CSE463: Neural Networks

Deep Neural Networks

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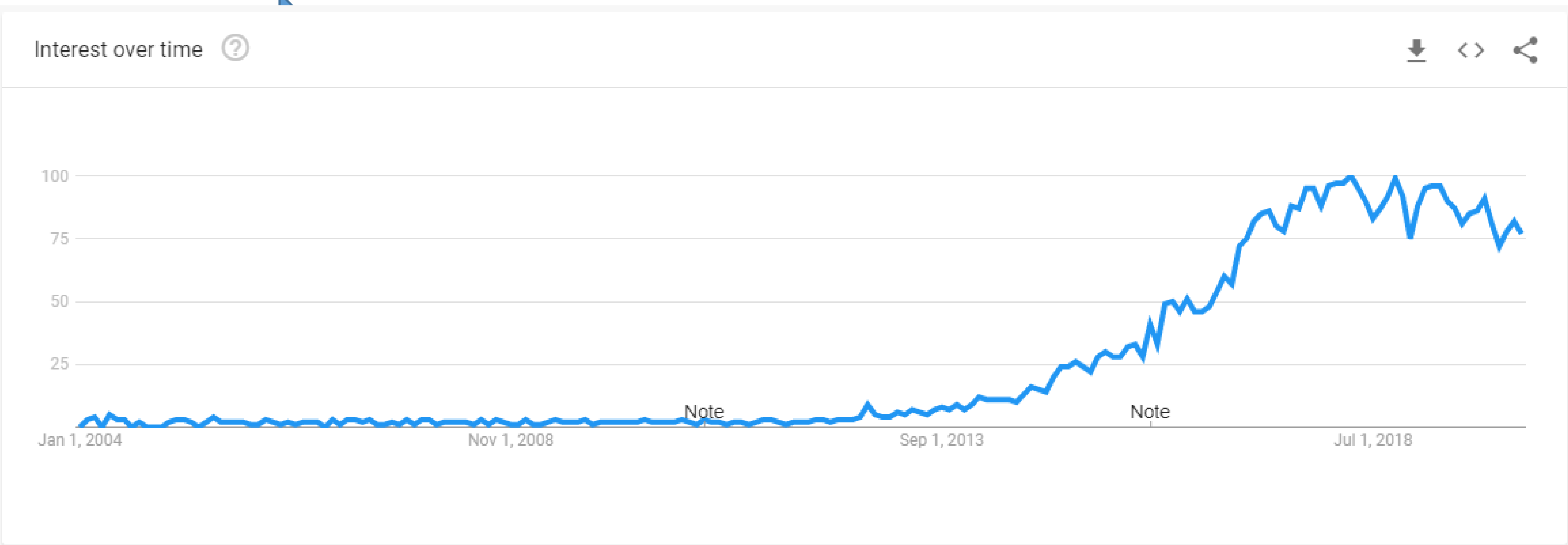
Deep learning attracts lots of attention.

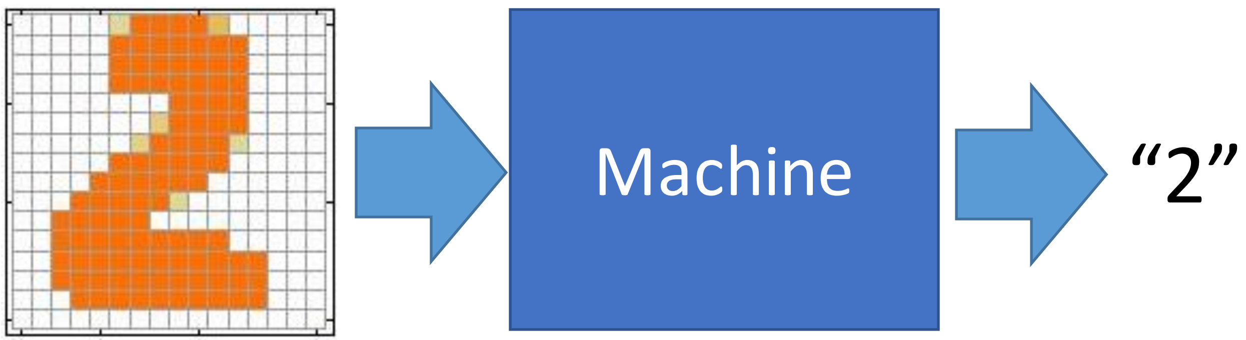
- Google Trends

Deep learning obtains many exciting results.



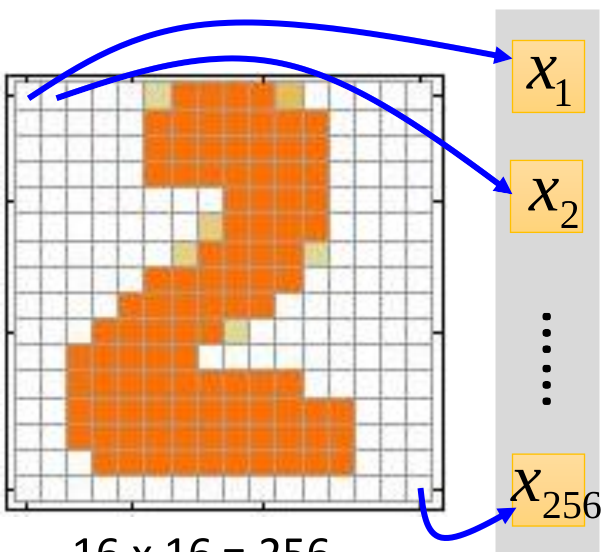
Example Application

- Handwriting Digit Recognition



Handwriting Digit Recognition

Input

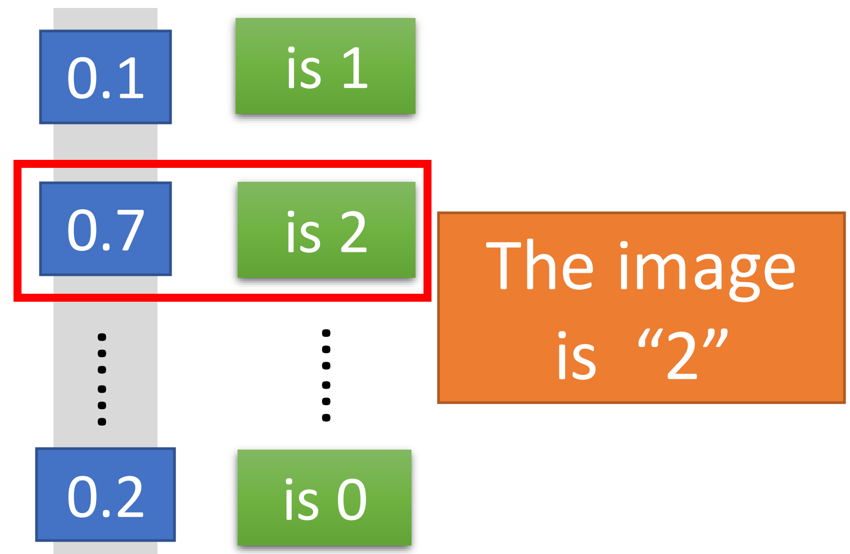


16 x 16 = 256

Ink → 1

No ink → 0

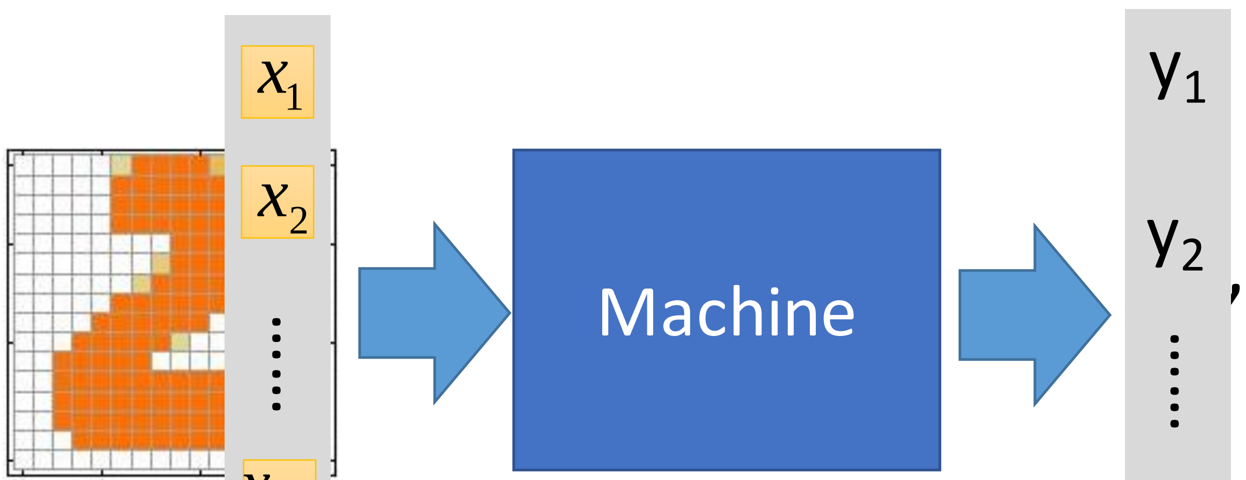
Output



Each dimension represents the confidence of a digit.

