{\text{greatest multiple of divisor} <= \text{dividend}}$

Quiz: $150 \% 11 = ?$

$150 -$ (greatest multiple of 11 $<= 150$)

$150 - 143 = \boxed{7}$

Quiz: $100 \% 7 = ?$

$100 -$ (greatest multiple of 7 $<= 100$)

$100 - 98 = \boxed{2}$

Quiz: $-40 \% 7 = ?$

$-40 - (\text{greatest multiple of } 7 <= -40)$

$-35 <= -40$ ×

$-42 <= -40$

$-40 - (-42)$

$= -40 + 42 = \boxed{2}$

Quiz: $-60 \% 9$

$-60 - (\text{greatest multiple of } 9 <= -60)$

$-60 - (-63)$

$= -60 + 63 = \boxed{3}$

Conceptually, remainders can't be <u>-ve</u>

In languages

	$(a\%p)$ Python	$(a\%p)$ Java / C++ / C# / JS
$-40\%7$	2	-5
$-60\%9$	3	-6

$+7$ $(-5+7)$

$+9$ $(-6+9)$

Conceptually this is wrong.

```
if(a < 0) {
  // Correct a%p
  = a%p + p
}
```

Why? → Extra content

↳ Doubt session

Why % operator? - Limits the input data to a given range

501, 839, 437, 606 $\% 10 = [9, 7, 1, 6]$

Value range : [0 9]

10 → 0, 11 → 1, 12 → 2

$-\infty$ to $+\infty$ $\% 10 = [0 \quad 9]$

$-\infty$ to $+\infty$ $\% p = [0 \quad p-1]$

~~p, p+1~~ X

WHY USEFUL?

- Hashing { Coming DSA }
- Consistent Hashing { LLD, HLD }

Modular arithmetics

Different arithmetic operators $\{+, -, *, /, \%\}$

① $(a+b)\%p = (a\%p + b\%p)\%p$

$(a+b)\%p$ | $(a\%p + b\%p)\%p$

a	b	p
8	6	10

$(8+6)\%10$

$14\%10$

$= 4$

$8\%10 + 6\%10$

$= 8 + 6$

$= (14)\%10$

$= 4$

7 9 6

$(7+9)\%6$

$= 16\%6$

$= 4$

$(7\%6 + 9\%6)\%6$

$= (1 + 3)\%6$

$= 4\%6$

$= 4$

② $(a*b)\%p = (a\%p * b\%p)\%p$

$(a*b)\%p$ | $(a\%p * b\%p)\%p$

a	b	p
8	6	10

$(8\times6)\%10$

$= 48\%10$

$= \boxed{8}$

$8\%10 * 6\%10$

$8 * 6$

$= (48)\%10$

$= 8$

7 9 6

$(7\times9)\%6$

$= 63\%6$

$= 3$

$(7\%6 * 9\%6)\%6$

$= (1 * 3)\%6$

$= 3$

$[0 \quad p-1]$

③ $(a-b)\%p$
④ $(a/b)\%p$ } ⇒ Will discuss it in advanced module + {Inverse modulo}

Qn: ① $\boxed{(a\%p)}\%p = a\%p$ ✓

$([0 \quad p-1])\%p$

$[0 \quad p-1]$

$[0 \quad p-1]$

② $(\underbrace{a\%p}_{x} * \underbrace{b}_{y})\%p = (a*b)\%p$ ✓

$(x*y)\%p = (x\%p * y\%p)\%p$

$= ((a\%p)\%p * b\%p)\%p$

$= (a\%p * b\%p)\%p$

$= \boxed{(a*b)\%p}$

Quiz: Not divisible by 3

(Sum of digits) % 3 == 0

Divisibility Rules

% 3 → Sum of digits should be divisible by 3

% 9 → Sum of digits should be divisible by 9

% 4 → Last 2 digits should be divisible by 4

% 8 → HW / ToDo

Proof % 3 : $(2475)\%3$

$\hookrightarrow (2\times10^3 + 4\times10^2 + 7\times10^1 + 5\times10^0)\%3$

$(a + b + c + d)\%3$

$\hookrightarrow (a\%3 + b\%3 + c\%3 + d\%3)\%3$

$\hookrightarrow ((2\times10^3)\%3 + (4\times10^2)\%3 + (7\times10^1)\%3 + (5\times10^0)\%3)\%3$

