<u>CS 600 A - Advanced Algorithms – Homework 12</u>

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Question 26.6.7: Convert the following linear program into standard form, and solve using Simplex Method

minimize: $z = 3y_1 + 2y_2 + y_3$

subject to: $-3y_1 + y_2 + y_3 \geq 1$

$2y_1 + y_2 - y_3 \geq 2$

$y_1, y_2, y_3 \geq 0$

Answer: The problem will be adapted to the standard linear programming model, adding the slack, surplus and / or artificial variables in each of the constraints:

- **Constraint 1:** It has a sign "≥" (greater than or equal) so the surplus variable S_1will be subtracted and the artificial variable A_1 will be added.
- **Constraint 2:** It has a sign "≥" (greater than or equal) so the surplus variable S_2will be subtracted and the artificial variable A_2 will be added.

The problem has artificial variables so we will use the **two-phase method**. In the first phase, the objective function seeks to **minimize the sum of the artificial variables**.

The problem is shown below in standard form. The coefficient 0 (zero) will be placed where it corresponds to create our table:

Objective Function:

Minimize: $Z = 0X_1 + 0X_2 + 0X_3 + 0S_1 + 0S_2 + 1A_1 + 1A_2$

Subject to:

$-3X_1 + 1X_2 + 1X_3 - 1S_1 + 0S_2 + 1A_1 + 0A_2 = 1$

$2X_1 + 1X_2 - 1X_3 + 0S_1 - 1S_2 + 0A_1 + 1A_2 = 2$

$X_1, X_2, X_3, S_1, S_2, A_1, A_2 \geq 0$

Initial Table:

Table 1	**Cj**	0	0	0	0	0	1	1	
Cb	**Base**	**X1**	**X2**	**X3**	**S1**	**S2**	**A1**	**A2**	**R**
1	**A1**	-3	**1**	1	-1	0	1	0	1
1	**A2**	2	1	-1	0	-1	0	1	2
	Z	-1	2	0	-1	-1	0	0	3

Enter the variable $\mathbf{X_2}$ and the variable $\mathbf{A_1}$ leaves the base. The pivot element is **1**

Iteration 1:

Table2	Cj	0	0	0	0	0	1	1	
Cb	**Base**	**X1**	**X2**	**X3**	**S1**	**S2**	**A1**	**A2**	**R**
0	**X2**	-3	1	1	-1	0	1	0	1
1	**A2**	**5**	0	-2	1	-1	-1	1	1
	Z	5	0	-2	1	-1	-2	0	1

Enter the variable $\mathbf{X_1}$ and the variable $\mathbf{A_2}$ leaves the base. The pivot element is **5**

Iteration 2:

Table3	Cj	0	0	0	0	0	1	1	
Cb	**Base**	**X1**	**X2**	**X3**	**S1**	**S2**	**A1**	**A2**	**R**
0	$\mathbf{X_2}$	0	1	-1/5	-2/5	-3/5	2/5	3/5	8/5
0	$\mathbf{X_1}$	1	0	-2/5	1/5	-1/5	-1/5	1/5	1/5
	Z	0	0	0	0	0	-1	-1	0

The iterations of the first phase are finished and there is some possible solution to the problem. We eliminate the artificial variables and go to the second phase:

Initial Table - Phase II

Table3	Cj	3	2	1	0	0	
$\mathbf{C_b}$	**Base**	$\mathbf{X_1}$	$\mathbf{X_2}$	$\mathbf{X_3}$	$\mathbf{S_1}$	$\mathbf{S_2}$	**R**
2	$\mathbf{X_2}$	0	1	-1/5	-2/5	-3/5	8/5
3	$\mathbf{X_1}$	1	0	-2/5	1/5	-1/5	1/5
	Z	0	0	-13/5	-1/5	-9/5	**19/5**

The optimal solution is Z = 19/5

$\mathbf{X_1 = 1/5, X_2 = 8/5, X_3 = 0, S_1 = 0, S_2 = 0}$

Question 21.5.13: Give a linear programming formulation to find the minimum spanning tree of a graph. Recall that a spanning tree T of a graph G is a connected acyclic subgraph of G that contains every vertex of G . The minimum spanning tree of a weighted graph G is a spanning tree T of G such that the sum of the edge weights in T is minimized.

Answer: To formulate the minimum spanning tree (MST) problem as a linear programming (LP) problem, we can use the following approach. Let's consider a weighted, undirected graph G with n vertices and m*m* edges, and let wij be the weight of the edge (i,j)

Decision Variables: Let xij be a binary decision variable representing whether the edge (i, *j*) is included in the minimum spanning tree. It takes the value 1 if the edge is included and 0 otherwise.

Objective Function: The objective is to minimize the total weight of the edges in the spanning tree. The objective function is the sum of the weights of the selected edges:

Minimize $\sum_{(i,j) \in E} w_{ij} \cdot x_{ij}$

Subject to the following constraints:

- Connectivity Constraint: Ensure that the resulting subgraph is connected. This can be achieved by requiring that there is a path between every pair of vertices in the selected edges.

$$\sum_{j:(i,j)\in E} x_{ij} + \sum_{j:(j,i)\in E} x_{ji} \geq 1 \text{ for all } i \in V$$

- Acyclicity Constraint: Ensure that the resulting subgraph is acyclic, meaning it is a tree.

$$\sum_{(i,j)\in S} x_{ij} \leq |S| - 1 \text{ for all } S \subseteq V, 2 \leq |S| \leq n$$

1. Binary Decision Variable Constraint: Ensure that the decision variables are binary.

$$x_{ij} \in \{0,1\} \text{ for all } (i,j) \in E$$

The first set of constraints ensures connectivity, the second set ensures acyclicity, and the third set ensures that the decision variables are binary.

This linear programming formulation captures the essence of the minimum spanning tree problem. Solving this linear program will give the optimal selection of edges that form the minimum spanning tree for the given weighted graph.

Question 21.5.27 A political candidate has hired you to advise them on how to best spend their advertising budget. The candidate wants a combination of print, radio, and tele- vision ads that maximize total impact, subject to budgetary constraints, and avail- able airtime and print space.

Design and solve a linear program to determine the best combination of ads for the campaign.

Answer: Let there be 'x' ads of radio, 'y' ads of print and 'z' ads of tv

The ***total impact*** would be:

impact = (x*a + y*b + z*c) which is to be ***maximized***

Let total budget be 'B'

Then the ***total cost*** for all the ads would be:

cost = (10000*x + 70000*y + 110000*z) which should be **<= *total budget***

(10000*x + 70000*y + 110000*z) <= B

where B is the total budge

There is a bound to maximum number of each type of ads which gives

x <= 25, y <= 7, z <= 15

The Linear Program would become

MAXIMIZE: impact = (x*a + y*b + z*c)

SUBJECT TO: (10000*x + 70000*y + 110000*z) <= B

BOUNDS: x <= 25, y <= 7, z <= 15

This LP would be solveable if numerical values of (a,b,c,B) are given