Example Application

- Handwriting Digit Recognition



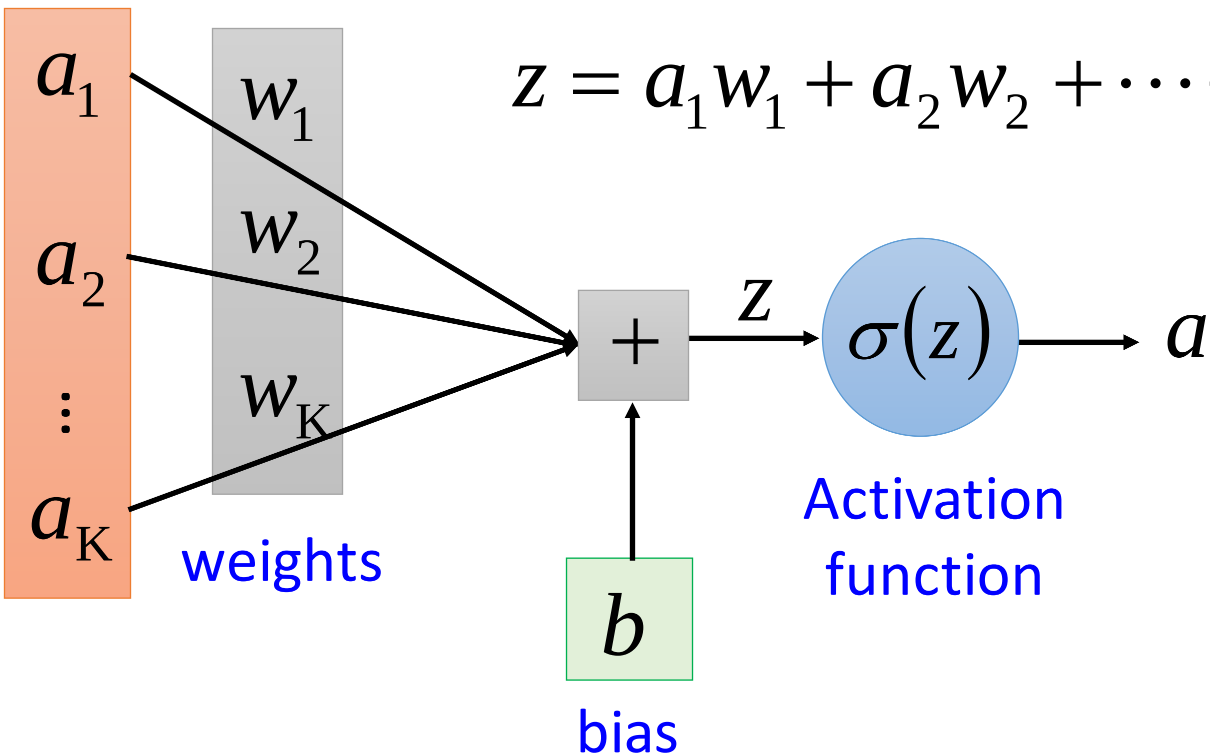
$$f: R^{256} \rightarrow R^{10}$$

In deep learning, the function f is represented by neural network

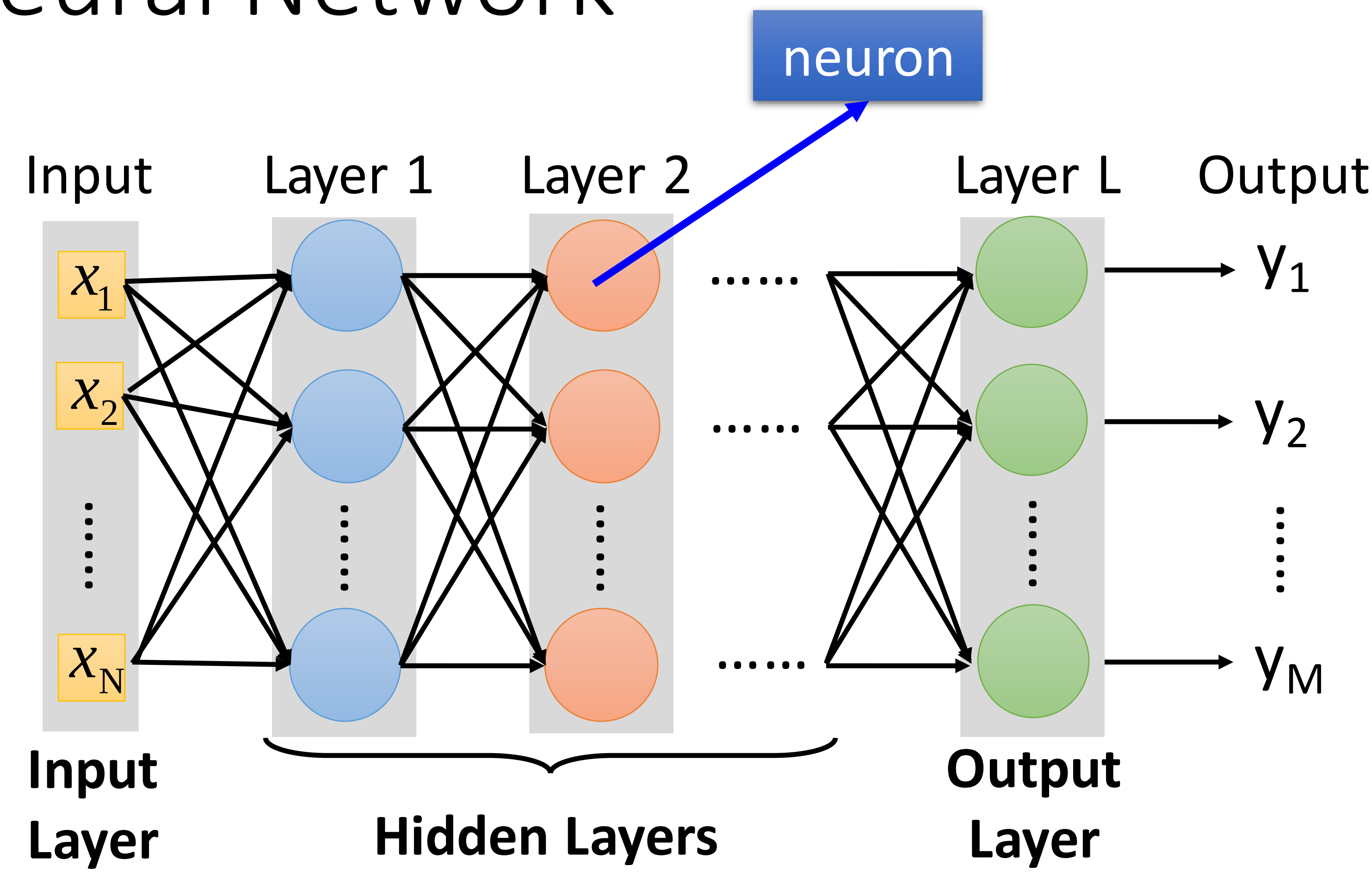
Element of Neural Network

Neuron $f: R^K \rightarrow R$

$$z = a_1 w_1 + a_2 w_2 + \cdots + a_K w_K + b$$

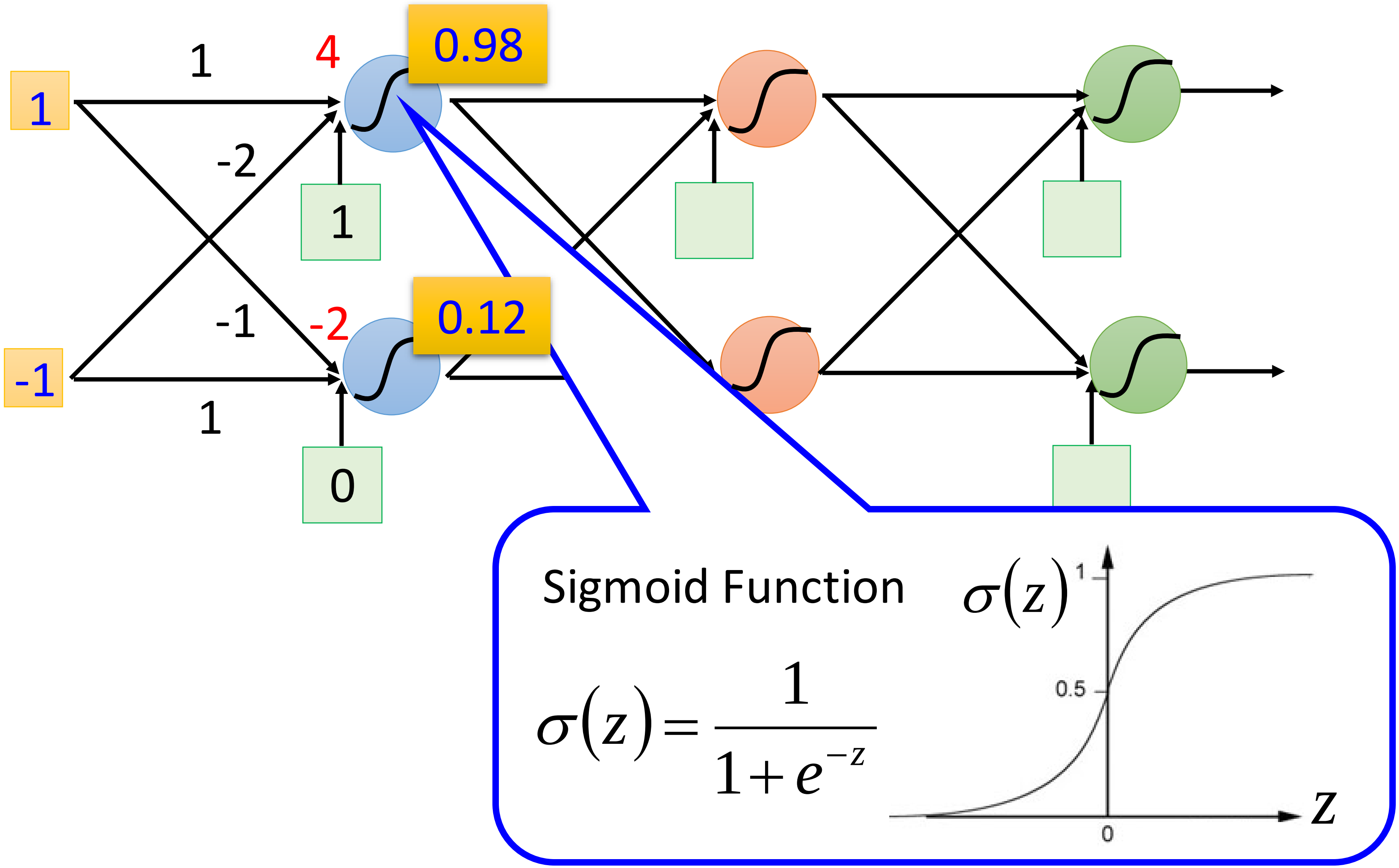


Neural Network

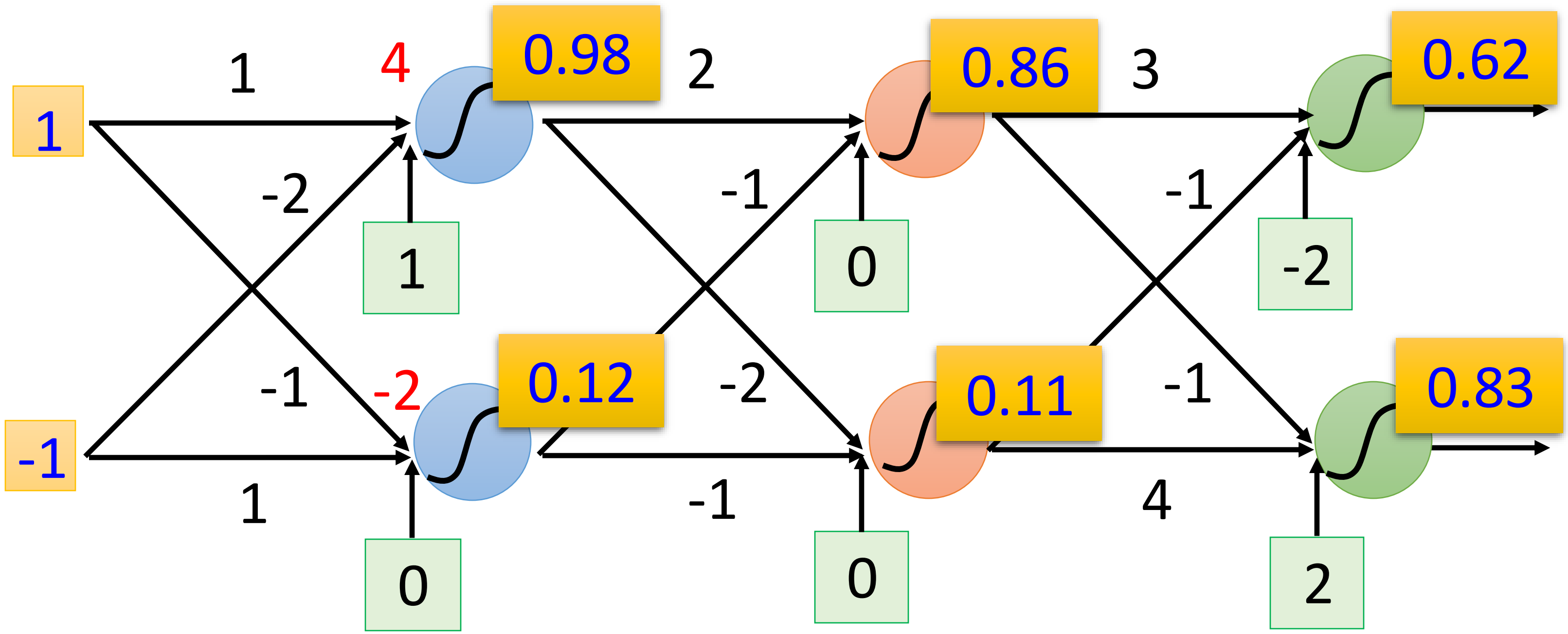


Deep means many hidden layers

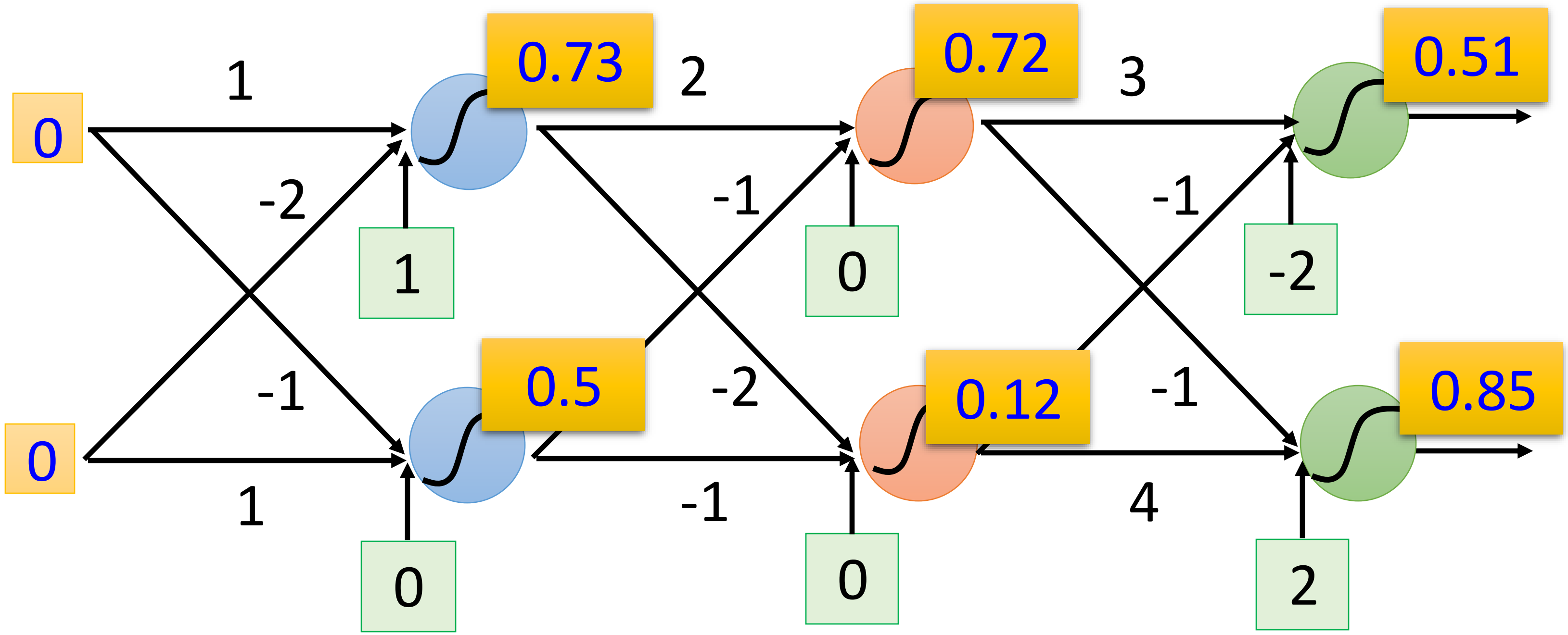
Example of Neural Network



Example of Neural Network



