# Analytical Dynamics
# Project 1
# Newton's Canon Ball Dynamics Simulation

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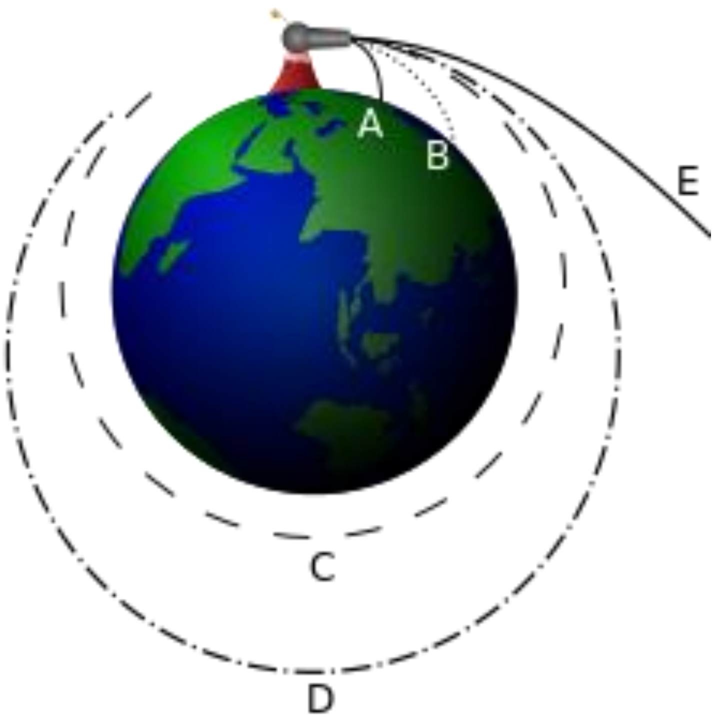
# Outline

- Problem Statement
- Equation of Motion (EoM)
- Conversion to reduced order EoM
- Differential Equation Solver
- Simulation Results
- EoM in Polar form

# Problem Statement

- To simulate dynamics of Newton’s canon ball and study the effect of velocity and height of canon on the path of canon ball.



# Equation of Motion (EoM)

Coupled 2nd order equations:

$$\boxed{\frac{-GMx}{x^2+y^2}\left\{\frac{x\,i+y\,j}{(x^2+y^2)^{1/2}}\right\} = m\ddot{x}\,i + m\ddot{y}\,j}$$

$$\frac{-GMx}{(x^2+y^2)^{3/2}} = \ddot{x}$$

$$\frac{-GMy}{(x^2+y^2)^{3/2}} = \ddot{y}$$

# Reduced order EoM

$$\boxed{\begin{gathered}\dot{x} = x_1 = \int \ddot{\mathrm{x}}\, dt \\ \dot{x_1} = \ddot{x} = \frac{-GMx}{(x^2 + y^2)^{3/2}} \\ \dot{y} = y_1 = \int \ddot{y}\, dt \\ \dot{y_1} = \ddot{y} = \frac{-GMy}{(x^2 + y^2)^{3/2}}\end{gathered}}$$

