HYPOTHESIS TESTING

Question 1: What is a null hypothesis (H_0) and why is it important in hypothesis testing?

Ans. The null hypothesis (H_0) is a statement that there is no effect, no difference, or no relationship between the parameters of interest. It represents the "status quo" or the widely accepted belief. For example, $H_0: \mu = 10$ (the population mean is 10).

Importance of H_0

H_0 is crucial because hypothesis testing is a proof by
contradiction. The entire process revolves around testing the assumption that H_0 is true. We collect sample data and use statistical methods to determine if there is enough evidence to reject H_0 in favor of an alternative hypothesis (H_a or H_1). If the sample evidence is very unlikely to occur under the assumption that
H_0 is true, we reject H_0.

Question 2: What does the significance level (α) represent in hypothesis testing?

Ans. The significance level (α) is the maximum probability of committing a Type I error that a researcher is willing to accept.

- It is the probability of rejecting the null hypothesis (H_0) when it is actually true.
- Common values for α are 0.05 (5%), 0.01 (1%), or 0.10 (10%).

- If the p-value of the test is less than or equal to α ($p \le \alpha$), we reject H_0.
- α defines the critical region (or rejection region) in the sampling distribution.

Question 3: Differentiate between Type I and Type II errors. **Ans.**

Feature	Type I Error (α)	Type II Error (β)
Definition	Rejecting the null hypothesis (H_0) when it is actually true.	Failing to reject the null hypothesis (H_0) when it is actually false.
Mistake	False Positive	False Negative
Consequence	Concluding an effect/difference exists when it doesn't.	Failing to detect an effect/difference that does exist.
Probability	The significance level (α)	Denoted by β

Question 4: Explain the difference between a one-tailed and two- tailed test. Give an example of each.

Ans. The difference lies in the alternative hypothesis (H_a) and the rejection region.

One-Tailed Test (Directional)

- Alternative Hypothesis (H_a): Specifies a direction (either greater than $>$ or less than $<$ the hypothesized value).
- Rejection Region: Located entirely in one tail (either the right or the left) of the sampling distribution.
- Example: Testing if the average product lifespan is greater than 5 years ($H_a: \mu > 5$).

Two-Tailed Test (Non-Directional)

- Alternative Hypothesis (H_a): States that the parameter is simply not equal to the hypothesized value ($\neq$).
- Rejection Region: Split between both tails (the far right and the far left) of the sampling distribution.
- Example: Testing if the average salary is different from $$$50,000 ($H_a: \mu \neq \$50,000$).

Question 5: A company claims that the average time to resolve a customer complaint is 10 minutes. A random sample of 9 complaints gives an average time of 12 minutes and a standard deviation of 3 minutes. At $\alpha = 0.05$, test the claim.

Ans. This is a t-test for a single mean since the population standard deviation (σ) is unknown and the sample size is small ($n < 30$).

- Given: $\mu_0 = 10$, $n = 9$, $\bar{x} = 12$, $s = 3$, $\alpha = 0.05$.
- Degrees of Freedom (df): $n - 1 = 9 - 1 = 8$.

1. **State Hypotheses**

- $H_0: \mu = 10$ (The average time is 10 minutes)
- $H_a: \mu \neq 10$ (The average time is different from 10 minutes) $\rightarrow$ Two-tailed test

2. Calculate the Test Statistic (t)

The formula for the t-test statistic is:

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

$$t = \frac{12 - 10}{3 / \sqrt{9}} = \frac{2}{3 / 3} = \frac{2}{1} = 2.0$$

3. Determine the Critical Value (or P-value)

- For a two-tailed t-test with $df = 8$ and $\alpha = 0.05$, the critical t-values are found to be ± 2.306.

4. Make a Decision

- Critical Value Method: Since $|t_{\text{calc}}| = 2.0$ is less than $|t_{\text{crit}}| = 2.306$, the test statistic falls in the non-rejection region.
- Decision: Do not reject H_0.

5. Conclusion

At the 0.05 significance level, there is not enough evidence to reject the company's claim that the average time to resolve a customer complaint is 10 minutes.

Question 6: When should you use a Z-test instead of a t-test?

Ans. A Z-test for a mean should be used instead of a t-test when at least one of the following two conditions is met:

1. The population standard deviation (σ) is known.

- *Note:* The t-test is used when σ is unknown and the sample standard deviation (s) is used as an estimate.

2. The sample size (n) is large ($n \ge 30$).

   - Due to the Central Limit Theorem, when n is large, the sampling distribution of the mean approximates a normal distribution, and the t-distribution becomes very close to the standard normal (Z) distribution, even if σ is unknown.