Example of Neural Network

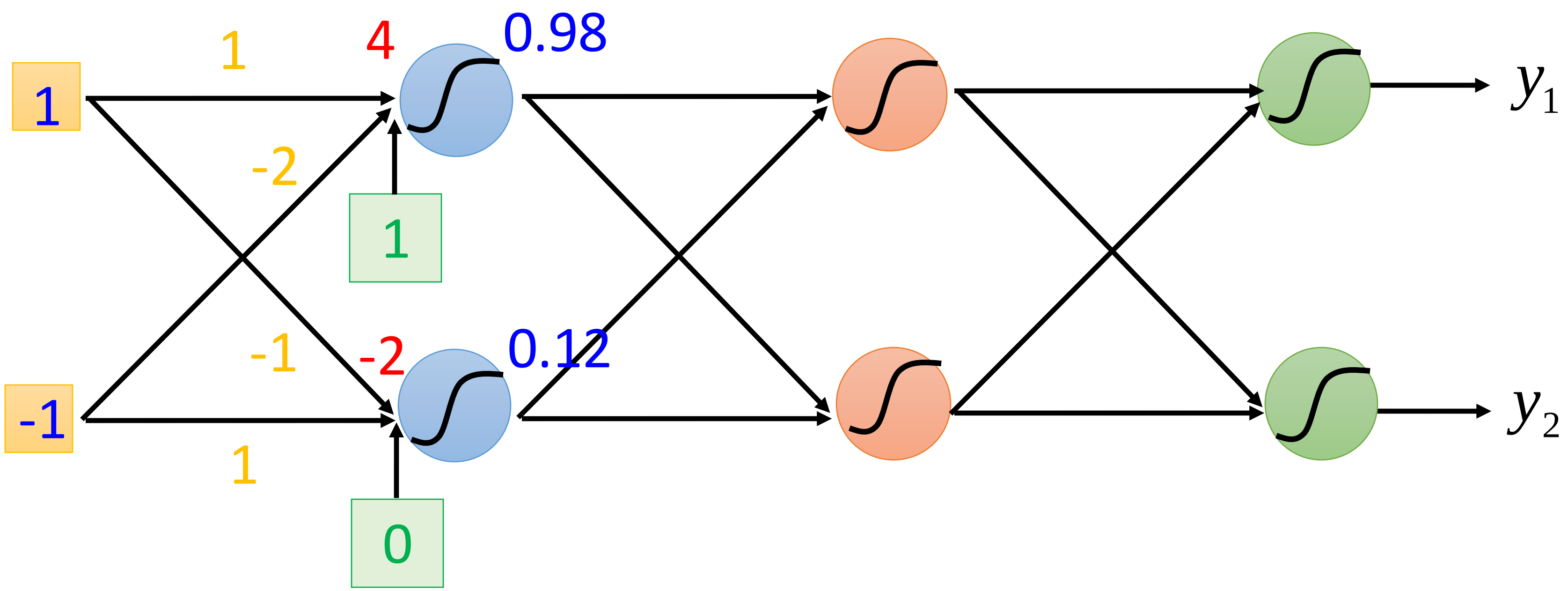


$$f: R^2 \rightarrow R^2$$

$$f\left(\begin{bmatrix}1\\-1\end{bmatrix}\right)=\begin{bmatrix}0.62\\0.83\end{bmatrix} \quad f\left(\begin{bmatrix}0\\0\end{bmatrix}\right)=\begin{bmatrix}0.51\\0.85\end{bmatrix}$$

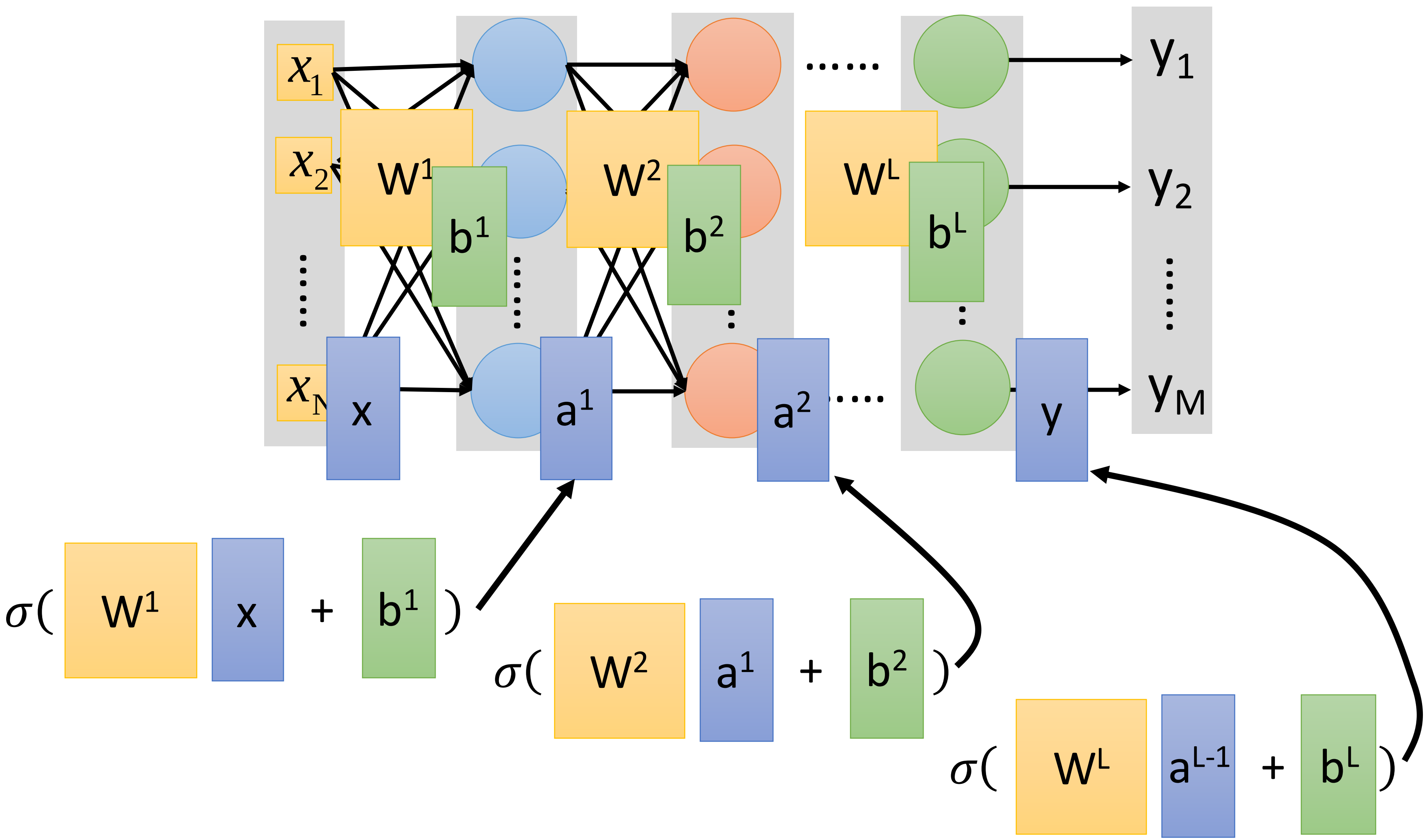
Different parameters define different function

Matrix Operation

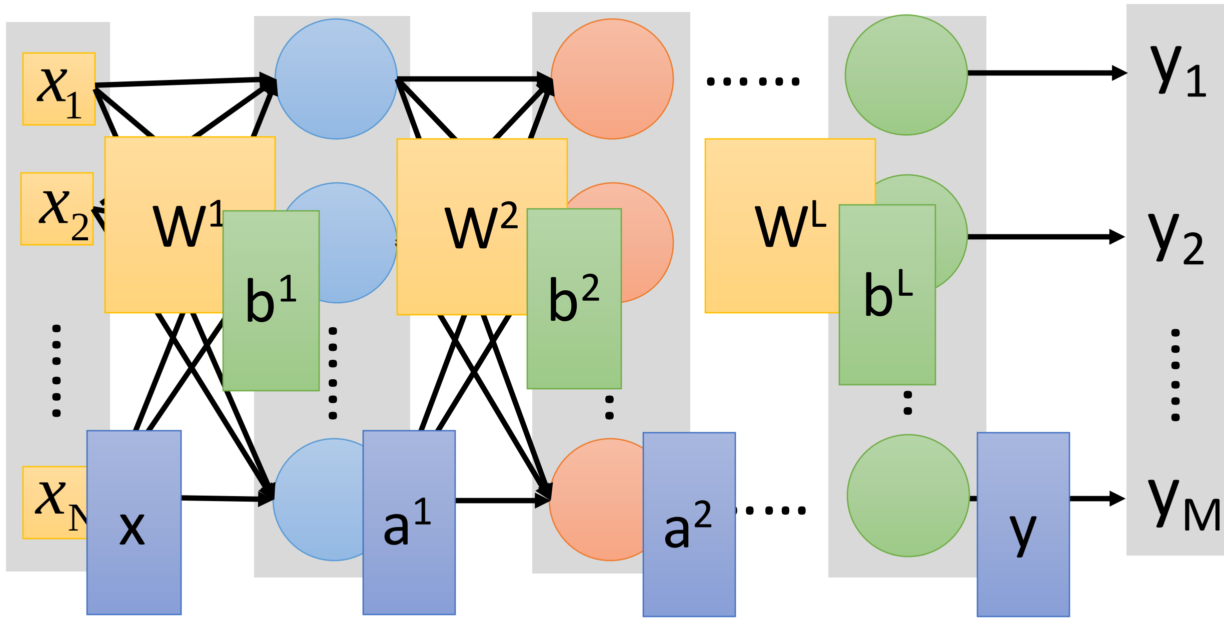


$$\sigma\Big(\underbrace{\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\begin{bmatrix} 4 \\ -2 \end{bmatrix}}\Big) = \begin{bmatrix} 0.98 \\ 0.12 \end{bmatrix}$$

