Exercise 1

1. Sol^n for Berlin tram routes is:

we have colours 1, 2, 3, 4, 5, 6, 7, ... (say)

Colour	allotted to route
1	M1, M2, 16, 18, 37, 61
2	M4, M17, 50, 62
3	M5, 12, 21, 63
4	M6, 60,
5	M8, 27, 68
6	M10, 67
7	M13

So, minimum colours required for colouring the map are ~~seven~~ 7.

2.

Procedure :

from the given sheet we can clearly see that a group of routes that intersect with each other so they need to be allotted different colours such group of routes are M2, M4, M5, M6, M8, M10

so colours	routes
1	M2
2	M4
3	M5
4	M6
5	M8
6	M10

Now we will see for route M1 as M1 intersect with M5, M8, M10 so we can't assign colour 3, 5, 6 to M1 now let us say we have given colour 1 to M1

Moving on to route 12 as route 12 intersect with M1, M2, M4, M8, M10 so we can't assign colours 1, 2, 5, 6 to ~~R4~~ route 12, let us say we have assigned colour 3 to route 12

Moving on to route M13 we see that route M13 intersect with M1, M2, M4, M5, M6, M8, M10 so we can't assign colour 1, 2, 3, 4, 5, 6 to M13 therefore we will assign a new colour to M13 ie, (7 say)

Repeating the above procedure we will see that minimum no. of colours used are 7

Below is the rough idea of what I have written above :

M2 → 1 M4 → 2 M5 → 3 M6 → 4 M8 → 5 M10 → 6

M1 → 3✗ 5✗ 6✗ — 1 ✓

M12 → 1✗ 2✗ 5✗ 6✗ — 3 ✓

M13 → 7 ✓

16 → 3✗ 4✗ 5✗ 7✗ 1 ✓

M17 → 3✗ 4✗ 5✗ 7✗ 1✗ 2 ✓

18 → 4✗ 5✗ 1 ✓

21 → 5✗ 6✗ 7✗ 1✗ 2✗ 3 ✓

27 → 2✗ 4✗ 3✗ 1✗ 2✗ 3✗ 5 ✓

37 → 5✗ 2✗ 3✗ 5✗ 1 ✓

50 → 1✗ 7✗ 2 ✓

60 → 2✗ 3✗ 5✗ 1✗ 4 ✓

61 → 4 ✗ 1 ✓

62 → 4 ✗ 1 ✗ 2 ✓

63 → 4 ✗ 1 ✗ 2 ✗ 3 ✓

67 → 2 ✗ 3 ✗ 5 ✗ 1 ✗ 4 ✗ 6 ✓

68 → 4 ✗ 1 ✗ 2 ✗ 3 ✗ 6 ✗ 5 ✓

Exercise 2: ① Bin Packing:

we are given items (1, 2, ..., n) with weights $w_1, w_2, w_3, \ldots, w_n$

Bin Capacity is B

first, arrange the weights of items in decreasing order $w_n, w_{n-1}, \ldots w_1$ (say)

Then allocate them into different bins with capacity B

say for w_n, allot w_n in bin b_1 (say) now to allot w_{n-1} check $w_n + w_{(n-1)} \leq B$ if this holds then allocate w_{n-1} in bin B_1 otherwise create new bin b_2 and allocate w_{n-1} in bin B_2.

Now for weight $w_{(n-2)} < w_{n-1} < w_n$ check whether it can be allocated in bin b_1 following the condition $w_n + w_{n-2} \leq B$ or can be allocated in bin B_2 and if not then create new bin b_3 for allocation of w_{n-2}.

Repeat above procedure for other weights of items.

(2)

let weights of items be 5, 4, 3, 3, 3 2, 3, 2, 3 and bin capacity be 10

Applying FFD algorithm

arrange weights in decreasing order 5 4 3 3 3 3 3 2 2

Bin 1 5 4

Bin 2 3 3 3

Bin 3 3 3 2 2

Bin 4 2

if we modify above arrangement we can clearly see we get better optimal solution than that we got from FFD algorithm

Bin - 1 5 3 2

Bin - 2 4 3 3

Bin - 3 3 3 2 2

Clearly, we can see in our modified procedure of bin packing we have used only three bins and with more accuracy as in 1st procedure Bin 1,3 1 spare left & Bin 4 8 spare left.

4) Given : we have n routes and there is a route that intersects the other $n-1$ routes,
we have asked whether we need n colours to represent map. True or false

False.

Counterexample :

we have 5 routes (say)

$M1$, $M2$, $M3$, $M4$, $M5$

and $M1$ intersects with all other routes

but as per our scenario $M2$ & $M4$ doesnot intersect (say) and $M3$ & $M5$ doesnot intersect. (say)

then we can assign colours in such a way

colours	routes	
1	$M1$	
2	$M2$	$M4$
3	$M3$	$M5$

colours used are 3. whereas routes are 5.