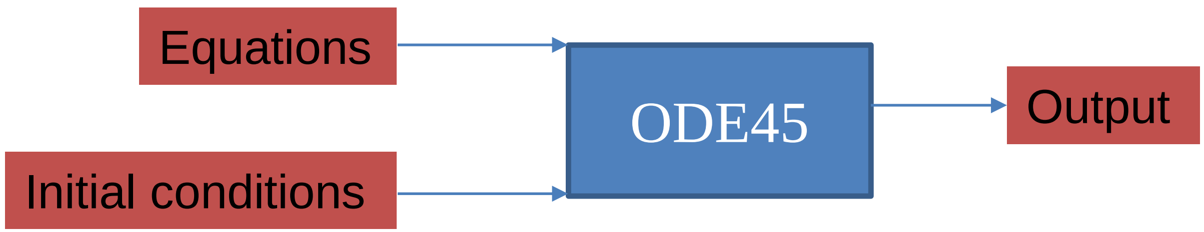
$$x_1 = f(x, y)$$
$$y_1 = f(x, y)$$

# Differential Equation Solver

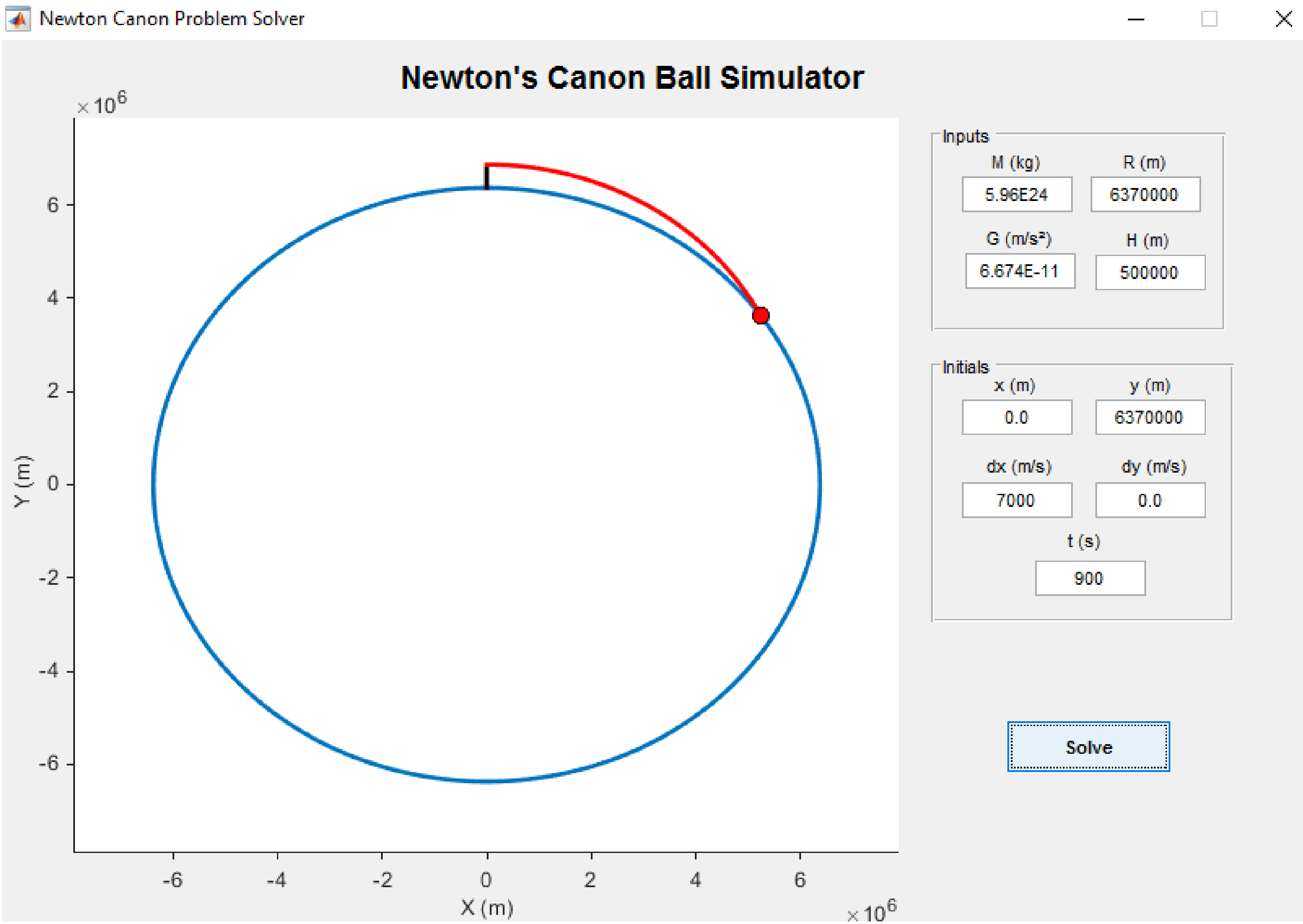
**ODE45:**

- This function implements a Runge-Kutta method
