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Mr. Williams.

Dept. of Mech. Eng.  
IIT Delhi

15/05/2007

**Sub:** Syllabus for the written comprehensive examination of  
Mr. Suril V. Shah (2006MEZ8020) to be held in July/August 2007.  
(Topic: Dynamics and Control of Hopping Robots).

**1. Linear Algebra**

Vector spaces; Matrix algebra; Singularity; Condition numbers; LU,  $LL^T$ , QR, SVD decompositions; Sensitivity analysis; Eigen value problems

**2. Differential Calculus**

Differential equations—linear and non-linear; Analytical and numerical solutions (Runge-Kutta, Adams-Bashforth and others); Stability of numerical solutions; Error and tolerances in numerical methods.

**3. Kinematics and Dynamics**

Linkages; Degree of freedom; Constraints; Types of chains; Loop equations; Graphical and analytical methods for analysis and synthesis of mechanisms; Holonomic and non-holonomic constraints; Lagrange, Newton-Euler, Kane's, and other multibody dynamics formulations.

**4. Mechanical Design**

Stress; Strain; Failure theories (static and fatigue); Material selection; Design of structures, gears; Selection of belts, chains, and bearings.

**5. Mechanical Vibration**

Single and multi degree-of-freedom vibratory systems; Natural, forced, and damped frequencies; Numerical methods; Modal analysis; Continuous systems.

**6. Robotics**

Types of joints; Serial, parallel, walking, and hopping robots; Gaits in walking robots; Forward and inverse kinematics; Jacobian; Singularity and workspace analysis

**7. Control Theories**

Necessity of control; Transfer function, poles, and zeros; Stability criteria; PD, PI, and PID controls; State space form; Non-linear controls.

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15/05/2007

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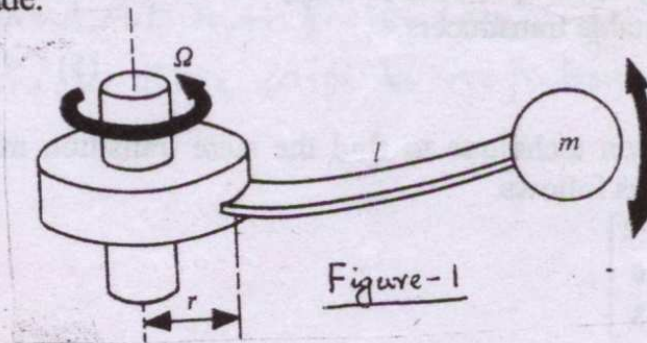
Prof. K. Gupta (Chairman); Prof. S. Mukherjee (Dept. Expert);

Dr. I.N. Kar (Outside Dept. Expert)

Tue 15/05/2007



1(a) Figure-1 shows the model of a turbine rotor-blade where the rotor of radius  $r$  spins about its axis at a speed  $\Omega$  and the blade modeled as a cantilevered beam of length  $l$  and mass  $m$  supposed to be concentrated at the end vibrates. Find the natural frequency of vibration of the blade. (5)



wp3  
3EI

1(b) To have an idea of the time to rebound in a fractional gravity situation a simple experiment is conducted by modeling the robot-pedestal system by a single spring-mass-damper (mass, ' $m$ ', damping coefficient, ' $d$ ' and stiffness ' $k$ ') system and letting it slide from rest over a smooth surface inclined at an angle of  $30^\circ$  from a distance  $s$  as shown in figure-2.

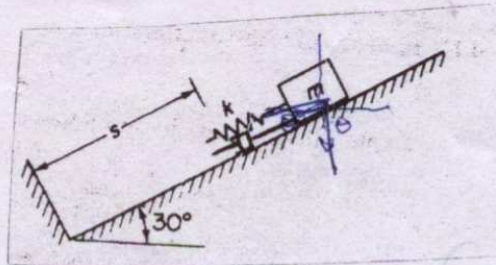


Figure-2.