Neural Network



Neural Network



$$y = f(x)$$

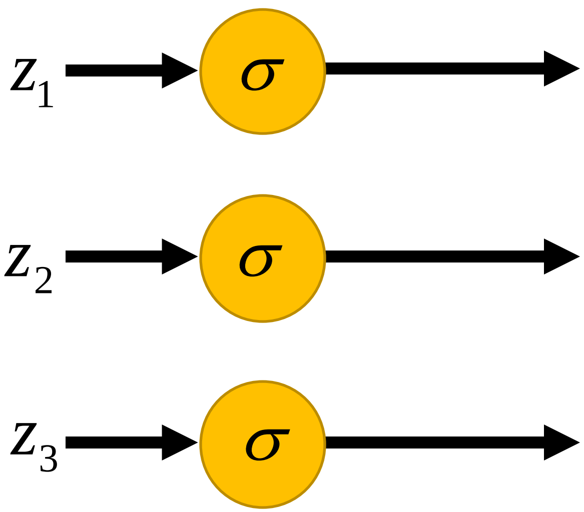
Using parallel computing techniques to speed up matrix operation

$$= \sigma(W^L \cdots \sigma(W^2 \sigma(W^1 x + b^1) + b^2) \cdots + b^L)$$

Softmax

- Softmax layer as the output layer

Ordinary Layer



$$y_1 = \sigma(z_1)$$

$$y_2 = \sigma(z_2)$$

$$y_3 = \sigma(z_3)$$

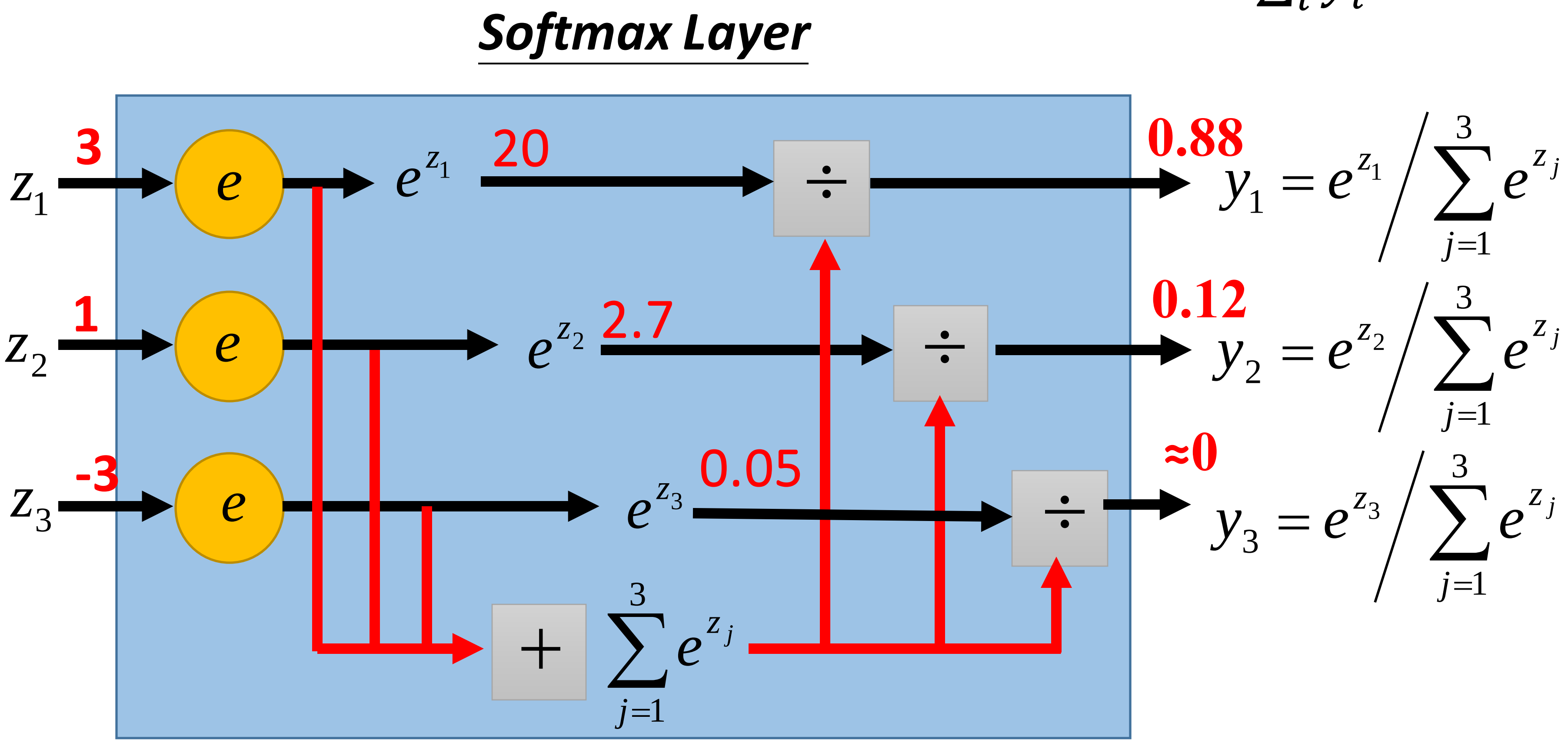
In general, the output of network can be any value.

May not be easy to interpret

Softmax

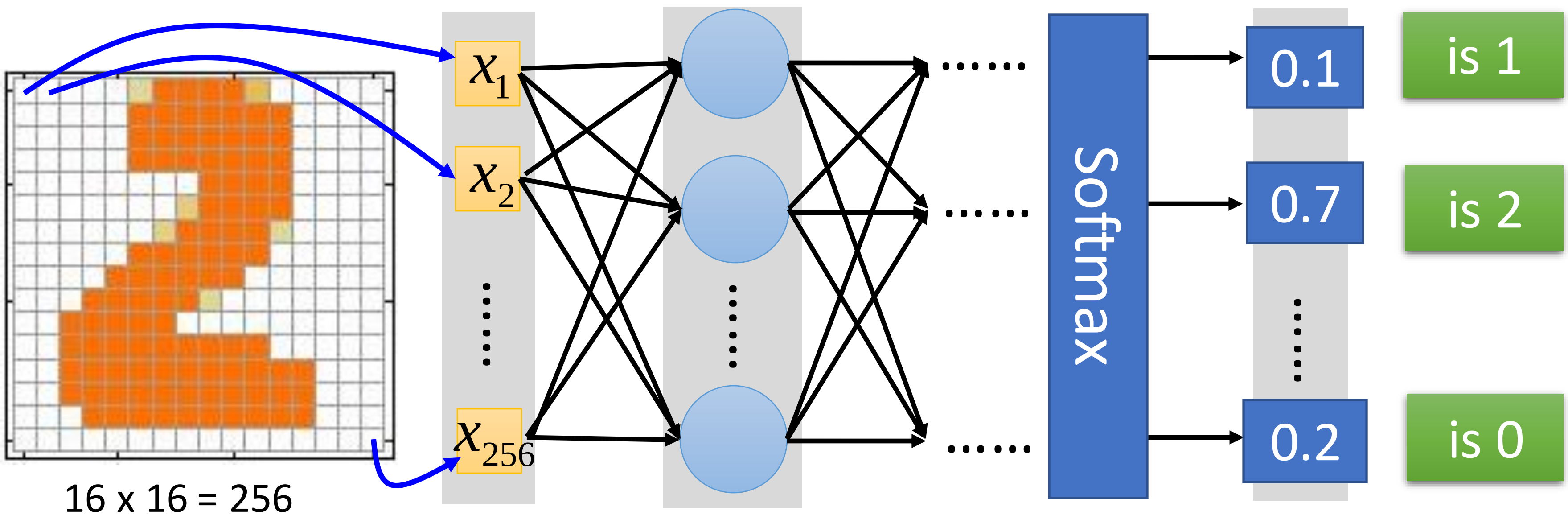
- Softmax layer as the output layer

Probability:
- $1 > y_i > 0$
- $\sum_i y_i = 1$



How to set network parameters

$$\theta = \{W^1, b^1, W^2, b^2, \cdots W^L, b^L\}$$



16 x 16 = 256

Ink → 1

No ink → 0

Set the network parameters θ such that ……

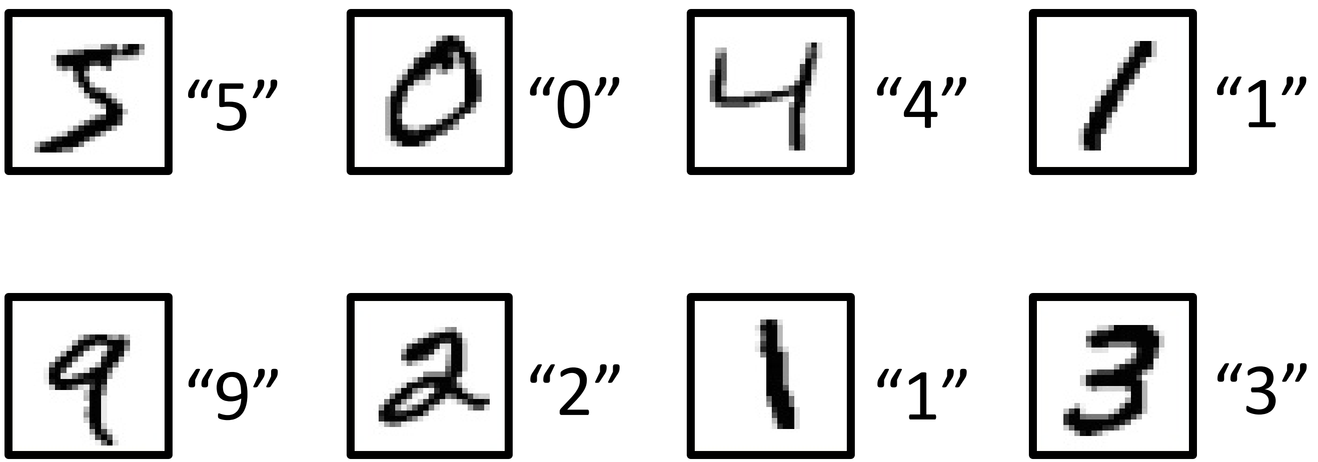
Input m value

How to let the neural network achieve this

Input:  y_2 has the maximum value

Training Data

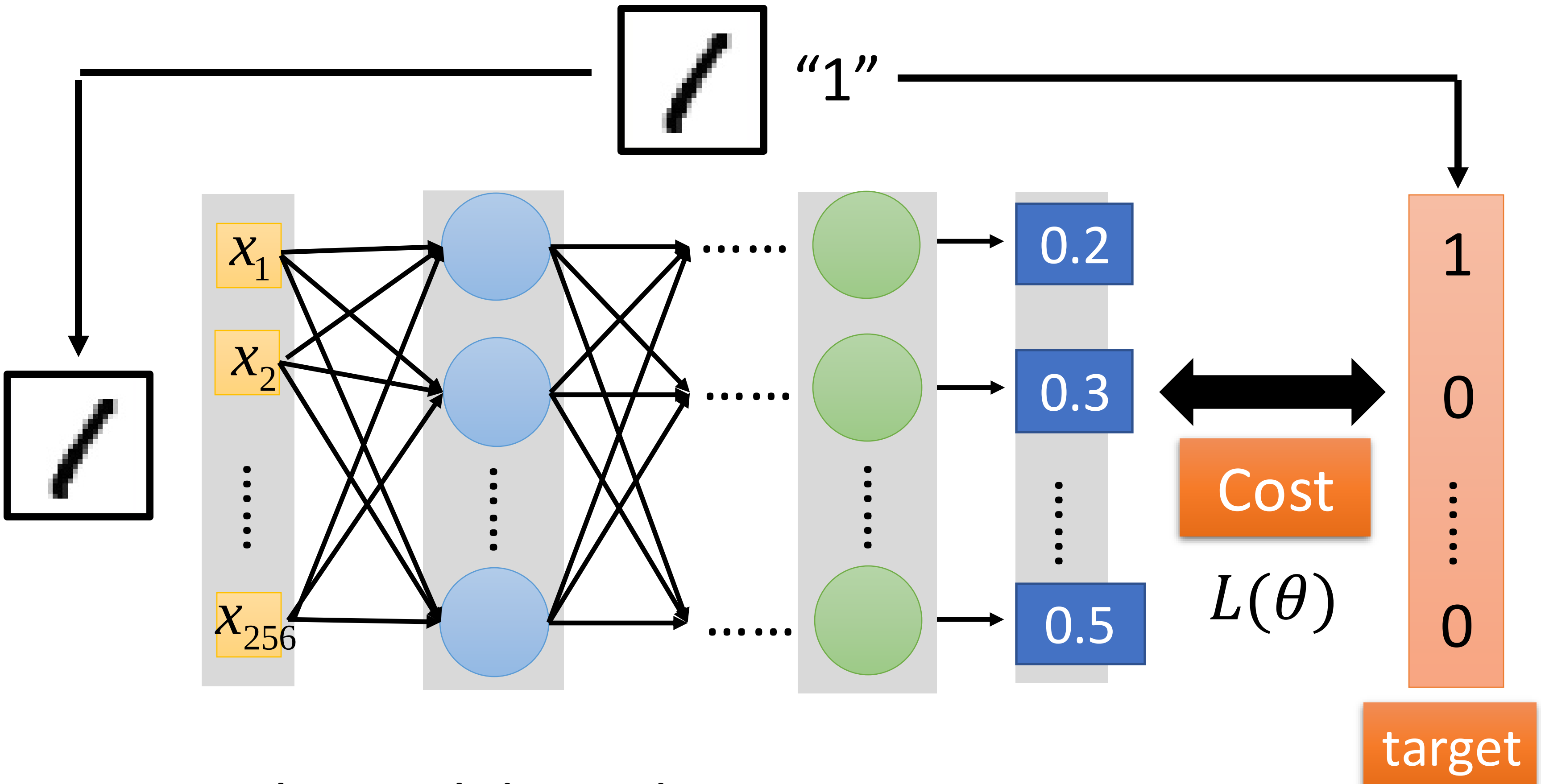
- Preparing training data: images and their labels



Using the training data to find the network parameters.