- Syntax

*[t,x] = ode45(@f_name, t_span, x_init, options)*

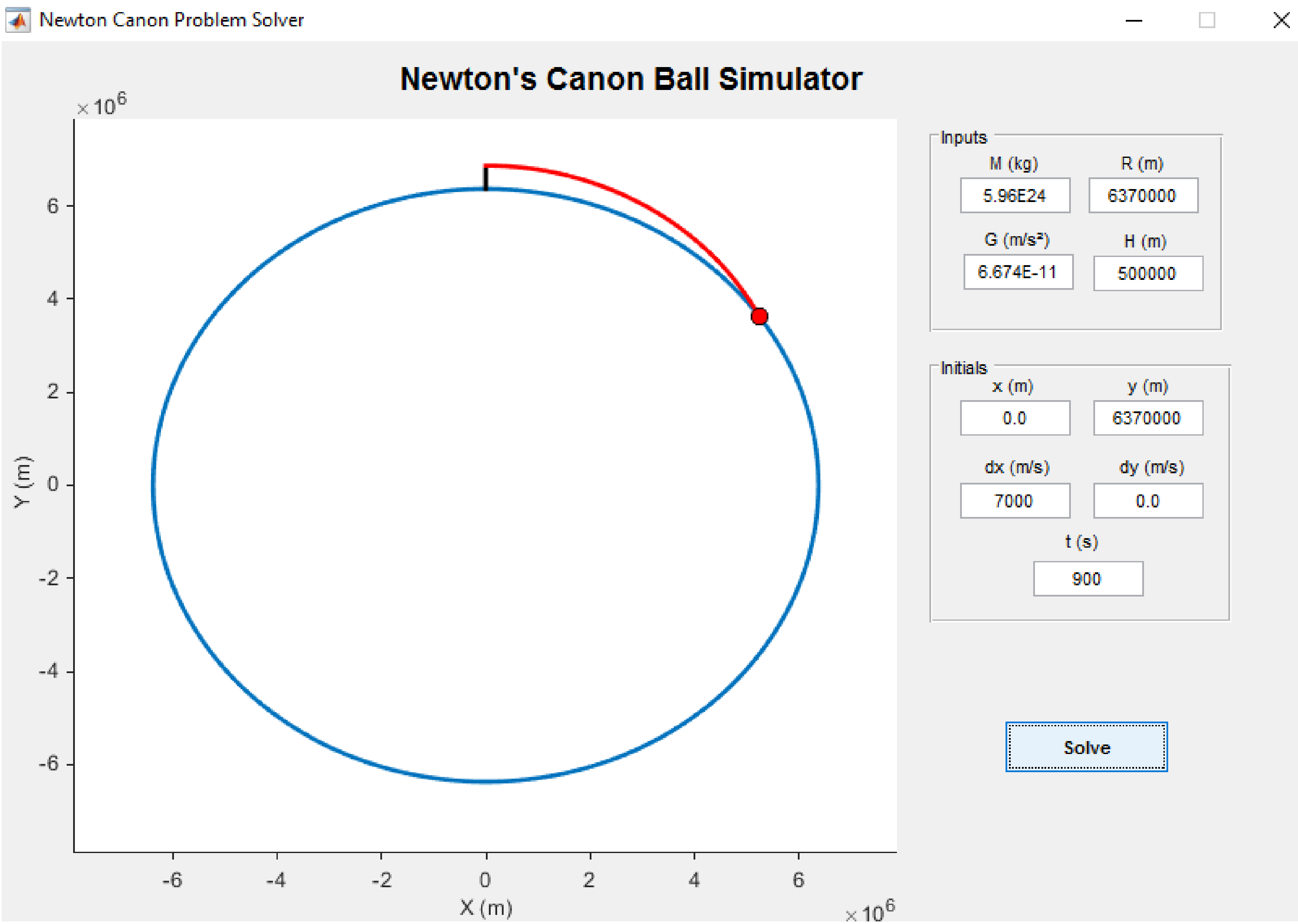


# MATLAB GUI

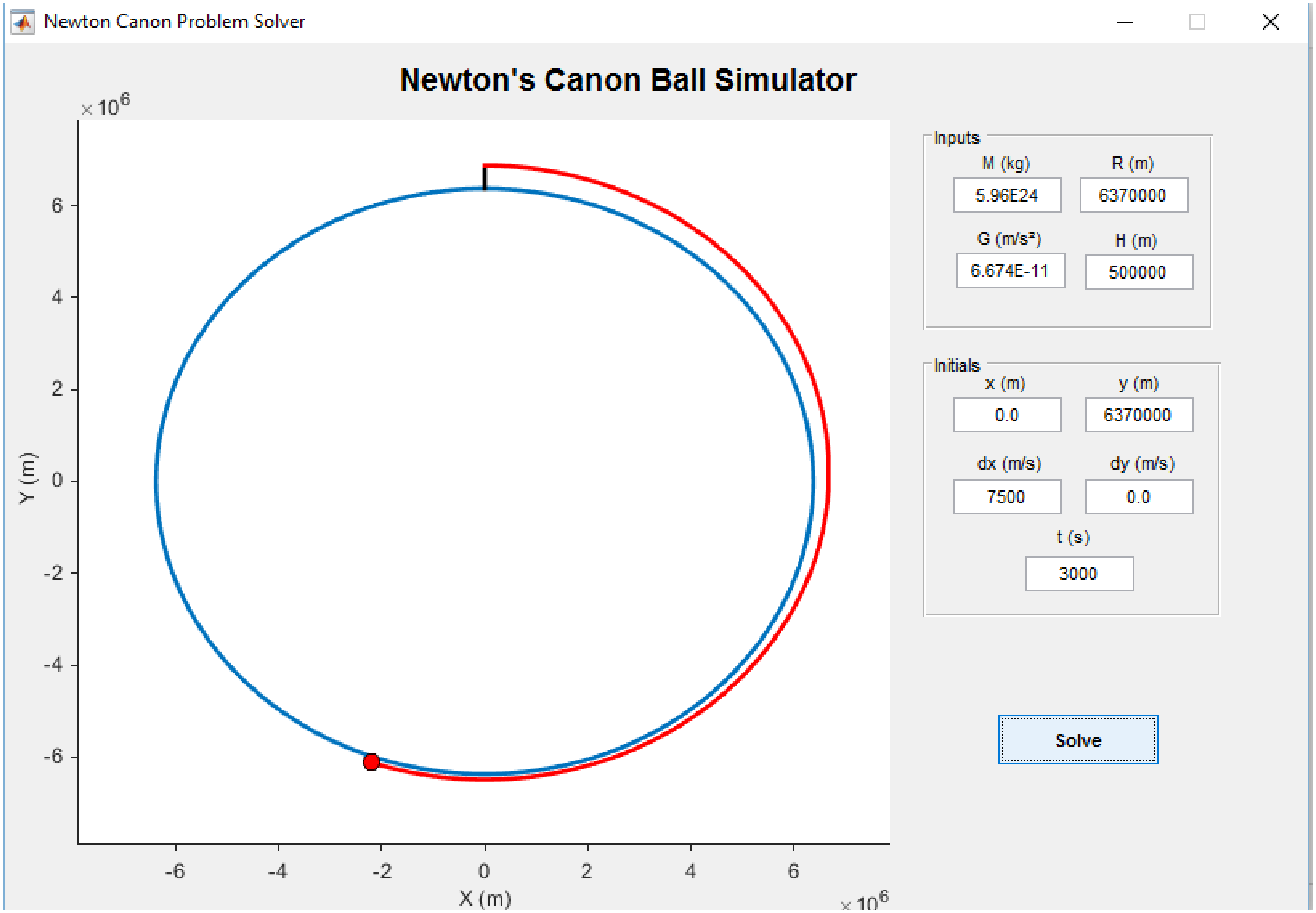


Newton Canon Problem Solver
Newton's Canon Ball Simulator
×10^6
Y (m)
X (m)
6
4
2
0
-2
-4
-6
Inputs
M (kg)
5.96E24
R (m)
6370000
G (m/s²)
6.674E-11
H (m)
500000
Initials
x (m)
0.0
y (m)
6370000
dx (m/s)
7000
dy (m/s)
0.0
t (s)
900
Solve

