ES331 Assignment 2 - Image Restoration

S Deepak Narayanan, 16110142, deepak.narayanan@iitgn.ac.in

Introduction

The main aim of the assignment is to reverse the effect of blur and additive gaussian noise on the input image using a Wiener filter. We perform this by designing a Wiener Filter that can reverse this effect.

A brief overview of the procedure and theory behind the working of Wiener Filter

The input to the filter is an clear image. Then we simulate a blur on it by convolving the image with a Gaussian Kernel. Then, by sampling from a normal distribution, we add additive noise to the blurred image. Now we obtain an image that is both blurred as well as noisy after the addition of the Gaussian noise.

We have been given an input signal $I(x)$, a filter that it is convolved with $H(x)$, a noisy signal that would be added to this convolved signal $N(x)$. We obtain the corrupted image as $Y(x) = H(x) * I(x) + N(x)$.

Wiener Filter is a filter G, that when convolved with Y, returns I. This can be viewed as passing the signal through a linear time invariant system. One obvious solution that comes to mind without making use of the filters is the following. Taking the discrete time Fourier transform of Y, we obtain in frequency domain,

$$Y(f) = H(f).I(f) + N(f) \implies I(f) = \frac{Y(f) - N(f)}{H(f)}$$

We can then take the inverse time Fourier transform of I to get back the original image I. This almost always never works because, it is suitable only in the cases when noise is very low or zero. This is called as inverse filtering. This can be seen from the spectrum of the image in grayscale form.

Wiener Filter tries to, in essence, do the following. Given the image $Y(x)$ as an input, it tries to retrieve the original image $I(x)$, by performing $G(x) * Y(x)$. We need to minimize $\mathbb{E}[|G(x) * Y(x) - I(x)|^2]$, which, in the frequency domain becomes $\mathbb{E}[|G(f).Y(f) - I(f)|^2]$. Upon optimizing this function, we get,

$$G(f) = \frac{H^*(f)S(f)}{|H(f)|^2 S(f) + N(f)}$$

where H* is the complex conjugate of H, S is the spectrum of the source image that we're considering.

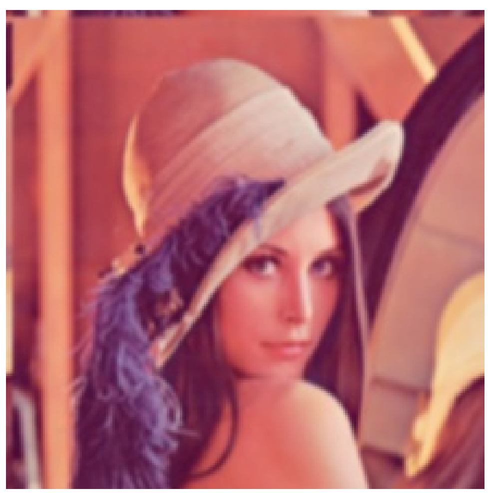
Results

I have used the Peak Signal to Noise Ratio (PSNR) as a metric to evaluate the performance of my implementation of the Wiener Filter. I am attaching the images that I obtained as a result of my implementation of the Wiener Filter.

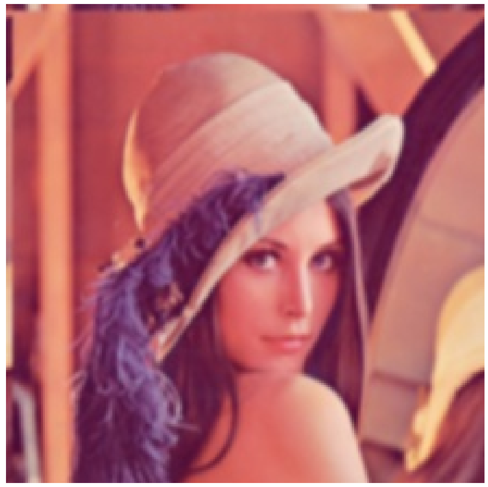
These set of images were obtained using a filter of sigma 1 for blurring and varying different values of additive noise's standard deviation from 10^{-4} to 1.