Find the time elapsed between the events of contact of the spring and its take off or rebound. Assume  $m=1\text{kg}$ ,  $d=0\text{ N/m/sec}$ ,  $k=1\text{N/m}$  and  $s=1\text{m}$ . (5)

Find the rebound time and the height of rebound if the landing mechanism has a viscous damping coefficient of  $0.1\text{N/m/sec}$ . (5)

2. Solve the following simultaneous linear differential equations

(a)  $\frac{dx}{dt} + 5x - 2y = t$ , given that  $x=y=0$  when  $t=0$  (5)

$\frac{dy}{dt} + 2x + y = 0$

(b) What is meant by a linear differential equation? Write the Leibnitz and Bernoulli's linear forms. Find the expression of the integrating factor to solve the Leibnitz linear form. (1+1+3)

$(1+x)^2 = 1 + 2x + x^2$



(c) Solve the second order differential equation given by  $y'' = xy'^2 - y^2$  for  $x=0.2$  using Runge-Kutta method. Initial conditions are given by  $y=1$  and  $y'=0$  when  $x=0$ . (10)

3(a) Suppose an experiment is performed to adjust the speeds of two D.C. motors at a ratio, without attaching them physically. Draw a control scheme to perform this experiment. Assume suitable transducers. (5)

3(b) Use Cayley Hamilton technique to find the state transition matrix of the system expressed in state-space as follows. (10)

$$\dot{x} = Ax \text{ where } A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

$$D = (-2 + j3)$$

$$D = (-2 - j3)$$

$$D + 2 - j3$$

$$D + 2 + j3$$

$$y = \frac{1}{2} \left[ -4.23 - 0.26e^{-2.6} - 3.73e^{-3.73} + 5.633 - 21.15t + 5(1)e^{-0.26} + 5(2)e^{-3.73} \right] - t$$

$$0.77 - 22.15t + 4.74e^{-0.26} + 1.27(2e^{-3.73})$$

$$\frac{1}{2} =$$





in MATLAB

Develop a computer code for finding the motion of the mass <sup>in question 1(b)</sup> ~~after~~ between the instants of contact with the wall and rebounds, ~~How~~ and the rebounds it undergoes with different damping  $\gamma$  coefficients. Is it possible to find out a stiffness-damping combination <sup>5</sup> for which there will be no rebound?

1.2.2.1





Duration: 3 hours

Marks: 100

Part A (50 marks)

A1. Answer the following:

[10×2 = 20]

- |   |   |
|---|---|
| a. Define a vector space in linear algebra?                           | f. Define Jacobian of a robot, and what it signifies when it is singular? |
| b. Define holonomic and non-holonomic systems?                        | g. Draw S-N diagram of iron alloys used in fatigue failure.               |
| c. What are the static theories of failures?                          | h. Define generalized coordinates and generalized forces in dynamics      |
| d. What is QR decomposition of a matrix, and its applications?        | i. Grashof's criterion and Kutzbach-Grubler's formula                     |
| e. What is Adams-Bashforth method, and where it can be used suitably? | j. Why one should use DeNOC-based dynamics over other formulations?       |

A2. For the elbow manipulator shown in Figure 1,

[15]

- Find its DH parameters. Evaluate its Jacobian matrix. Determine the singularity conditions and sketch the corresponding configurations of the manipulator.
- Formulate the dynamics problem to find its equations of motion using the Euler-Lagrange equations.

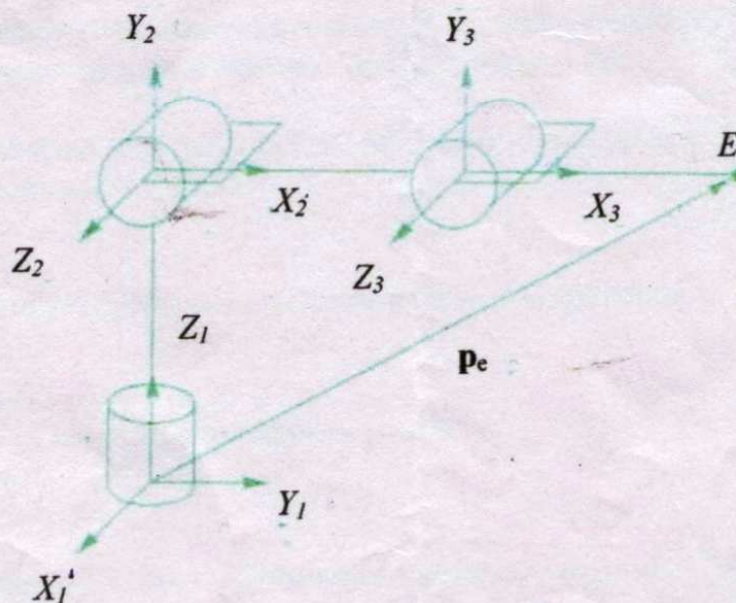


Figure 1 An elbow manipulator

$$\begin{bmatrix} \cos\theta_1 & 0 & 0 \\ \sin\theta_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \\ 0 \\ 0 \end{bmatrix}$$