Cost

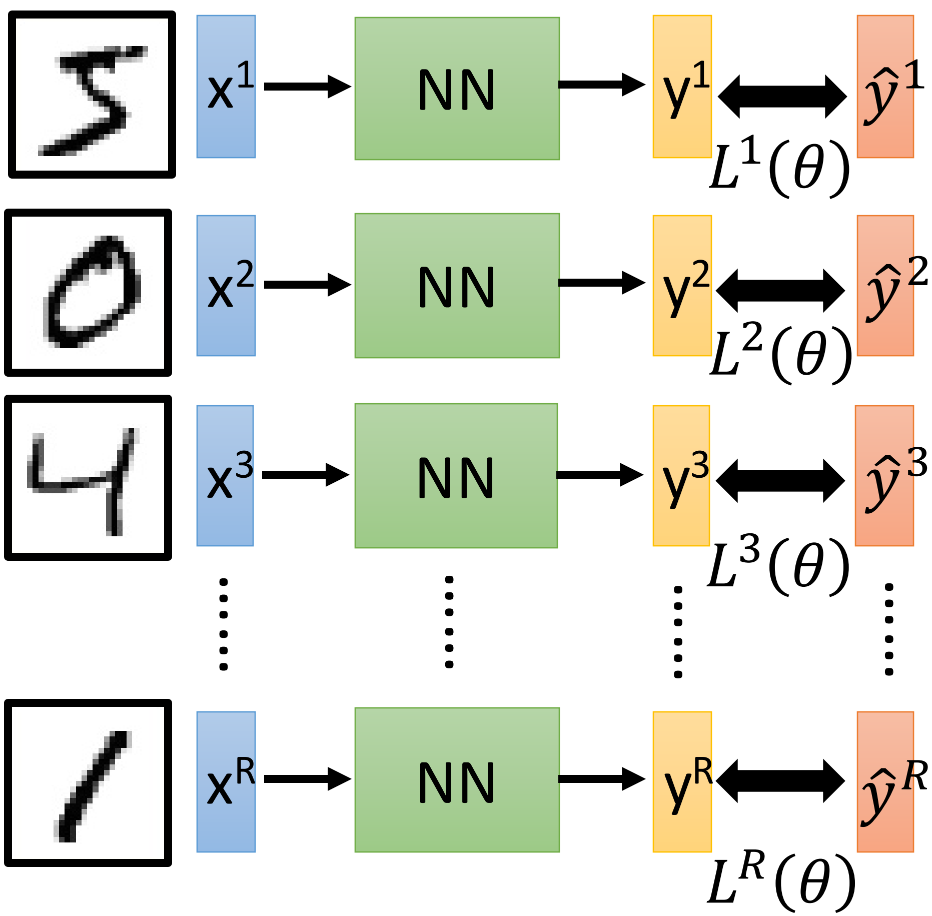
Given a set of network parameters θ, each example has a cost value.



Cost can be Euclidean distance or cross entropy of the network output and target

Total Cost

For all training data …



Total Cost:

$$C(\theta) = \sum_{r=1}^{R} L^r(\theta)$$

How bad the network parameters θ is on this task

Find the network parameters θ^* that minimize this value

Backpropagation

- A network can have millions of parameters.
  - Backpropagation is the way to compute the gradients efficiently (not today)
  - Ref: http://speech.ee.ntu.edu.tw/~tlkagk/courses/MLDS_2015_2/Lecture/DNN%20backprop.ecm.mp4/index.html
- Many toolkits can compute the gradients automatically

Ref: http://speech.ee.ntu.edu.tw/~tlkagk/courses/MLDS_2015_2/Lecture/Theano%20DNN.ecm.mp4/index.html

Why Deep?

Deeper is Better?

Layer X Size	Word Error Rate (%)
1 X 2k	24.2
2 X 2k	20.4
3 X 2k	18.4
4 X 2k	17.8
5 X 2k	17.2
7 X 2k	17.1

Not surprised, more parameters, better performance

Seide, Frank, Gang Li, and Dong Yu. "Conversational Speech Transcription Using Context-Dependent Deep Neural Networks." *Interspeech*. 2011.

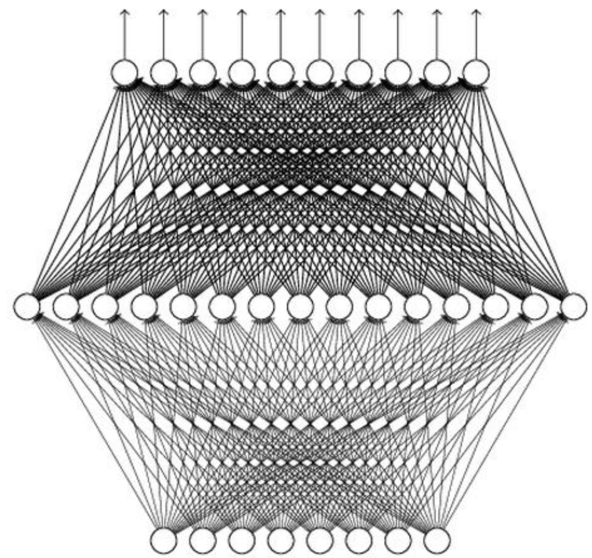
Universality Theorem

Any continuous function f

$$f : R^N \rightarrow R^{\mathrm{M}}$$

Can be realized by a network with one hidden layer

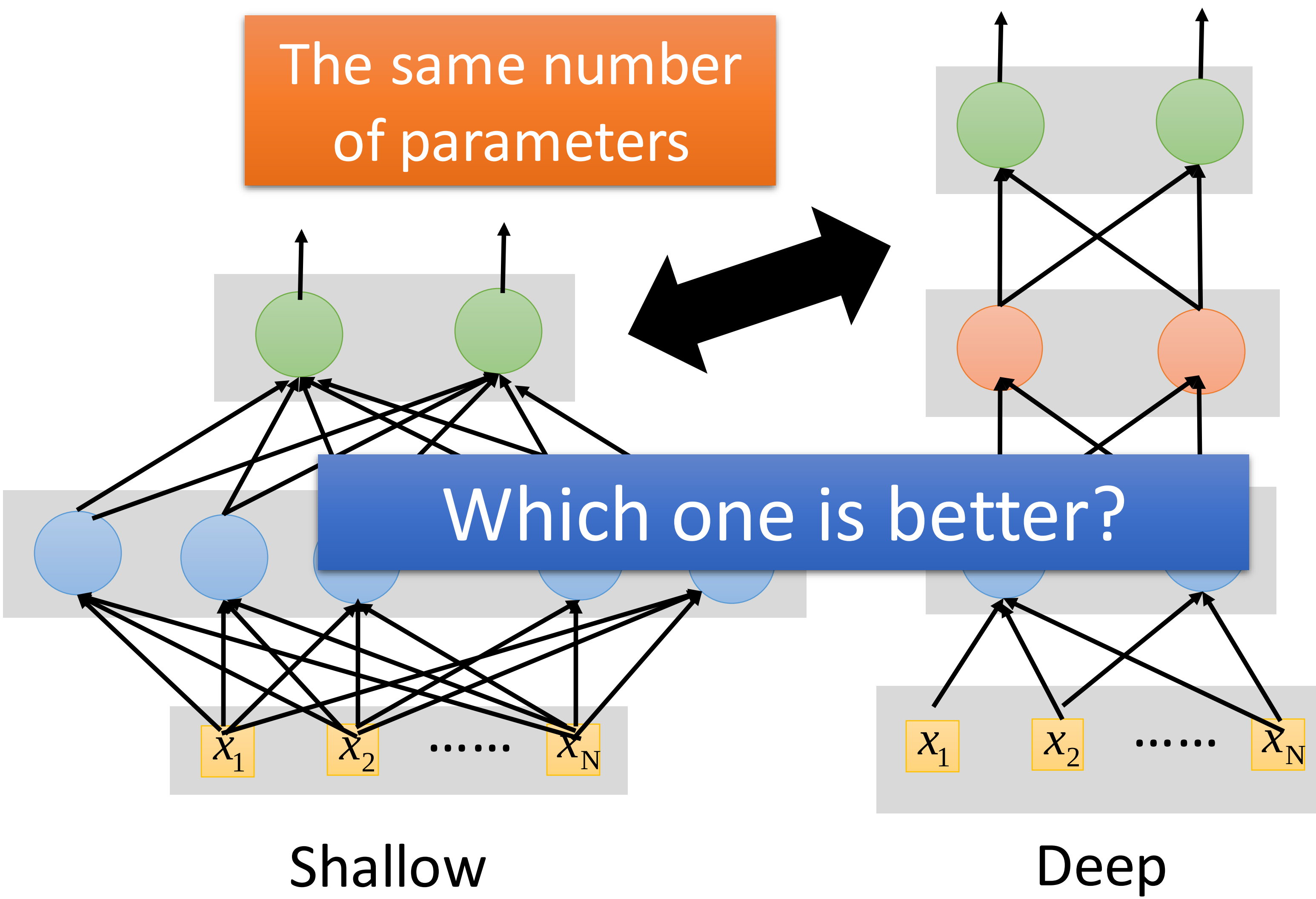
(given **enough** hidden neurons)



Reference for the reason: http://neuralnetworksanddeeplearning.com/chap4.html

Why "Deep" neural network not "Fat" neural network?

Fat + Short v.s. Thin + Tall



Fat + Short v.s. Thin + Tall

Layer X Size	Word Error Rate (%)	Layer X Size	Word Error Rate (%)
1 X 2k	24.2		
2 X 2k	20.4		
3 X 2k	18.4		
4 X 2k	17.8		
5 X 2k	17.2	1 X 3772	22.5
7 X 2k	17.1	1 X 4634	22.6
		1 X 16k	22.1

Seide, Frank, Gang Li, and Dong Yu. "Conversational Speech Transcription Using Context-Dependent Deep Neural Networks." *Interspeech*. 2011.

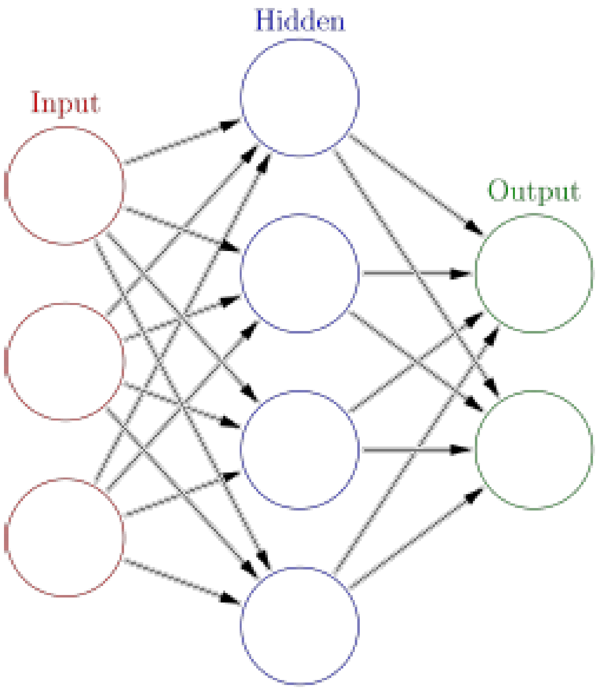
Why Deep?