$\hookrightarrow (2+4+7+5)\%3$

$\hookrightarrow$ (Sum of digits) % 3

Observation (3)

$10^0 \% 3 = 1$

$10^1 \% 3 = 1$

$10^2 \% 3 = 1$

$10^3 \% 3 = 1$

$10^x \% 3 = 1$

Obs 2 (9)

$10^0 \% 9 = 1$

$10^1 \% 9 = 1$

$10^2 \% 9 = 1$

$10^3 \% 9 = 1$

$10^x \% 9 = 1$

Obs 3 (4)

$10^2 \% 4 = 0$

$10^3 \% 4 = 0$

$10^4 \% 4 = 0$

$(2457)\%4$

$\hookrightarrow (2400 + 57)\%4$

$\hookrightarrow (2400\%4 + 57\%4)\%4$

$= 0 + 57\%4$

$\hookrightarrow$ Last 2 digits should be divisible by 4

Qn: Given a, n, p. Calc $(a^n) \% p$ w/o any in-built functions

Constraints: $1 <= a <= 10^9$

$2 <= p <= 10^9$

$1 <= n <= 10^5$

Eg: $a = 3, n = 4, p = 7$

$\hookrightarrow 3^4 \% 7 = 81 \% 7 = \boxed{4}$

$\boxed{a << n = a * 2^n}$ $!= a^n$ ✗

```
powmod(int a, int n, int p) {
    // Version 1
    for(int i=1; i<=n; i++) {
        a = a*a        X
    }
    return a%p
```

a^4

a, n=4, p

a = a*a

i	i <= 4	
		$a = a*a = a^2$
1	✓	$a = a*a = a^2 \times a^2 = a^4$
2	✓	$a = a*a = a^4 \times a^4 = a^8$
3	✓	$a = a*a = a^8 \times a^8 = \boxed{a^{16}}$
4	✓	$a = a*a$

// Version 2 ✓

TC: O(N)
SC: O(1)

```
long ans = 1
for (i=1; i<=n; i++) {
    ans = (ans * a) % p
}
return ans % p
```

$10^9 * 10^9 \Rightarrow \boxed{10^{18}}$ → long → $10^{18} \% p$ → $[0 \quad 10^9]$

$\% p$ → optional

Issues in the code

$(a^n)\% p$

a	n	p	
2	30	45	$(2^{30})\%45$ ✓
2	60	41	$(2^{60})\%41$ ✓ → long
2	100	7	$(2^{100})\%7$ → int ✗, long ✗

ans *= (a % p)

ans = ans × (a%p)

Dry Run   a, p, n=5

ans	i	i<=5
1	1	✓
$a\%p$	2	✓
$a^2\%p$	3	✓
$a^3\%p$	4	✓
$a^4\%p$	5	✓
$\boxed{a^5\%p}$	6	✗

→ $\boxed{a^5\%p}$

ans = (ans * a) % p

ans = (1 * a) % p = a % p

ans = (a%p * a) % p
= $a^2 \% p$

ans = (a^2 % p * a) % p
= $a^3 \% p$

ans = (a^3 % p * a) % p
= $a^4 \% p$

ans = (a^4 % p * a) % p
= $a^5 \% p$

Break till 8.50 am

Surface - Come with me

Qn: Given 1 number in a[ ] format. Calc a[ ] % p

(Google)