A3. For the given algebraic equation:

[15]

$$\begin{bmatrix} \frac{52}{3} & -3 & -2 \\ -3 & \frac{20}{3} & -6 \\ -2 & -6 & \frac{40}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

- If one of the three eigenvalues of the associated matrix is 2.2755, find the other two [Hint: Try without solving cubic polynomials]
- Can you identify any specific property (ies) of the associated matrix?
- Applying a suitable decomposition technique to solve for the unknowns.

(c) 2.2555  
16.8354  
18.2225

$x_1 = 0.5701$   
 $x_2 = 1.6824$   
 $x_3 = 0.9176$

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**ME Dept., IIT Delhi**  
**Take Home Comprehension Examination for**  
**Mr. Suril V. Shah (2006MEZ8020)**  
**25.10.2007**  
**[Submit separate reports for Parts A and B]**

Duration: 48 hours (Report Submission: 29.10.07; 10am)

Marks: 100

**Part A (50 marks)**

A1. Write a MATLAB program to find the joint torques of the elbow manipulator in Question A2 of your written comprehensive examination using the expressions developed there. Take the joint trajectories as

$$\theta_i = \theta_i(0) + \frac{\theta_i(T) - \theta_i(0)}{T} \left[ t - \frac{T}{2\pi} \sin\left(\frac{2\pi}{T}t\right) \right]$$

where  $\theta_i(0) = 0$ ,  $\theta_i(T) = \pi/2$ , for  $i = 1, 2, 3$ , and  $T = 10$  sec. Plot the joint trajectories, their first and second derivatives, and the joint torques. Assume any numerical values required for the geometrical and inertia parameters of the manipulator.

A2. Using MATLAB commands check the answers of Question A3 of your writing comprehensive examination. Write the commands used to find the answers.



**ME Dept., IIT Delhi**  
Written Comprehension Examination for  
Mr. Ali Mr. Ali Rahmani Hanzaki (2004MEZ8136)  
30.09.2005

Duration: 2 hours

Marks: 120

Part A (75 marks)

1. Define the following: [10×3 = 30]
  - a. Ackerman condition in automobile steering system
  - b. Equivalent linkages in mechanisms
  - c. Rank, nullity, and singularity of a matrix
  - d. SVD decomposition of a matrix, and its applications
  - e. Generalized mass in vibration
  - f. Controllability and observability in control theory
  - g. S-N diagram in fatigue failure
  - h. Generalized coordinates and generalized forces in dynamics
  - i. Grashof's criterion and Kutzbach-Grubler's formula
  - j. State-space form and its applications
2. Figure below shows a four-bar mechanism. Link 2 rotates at a constant rotational speed of 120 rad/sec in the counterclockwise direction. Also indicated in the figure are the results of the acceleration analysis. Pertinent properties of links 3 and 4 are listed below

	Link 3	Link 4
Mass (kg)	0.50	0.80
Inertia about G (kg-m <sup>2</sup> )	$2.52 \times 10^{-3}$	$8.00 \times 10^{-4}$

Determine the magnitude and sense of the torque to be applied to link 2 from base link 1 to overcome the inertias of links 3 and 4. Also, given the link lengths,  $r_1 = 9.0$  cm;  $r_2 = 2.0$  cm;  $r_3 = 13.0$  cm;  $r_4 = 7.0$  cm [30]