The below image is the input image.

The image after blurring is,



The blurred image after adding Gaussian Noise of standard deviation 10^{-4} is below. The PSNR value with the input image is **17.825860729133186**



The final restored image is below. The PSNR value with the original input image is **30.645904003614746**

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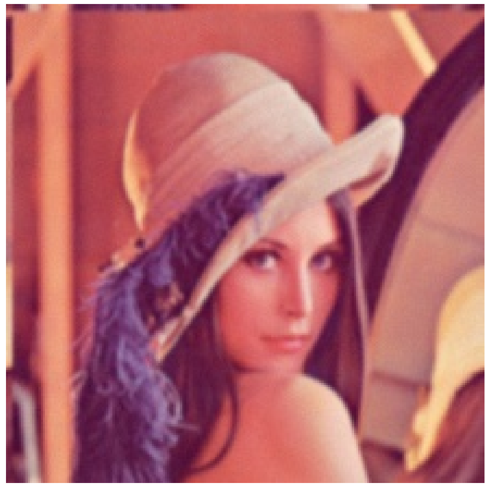
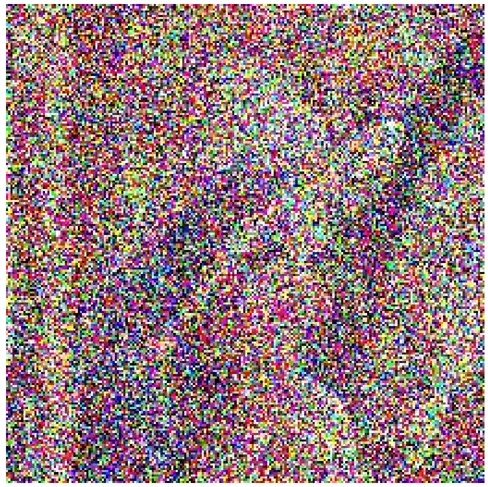
The image after adding noise of sigma 10^{-2} is below. The PSNR value with the input image is **17.79949515591459**.

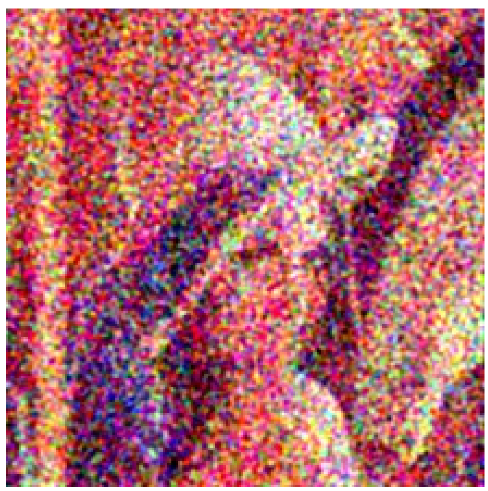


The image after restoration is below. The PSNR value with the original image is **26.403225198155145**.

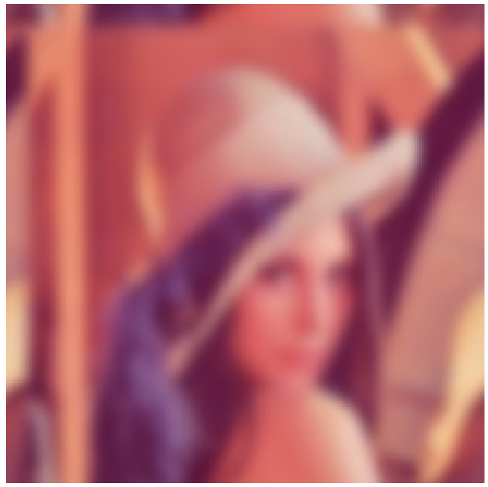
The blurred images after adding Gaussian Noise of standard devation 1 is below. The PSNR value is **6.834439440095068**.