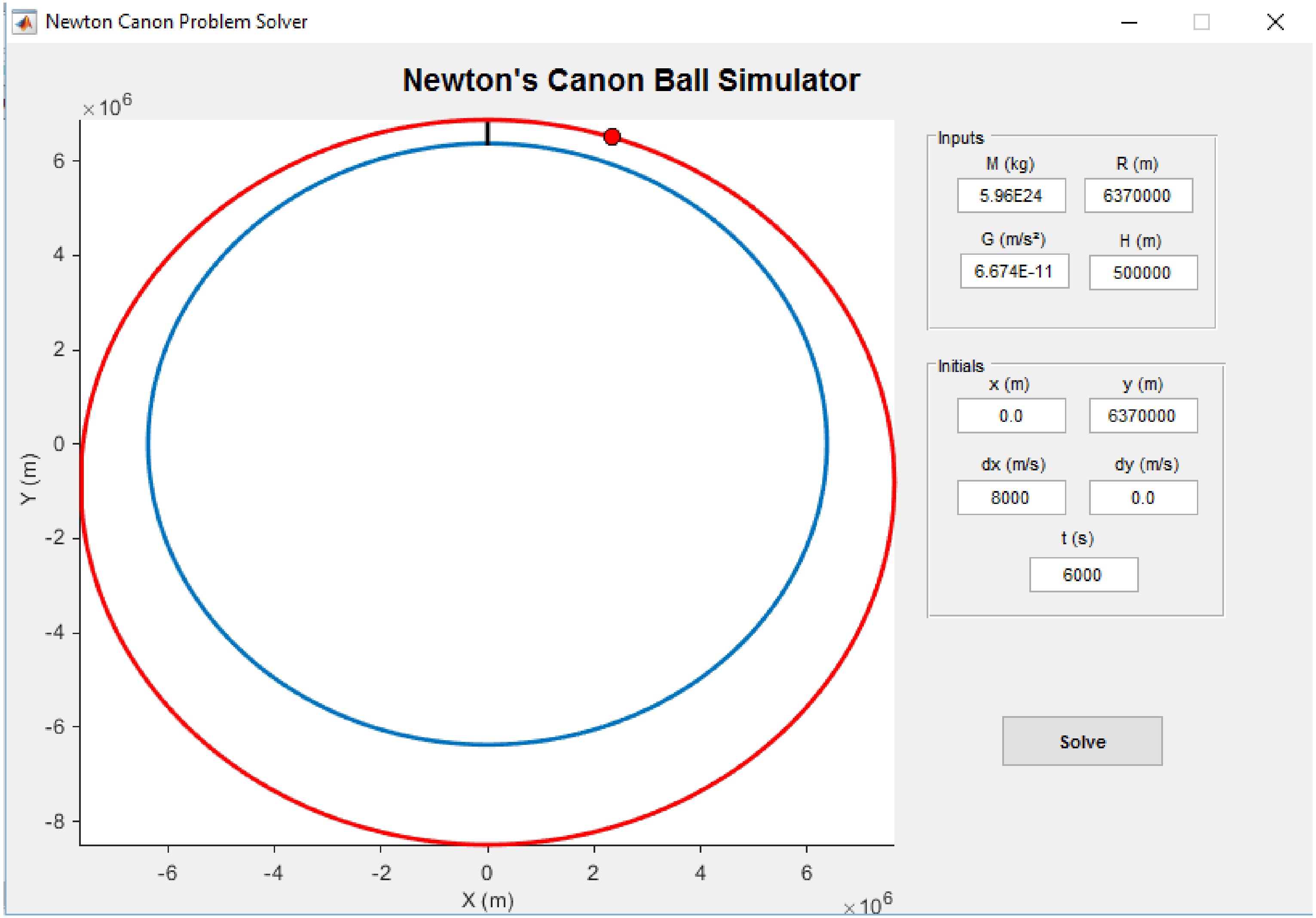
# Simulation Results (v = 7000 m/s)



