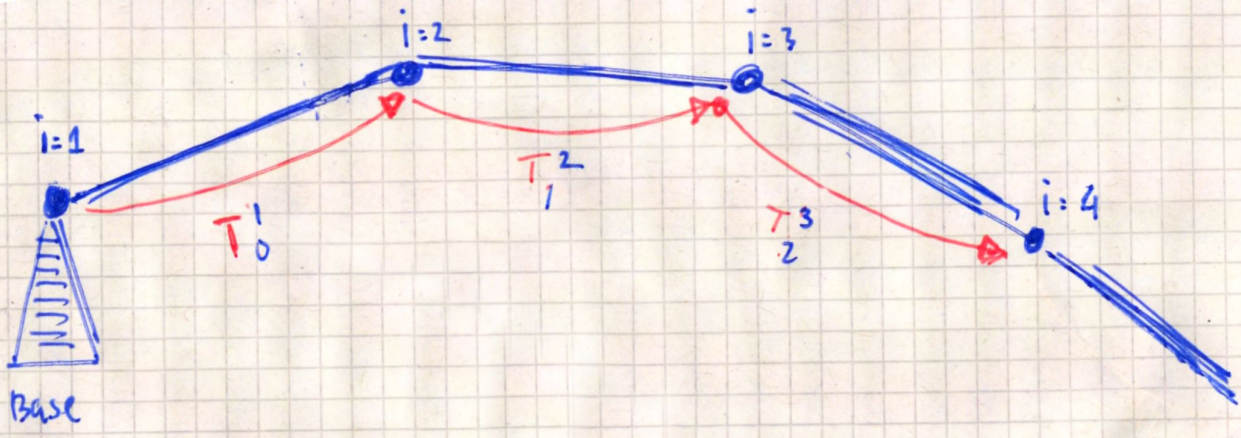

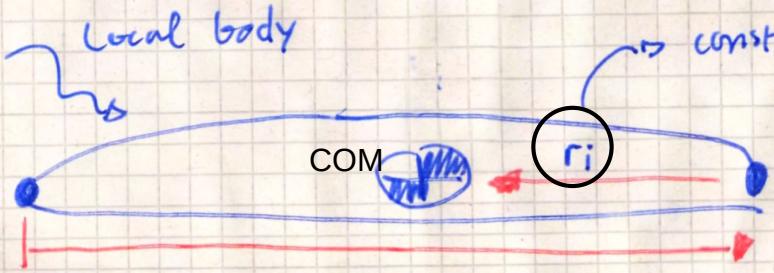

p_i

$$P_i = \begin{bmatrix} a_i \\ d \sin\alpha_i \\ d\cos\alpha_i \end{bmatrix} \Rightarrow \text{constant (local frame)}$$

$z = [0\ 0\ 1]^T$

FORWARD : $1 \le i \le n$ where n is the number of DOFs

$$w_i = (R_{i-1}^i)^T (w_{i-1} + \dot\theta_i z)$$

$$\dot w_i = (R_{i-1}^i)^T (\dot w_{i-1} + \ddot\theta_i z + w_{i-1} \times \dot\theta_i z)$$

$$\dot v_i = (R_{i-1}^i)^T \dot v_{i-1} + \dot w_i \times p_i + w_i \times (w_i \times p_i)$$

Backward $n \ge i \ge 1$

$$\dot v_{ci} = \dot v_i + \dot w_i \times r_i + w_i \times w_i \times r_i$$

$$F_i = m_i \dot v_{ci}$$

$$N_i = I_i \dot w_i + w_i \times I_i w_i$$

$$n_i = R_i^{i+1} \left\{ n_{i+1} + \left(R_i^{i+1}\right)^T p_i \right) \times f_{i+1} + (r_i + p_i) \times F_i \Big\} + N_i$$

$$f_i = R_i^{i+1} f_{i+1} + F_i$$

Finally :

$$Q_i = f_i \left(R_{i-1}^i\right)^T z \quad \text{For translational joint}$$

$$Q_i = n_i (R_{i-1}^i)^T z \quad \text{For rotational joint}$$