

## Written Comprehensive (Closed-book) Examination

Mr. Arun Dayal Udai (2010MEZ8088)  
November 21, 2011 (Monday), 9:30-12:30

Duration: 3 hours

Marks: 100

1. (a) What is SVD decomposition of a matrix? What are the methods to find it?  
(b) Showing the steps find  $LL^T$  (Cholesky) decomposition of the following matrix:

$$A = \begin{bmatrix} 9 & -1 & 2 \\ -1 & 8 & 5 \\ 2 & 5 & 7 \end{bmatrix}$$

- (c) Find the solution for the 2<sup>nd</sup> order differential equation using Runge-Kutta method for the value of  $x = 2$ .

$$y'' - 3y' + 2y = 0 \text{ with initial conditions } y(0) = 4, y'(0) = 5$$

[5+10+10 = 25]

2. (a) Mention at least three different ways to specify three-dimensional rotations? Using sketch explain the physical interpretations of the parameters used to represent rotations.  
(b) Formulate the dynamics problem of a planar four-bar linkage, i.e., write down the equations of motion [Assume all the parameters necessary to derive the equations]  
(c) For the problem in (b) how the DAE and ODE formulations will look like.

[5+10+10 = 25]

3. Derive the Jacobian matrix of a 3-link spatial articulated arm with all revolute joints [Hint: Write the DH parameters first] [20]

4. (a) What are the natural constraints for a round peg sliding into a round hole? What the artificial constraints for the above operation?

- (b) Determine the stability of the following linear system by Lyapunov's method:

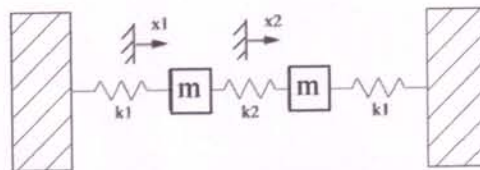
$$\dot{x}_1 = -x_1 - 2x_2$$

$$\dot{x}_2 = x_1 - 4x_2$$

[Hint: Assume a suitable Lyapunov function. You may need a positive definite matrix.]

[5+10 = 15]

5. Consider the system shown below with 2 masses and 3 springs. The masses are constrained to move only in the horizontal direction (they can't move up and down):



[Source: <http://lpsa.swarthmore.edu/MtrxVibe/EigApp/EigVib.html#Example>]

Write the equations of motion of the above 2-DOF system. Formulate it as an eigenvalue problem to find the solutions for  $k_1=k_2=m=1$ .

[15]

... END ...

**Take-home Comprehensive Examination**

Mr. Arun Dayal Udai (2010MEZ8088)

November 21-22, 2011

[To be handed over after the end of written comprehensive]

**Submission: November 23, 2011 (Wed), 10am**

**Marks: 100**

For Question 3 of your written comprehensive, do the following:

1. Write a MATLAB program to evaluate the Jacobian matrix for some numerical values of the DH parameters.
2. Find its inverse dynamics results for some input trajectory using any existing algorithm (like RoboAnalyzer or ReDySim). [Any discussion with anybody is permissible]
3. Verify the results of item 2 above using SimMechanics of MATLAB.

Submit a hand-written report only with the relevant printout of the plots.

... END ...