Question 7: The productivity of 6 employees was measured before and after a training program. At $\alpha = 0.05$, test if the training improved productivity.

Ans. This is a Paired Samples t-test (Dependent Samples t-test)
because the same employees are measured *before* and *after* the training. We analyze the difference (d) between the paired observations.

Employee	Before (XB)	After (XA)	Difference (d=XA −XB)	d2
1	50	55	5	25
2	60	65	5	25
3	58	59	1	1
4	55	58	3	9

Employee	Before (XB)	After (XA)	Difference (d=XA −XB)	d2
5	62	63	1	1
6	56	59	3	9
Sum ($\sum$)			18	70

- Given: $n = 6$, $\alpha = 0.05$.
- Degrees of Freedom (df): $n - 1 = 6 - 1 = 5$.

Calculations

- Mean Difference ($\bar{d}$): $\bar{d} = \frac{\sum d}{n} = \frac{18}{6} = 3$
- Standard Deviation of Differences (s_d):

$$s_d = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}} = \sqrt{\frac{70 - \frac{(18)^2}{6}}{5}} = \sqrt{\frac{70 - \frac{324}{6}}{5}}$$

$$s_d = \sqrt{\frac{70 - 54}{5}} = \sqrt{\frac{16}{5}} = \sqrt{3.2} \approx 1.7889$$

1. State Hypotheses

We are testing if the training improved productivity, meaning "After" should be greater than "Before", or $d = X_A - X_B$ should be greater than 0.

- $H_0: \mu_d \le 0$ (Training did not improve productivity)

- $H_a: \mu_d > 0$ (Training improved productivity)
  $\rightarrow$ Right-tailed test

2. **Calculate the Test Statistic (t)**

The formula for the paired t-test statistic is:

$$t = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}}$$ (Assuming $\mu_{d0} = 0$)

$$t = \frac{3 - 0}{1.7889 / \sqrt{6}} = \frac{3}{1.7889 / 2.4495} = \frac{3}{0.7303} \approx 4.108$$

3. **Determine the Critical Value**

- For a right-tailed t-test with $df = 5$ and $\alpha = 0.05$, the critical t-value is $t_{\text{crit}} = 2.015$.

4. **Make a Decision**

- Critical Value Method: Since $t_{\text{calc}} = 4.108$ is greater than $t_{\text{crit}} = 2.015$, the test statistic falls in the rejection region.
- **Decision:** Reject H_0.

5. **Conclusion**

At the 0.05 significance level, there is sufficient evidence to conclude that the training program improved employee productivity.

Question 8: A company wants to test if product preference is independent of gender.

Gender	Product A	Product B	Total
Male	30	20	50
Female	10	40	50
Total	40	60	100

At $\alpha = 0.05$, test independence.

Ans. This is a Chi-Square Test for Independence (χ^2).

1. **State Hypotheses**

- H_0: Product preference is independent of gender.
- H_a: Product preference is dependent on gender.

2. **Calculate Expected Frequencies** (E)

The expected frequency for each cell is $E = (\text{Row Total} \times \text{Column Total}) / \text{Grand Total}$.

- $E_{\text{Male, A}} = (50 \times 40) / 100 = 20$
- $E_{\text{Male, B}} = (50 \times 60) / 100 = 30$
- $E_{\text{Female, A}} = (50 \times 40) / 100 = 20$
- $E_{\text{Female, B}} = (50 \times 60) / 100 = 30$

3. **Calculate the Test Statistic** (χ^2) The formula is:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Where O is the Observed frequency and E is the Expected frequency.

Observed (O)	Expected (E)	O−E	(O−E)2	(O−E)2/E
30	20	10	100	$100 / 20 = 5.0$
20	30	-10	100	$100 / 30 \approx 3.333$
10	20	-10	100	$100 / 20 = 5.0$
40	30	10	100	$100 / 30 \approx 3.333$
Sum ($\sum$)				$\mathbf{16.666}$

$$\chi^2_{\text{calc}} = 5.0 + 3.333 + 5.0 + 3.333 = 16.666$$

4. Determine the Critical Value

- Degrees of Freedom (df): $df = (r - 1)(c - 1) = (2 - 1)(2 - 1) = 1$.
- For χ^2 test with $df = 1$ and $\alpha = 0.05$, the critical value is $\chi^2_{\text{crit}} = 3.841$.

5. Make a Decision

- Critical Value Method: Since $\chi^2_{\text{calc}} = 16.666$ is greater than $\chi^2_{\text{crit}} = 3.841$, the test statistic falls in the rejection region.
- Decision: Reject H_0.

6. Conclusion

At the 0.05 significance level, there is sufficient evidence to conclude that product preference is dependent on gender.