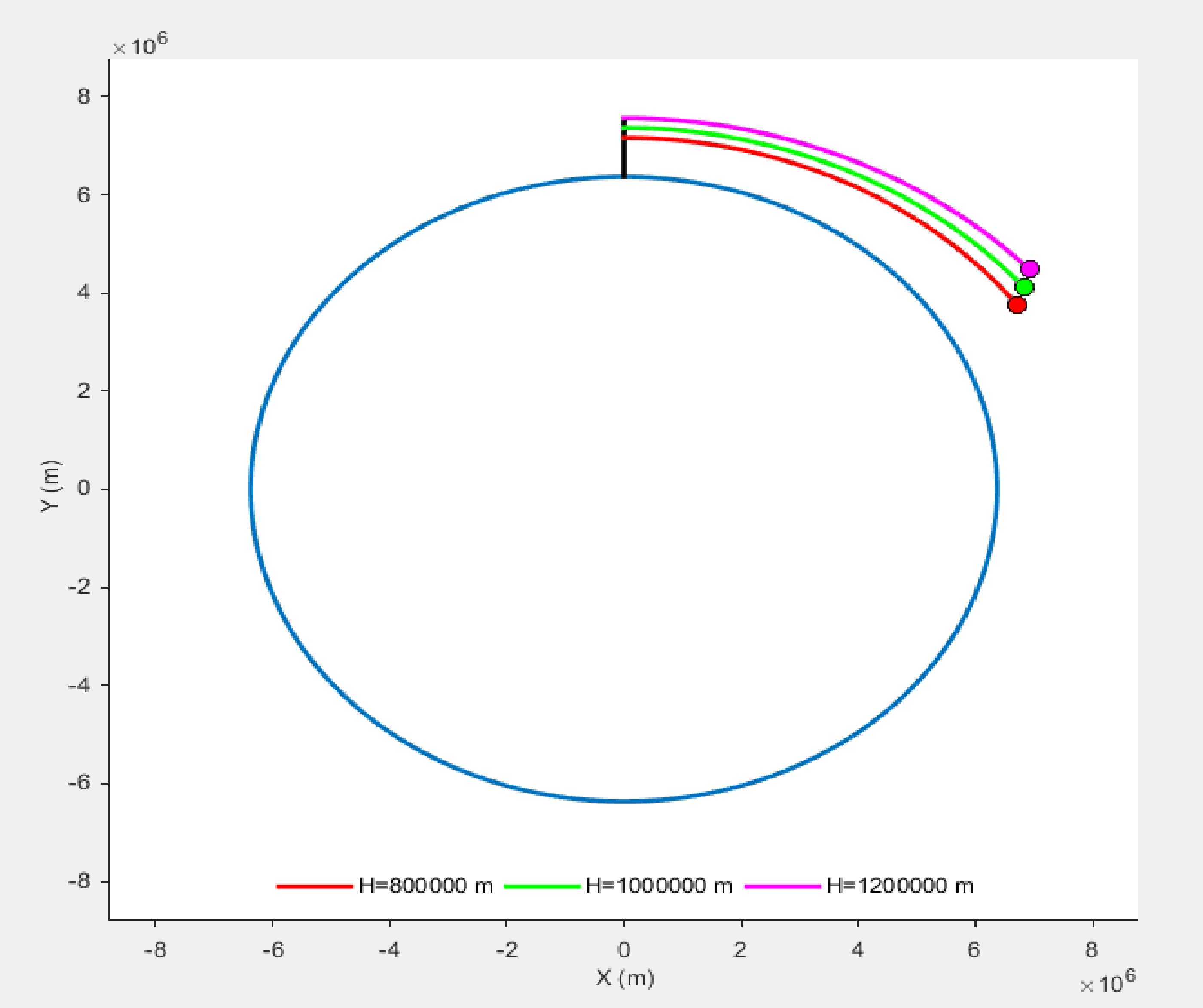
# Simulation Results (v = 7500 m/s)



# Simulation Results (v = 8000 m/s)



# Simulation Results – Height Variation



# Polar Form

$$\mathrm{T} = \frac{1}{2} m\left(\dot{r}^2 + r^2\,\dot{\theta}^2\right)$$

$$V = -\frac{GMm}{r}$$

$$L = T - V$$

Applying Lagrangian equation of motion:

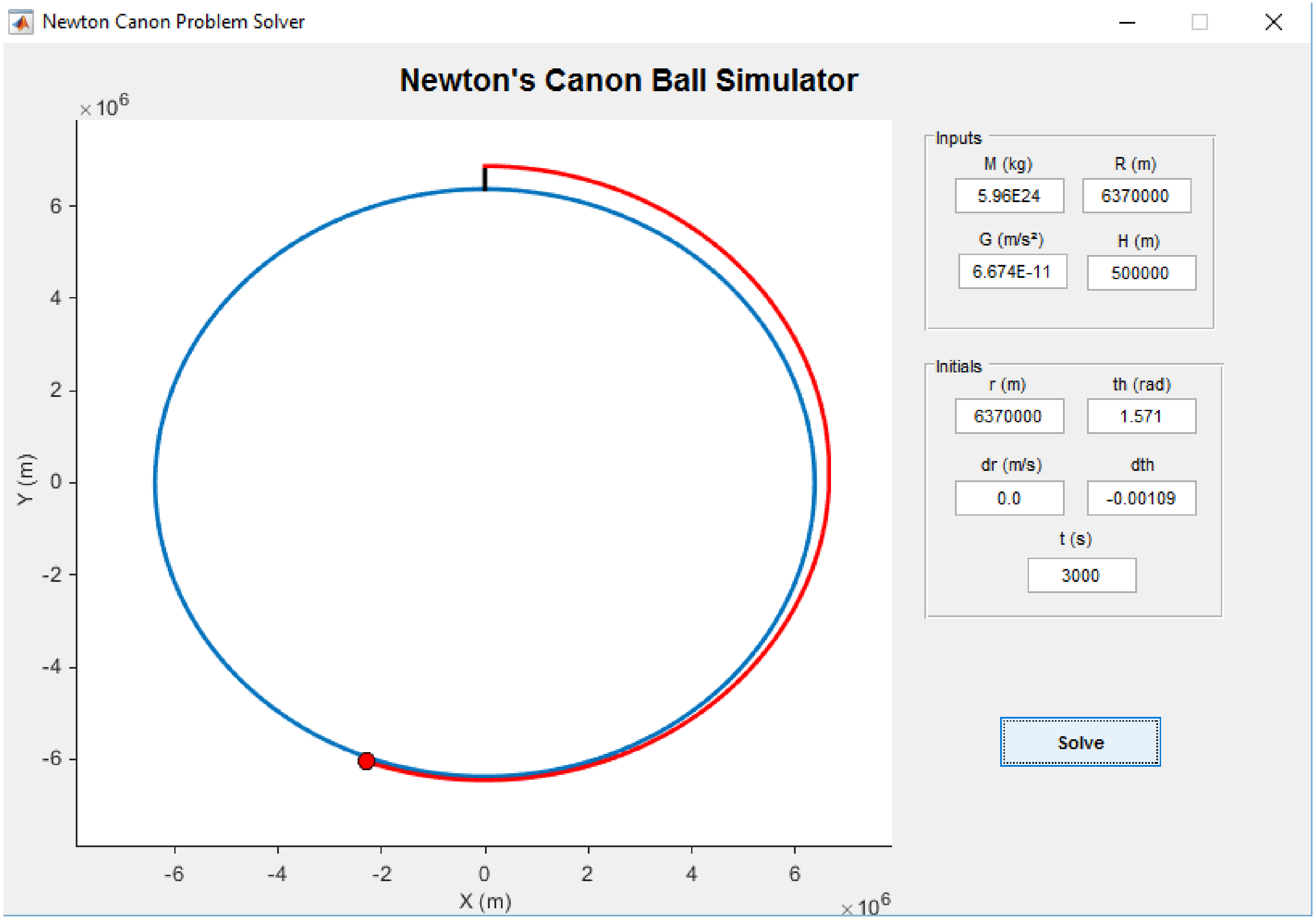
$$m\ddot{r} - mr\dot{\theta}^2 + \frac{GMm}{r^2} = 0 \qquad \ldots(1)$$