Shallow Neural Network

Shallow Neural Network is similar to most of the conventional supervised models

Pros:
1.Easy to train and test
2.Able to approach any continuous function

Cons:
1.Performance depends on well-designed features
2.Difficult to generalize the prediction

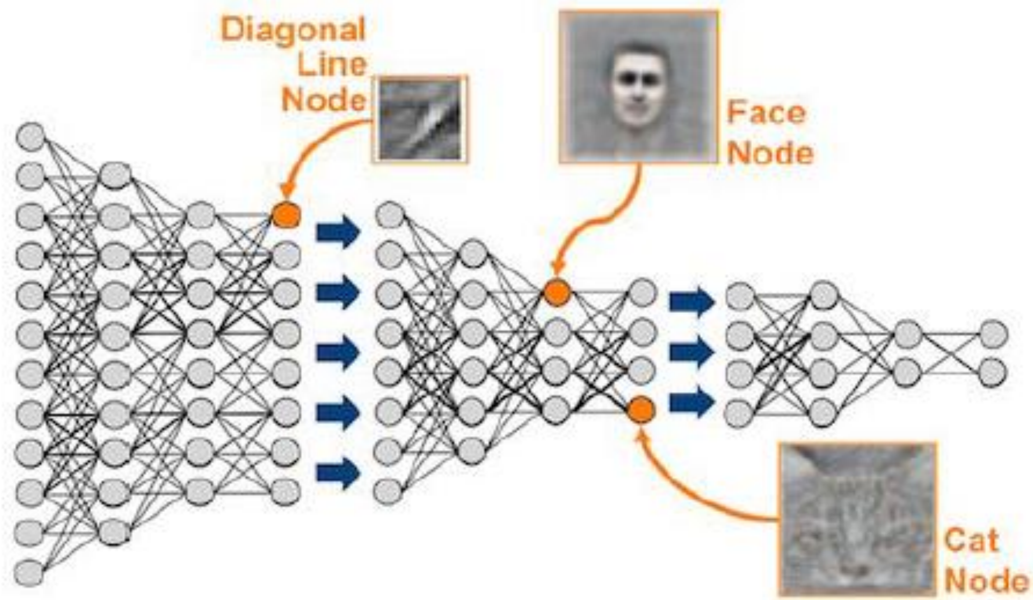


Why Deep?

Deep Neural Network

Pros:

1. Automatically learn the High-level features representation
2. Modularity: DNN can be composed like LEGO bricks
3. Able to do **Transfer Learning**

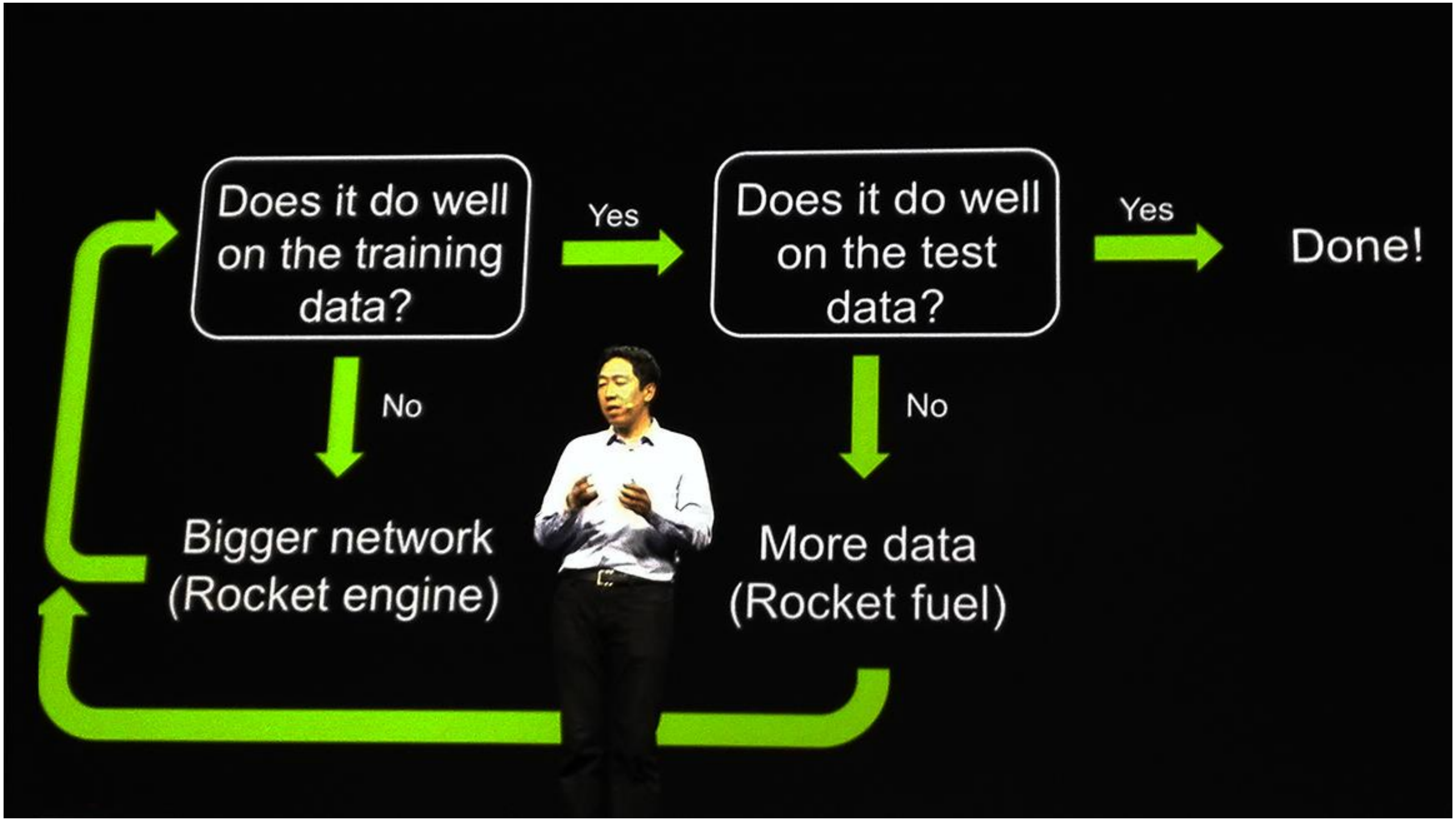


Cons:

1. Requires tons of data for training
2. Expensive computation power
   for training and testing (no CV)

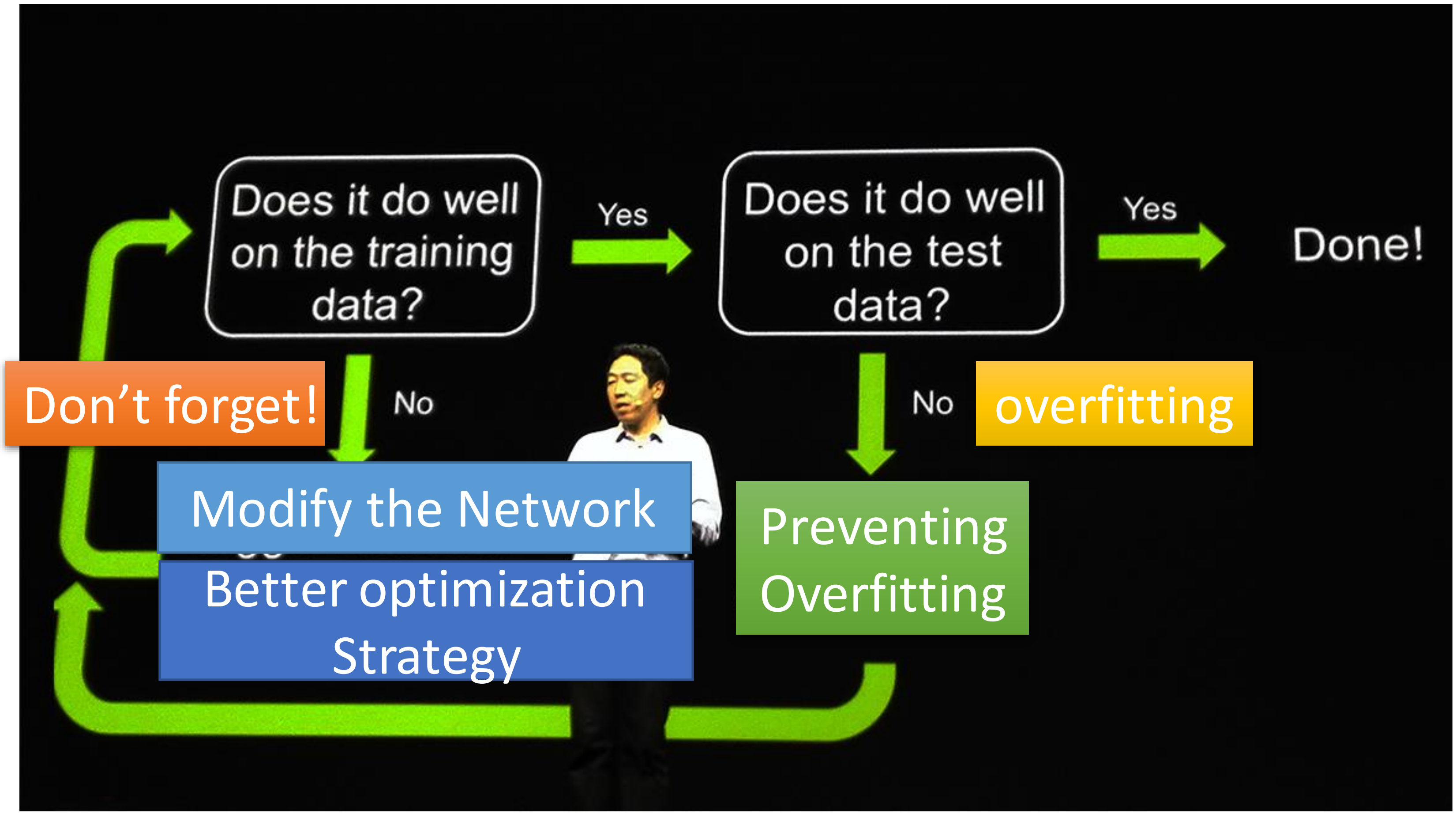
Tips for Training DNN

Recipe for Learning



http://www.gizmodo.com.au/2015/04/the-basic-recipe-for-machine-learning-explained-in-a-single-powerpoint-slide/

Recipe for Learning



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Recipe for Learning

Modify the Network

- New activation functions, for example, ReLU or Maxout

Better optimization Strategy

- Adaptive learning rates

Prevent Overfitting

- Dropout

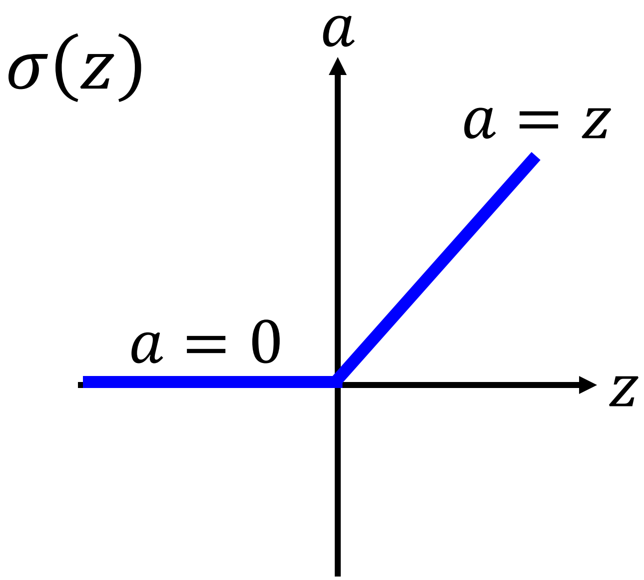
Only use this approach when you already obtained good results on the training data.

