3.a)

If f(n) is O(g(n)) then g(n) is O(f(n))

<u>Counterexample to prove this is false:</u>

Let f(n) = 1
g(n) = n

f(n) is O(g(n))

implies 0 <= 1<=cn
for c=1, n_0=1 the above condition holds good

g(n) is O(f(n)) implies 0<= n <= c

As we cant find a constant c in this case which holds for all n >= n_0
g(n) is O(f(n)) does not hold good.

3.b)

If f(n) is Θ(g(n)) then lg f(n) is Θ(lg g(n))

<u>Counterexample to prove this is false:</u>

Let f(n)=2
g(n)=1

f(n) is Θ(g(n))

implies 0<=c_1<=2<=c_2
The above case can be satisfied for some value of $c_1 = 1$ and c_2 = 3

lg f(n) is Θ(lg g(n))

implies 0<=$c_1$lg g(n) <= lg f(n) <=$c_2$lg g(n)

=> 0<= 0<=1<=0
which cannot be true.

3.c)

If f(n) is Θ(g(n)) then $2^{f(n)}$ is Θ ($2^{g(n)}$)

Counterexample to prove this is false:

Let f(n)=2n, g(n)=n

f(n) is Θ(g(n))

implies 0<=c_1 g(n) <= f(n) <=c_2 g(n)
=> 0<= c_1n<=2n<= c_2n

The above case holds for c_1=1, c_2=3, n_0=1

$2^{f(n)}$ is Θ ($2^{g(n)}$)

implies 0<=c_1 $2^{g(n)}$ <= $2^{f(n)}$ <=c_2 $2^{g(n)}$
=> 0<=$c_1 2^n$<=2^{2n}<=c_2 2^n
=> c_2>= 2^n for all n>=n_0 which is not possible.

3.d)

If f(n) is O(g(n)) then g(n) is Ω(f(n))

This is true.

f(n) is O(g(n))

implies 0<=f(n)<=cg(n)

Let c'=1/c

=> 0<=c'f(n)<=g(n)

=> g(n) is Ω(f(n))

3.e)

f(n) is Θ(f(n/2))

Counterexample to prove this is false:

Let f(n)= 2^n

f(n) is Θ(f(n/2))

implies 0<=c_1 (f(n/2))<= f(n) <=c_2 (f(n/2))

=> 2^n <= $c_2 2^{n/2}$
=> c_2>= $2^{n/2}$ for all n>=n_0 which is not possible.

3.f)

If g(n) is o(f(n)) then f(n) + g(n) is Θ(f(n))

This is true.

g(n) is o (f(n))

=> g(n)<cf(n)

f(n) <= f(n) + g(n) <= f(n) +cf(n)
=> f(n) <= f(n) + g(n) <= c'f(n)   (c'=c+1)
=> f(n) + g(n) is Θ(f(n))

3.g)

f(n) + g(n) is Θ(max(f(n),g(n)))

This is true.

f(n) + g(n) is Θ(max(f(n),g(n)))

=> $c_1$max(f(n),g(n))<= f(n)+g(n) <= $c_2$max(f(n),g(n))

$c_1$max(f(n),g(n))<= f(n)+g(n) holds good for c_1=1
as the summation of 2 numbers is always greater than or equal to the maximum of the two values.

f(n)+g(n) <= $c_2$max(f(n),g(n)) holds good for c_2=2
=> (f(n)+g(n))/2 <= max(f(n),g(n))
as the average of 2 numbers is always less than or equal to the maximum of the two values.

Hence f(n) + g(n) is Θ(max(f(n),g(n))) is true.