$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} = 0 \qquad \ldots(2)$$

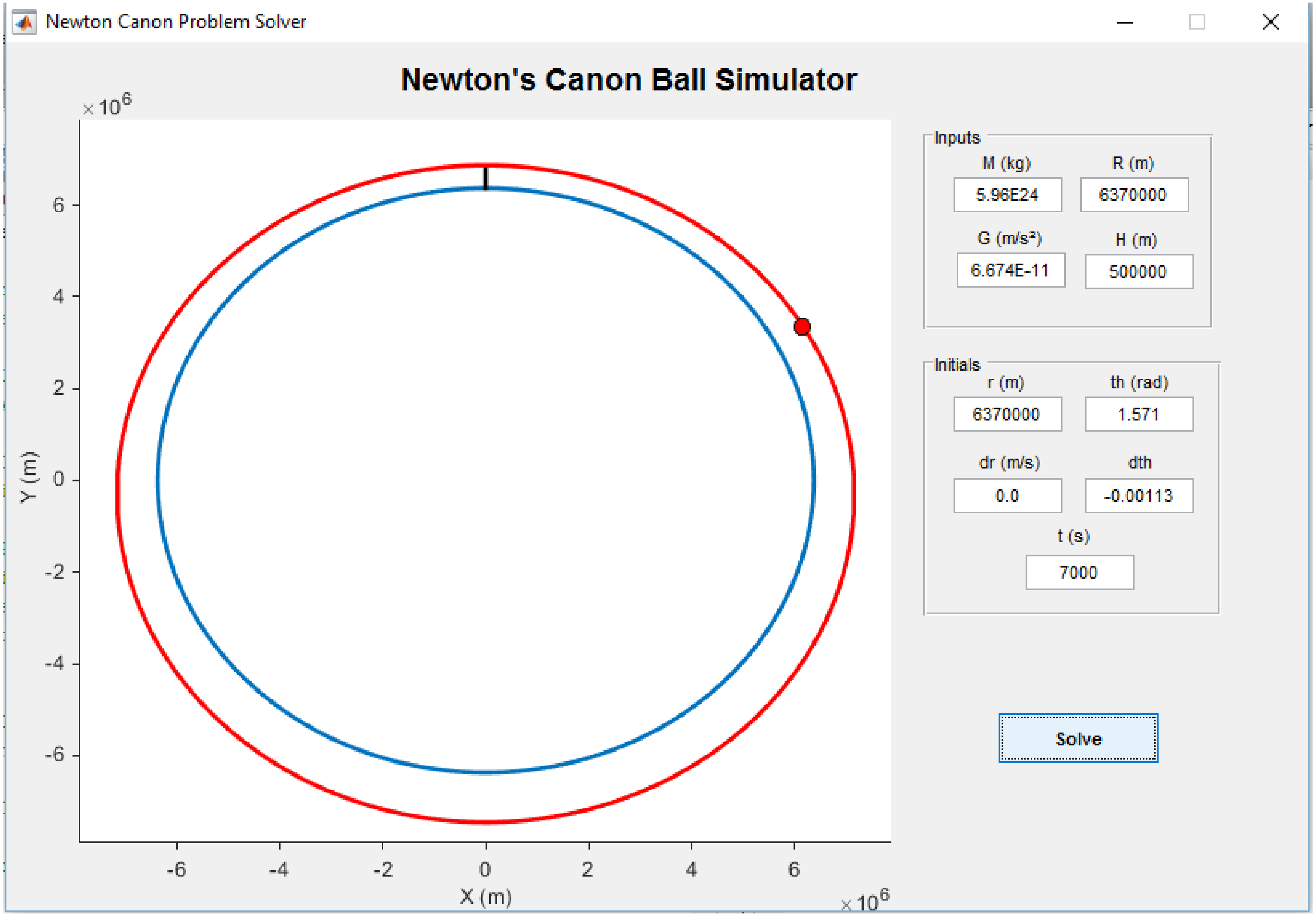
# 2nd order ODE

$$\boxed{\begin{aligned} \ddot{r} &= r\dot{\theta}^2 - \frac{GM}{r^2} \\ \ddot{\theta} &= -\frac{2\dot{r}\dot{\theta}}{r} \end{aligned}}$$

# Simulation Results (v = 6900 m/s)



# Simulation Results (v = 7200 m/s)



Thank You