Tips for Training DNN

New Activation Function

ReLU

- Rectified Linear Unit (ReLU)

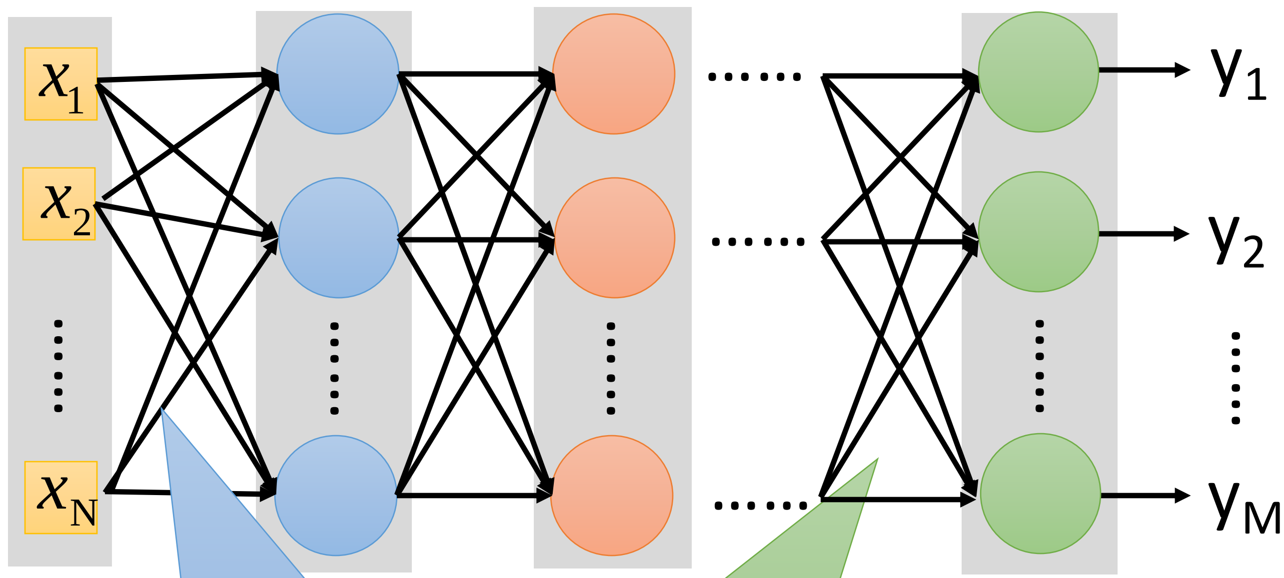


[Xavier Glorot, AISTATS'11]
[Andrew L. Maas, ICML'13]
[Kaiming He, arXiv'15]

Reason:

Vanishing gradient problem

Vanishing Gradient Problem



Smaller gradients

Learn very slow

Almost random

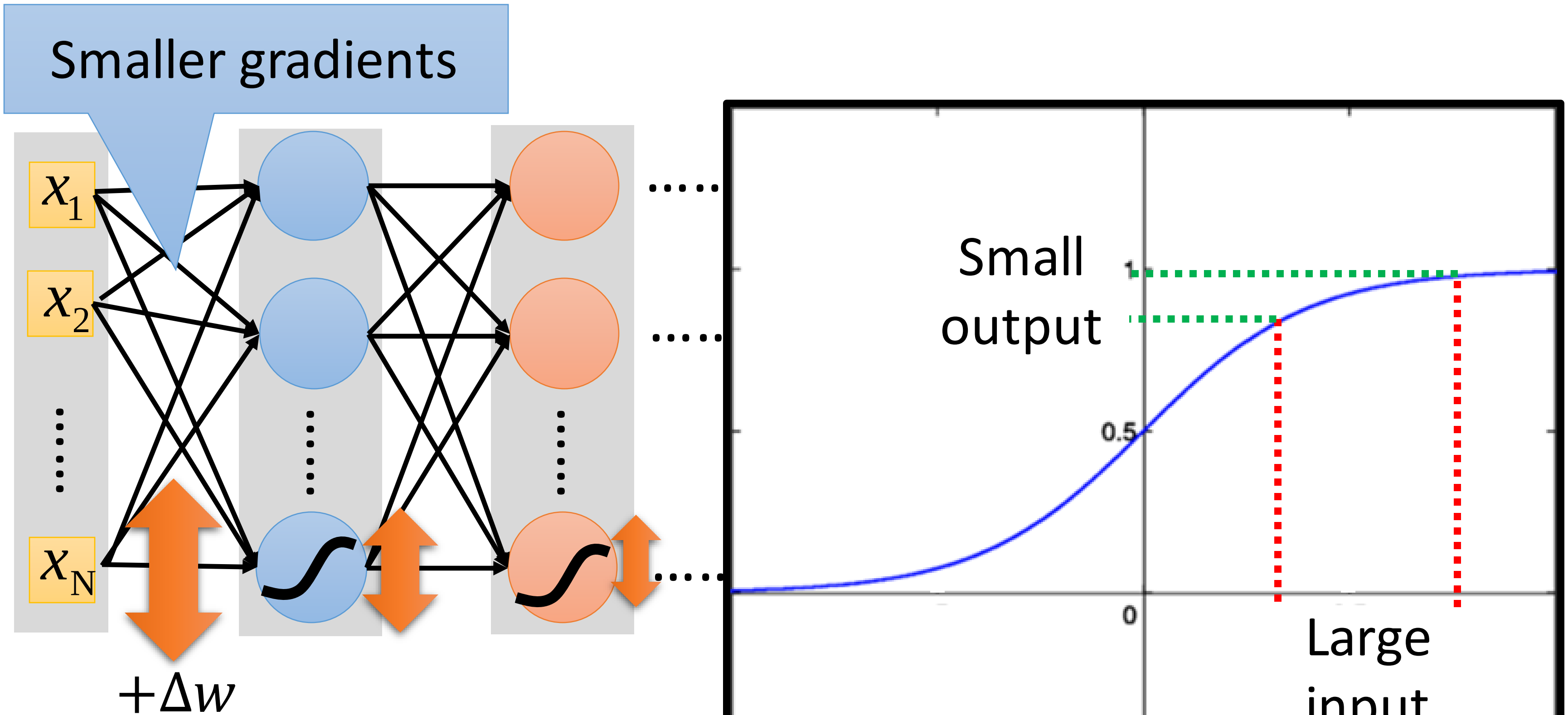
Larger gradients

Learn very fast

Already converge

based on random!?

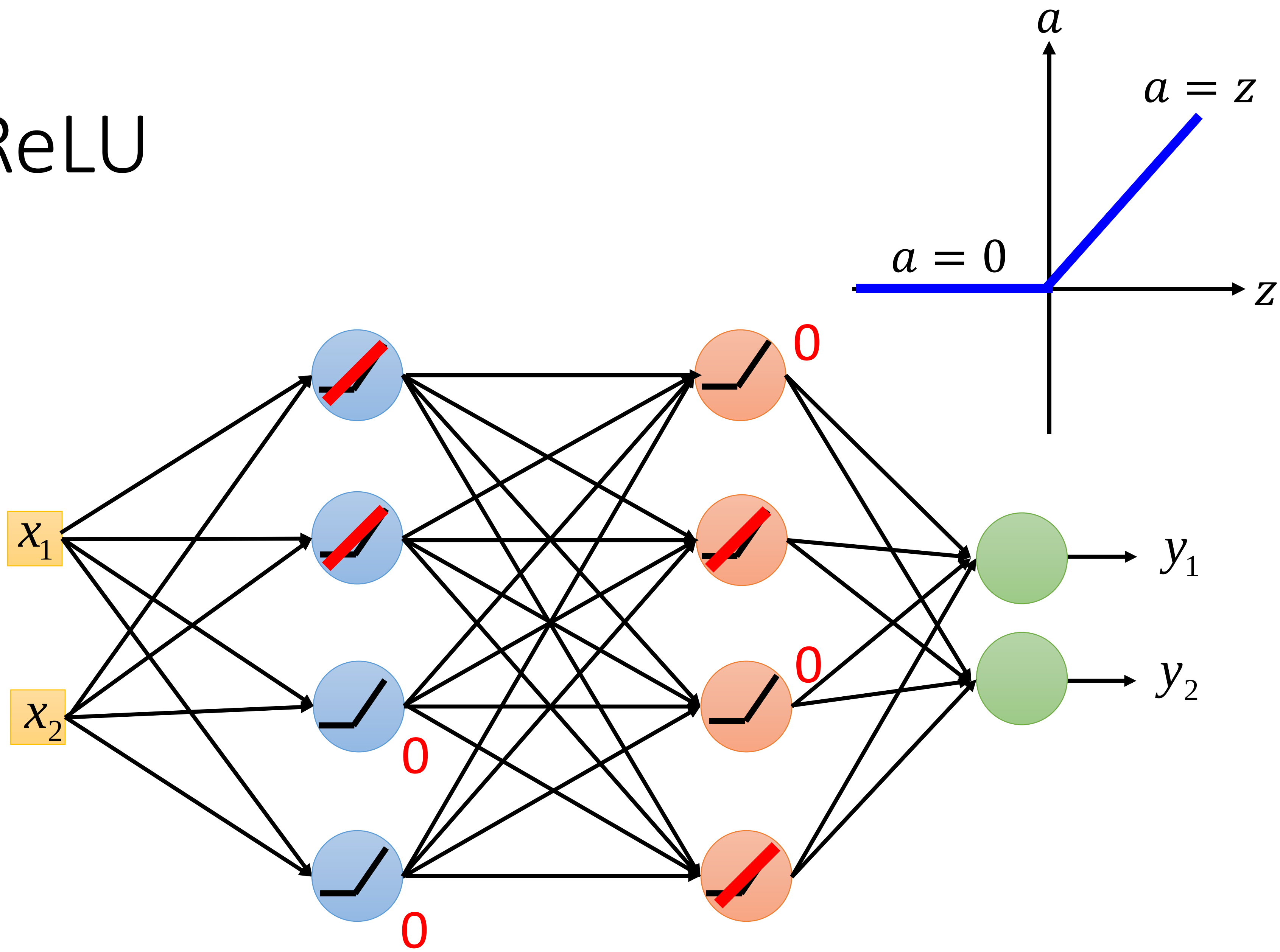
Vanishing Gradient Problem



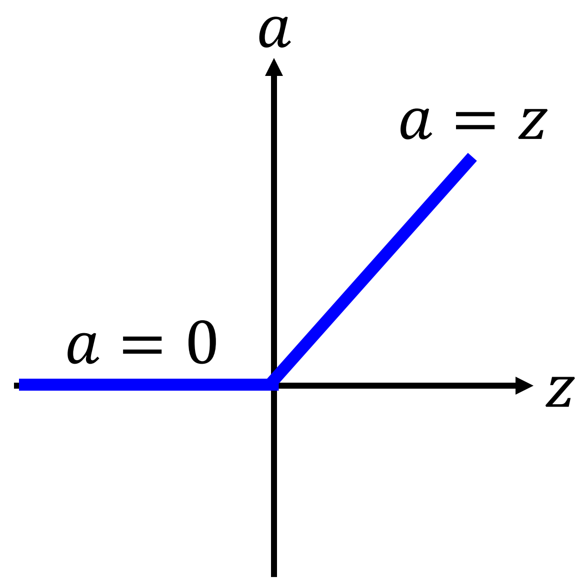
Intuitive way to compute the gradient …

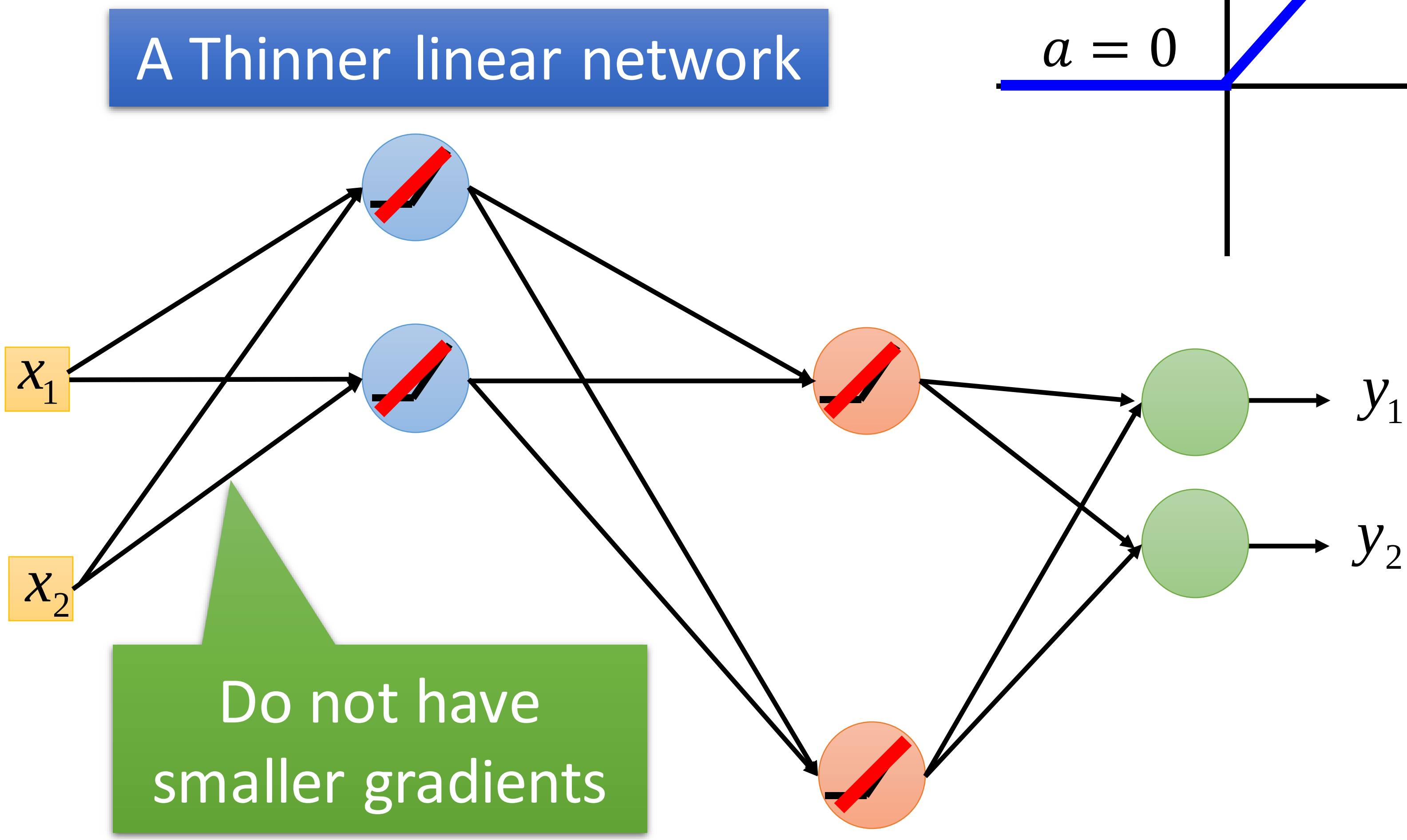
$$\frac{\partial C}{\partial w} =? \ \frac{\Delta C}{\Delta w}$$