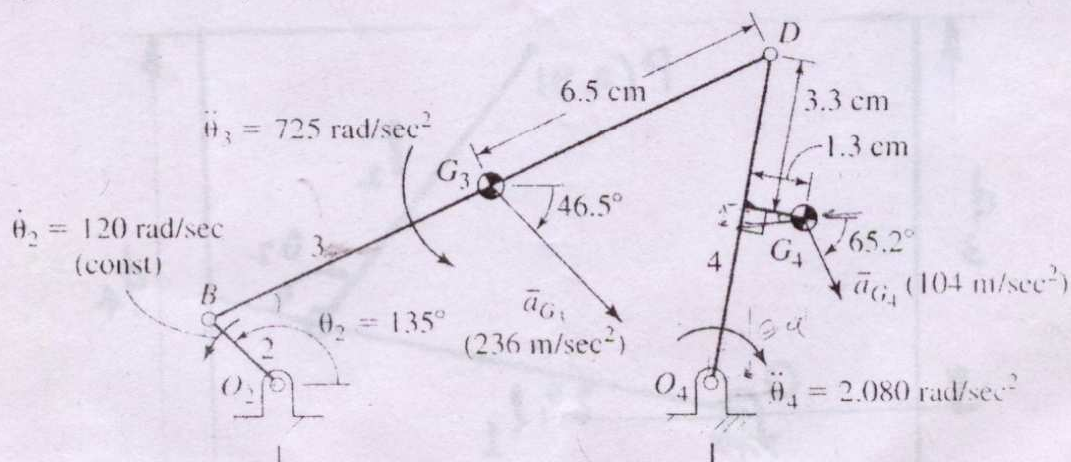


Figure: Four-bar linkage

3. Consider the a dynamical system whose equations of motion is given by  $m\ddot{x} + b\dot{x} + kx = f$ . [15]
  - a. Solve for  $x$  when  $m = 1$ ,  $b = 4$ ,  $k = 3$ , and  $f = 5\sin 2t$
  - b. Propose a PD control law, and find out the relation between the two gains so that the closed-loop system is critically damped.



Part B (45 marks)

1. a. How can one convert a constrained optimization into an unconstrained optimization. Explain with an example. Don't solve the optimization problem.

b. Find maxima of function  $(\cos^2\theta - \sin^2\theta)$

2. a. Can redundancy in manipulators help in arriving at optimum designs?

b. Figure below shows a two-link planar manipulator. Write down forward and inverse kinematic relations for the same. (Hint: Forward kinematics is to write  $x$  and  $y$  coordinates of Point  $P$  in terms of  $\theta_1$  and  $\theta_2$ . Inverse kinematics is to write  $\theta_1$  and  $\theta_2$  in terms of  $x$  and  $y$ )

c. As shown in the figure, the manipulator is constrained to move within rectangular boundaries. Formulate an optimization problem to find out the furthest point from  $O$  which end-effector can reach.

d. Is this a linear or nonlinear optimization problem?

e. Explain how can one solve this optimization problem. (Do not solve. Only explain the methodology).