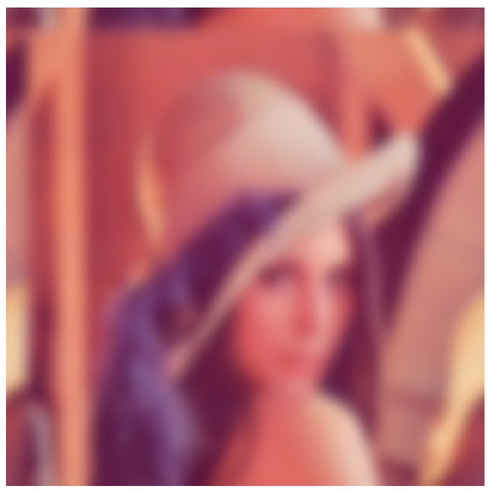
The final restored image is below. The PSNR value is **5.554869100241171**



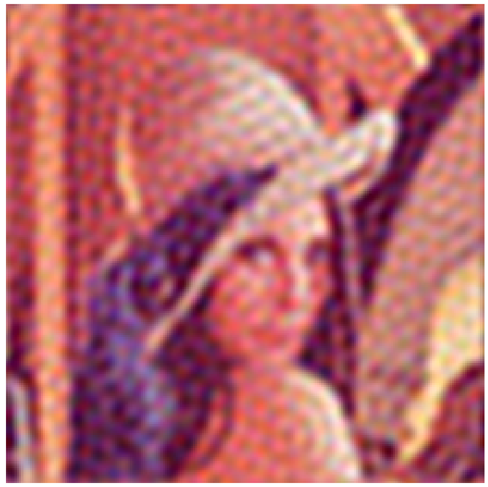
Now, we set the sigma value to be 3.5 for Gaussian blur. The blurred image is below.



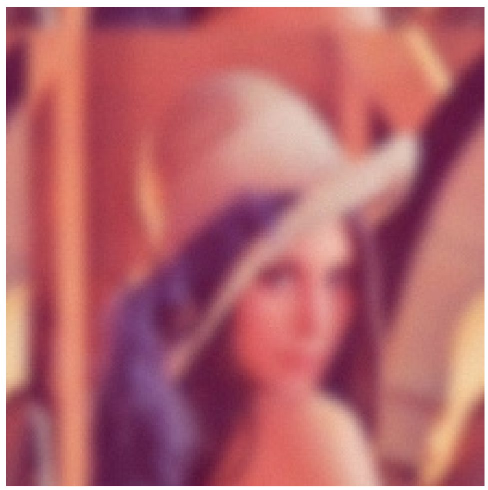
The image with additive noise of sigma = 10^{-4} is below. For additive noise of sigma 10^{-4} the noisy image with blurring has PSNR **14.566342595041789**.



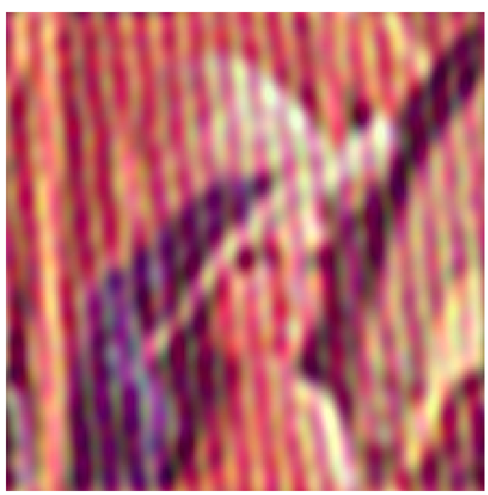
The final restored image is below. The PSNR is **20.28601649195224**