ReLU



ReLU


a
$a = z$
$a = 0$
z


A Thinner linear network
x_1
x_2
y_1
y_2
Do not have smaller gradients

Not the whole story ……

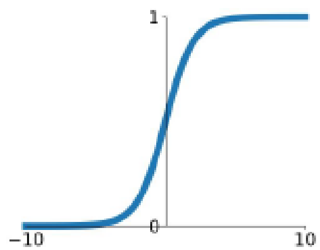
- Adagrad [John Duchi, JMLR'11]
- RMSprop
  - https://www.youtube.com/watch?v=O3sxAc4hxZU
- Adadelta [Matthew D. Zeiler, arXiv'12]
- Adam [Diederik P. Kingma, ICLR'15]
- AdaSecant [Caglar Gulcehre, arXiv'14]
- "No more pesky learning rates" [Tom Schaul, arXiv'12]

Notes:

Last time: Activation Functions

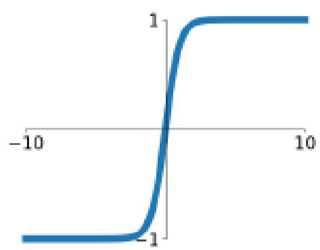
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



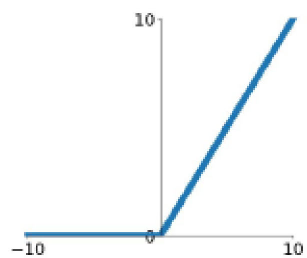
tanh

$\tanh(x)$



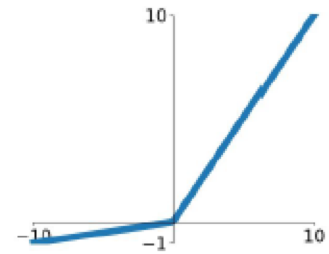
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

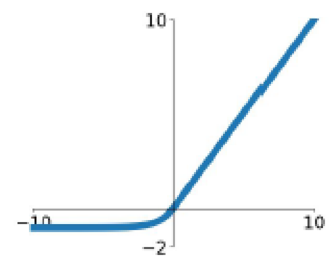


Maxout

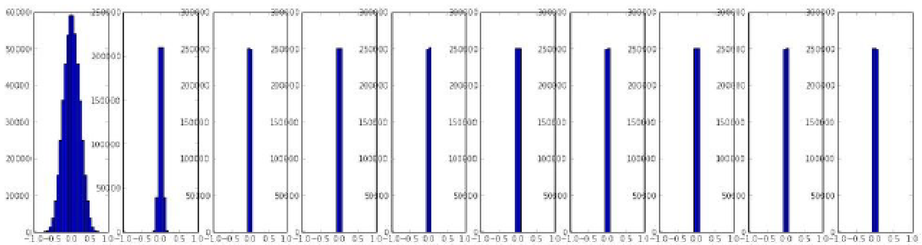
$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

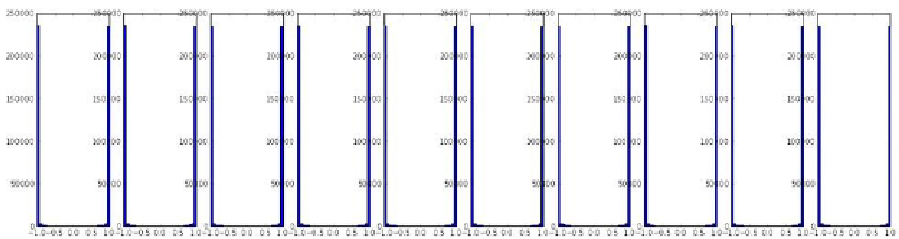
$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$



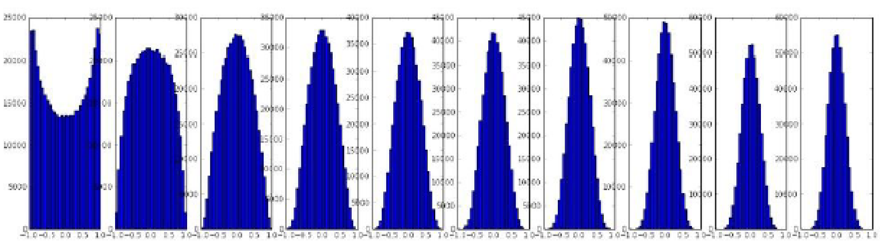
Last time: Weight Initialization



Initialization too small:
Activations go to zero, gradients also zero,
No learning

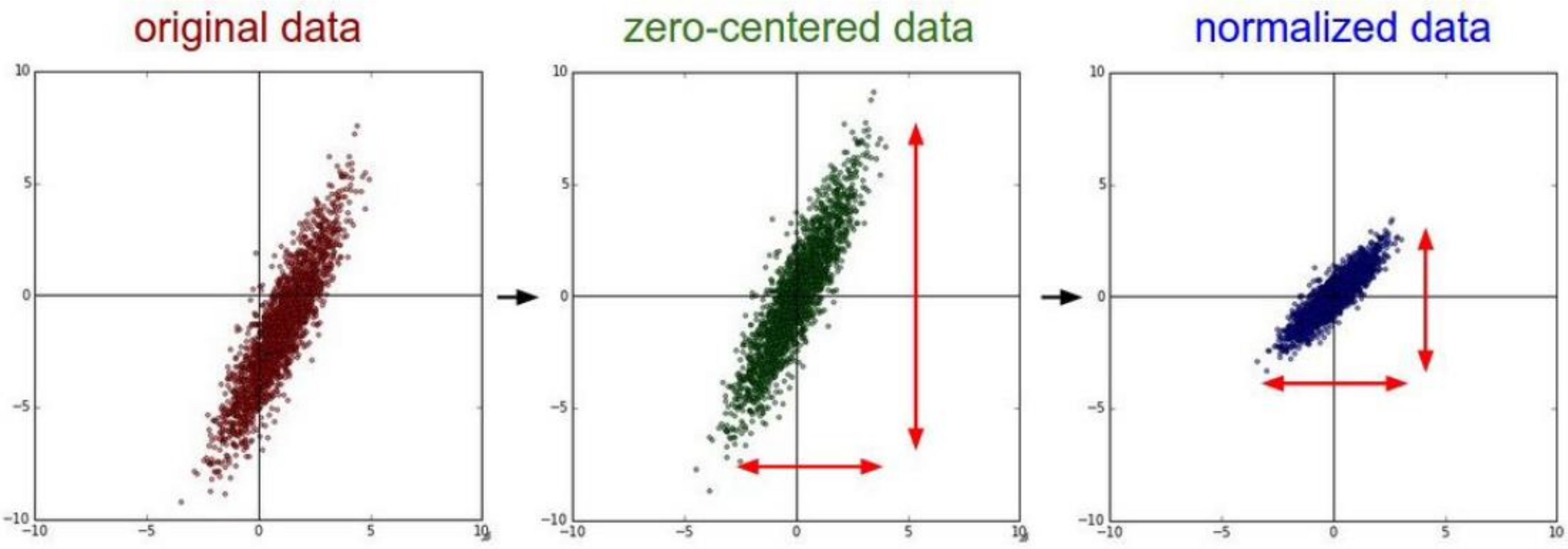


Initialization too big:
Activations saturate (for tanh),
Gradients zero, no learning



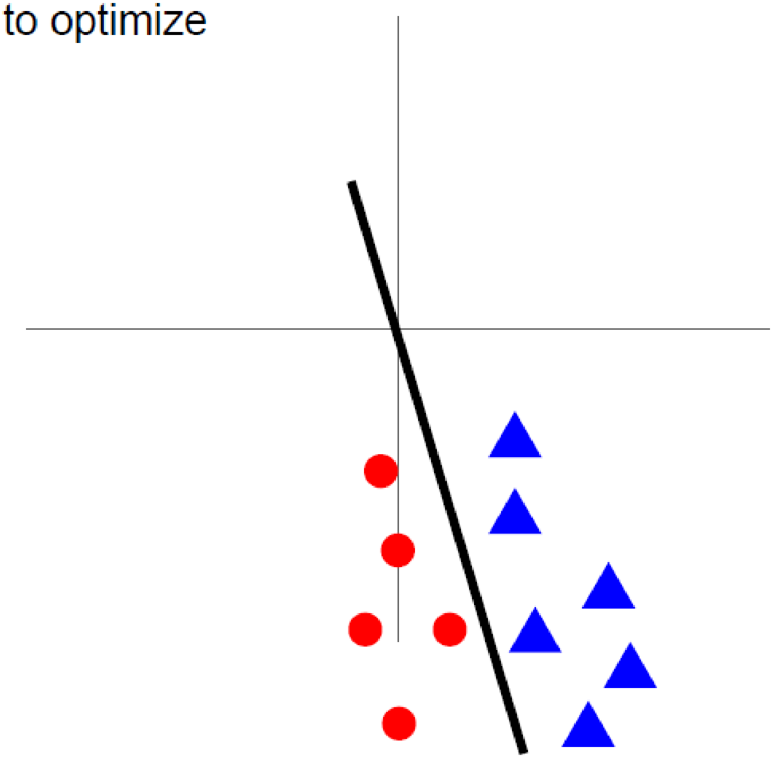
Initialization just right:
Nice distribution of activations at all layers,
Learning proceeds nicely

Last time: Data Preprocessing

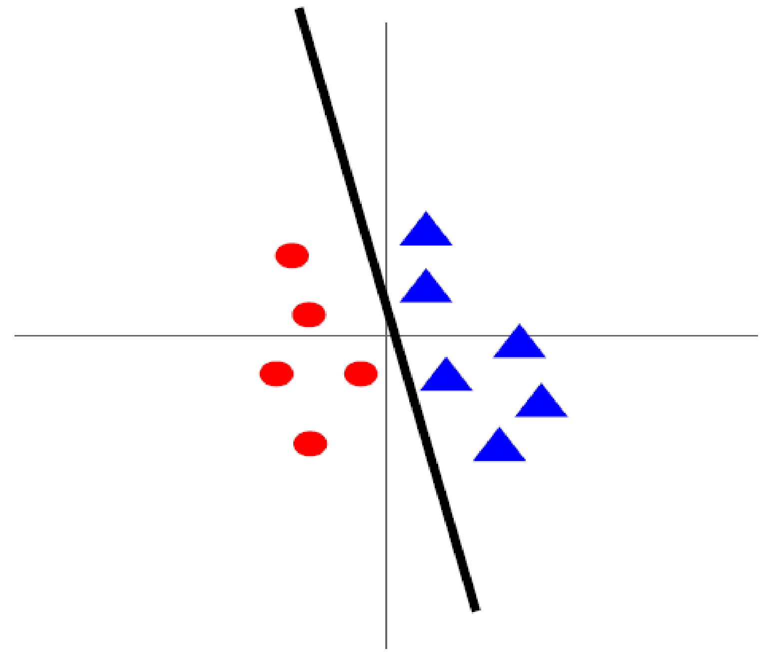


Last time: Data Preprocessing

Before normalization: classification loss very sensitive to changes in weight matrix; hard to optimize



After normalization: less sensitive to small changes in weights; easier to optimize



Last time: Batch Normalization

Input: $x : N \times D$

Learnable params:
$\gamma, \beta : D$

Intermediates: $\mu, \sigma : D$
$\hat{x} : N \times D$

Output: $y : N \times D$

$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{i,j} - \mu_j)^2$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Model Validation