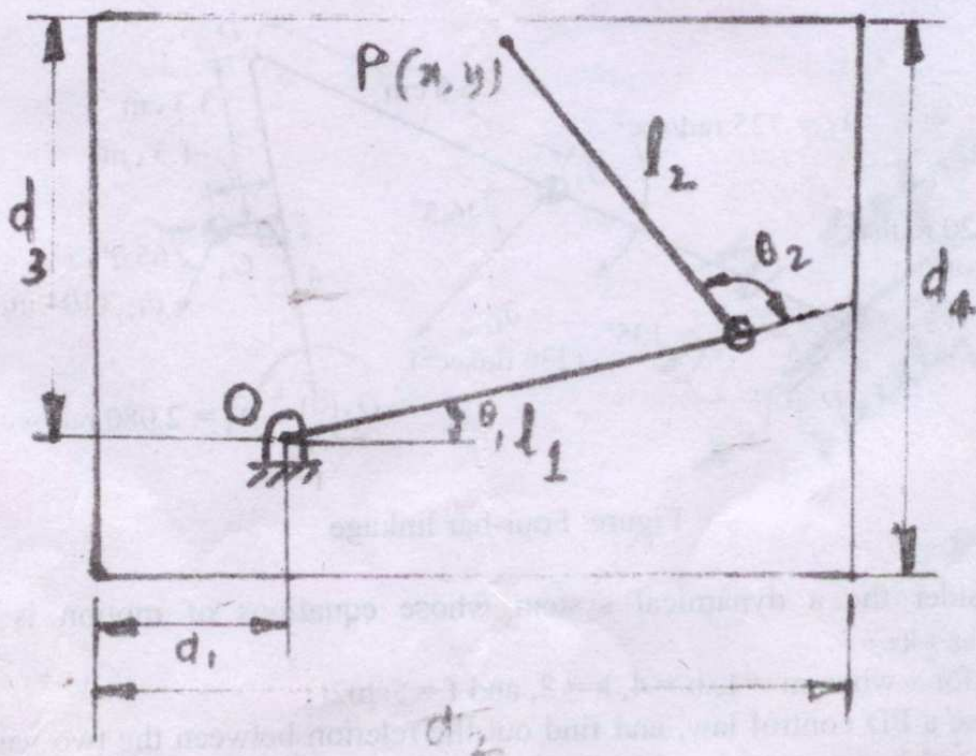


Figure: Two-link manipulator



**ME Dept., IIT Delhi**  
Take Home Comprehension Examination for  
Mr. Ali Mr. Ali Rahmani Hanzaki (2004MEZ8136)  
30.09.2005

Duration: 24 hours

Marks: 120

Part A (60 marks)

Write a Matlab program to find the results of Problem 2 in Part A for the complete rotation of the crank.

Part B (60 marks)

Give solution for Problem 2 of Part B of the written examination

- (a) by using a suitable optimization methods, and
- (b) using Matlab



Written Comprehensive Examination of

Mr. Himanshu Chaudhary

(2003ME20016)

Duration: 3hrs

Marks: 180

- 1.a) What is the Kutzbach-Grübler formula to obtain the degree of freedom (dof) of a mechanical system? Find the dof of the following systems:

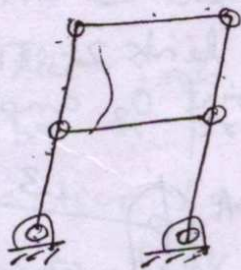


Fig. 1

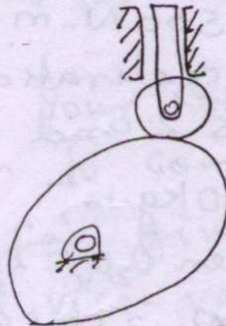


Fig. 2

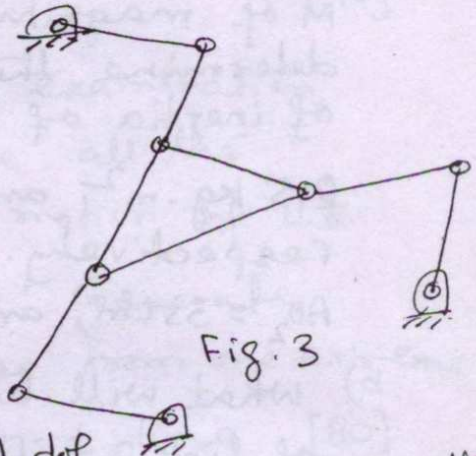


Fig. 3

Mention alternate ways to find dof. using any of these verify your dof results for the above systems. [20]

2. For the 3-member truss (Fig. 4), find the reaction forces in locations 1 and 3. Formulate the problem through Finite Element Analysis, which can then be solved using a computer to verify your results. [45]

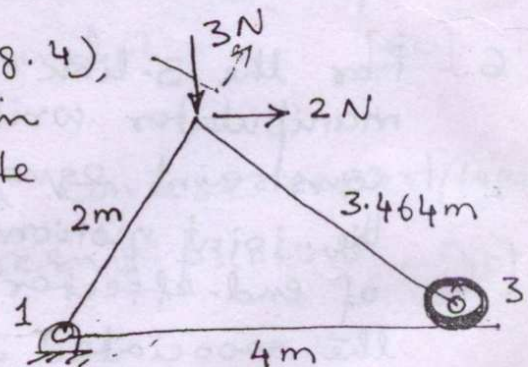


Fig. 4

3. a) Prove the solutions of the following set of equations

$$3.000x + 4.127y = 15.41$$

$$1.000x + 1.374y = 5.147$$

are  $x = 13.6658$  and  $y = -6.2$ .