↳ Each a[i] represents a single digit

Constraints

$1 <= N <= 10^5$

$0 <= a_i <= 9$

$2 <= p <= 10^9$

N = 5

a[5]: | 6 | 2 | 3 | 4 | 3 |

p = 49

⇒ (62343) % 49

= $\boxed{15}$

N = 4

a[4] = | 2 | 4 | 3 | 7 |

p = 16

(2437) % 16

= $\boxed{5}$

ideas

① Convert a[ ] → number & take % p

N = 2 $\quad$ 9 9 $= 10^2 - 1$

N = 3 $\quad$ 9 9 9 $= 10^3 - 1$

N = 4 $\quad$ 9 9 9 9 $= 10^4 - 1$

$N = 10^5$ $\quad$ - - - - - - - ... - - - $= \boxed{10^{10^5} - 1}$

Not possible to store in int / long

② Divisibility rules ✗

③ Hint

↳ Split the no. digit by digit

↳ Calc the value digit by digit

$N=5,\ a[5]$

$p=49$

0	1	2	3	4
6	2	3	4	3

$$\left.\begin{aligned} &+(3\times10^{0})\%p \\ &+(4\times10^{1})\%p \\ &+(3\times10^{2})\%p \\ &+(2\times10^{3})\%p \\ &(6\times10^{4})\%p \end{aligned}\right\}\%p$$

$$\Rightarrow \left[6\times10^{4} + 2\times10^{3} + 3\times10^{2} + 4\times10^{1} + 3\times10^{0}\right]\%p$$

$(3\times10^{0})\%p$

$3\times t = 1$

$t=(t\times10)\%p$

$(4\times10^{1})\%p$

$4\times t = 10\%p$

$t=(t\times10)\%p$

$\Rightarrow (10\%p * 10)\%p$

$(3\times10^{2})\%p$

$3\times t = 10^{2}\%p$

$t=(t\times10)\%p$

$= (10^{2}\%p \times 10)\%p$

$(2\times10^{3})\%p$

$2\times t = 10^{3}\%p$

$t=(t\times10)\%p$

$t=(10^{3}\%p * 10)\%p$

$(6\times10^{4})\%p$

$6\times t = 10^{4}\%p$

↳ add all purple boxes

```
long arrmod(int a[], int N, int p) {
    long sum = 0
    long t = 1
    for(i = n-1; i >= 0; i--) {
        sum = (sum + a[i] * t) % p
        t = (t * 10) % p
    }
    return sum
}
```

$(t \cdot 10) \% p$: $[0 \quad p-1] \times 10 \approx 10p$

$p = 10 \times 10^9$

$= 10^{10}$

overflow

$\therefore$ use long

$$\text{sum} = (\underbrace{\text{sum}}_{[0 \quad p-1]} + \underbrace{a[i]}_{9} * \underbrace{t}_{[0 \quad p-1]}) \% p$$

$$\approx p + 9p = 10p$$

$$= \boxed{10^{10}} \rightarrow \text{overflow}$$

$\therefore$ use long

Doubts:

```
long ans = 0
long pow = 1
for (i = 0 ; i < n ; i++) {
    ans = a[i] * pow
    pow = (pow x 10) % p
}
ans % p
```

$1 \to 10 \to 10^2 \to 10^3 \to 10^4$

$i = 18$ 10^{18}

$i = 19$ 10^{19}

$i = 20$ 10^{20}

$\% p$ $\boxed{10^9 + 7 = \text{prime}}$

Python v/s C++/Java/C#

Rem = Dividend − Quotient × Divisor

(a/b) : integer division

$100 \% 7 \Rightarrow 100 - 7 * (100/7)$

$100 - 7 * 14 = \boxed{2}$

$\to -5$

$-40 \% 7 \Rightarrow -40 - 7 * (-40/7)$

$= -40 - (-35) = \boxed{-5}$

% in python :-

Rem = Dividend − divisor * quo floor(14. - - - -)

$100\%7 \Rightarrow 100 - 7 * (100/7)$

$= 100 - 7 * 14$

$= \boxed{2}$

floor(−5. - - - -)

= −6

$-40\%7 = -40 - 7 * (-40/7)$

$= -40 - 7 * (-6)$

$= -40 + 42$

$= \boxed{2}$

floor(40/7)

Repeat + Missing

[1 . . . n]   R

M

Sum N terms!

$\frac{N(N+1)}{2}$

$\frac{N(N+1)}{2} - M + R$

$(\text{Sum N terms})^2$

$\frac{N(N+1)(2N+1)}{6}$

$\frac{N(N+1)(2N+1)}{6} - M^2 + R^2$