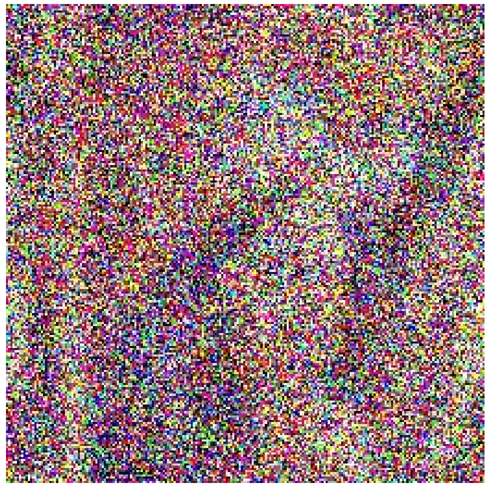
Now. with additive noise of sigma $= 10^{-2}$ is below. The PSNR is **14.553647734216913**.



The final restored image is below. The PSNR is **20.023417313887116**

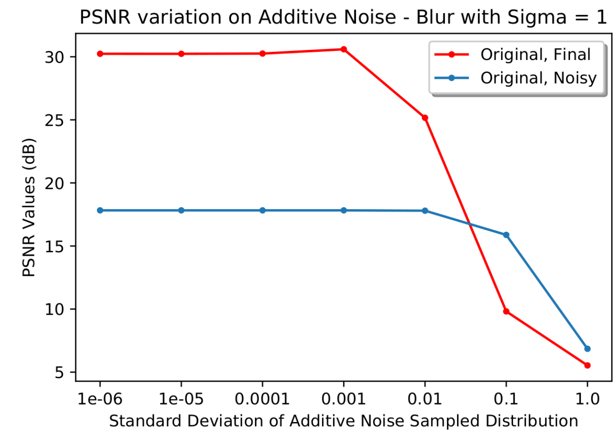


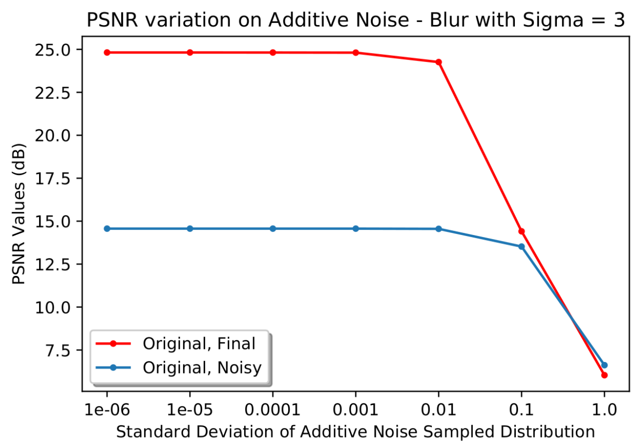
The image with additive noise of sigma = 1 is below. The PSNR is **6.630005331106901**.



The final restored image is below. The PSNR is **6.022395169247561**

Here I am attaching a few plots that depict the variation of the PSNR as we add additive noise, showing a clear decrease, causing the reversal to be poorer.





Conclusions

1. The image is extremely sensitive to the additive noise added as can be seen in the result images above.
2. The PSNR values are showing a positive trend as they are increasing with respect to the noisy image with blur and for the final image. Also, we can see the performance as the blur and the amount of noise added changes.
3. Though an ideal PSNR of 30 to 40 would be expected, I was able to get only in the range from 20 to 30.
4. The PSNR Value is also a function of the amount of blurring as the plots above illustrate. They are decreasing as the amount of blurring increases.
5. Wiener filter is a very fundamental filter. There are other deconvolution methods like Lucy Richardson that do a better job at this image restoration process.

References

1. Wiener N: *'The interpolation, extrapolation and smoothing of stationary time series'*, Report of the Services 19, Research Project DIC-6037 MIT, February 1942.

2. Wikipedia Link to Wiener Filter here.

3. Wikipedia Link to Wiener Deconvolution here.

4. Blog Post by Vladimir Yuzhikov here.