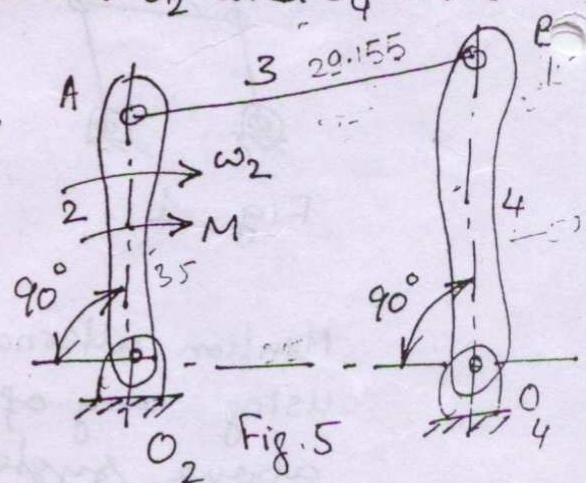
- b) Solve  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 5\sin 2x$  [15]



4. Using LU decomposition technique solve the following equation [20]

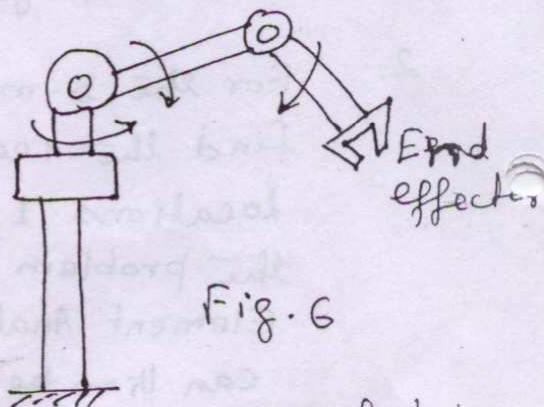
$$Ax = b, \text{ where } A = \begin{bmatrix} 2 & 4 & -2 \\ 1 & -1 & 5 \\ 4 & 1 & -2 \end{bmatrix}; b = \begin{bmatrix} 6 \\ 0 \\ 2 \end{bmatrix}; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

5. a) Figure 5 shows a mechanism where link 2 is driving link 4 with the help of a rigid, uniform coupler of negligible mass. The angular velocity  $\omega_2$  at the instant shown is 10 rad/s. If a moment  $M$  of magnitude 500 N-m is applied on 2, as shown, determine the acceleration of link 2. The moments of inertia of links 2 and 4 about  $O_2$  and  $O_4$  are  $0.5 \text{ kg-m}^2$  and  $1.0 \text{ kg-m}^2$ , respectively. Given  $O_2O_4 = 25 \text{ cm}$ ,  $AO_2 = 35 \text{ cm}$ , and  $BO_4 = 50 \text{ cm}$ .



- b) What will be the acceleration of link 2 if the coupler has a mass 2 kg and the mechanism is in a horizontal plane? [30]

6. For the 3-link robot manipulator write the velocity constraint equations relating the joint motions with ~~that~~ those of end-effector. Based on the associated Jacobian can you identify its singular configurations? What is the correlation of these configurations with the condition number of the Jacobian. Explain with proper definitions and Justifications. [30]



7. Formulate an optimization problem to minimize the forces transmitted from  $O_2$  and  $O_4$  to the ground (Fig. 5). What methods are suitable to solve the problem? [20]



Take Home Comprehensive Exam.  
of Mr. Himanshu Chauhan  
(2003ME20016)

11/10/2004

Marks: 220

Duration: 72 hrs  
(Submission: 15/10/04)

1. For the Prob. 2 of your written examination, write a computer program (MATLAB or other) to verify your results. [40]
2. For the Prob. 5 of your written examination, write a program to compute all the reaction forces for given motion of the input link and vice-versa. Generate results when link 2 traverses from one extreme to the other with  $\theta_2 = \sin \omega_2 t$ ,  $t$ : time. [80]  
Verify your results of 5(a) and (b).
3. For Prob. 7 of your written examination, what is the minimum force transmitted (Shaking force) to the ground? [60]

For all the problem, submit a concise report (typed) along with the program and result printout. [40]

— END —